THE COULOMB SUM RULE IN NUCLEI

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FOR THE E05-110 COLLABORATION.
COULOMB SUM RULE

Inclusive electron scattering cross-section:

\[
\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[ \frac{q^4}{|q|^4} R_L(\omega, |q|) + \left( \frac{q^2}{2|q|^2} + \tan^2 \theta \right) R_T(\omega, |q|) \right]
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- Scattering response due to **charge** properties
- Scattering response due to **magnetic** properties
COULOMB SUM RULE

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Coulomb Sum Rule definition:

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S_L(|q|) = \int_{\omega^+}^{q} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}
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Scattering response due to charge properties

Scattering response due to magnetic properties

If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.
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At small $|q|$, $S_L$ will deviate from unity due to long range nuclear effects, Pauli blocking. (directly calculable, well understood).
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At large $|q| \gg 2k_f$, $S_L$ should go to 1. Any significant* deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

*Short range correlations will also quench $S_L$, but only by < 10%
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COULOMB SUM RULE

- Long standing issue with many years of theoretical interest.
- Even most state-of-the-art models cannot predict existing data.
- New precise data at larger $|q|$ would provide crucial insight and constraints to modern calculations.

\[
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Quasi-elastic scattering at intermediate $Q^2$ is the region of interest for our experiment:

- Nuclei investigated:
  - $^4$He
  - $^{12}$C
  - $^{56}$Fe
  - $^{208}$Pb

We want to integrate above the coherent elastic peak:

Quasi-elastic is "elastic" scattering on constituent nucleons inside nucleus.
First group of experiments from Saclay, Bates, and SLAC show a quenching of $S_L$ consistent with medium modified form-factors.

$$S_L(|q|) = \int_{\omega^+}^{|q|} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

$|q_{eff}|$ is $|q|$ corrected for a nuclei dependent mean coulomb potential.

Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.
PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of $S_L$ consistent with medium modified form-factors.
- Very little data above $|q|$ of 600 MeV/c, where the cleanest signal of medium effects should exist!
- Saclay, Bates limited in beam energy reach up to 800 MeV.
- SLAC limited in kinematic coverage of scattered electron at $|q|$ below 1150 MeV/c.

$|q_{\text{eff}}|$ is $|q|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.
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EXPERIMENTAL DESIGN

- Need $R_L$ → Use Rosenbluth separation!
  
  $$S_L(|q|) = \int_{\omega^+}^{\omega^-} d\omega \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}(Q^2) + N\tilde{G}_{En}(Q^2)}$$

  Slope = $\frac{Q^4}{\tilde{q}^4} R_L$
  
  Intercept = $\frac{Q^2}{2\tilde{q}^2} R_T$

- Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of $R_L$!

- Need data for each angle at a constant $|q|$ over an $\omega$ range starting above the elastic peak up to $|q|$. When running a single arm experiment with fixed beam energy and scattering angle, $|q|$ is NOT constant over your momentum acceptance.

- Need to take data at varying beam energies, and “map-out” $|q|$ and $\omega$ space.
If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c

CSR calculated at constant $|q|$!!

$$S_L(|q|) = \int_{\omega^+}^{\omega^-} |q| \frac{R_L(\omega, |q|)}{Z^2 \tilde{G}_{Ep}^2(Q^2) + N \tilde{G}_{En}^2(Q^2)}$$
If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c

Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$.
EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c
- Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$.
- $|q|$ can be selected between 550 and 1000 MeV/c

Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.
EXPERIMENTAL DESIGN

If one wants to measure from 100 to 600 MeV at constant $|q| = 650$ MeV/c:

- Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$. $|q|$ can be selected between 550 and 1000 MeV/c.

Repeat this "mapping" for 60, 90, and 120 degree spectrometer central angles.

The Coulomb sum rule in nuclei: $q$ vs. $\omega$ coverage for 15 degree Iron data.
EXPERIMENTAL SPECIFICS

- E05-110:
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: $^4\text{He}$, $^{12}\text{C}$, $^{56}\text{Fe}$, $^{208}\text{Pb}$.

Each data line represents a constant beam-energy
ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

- Blue histograms are reconstructed data.
- Red histograms are monte-carlo:
  - Event sample generated from expected XS calculations (Fourier-Bessel fit to world data)
  - Radiative effects (internal, external, vertex) are handled, including exact bremsstrahlung distributions.
  - Resolution effects are applied by calculating the expected material effects of tracks passing through the VDC chamber materials.
ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

$^{12}$C elastic XS at 1260 MeV, 15 degrees

< 1 % deviation from calculation

Run: 3503  BeamE: 1259.66 MeV  Angle: 15 deg  dp-peak: -0.0337

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ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

$^{12}$C elastic XS at 400 MeV, 26 degrees

$< 1 \%$ deviation from calculation

Run: 4970  BeamE: 400.99 MeV  Angle: 26 deg  dp-peak: -0.0045
ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

$^{12}\text{C}$ elastic XS at 400 MeV, 35 degrees

< 1% deviation from calculation
ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

$^{12}$C elastic XS at 400 MeV, 45 degrees

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Run: 4986  BeamE: 401.16 MeV  Angle: 45 deg  dp-peak: -0.0045

< 3 % deviation from calculation
COMPARISON TO WORLD DATA

- By using our available $\omega / |q|$ space over 4 angles, we can interpolate to any $\omega / |q|$, and use Rosenbluth fits to calculate at a given angle.

Iron Comparisons

- **56Fe**
  - Beam-E: 3300 MeV
  - Angle: 15 deg
  - World Data: Chen 1990
  - Hall-A CSR interpolated

- **56Fe**
  - Beam-E: 2020 MeV
  - Angle: 20 deg
  - World Data: Day 1998
  - Hall-A CSR interpolated

- **56Fe**
  - Beam-E: 440 MeV
  - Angle: 90 deg
  - World Data: Meziani 1984
  - Hall-A CSR interpolated
By using our available $\omega / |q|$ space over 4 angles, we can interpolate to any $\omega / |q|$, and use Rosenbluth fits to calculate at a given angle.
COMPARISON TO WORLD DATA

- By using our available $\omega / |q|$ space over 4 angles, we can interpolate to any $\omega / |q|$, and use Rosenbluth fits to calculate at a given angle.
CSR CARBON

$^{12}\text{C}$ at $|q| = 650$ MeV

CSR = 0.86 +/- 0.06
CSR CARBON

$^{12}\text{C at } |q| = 650 \text{ MeV}$

$R_L (\text{GeV}^2)$

CSR = 0.86 +/- 0.06

[Graph showing data points and error bars]
Recent efforts:
- Our understanding of the behavior of the response of the HRS at very low central momentum settings has improved.
- Good understanding of Elastic XS's.
- Good agreement with world data provides confidence in XS extraction, radiative corrections, efficiencies, and interpolation methodology.

Looking ahead:
- LHRS/ RHRS (redundant measurements) comparisons need to be revisited with latest updates to low momentum HRS response.
- The Iron and Carbon CSR is very close to completion (expected this year).
  - Following those, we need to focus on extended target extraction (including Helium, see Kai Jin's talk next).
THANK YOU!!!


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PhD Students   Spokespersons   Run Coordinators