Coulomb Sum Rule \(^{(4}\text{He target})\)

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Coulomb Sum Rule

\[
\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{Mott}\left[ \frac{G_E^2(Q^2)}{1 + \tau} + \frac{\tau G_M^2(Q^2)}{\varepsilon(1 + \tau)} \right]
\]

\[
\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{Mott}\left[ \frac{Q^4}{q^4} R_L(|\vec{q}|,\omega) + \frac{Q^2}{2q^2\varepsilon} R_T(|\vec{q}|,\omega) \right]
\]

- Virtual photon polarization: \(\varepsilon(|\vec{q}|,\omega) = [1 + \frac{2\vec{q}^2}{Q^2} \tan^2 \frac{\theta}{2}]^{-1}\)
- Mott cross-section: \(\sigma_M = \frac{\alpha^2 \cos^2(\frac{\theta}{2})}{4E^2 \sin^4 \frac{\theta}{2}}\)
- Four momentum square: \(Q^2 = \vec{q}^2 - \omega^2 = 4EE' \sin^2(\frac{\theta}{2})\)
- \(\tau = \frac{Q^2}{4M}\)

Rosenbluth separation

\[S_L(|\vec{q}|) = \int_{\omega+} d\omega \frac{R_L(\omega,|\vec{q}|)}{ZG_E^2(Q^2) + NG_E^2(\omega)}\]

At large \(|q| >> 2k_f\), \(S_L\) should go to 1. Any significant deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!
Experiment Setup

- $550 \text{ MeV/c} \leq |q| \leq 1000 \text{ MeV/c}$
- Beam energy: $0.4 - 4 \text{ GeV}$
- Scattered electron energy: $0.1 - 4 \text{ GeV}$
- Scattering angle: $15^\circ, 60^\circ, 90^\circ$ and $120^\circ$ (The number of kinematics is larger)
- LHRS and RHRS independent (redundant) measurements for most settings
- Targets: $^4\text{He}, ^{12}\text{C}, ^{56}\text{Fe}, ^{208}\text{Pb}$
Kinematics

4He kinematics in $E$ (beam energy) and $P_0$ (HRS central momentum)

A lot of kinematics at low momentum.

Runs at large angle (90deg, 120deg) reach very low central momentum $P_0$.

4He kinematics in $\omega$ and $q$

The spectrums are not along constant $q$: interpolation between spectrums is necessary!
Extended Targets

Loop1 (high pressure $^4$He gas): racetrack, 10 cm, 0.12 g/cm$^3$ 6.3K, 1.4 MPa

Loop3, top (Pb foil kept in liquid $\text{H}_2$): 15 cm

Loop3, bottom (liquid $\text{H}_2$): 15 cm, 0.0723 g/cm$^3$ 7.0 K, 170 psi.
Acceptance at Low Momentum

The low momentum acceptance is important for CSR experiment:

- The HRS behavior is different when $P_0 < 450$ MeV:

  $P_0 > 450$ MeV, field ratio between magnets kept nearly constant so optics property does not change with $P_0$.
  $P_0 < 450$ MeV, field ratio between magnets change with $P_0$ by $\sim 3\%$.

- The multiple scattering is inverse-proportional to particle's momentum:

  $$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{x/X_0} \left[ 1 + 0.038 \ln(x/X_0) \right]$$

  at very low momentum ($<200$ MeV), the effect of multiple scattering is big.
Extended Target Acceptance

Acceptance is a function \( A(\delta, \theta_{tg}, \phi_{tg}, y_{tg}) \) which tells the possibility of a particle passes through spectrometer and reaches detectors. The main difference between acceptance used for foil target (\(^{12}\text{C}\) and \(^{56}\text{Fe}\)) and extended target (\(^{4}\text{He}\) and \(^{1}\text{H}\)) is the extra dimension \( y_{tg} \). The extend target acceptance is extracted from Monte-Carlo simulation program SAMC.

A comparison between data and simulation on optics target (sieve slit not inserted) is done to check the performance of monte-carlo:

- Events generated outside target, and include multiple scattering, energy loss before and after main scattering;
- Radiative effects includes internal and external bremsstrahlung are included;
- Events are weighted by cross sections calculated Sum-Of-Gauss fit to world data.

![Graph showing comparison between data and simulation for optics target, E=1102 MeV, θ=15 deg](image)

![Graph showing comparison between data and simulation for optics target, E=399 MeV, θ=35 deg](image)
Extended Target Acceptance

-1 cm < ytg < 1 cm

1 cm < ytg < 3 cm

3 cm < ytg < 5 cm

5 cm < ytg < 7 cm

- Monte-Carlo simulations with \( P_0 \) from 450 MeV to 100 MeV;
- Acceptance sum over \( \theta_{tg} \) and \( \phi_{tg} \), bin with \( dp \) and \( ytg \);
- An interpolation is done between acceptance at different \( P_0 \);
There are many LH2 elastic runs at different angles, with momentum down to ~230 MeV. They can be used to check the quality of acceptance and other correction of data at very low mom. The Monte-Carlo is weighted by cross sections calculated from proton elastic form factor fit of world data. The comparison between data and simulation is absolute.
LH2 elastic run at 90 deg, $E=646.5$ MeV, $P_0=383.2$ MeV

$Y_{\text{Data}}/Y_{\text{MC}} = 0.9753 \pm 0.0259$

cross section $= 7.316\times10^{-33} \text{ cm}^2$

Cuts:
PID cut,
$|\delta| < 3.5\%$, $|\theta_{tg}| < 40\text{mrad}$, $|p_{htg}| < 20\text{mrad}$

$|y_{tg}| < 0.04$
LH2 Elastic run at 120 deg, E=399.5 MeV, P=237.7 MeV

\[ \frac{Y_{\text{Data}}}{Y_{\text{MC}}} = 0.9716 \pm 0.0205 \]

cross section = 1.129e-32 cm²

Cuts:
PID cut, 
\[ |\delta| < 3.5\%, |\theta tg| < 40\text{ mrad}, |phtg| < 20\text{ mrad}, |ytg| < 0.04 \]
Window Subtraction

Use SAMC to simulate the external radiation effect of different aluminum thickness and He4 gas before or after scattering on windows. The simulation runs are weighted by F1F209 fitting.

The radiation factor was put into the following formula:

\[ Y_{corrected} = Y_{cryotarget} - Y_{dummy} \cdot \frac{T_{wall}}{T_{dummy}} \cdot \frac{R_{wall}}{R_{dummy}} \]

He4 target
He4 density=0.12g/cm^3

dummy target
each foil thickness:0.259g/cm^2

entr window: 0.073 g/cm^2
exit window: 0.076 g/cm^2

window subtraction: He4=4174, Dum=4179

Red: cryo target
Blue: dummy(normalized)
$^4$He raw and radiative corrected spectrum at 15 deg.

$^4$He raw and radiative corrected spectrum at 90 deg.
Interpolation

- The $R_L$ should be integrated along a constant $|q|$
- The measured spectrum is along constant beam energy not constant $|q|$
- Interpolation between measured spectrum in $(|q|, \omega)$ is necessary
- The interpolation method should following paths that passes through correspond features in each spectrum.
- Two interpolation methods are used:
  - (1) $y^*$ interpolation, $y^*$ is quasi-elastic scaling variable:
    \[
    \omega + M_A = \left( y^2 + 2y|q| + M^2 + q^2 \right)^{1/2} + \left( y^2 + M^2_{A-1} \right)^{1/2}
    \]
  - (2) $W$ interpolation, $W$ is invariant mass.

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$y^*$ interpolation aligns the quasi-elastic peak better, while $W$ interpolation aligns the dip region better.
Interpolation

\[ \text{Green: W interpolation} \]
\[ \text{Red: } y^* \text{ interpolation} \]
Rosenbluth Separation

\[ \frac{d^2 \sigma}{d\Omega d\omega} \frac{\epsilon}{\sigma_{Mott}} = \epsilon \frac{Q^4}{q^4} R_L(|\tilde{q}|, \omega) + \frac{Q^2}{2q^2} R_T(|\tilde{q}|, \omega) \]

XS at same |q| and \( \omega \) but different angles from interpolation are plotted together and fitted with linear fit:

Slope = \( \frac{Q^4}{q^4} R_L \)

Intercept = \( \frac{Q^4}{2q^2} R_T \)

4He target, \( q=660 \text{ MeV} \), \( \omega=270 \text{ MeV} \)
Summary

Recent work:
• Get extended target acceptance at low momentum, and check with LH2 elastic;
• Extract $^4$He spectrums for 4 angles;

Plans:
• Interpolation and LT separation;
• Compare with world data;
$^4$He raw and radiative corrected spectrum at 90 deg.

$^4$He raw and radiative corrected spectrum at 120 deg.