Quantum correlations functions overview
Motivations

hadrons as emergent phenomena of QCD

quarks and gluons
Motivations

hadrons as **emergent phenomena** of QCD

nucleon structure  quarks and gluons
Motivations

hadrons as emergent phenomena of QCD

nucleon structure  quarks and gluons  hadronization
Motivations

electron \rightarrow \text{proton}
Motivations
Motivations
Motivations

Where are the quarks and gluons?
Motivations

- Quark and gluon d.o.f. cannot be measured directly.
Motivations

- Quark and gluon d.o.f. cannot be measured directly

- Experimental measurements can be interpreted in terms of quark and gluon d.o.f.
Motivations

The interpretation relies on:

- QCD factorization theorems (theory)
- Experimental cross section measurements
- Global QCD analysis (Bayesian inference)
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- **QCD factorization theorems (theory)**
- **Experimental cross section measurements**
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Motivations

The strategy:
Motivations

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1. Define nucleon structure/hadronization objects in QFT
Motivations

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2. Identify cross sections that factorize in terms of such QFT objects
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The strategy:

1. Define **nucleon structure/hadronization** objects in QFT
2. Identify cross sections that **factorize** in terms of such QFT objects
3. Perform a **global QCD analysis**
Motivations

What do we mean by “structure of nucleon”? e.g.
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\[ f_{j/h}(\xi) = \int \frac{dw^-}{2\pi} e^{-i\xi P^+ w^-} \left\langle P \right| \bar{\psi}_j(0, w^-, 0_T) \frac{\gamma^+}{2} \psi_j(0) \left| P \right\rangle \]

Not currently computable from first principles
Needs to be inferred from data
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\[ d_{h/j}(\zeta) = \frac{\text{Tr}_{\text{color}, \text{Dirac}}}{4N_{c,j}} \sum_X \zeta \int \frac{dw^+}{2\pi} e^{i(p^-/\zeta)w^+} \]

\[ \times \gamma^- \langle 0 | \bar{\psi}_j(0, w^+, 0_T) | p_h, X \rangle \langle p_h, X | \psi_j(0) | 0 \rangle \]

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Acronyms for 1D distributions

\( f_{j/h}(\xi) \): "Parton Distribution Functions"

\( d_h/j(\zeta) \): "Fragmentation Functions"
Motivations

Acronyms for 1D distributions

- $f_{j/h}(\xi)$: “Parton Distribution Functions”
- PDFs
Motivations

Acronyms for 1D distributions

- $f_{j/h}(\xi)$: “Parton Distribution Functions”
  - PDFs

- $d_{h/j}(\zeta)$: “Fragmentation Functions”
  - FFs
Motivations

What do we mean by “factorization”? e.g DIS
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\[ F_2(x, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} \quad C_2(\xi, \mu) \quad f_j \left( \frac{x}{\xi}, \mu \right) \]
Motivations

What do we mean by “factorization”? e.g. DIS

\[ F_2(x, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} C_2(\xi, \mu) f_j \left( \frac{x}{\xi}, \mu \right) \]

- \( C_2 \) is calculable in perturbative QCD
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- \( C_2 \) is calculable in perturbative QCD
- \( f_j \) cannot be solved in closed form

→ inverse problem
Motivations

Another example: SIDIS

\[ F_{1}^{h}(x, z, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta} \ C_1(\xi, \zeta, \mu) \ f_j \left( \frac{x}{\xi}, \mu \right) \ d_j \left( \frac{z}{\zeta}, \mu \right) \]
Motivations

Another example: SIDIS

\[ F^h_1(x, z, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta} \left( C_1(\xi, \zeta, \mu) \right) \left( f_j \left( \frac{x}{\xi}, \mu \right) \right) \left( d_j \left( \frac{z}{\zeta}, \mu \right) \right) \]

\[ C_1 \] is calculable in perturbative QCD
Motivations

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\[ F_1^h(x, z, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta} C_1(\xi, \zeta, \mu) f_j \left( \frac{x}{\xi}, \mu \right) d_j \left( \frac{z}{\zeta}, \mu \right) \]

- \( C_1 \) is calculable in perturbative QCD
- \( f_j \) and \( d_j \) cannot be solved in closed form

\[ \rightarrow \text{inverse problem} \]
Motivations

**Universality** $\rightarrow$ the predictive power of QCD
Motivations

Universality $\rightarrow$ the predictive power of QCD

$$\sigma_{l+P \rightarrow l+X}^{\text{EXP}} = C_{l+k \rightarrow l+X} \otimes f$$
Motivations

**Universality** → the predictive power of QCD

\[
\sigma_{l+P \rightarrow l+X}^{\text{EXP}} = C_{l+k \rightarrow l+X} \otimes f
\]

\[
\sigma_{l+P \rightarrow l+H+X}^{\text{EXP}} = C_{l+k \rightarrow l+k+X} \otimes f \otimes d
\]
Motivations

Universality $\rightarrow$ the predictive power of QCD

$$\sigma_{\text{EXP} \ l+P \rightarrow l+X} \ = \ C_{l+k \rightarrow l+X} \otimes f$$

$$\sigma_{\text{EXP} \ l+P \rightarrow l+H+X} \ = \ C_{l+k \rightarrow l+k+X} \otimes f \otimes d$$

$$\sigma_{\text{EXP} \ P+P \rightarrow l+\bar{l}+X} \ = \ C_{k+k \rightarrow l+\bar{l}+X} \otimes f \otimes f$$
Motivations

Universality $\rightarrow$ the predictive power of QCD

$$\sigma_{l+P \rightarrow l+X}^{\text{EXP}} = C_{l+k \rightarrow l+X} \otimes f$$

$$\sigma_{l+P \rightarrow l+H+X}^{\text{EXP}} = C_{l+k \rightarrow l+k+X} \otimes f \otimes d$$

$$\sigma_{P+P \rightarrow l+\bar{l}+X}^{\text{EXP}} = C_{k+k \rightarrow l+\bar{l}+X} \otimes f \otimes f$$

$$\sigma_{l+\bar{l} \rightarrow H+X}^{\text{EXP}} = C_{l+\bar{l} \rightarrow k+X} \otimes d$$
Global QCD analysis in a nutshell
Global QCD analysis in a nutshell

1. Parametrize $f$’s and $d$’s
Global QCD analysis in a nutshell

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\[
\begin{align*}
    f_j(\xi) &= N_j \xi^{a_j} (1 - \xi)^{b_j} P(\xi; \omega_j) \\
    d_j(\zeta) &= \tilde{N}_j \zeta^{\tilde{a}_j} (1 - \zeta)^{\tilde{b}_j} P(\zeta; \tilde{\omega}_j)
\end{align*}
\]
Global QCD analysis in a nutshell

1. Parametrize $f$’s and $d$’s

\[
\begin{align*}
 f_j(\xi) &= N_j \xi^{a_j} (1 - \xi)^{b_j} P(\xi; w_j) \\
 d_j(\zeta) &= \tilde{N}_j \zeta^{\tilde{a}_j} (1 - \zeta)^{\tilde{b}_j} P(\zeta; \tilde{w}_j)
\end{align*}
\]

\[p = (..., N_j, a_j, b_j, w_j..., \tilde{N}_j, \tilde{a}_j, \tilde{b}_j, \tilde{w}_j, ...)
\]
Global QCD analysis in a nutshell

2. Sample the Bayesian posterior distribution
Global QCD analysis in a nutshell

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$$\rho(p|\text{data}) \propto \mathcal{L}(p, \text{data})\pi(p)$$
Global QCD analysis in a nutshell

2. Sample the Bayesian posterior distribution

\[ \rho(p \mid \text{data}) \propto \mathcal{L}(p, \text{data}) \pi(p) \]

\[
E[\mathcal{O}] = \frac{1}{N} \sum_{k} O(p_k) \quad V[\mathcal{O}] = \frac{1}{N} \sum_{k} [O(p_k) - E[O]]^2
\]
Global QCD analysis in a nutshell

2. Sample the Bayesian posterior distribution

\[
\rho(p|\text{data}) \propto \mathcal{L}(p, \text{data}) \pi(p)
\]

\[
E[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(p_k) \quad \text{V}[\mathcal{O}] = \frac{1}{N} \sum_k [\mathcal{O}(p_k) - E[\mathcal{O}]]^2
\]

\[
\mathcal{O} = f, d, \sigma, ...
\]
Global QCD analysis in a nutshell
Global QCD analysis in a nutshell

Experiments

\( lN \)

\( NN \)

\( \bar{u} \)

\( lN \) → DIS

SIDIS

DY

SIA
Global QCD analysis in a nutshell
Global QCD analysis in a nutshell
Global QCD analysis in a nutshell

Experiments

$\bar{l}N$

$NN$

$\bar{u}u$

$\ell N$

DIS

SIDIS

DY

SIA

Exp $\neq$ Thy

Yes

No

“inference”

Parameters

PDF

FF

SIA

DY

SIDIS

DIS

Theory $=$ QCD
Global QCD analysis in a nutshell

Experiments

Theory = QCD

PDF

FF

Parameters

"inference"

Exp ? = Thy

No

Yes

Physics interpretation

\( lN \)

\( NN \)

\( \bar{u} \)

DIS

SIDIS

DY

SIA

\[ \text{Exp} \neq \text{Thy} \]
Global QCD analysis in a nutshell

Experiments

Optimize measurements

Theory=QCD

Parameters

“inference”

Yes

No

Exp ≠ Thy

DIS

SIDIS

DY

SIA

PDF

FF

PDF

FF

Theory=QCD

Exp = Thy

No

“inference”

Parameters

Optimize measurements

Yes

No

Physics interpretation
Bayesian inference
Bayesian inference

- Maximum likelihood (CJ, CT, MMHT,...)
Bayesian inference

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\[ E[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(p_k) \sim \mathcal{O}(p_0) \]
Bayesian inference

- Maximum likelihood (CJ, CT, MMHT,...)

\[ E[\mathcal{O}] = \frac{1}{N} \sum_{k} \mathcal{O}(p_k) \sim \mathcal{O}(p_0) \]

\[ V[\mathcal{O}] = \frac{1}{N} \sum_{k} [\mathcal{O}(p_k) - E[\mathcal{O}]]^2 \]

\[ = \text{hessian, lagrange} \]
Bayesian inference

- Data resampling (JAM, NNPDF)
Bayesian inference

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  + Generate $N$ resampled data

  $$\sigma_{i,k} = \sigma_i + R_{i,k}\delta\sigma_i$$
Bayesian inference

- Data resampling (JAM, NNPDF)
  
  + Generate $N$ resampled data
  
  + $\{p_k : 1...N\}$ from $N$ fits to resampled data

\[ \sigma_{i,k} = \sigma_i + R_{i,k} \delta \sigma_i \]
Bayesian inference

- Data resampling (JAM, NNPDF)
  
  + Generate $N$ resampled data
  
  + $\{p_k : 1...N\}$ from $N$ fits to resampled data
  
  + Use flat priors as guess for the $N$ fits

\[ \sigma_{i,k} = \sigma_i + R_{i,k} \delta \sigma_i \]
Bayesian inference

- Other approaches
Bayesian inference

- Other approaches
  - Hybrid Markov Chain (Gbedo, Mangin-Brinet)
Bayesian inference

**Other approaches**

- Hybrid Markov Chain (Gbedo, Mangin-Brinet)
- Nested sampling (JAM)
  → challenging for higher dimensions \( O(100) \)
Summary

- **The goal**: understand the structure of hadrons in terms of quarks and gluons
- **The challenge**: we cannot detect quarks and gluons in isolation
- **The method**: QCD factorization & global analysis