Quantum correlations functions overview

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A.I. for Nuclear Physics Workshop
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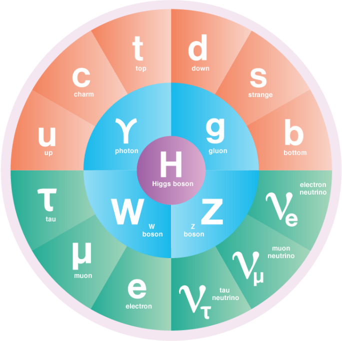
Motivations

hadrons as emergent phenomena of QCD



quarks and gluons

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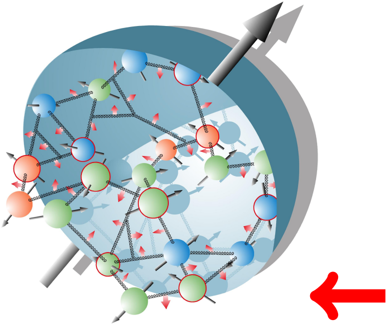
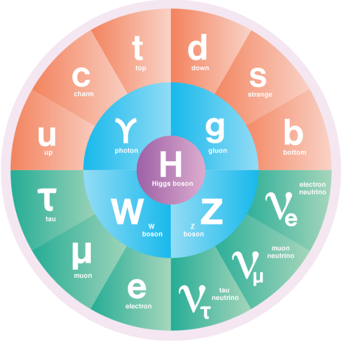


nucleon structure



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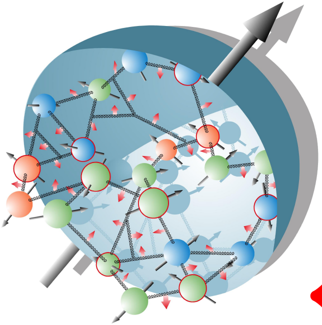

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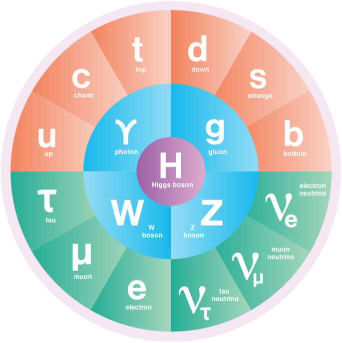

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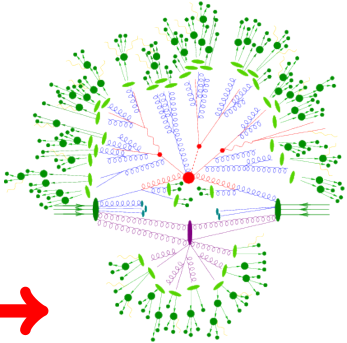
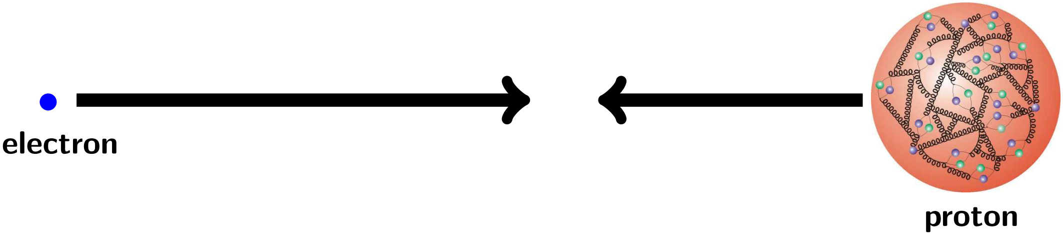


hadronization

Motivations



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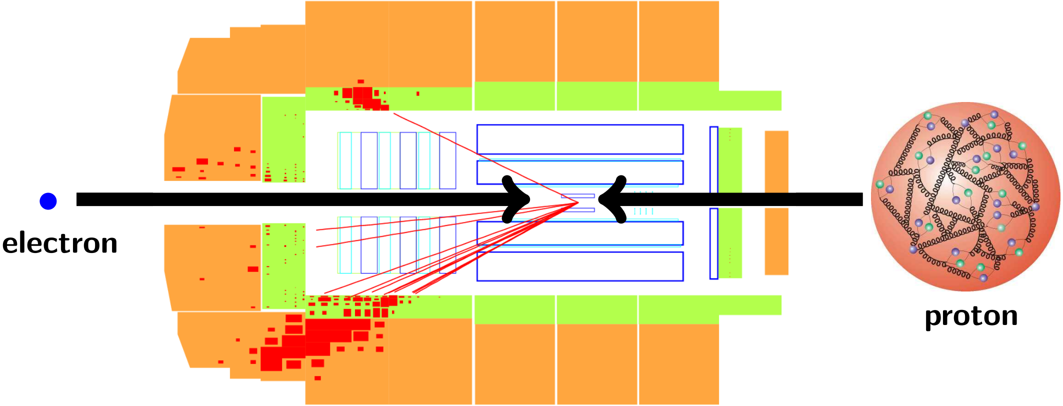

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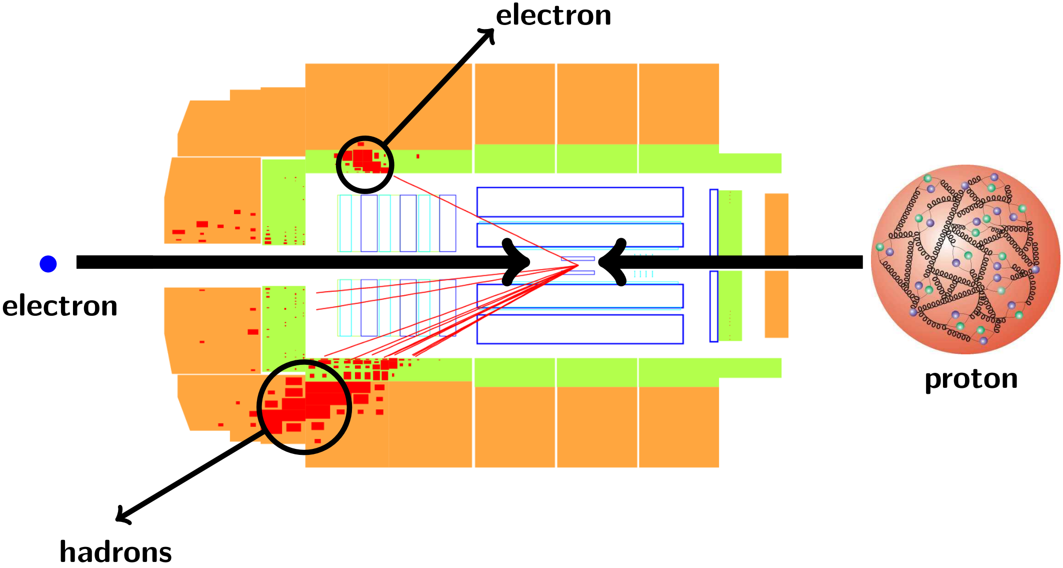

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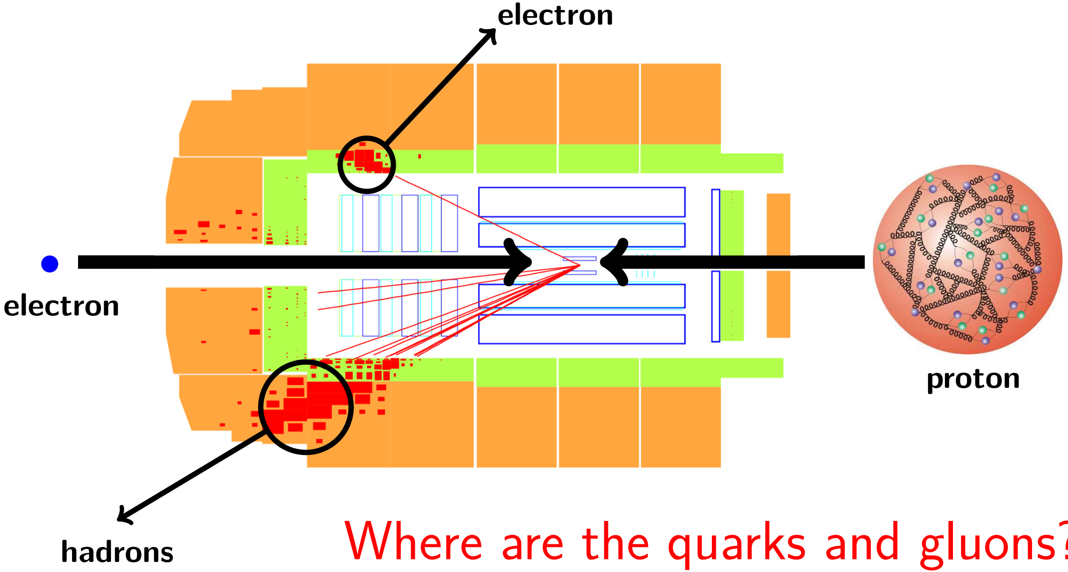

Where are the quarks and gluons?

Motivations

- Quark and gluon d.o.f. **cannot be measured** directly

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- Experimental measurements **can be interpreted** in terms of quark and gluon d.o.f.

Motivations

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- Experimental cross section measurements
- Global QCD analysis (Bayesian inference)

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2. Identify cross sections that **factorize** in terms of such QFT objects
3. Perform a **global QCD analysis**

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- $d_{h/j}(\zeta)$: "Fragmentation Functions"
  FFs

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$\rightarrow$ **inverse problem**

Motivations

Another example: SIDIS

$$F_1^h(x, z, Q) = x \sum_j e_j^2 \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta} \; C_1(\xi, \zeta, \mu) \quad f_j\left(\frac{x}{\xi}, \mu\right) \quad d_j\left(\frac{z}{\zeta}, \mu\right)$$

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Global QCD analysis in a nutshell

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$$f_j(\xi) = N_j \xi^{a_j} (1-\xi)^{b_j} P(\xi; \boldsymbol{w}_j)$$
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$$\boldsymbol{p} = (..., N_j, a_j, b_j, \boldsymbol{w}_j ..., \tilde{N}_j, \tilde{a}_j, \tilde{b}_j, \tilde{\boldsymbol{w}}_j, ...)$$

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$$\mathcal{O} = f, d, \sigma, ...$$

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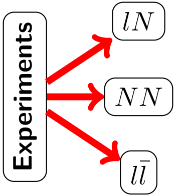

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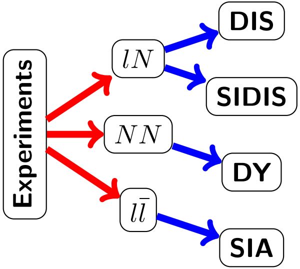

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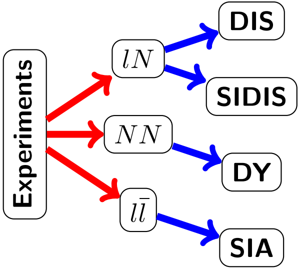

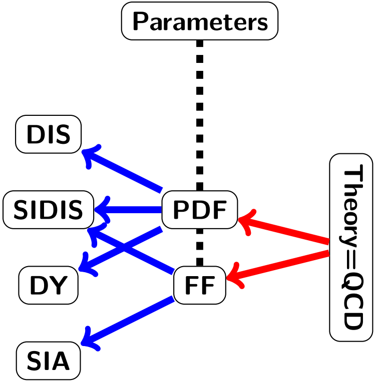

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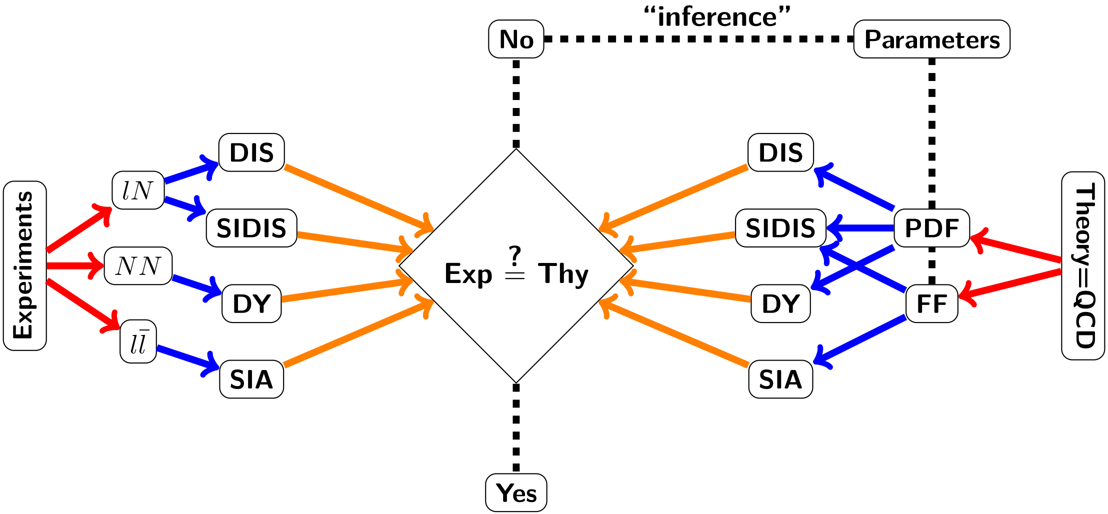

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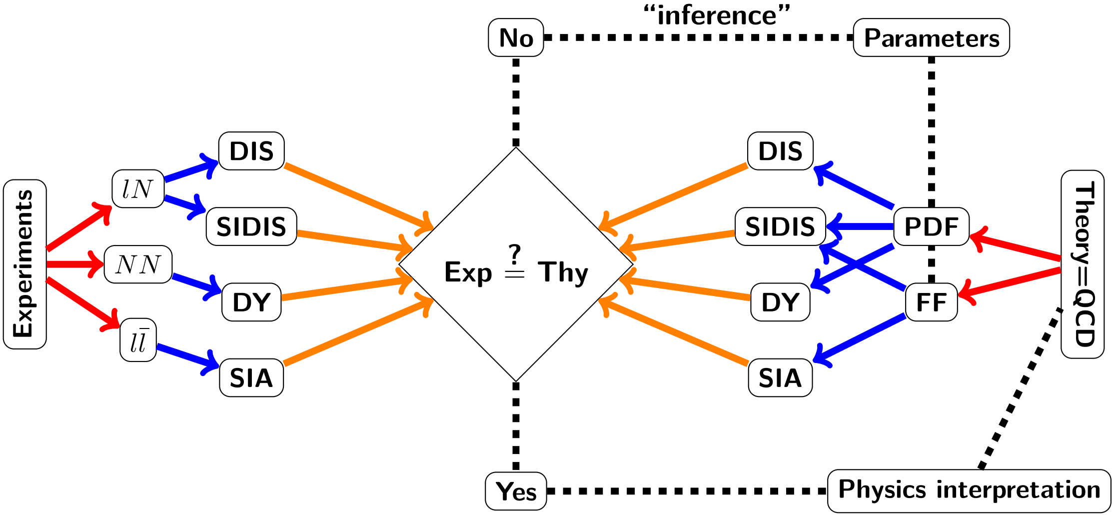

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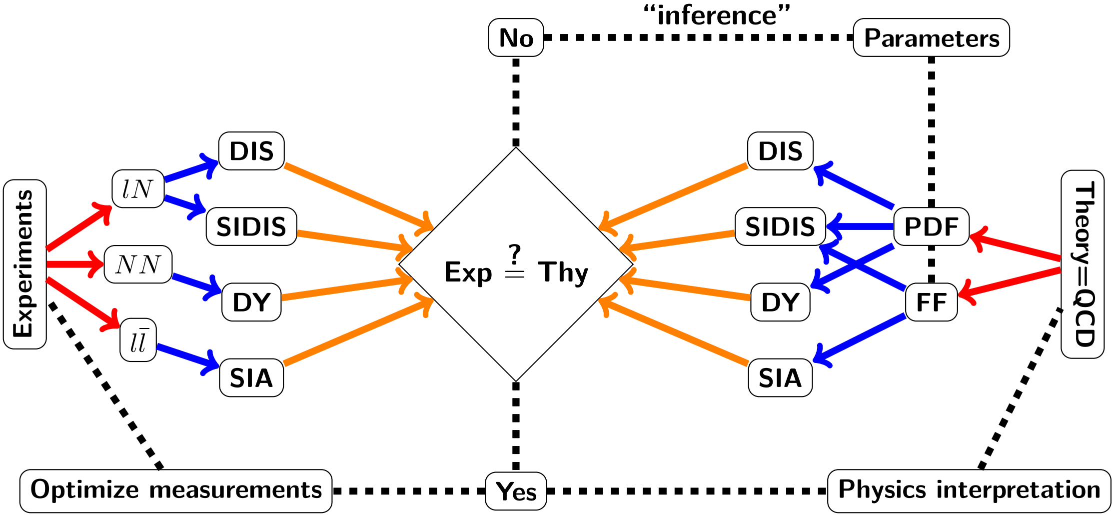

Bayesian inference

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$$= \text{hessian},\ \text{lagrange}$$

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  - Generate N resampled data $\sigma_{i,k} = \sigma_i + R_{i,k}\delta\sigma_i$
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  - Use flat priors as guess for the N fits

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Bayesian inference

- Other approaches
  - + Hybrid Markov Chain (Gbedo, Mangin-Brinet)
  - + Nested sampling (JAM)
    $\rightarrow$ challenging for higher dimensions $O(100)$

Summary

- **The goal**: understand the structure of hadrons in terms of quarks and gluons
- **The challenge**: we cannot detect quarks and gluons in isolation
- **The method**: QCD factorization & global analysis