AI for Jets in JAM

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There are three parts in my talk today:

- motivation,
- challenges,
- results.
In DIS, sensitivity to gluon only appears in NLO

\[ \gamma \pm \gamma q \rightarrow q, \quad (1) \]

while for Jet, gluon diagrams appears at LO

\[ g \rightarrow q, \quad q \rightarrow g \]
JAM 2015 fits [1] on $\Delta g$ has larger error bands than other flavors with the RHIC Jet $A_{LL}$ data, we can narrow down the uncertainty significantly.
The RHIC [2] measured Jet $A_{LL}$ is

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}},$$

where the denominator is the spin averaged cross section. Therefore we can fit spin averaged and spin dependent PDFs at the same time.
Why Mellin

DGLAP equations are convoluted in $x$ space

$$\frac{\partial q(x, t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int_x^1 P(\frac{x}{z}, \alpha_s(t)) q(z, t) \frac{1}{z} \, dz,$$

(3)

apply Mellin transformation $\int_0^1 x^{N-1} \, dx$ and the right side becomes nothing but a Mellin convolution

$$\frac{\partial \widetilde{q}(N, t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \widetilde{P}(N, \alpha_s(t)) \widetilde{q}(N, t),$$

(4)

and is no longer convoluted.
The single jet inclusive cross section is given by [3]

\[
\frac{E_J d^3 \sigma}{d^3 p_J} = \frac{1}{16\pi^2 s} \sum_{i,j,k,l} \int_0^1 \frac{dx}{x} \frac{dy}{y} f_i(x, \mu^2) f_j(y, \mu^2) \cdot 
\overline{\sum} |M_{ij \to kl}|^2 \frac{1}{1 + \delta_{kl}} \delta (\hat{s} + \hat{t} + \hat{u}),
\]

where \( f_i \) and \( f_j \) are the parton distribution functions.
Double Mellin Transformation

Writing equation (5) in a Mellin inverted form, we can single out the $x$ dependence of $f_i$ and $f_j$ from their shapes, the cross section can be written proportional to \( \frac{E_J d^3 \sigma}{d^3 p_J} \propto \sigma_1 + \sigma_2 \)

\[
\sigma_1 = \frac{-1}{2\pi^2} \text{Re} \left( \int_0^\infty \text{d}r_N \text{d}r_M \tilde{f}_i (M) e^{2i\phi} \tilde{f}_j (N) \int \text{d}x \text{d}y x^{-M} y^{-N} \mathcal{H} \right),
\]

\[
\sigma_2 = \frac{1}{2\pi^2} \text{Re} \left( \int_0^\infty \text{d}r_N \text{d}r_M \tilde{f}_i (M) \tilde{f}_j (N)^* \int \text{d}x \text{d}y x^{-M} y^{-N^*} \mathcal{H} \right),
\]

where $\mathcal{H}$ represents the perturbative part and we define Mellin tables as

\[
T_{M,N} = \int \text{d}x \text{d}y x^{-M} y^{-N} \mathcal{H}, \quad T_{M,N}^* = \int \text{d}x \text{d}y x^{-M} y^{-N^*} \mathcal{H}.
\]
Mellin Moments of PDFs

Generally when fitting PDFs, they were taken the form 
\[ f(x) = x^\alpha (1 - x)^\beta, \]
the Mellin transformation turns out to be simple

\[
\tilde{f}(N) = \int_0^1 x^{N-1} f(x) \, dx = B(N + \alpha, \beta + 1),
\]

the shape of the PDFs is now contained in the Beta function.
We need to compute the integrals over $x$ and $y$ for each $M$ and $N$ along the contour to construct Mellin tables.

There are $68 \times 68$ points in total for $M$ and $N$.

For all 6 channels and 4 components in each channel, we compute $68 \times 68$ integrals. And this is only one data point.

We have in total 196 data points.
Here we are showing the single inclusive Jet data from CDF 2002 to 2006 Run II 1.96 TeV data from [4]

Figure 1: different multipliers are applied to distinguish η bins
We have carried out a new global analysis using the multi step strategy as explained by Carlota
As we have seen in the previous slides, a lot of computation was put into constructing the Mellin tables. It would be great if we can somehow cleverly avoid this hurdle. And there is, it is machine learning.

We can let the machine learn the Mellin tables and once new data points are added, we will be able to find their Mellin tables by interpolation.

Also, by doing machine learning, we save the hard disk space, because we would store only one model for many data points.
$p_T$ bins

Using machine learning models to predict, we can also narrow down the $p_T$ bins \([5]\), which is very time consuming in previous method.

![Graph showing $p_T$ bins vs. $\chi^2$]

- $DØ$, 0.70 fb$^{-1}$
- $0.8 < |y_{jet}| < 1.2$
- $\chi^2 = 12.0/16$

(Data / Ansatz)
Now the challenge for Jet is, using double Mellin table, the values at large $M$ and $N$ are very suppressed, can be as small as $10^{-69}$. Learning such small values is a challenge for the neutral network.

![Graphs of rows and columns](image)
The work shown here is in collaboration with Nobuo Sato, Wally Melnitchouk and Carlota Andrés, thanks to Patrick Barry and Christopher Cocuzza for helpful discussions!
Reference


