Structure of TMD observables

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A.I. for nuclear physics

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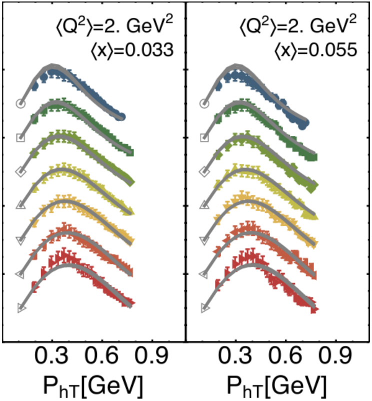
From data to partons

Semi-Inclusive Deep-Inelastic Scattering (SIDIS): $\ell(l)\, N(P) \to \ell(l')\, h(P_h)\, X$



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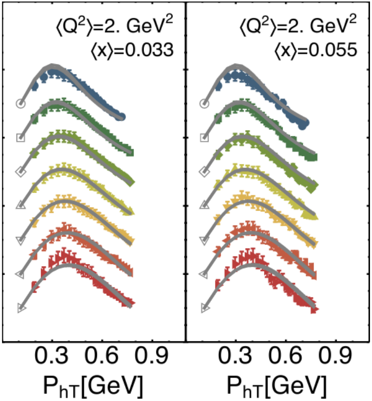

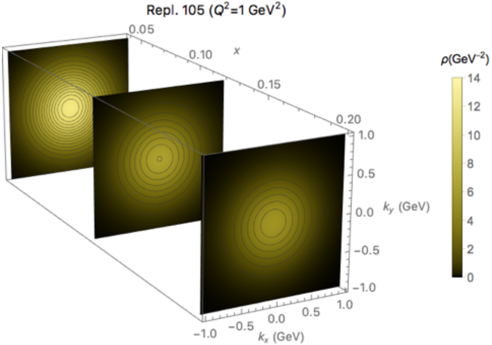

Quark tomography of the nucleon

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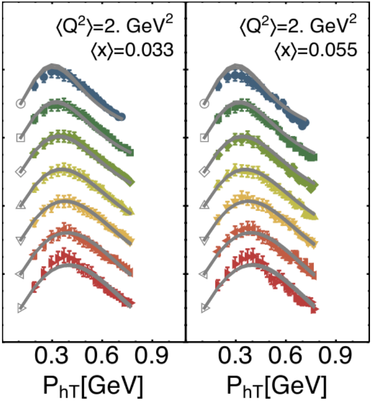
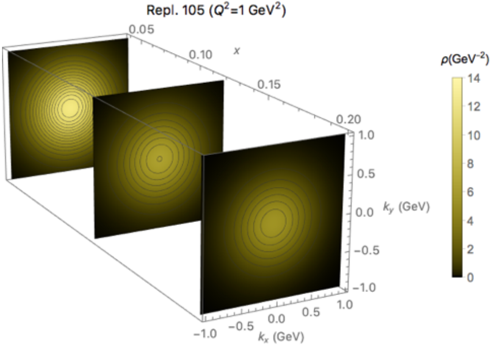


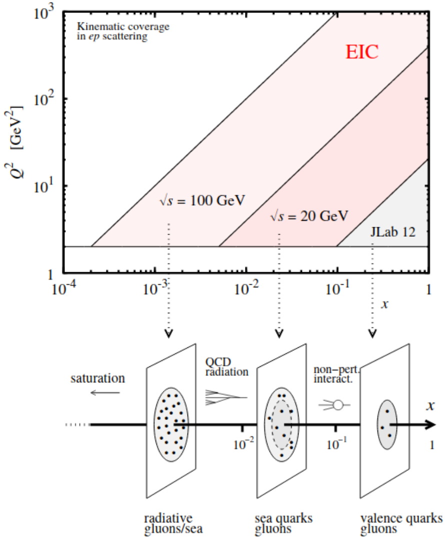


Quark tomography of the nucleon

How to access hadron tomography from experimental data?

Kinematic coverage for SIDIS

credit: C. Weiss



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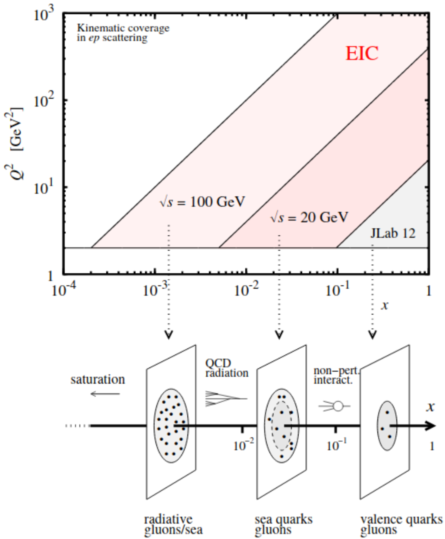

- JLab 12: valence structure

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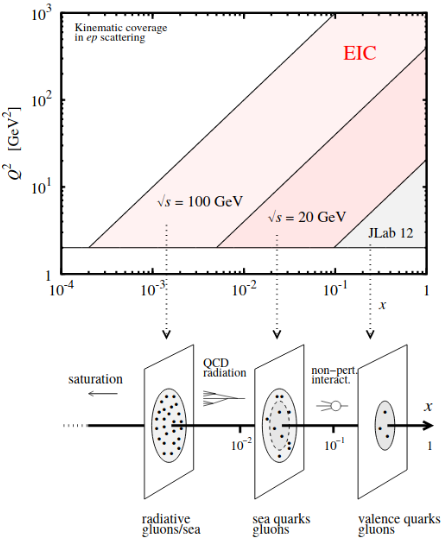

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- COMPASS: valence/sea quarks

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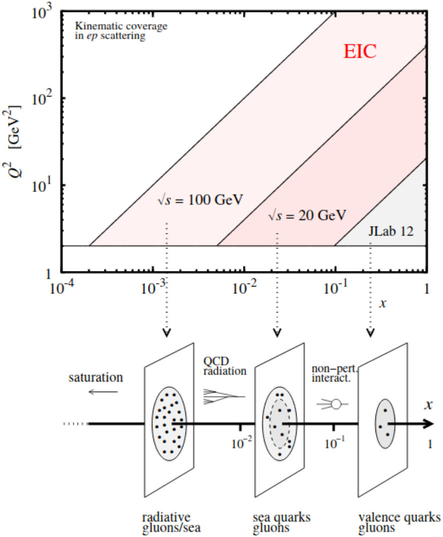

- JLab 12: valence structure
- COMPASS: valence/sea quarks
- EIC: sea quarks and (radiative) glue

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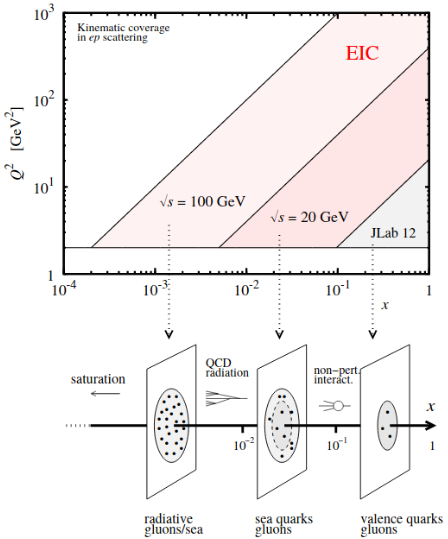
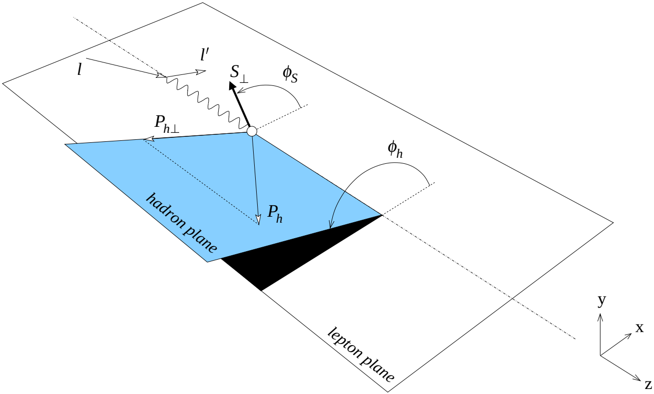


The Electron-Ion Collider (EIC) will greatly extend the kinematic reach of exisisting facilities for SIDIS

- JLab 12: valence structure
- COMPASS: valence/sea quarks
- EIC: sea quarks and (radiative) glue

SIDIS: ingredients

credit: A. Bacchetta



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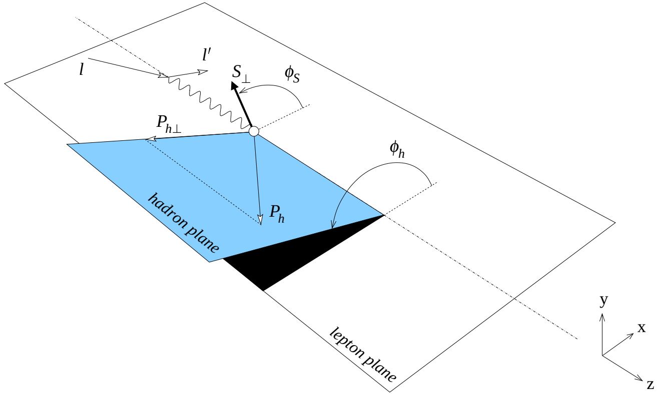

Ingredients to build the cross section:

- available four-vectors (momenta for leptons, hadrons, photon, spin of target hadron)

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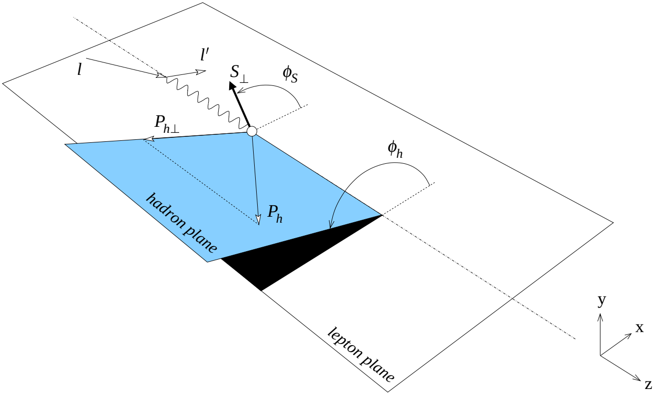


Ingredients to build the cross section:

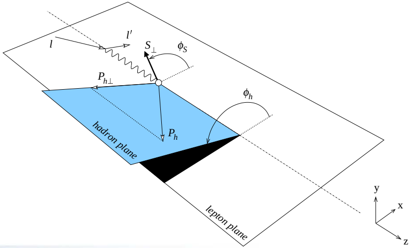
- available four-vectors (momenta for leptons, hadrons, photon, spin of target hadron)
- symmetries of the theory

SIDIS: Structure Functions

$$\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$ $\rightarrow$ 4D quantities

$+\,\epsilon\,F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\,\cos(\phi_h)\,F_{UU}^{\cos\phi_h} + \epsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$ $\rightarrow$ unpolarized



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$+\,S_{||}\left[\ldots 2\,\text{SFs}\ldots\right]+$

$+\,S_{||}\,\lambda_e\left[\ldots 2\,\text{SFs}\ldots\right]$

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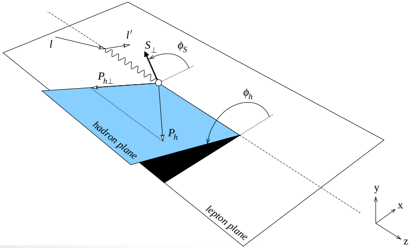

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18 structure functions in total!

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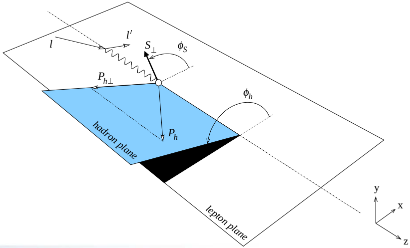

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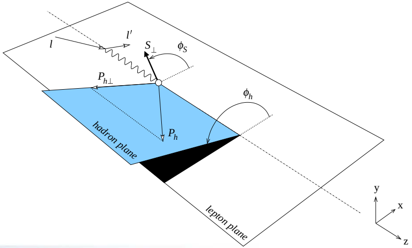
Experimental data for all of them



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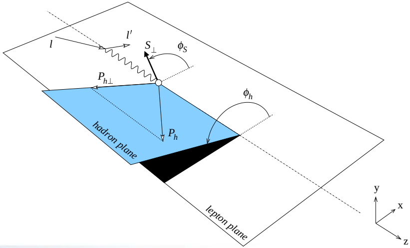
Theory? Connection to parton physics

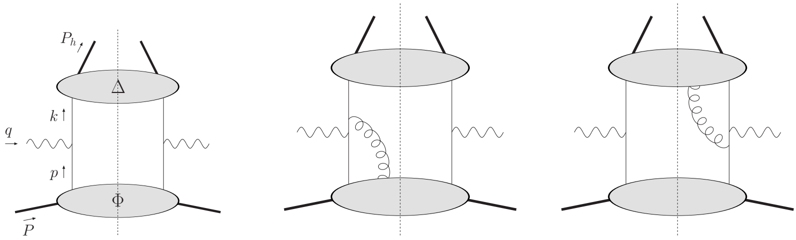


Diagrammatic approach

Tree level connection between structure functions and partonic quantities

SFs $\rightarrow$ tree-level convolutions of *(un)polarized* TMD PDFs and TMD FFs

$$F_{\ldots} \rightarrow x \sum_{a,f,D} e_a^2 \int d^2 p_T \, d^2 k_T \, \delta^{(2)}(p_T - k_T - P_{h\perp}/z) \, f^{a/N}(x, p_T^2) \, D^{a\rightarrow h}(z, k_T^2)$$



hep-ph/0611265

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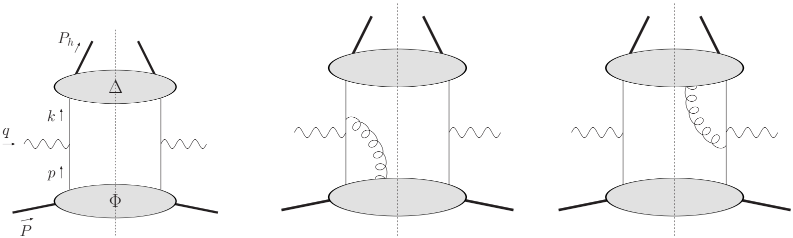
PDFs : parton distribution functions - hadron → parton transition



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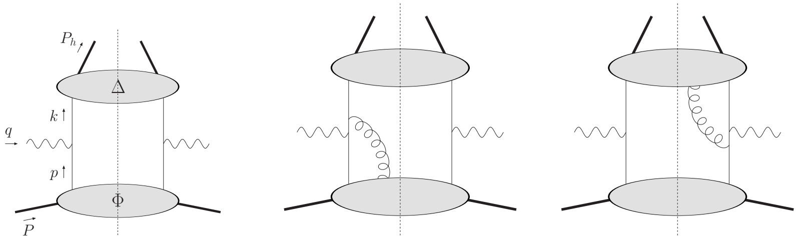

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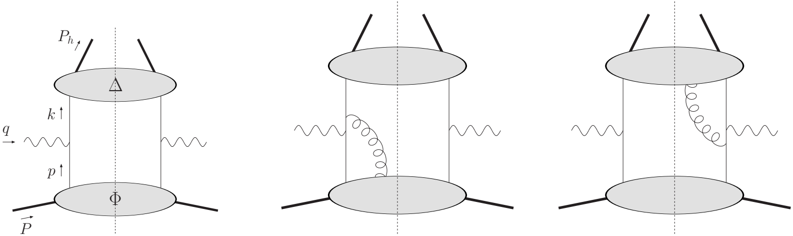
Separation of hard and soft physics $\rightarrow$ factorization (complete only for $F_{UU,T}$)



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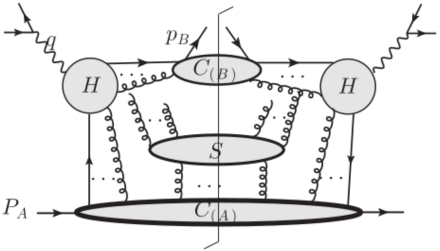
TMD factorization

Proper separation of perturbative and non-perturbative (= structure) physics

$$F_{UU,T}(x, z, P_{hT}^2, Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2)$$

$$\times\ x \int d^2 p_T\, d^2 k_T\, \delta^{(2)}(z k_\perp + P_\perp - P_{hT})\, f_1^{a/N}(x, p_T^2, Q^2)\, D_1^{a\to h}(z, k_T^2, Q^2)$$

picture from Collins pQCD book



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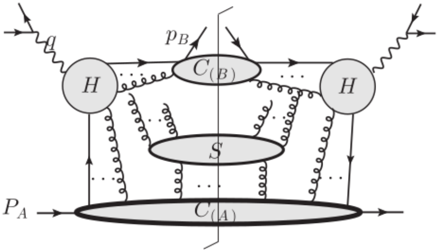
$$+\, Y_{UU,T}(x,z,P_{hT}^2,Q^2)$$



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$$+\, Y_{UU,T}(x,z,P_{hT}^2,Q^2) + \mathcal{O}(M^2/Q^2)$$

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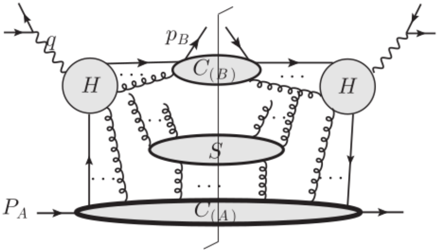

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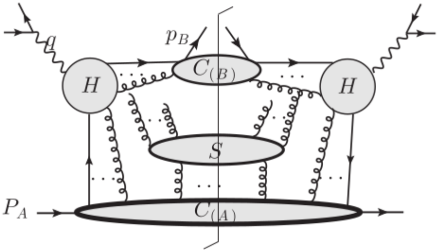
$$+ Y_{UU,T}(x,z,P_{hT}^2,Q^2) + \mathcal{O}(M^2/Q^2) \approx$$

low transverse momentum



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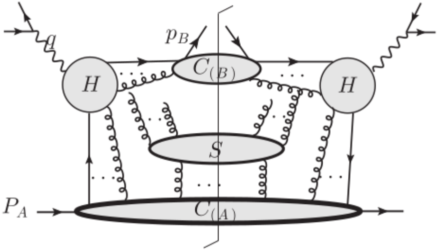
low transverse momentum

$$\approx \sum_a \mathcal{H}^a_{UU,T}(Q^2) \int_0^\infty db_T\, b_T\, J_0(b_T|P_{hT}|/z)\, \tilde{f}_1^{a/N}(x,b_T^2,Q^2)\, \tilde{D}_1^{a\to h}(z,b_T^2,Q^2)$$



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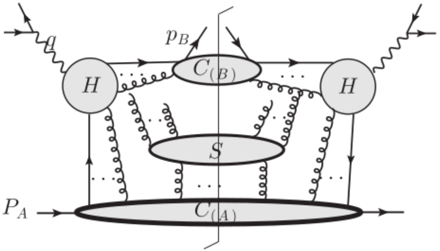
- $\mathcal{H}$: perturbative
- $\tilde{f}_1, \tilde{D}_1$: perturbative and non-perturbative



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Structure of TMDs

Fourier transform of a TMD distribution (b_T space):

$$\tilde{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \tilde{f}_1^a(x, b_T^2; \mu_i, \zeta_i)$$

$$\times \exp\left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\} \qquad \longrightarrow \mu \text{ evolution}$$

$$\times \left(\frac{\zeta_f}{\zeta_i} \right)^{-K(b_T;\mu_i)} \qquad \longrightarrow \zeta \text{ evolution}$$

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The input TMD: expanded at low b_T on the collinear distributions:

$$\tilde{f}_1^a(x, b_T^2;\, \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2; \mu_i, \zeta_i) \otimes f_1^b(z, \mu_i)$$

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$\gamma_F, K, C_{a/b}$: calculable in perturbation theory

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Structure of TMDs

Fourier transform of a TMD distribution (b_T space):

$$\tilde{f}_1^a(x, b_T^2;\, \mu_f, \zeta_f) = \tilde{f}_1^a(x, b_T^2;\, \mu_i, \zeta_i)$$

$$\times \exp\left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[\alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\} \qquad \longrightarrow \mu \text{ evolution}$$

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f_1, F_{NP}, g_K : non-perturbative contributions - fits to data

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$F_{UU,T}$ - for every experimental bin (e.g. $\mathcal{O}(\#) \sim 10^{4,5}$):

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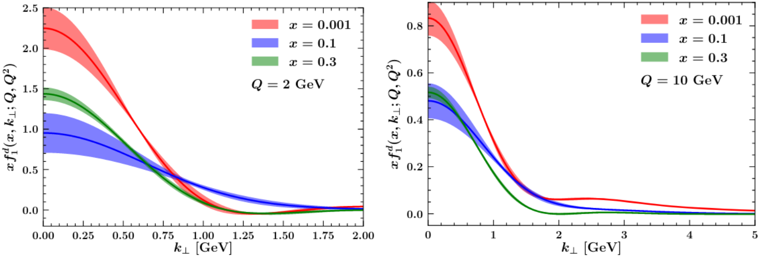
Extractions

$f_1^{a/h}(x, k_T^2, Q)$: probabily of finding quark/gluon with a fraction x of the hadron's momentum and a transverse momentum k_T at the energy/resolution scale Q

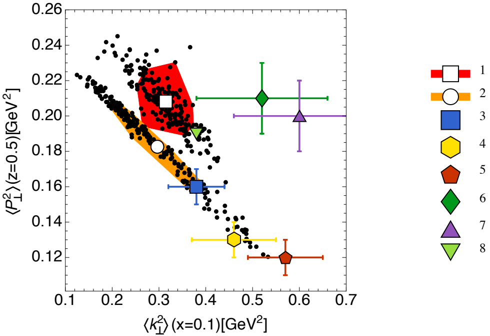


hadron structure in terms of quarks/gluons in 3D momentum space
(from PV19 fit - 1912.07550)

Extractions

SIDIS: convolution of hadron structure and hadronization

$$F_{UU,T} \approx \sum_a \mathcal{H}^a_{UU,T} \int_0^\infty db_T\, b_T\, J_0(b_T|P_{hT}|/z)\, \tilde{f}_1^{a/N}(x, b_T^2, Q^2)\, \tilde{D}_1^{a\to h}(z, b_T^2, Q^2)$$



Extractions of
TMD PDFs and FFs
(PV17 fit: 1703.10157)

$\langle k_\perp^2 \rangle$ and $\langle P_\perp^2 \rangle$

mapping quarks
in 3D momentum space

Backup

Twist 2 quark TMD PDFs

Twist-2 transverse momentum dependent PDFs
for a quark in a spin 1/2 hadron

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

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$f_{1T}^\perp$ and $h_1^\perp$ are time-reversal odd and are process dependent

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$f_{1T}^{\perp}$ and $h_1^{\perp}$ are time-reversal odd and are process dependent

The experimental confirmation of this calculable process dependence
(= sign flip between Semi-Inclusive DIS and Drell-Yan) is
a fundamental test for the symmetries of QCD

Twist 3 quark TMD PDFs

Twist-3 transverse momentum dependent PDFs
for a quark in a spin 1/2 hadron

quark pol.

nucleon pol.	U	L	T
U	$f^\perp$	$g^\perp$	h, e
L	$f_L^\perp$	$g_L^\perp$	h_L, e_L
T	f_T, $f_T^\perp$	g_T, $g_T^\perp$	h_T, $h_T^\perp$, e_T, $e_T^\perp$

Twist 3 quark TMD PDFs

Twist-3 transverse momentum dependent PDFs
for a quark in a spin 1/2 hadron

quark pol. / nucleon pol.

nucleon pol.	U	L	T
U	$f^{\perp}$	$g^{\perp}$	h, e
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L, e_L
T	f_T, $f_T^{\perp}$	g_T, $g_T^{\perp}$	h_T, $h_T^{\perp}$, e_T, $e_T^{\perp}$

Functions in black survive transverse momentum integration.
Functions in red are T-odd.
A similar decomposition at twist 2 and 3 exists also for FFs.