Structure of TMD observables

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From data to partons

Semi-Inclusive Deep-Inelastic Scattering (SIDIS): \( \ell(l) \, N(P) \rightarrow \ell(l') \, h(P_h) \, X \)
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Quark tomography of the nucleon
From data to partons

Semi-Inclusive Deep-Inelastic Scattering (SIDIS): $\ell(l) N(P) \rightarrow \ell(l') h(P_h) X$

Quark tomography of the nucleon

How to access hadron tomography from experimental data?
Kinematic coverage for SIDIS

credit: C. Weiss
Kinematic coverage for SIDIS

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JLab 12: valence structure
Kinematic coverage for SIDIS

- JLab 12: valence structure
- COMPASS: valence/sea quarks
Kinematic coverage for SIDIS

- JLab 12: valence structure
- COMPASS: valence/sea quarks
- EIC: sea quarks and (radiative) glue
Kinematic coverage for SIDIS

The Electron-Ion Collider (EIC) will greatly *extend the kinematic reach of existing facilities for SIDIS*

- **JLab 12**: valence structure
- **COMPASS**: valence/sea quarks
- **EIC**: sea quarks and (radiative) glue
SIDIS: ingredients

credit: A. Bacchetta
SIDIS: ingredients

Ingredients to build the cross section:

- available four-vectors (momenta for leptons, hadrons, photon, spin of target hadron)
SIDIS: ingredients

Ingredients to build the cross section:

- available four-vectors (momenta for leptons, hadrons, photon, spin of target hadron)
- symmetries of the theory
SIDIS: Structure Functions

\[
\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1 - \epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ \right.
\]

\[
F_{UU,T}(x, z, P_{h\perp}^2, Q^2) \quad \rightarrow \text{4D quantities}
\]

\[
+ \epsilon F_{UU,L} + \sqrt{2\epsilon(1 + \epsilon)} \cos(\phi_h) F_{UU}^{\cos\phi_h} + \epsilon \cos(2\phi_h) F_{UU}^{\cos^2\phi_h} \quad \rightarrow \text{unpolarized}
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+ \lambda_e \left[ \ldots 1 \text{SF} \ldots \right] \quad \rightarrow \text{polarized terms} \\
+ S_{\parallel} \left[ \ldots 2 \text{SFs} \ldots \right] + \\
+ S_{\parallel} \lambda_e \left[ \ldots 2 \text{SFs} \ldots \right] \\
+ S_{\perp} \left[ \ldots 6 \text{SFs} \ldots \right] \\
+ S_{\perp} \lambda_e \left[ \ldots 3 \text{SFs} \ldots \right] \end{array} \right\}$$
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18 structure functions in total!
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Experimental data for all of them
Theory? Connection to parton physics
Diagrammatic approach

Tree level connection between structure functions and partonic quantities
SFs $\rightarrow$ tree-level convolutions of (un)polarized TMD PDFs and TMD FFs

$$F_{\ldots} \rightarrow x \sum_{a,f,D} e_a^2 \int d^2p_T d^2k_T \delta^{(2)}(p_T - k_T - P_{h\perp}/z) f_{a/N}^{a/N}(x,p_T^2) D^{a-h}(z,k_T^2)$$

hep-ph/0611265
Diagrammatic approach

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PDFs: parton distribution functions - hadron $\rightarrow$ parton transition
Diagrammatic approach

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Separation of hard and soft physics $\rightarrow$ factorization (complete only for $F_{UU,T}$)

hep-ph/0611265
TMD factorization

Proper separation of perturbative and non-perturbative (= structure) physics

\[ F_{UU,T}(x, z, P_{hT}^2, Q^2) = \sum_a H_{UU,T}^a(Q^2) \]

\[ \times x \int d^2 p_T d^2 k_T \delta^{(2)}(z k_T + P - P_{hT}) f^{a/N}_1(x, p_T^2, Q^2) D^{a\rightarrow h}_1(z, k_T^2, Q^2) \]

picture from Collins pQCD book
TMD factorization

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\[ \times x \int d^2 p_T d^2 k_T \delta^{(2)}(zk_{\perp} + P_{\perp} - P_{hT}) f_1^{a/N}(x, p_T^2, Q^2) D_1^{a\rightarrow h}(z, k_T^2, Q^2) \]

\[ + Y_{UU,T}(x, z, P_{hT}^2, Q^2) \]

picture from Collins pQCD book
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\[
+ Y_{UU,T}(x, z, P_{hT}^2, Q^2) + O(M^2/Q^2)
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picture from Collins pQCD book
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\[ \approx \sum_a H_{UU,T}^a(Q^2) \int_0^\infty db_T b_T J_0(b_T|P_{hT}|/z) \tilde{f}_1^{a/N}(x, b_T^2, Q^2) \tilde{D}_1^{a\rightarrow h}(z, b_T^2, Q^2) \]

picture from Collins pQCD book
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- \( \mathcal{H} \): perturbative
- \( \tilde{f}_1, \tilde{D}_1 \): perturbative and non-perturbative

picture from Collins pQCD book
Structure of TMDs

Fourier transform of a TMD distribution ($b_T$ space):

$$\tilde{f}_1^a (x, b_T^2; \mu_f, \zeta_f) = \tilde{f}_1^a (x, b_T^2; \mu_i, \zeta_i)$$

$$\times \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\}$$

$$\times \left( \frac{\zeta_f}{\zeta_i} \right)^{-K(b_T;\mu_i)}$$

\[ \rightarrow \mu \text{ evolution} \]

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The input TMD: expanded at low $b_T$ on the collinear distributions:

$$\tilde{f}_1^a(x, b_T^2; \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2; \mu_i, \zeta_i) \otimes f_1^b(z, \mu_i)$$
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\(\gamma_F, K, C_{a/b} : \text{calculable in perturbation theory}\)
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{$f_1, F_{NP}, g_K$: non-perturbative contributions - fits to data}
Practical challenges

$F_{UU,T}$ - for every experimental bin (e.g. $O(\#) \sim 10^{4,5}$):

- integrals to match TMDs to PDFs in $b_T$ space, summed over flavors
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- separation among fragmentation regions
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Conceptual challenges:

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- separation between “low” and “large” transverse momentum regions
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$f_{1}^{a/h}(x, k_{T}^{2}, Q)$: probability of finding quark/gluon with a fraction $x$ of the hadron’s momentum and a transverse momentum $k_T$ at the energy/resolution scale $Q$.

Hadron structure in terms of quarks/ gluons in 3D momentum space (from PV19 fit - 1912.07550)
SIDIS: convolution of hadron structure and hadronization

\[ F_{UU,T} \approx \sum_a \mathcal{H}_{UU,T}^a \int_0^\infty db_T b_T J_0(b_T | P_{hT} | / z) \tilde{f}_1^a/N(x, b_T^2, Q^2) \tilde{D}_1^{a \rightarrow h}(z, b_T^2, Q^2) \]

Extractions of TMD PDFs and FFs (PV17 fit: 1703.10157)

\[ \langle k_T^2 \rangle \text{ and } \langle P_T^2 \rangle \]

mapping quarks in 3D momentum space
Backup
Twist-2 transverse momentum dependent PDFs for a quark in a spin 1/2 hadron

<table>
<thead>
<tr>
<th>nucleon pol.</th>
<th>U</th>
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<th>T</th>
</tr>
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<tbody>
<tr>
<td>U</td>
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<td>$h_1^+$</td>
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<tr>
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</tbody>
</table>

$f_{1T}^\perp$ and $h_1^+$ are time-reversal odd and are process dependent.
### Twist 2 quark TMD PDFs

Twist-2 transverse momentum dependent PDFs for a quark in a spin 1/2 hadron

<table>
<thead>
<tr>
<th>nucleon pol.</th>
<th>quark pol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>L</td>
</tr>
<tr>
<td>U</td>
<td>$f_1$</td>
</tr>
<tr>
<td>L</td>
<td>$g_{1L}$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^\perp$</td>
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$f_{1T}^\perp$ and $h_1^+$ are time-reversal odd and are process dependent.

The experimental confirmation of this calculable process dependence (\(=\) sign flip between Semi-Inclusive DIS and Drell-Yan) is a fundamental test for the symmetries of QCD.
**Twist 3 quark TMD PDFs**

Twist-3 transverse momentum dependent PDFs for a quark in a spin 1/2 hadron

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<td>$g^\perp$</td>
<td>$h, e$</td>
</tr>
<tr>
<td>L</td>
<td>$f_L^\perp$</td>
<td>$g_L^\perp$</td>
<td>$h_L, e_L$</td>
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<tr>
<td>T</td>
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Twist 3 quark TMD PDFs

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Functions in black survive transverse momentum integration. Functions in red are T-odd. A similar decomposition at twist 2 and 3 exists also for FFs.