Nucleon structure from Lattice QCD at nearly physical quark masses

Gunnar Bali for RQCD

with Sara Collins, Benjamin Gläßle, Meinulf Göckeler, Johannes Najjar, Rudolf Rödel, Andreas Schäfer, Wolfgang Söldner and André Sternbeck
Outline

- Importance of proton structure beyond QCD
- Lattice QCD set-up
- Mass: $\sigma$-terms
- Spin: The $\Delta q$'s and $g_A$
- Other couplings
- Momentum fraction: $\langle x \rangle_{u-d}$
- Summary
Protons in use e.g. at the LHC
What is known about parton distribution functions?

The $u$ and $d$ PDFs are well-known from experiment, e.g., at DESY. Strangeness and gluonic PDFs have much larger uncertainties.

Generated using

http://hepdata.cedar.ac.uk/pdfs

from the NNPDF2.3 data set.

NNPDF: R D Ball et al,
NPB 867 (13) 244
Nucleons as dark matter probes: XENON1T at Gran Sasso

y-scale of shaded areas depends on scalar couplings $m_q \langle N | \bar{q} q | N \rangle$. 
Proton structure calculations are...

- ... essential to constrain beyond-the-Standard-Model (BSM) dark matter candidates, relating predictions to experimental limits.
- ... important to predict cross-sections for processes on the quark-gluon level. Experiment e.g. unable to directly measure strangeness and gluon PDFs.
- ... needed to relate QCD to low energy effective theories that are also relevant for precision experiments.

Here I concentrate on

- How is the mass distributed among the partons? (scalar couplings)
- How is the spin distributed? (axial couplings)
- Proton-neutron transition couplings. \( (g_S, g_T, \tilde{g}_T, g_P, g_P^*) \)
- How is the momentum distributed? (moments of PDFs)
Lattice QCD

Typical values:
\[ a^{-1} = 2-5 \text{ GeV}, \quad Na = 2-7 \text{ fm} \]

Continuum limit: \( a \to 0, \ Na \text{ fixed} \)

Infinite volume: \( Na \to \infty \)

\[
\langle O \rangle = \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] \ O[U] e^{-S[U,\psi,\bar{\psi}]}
\]

"Measurement": average over a representative ensemble of gluon configurations \( \{U_i\} \) with probability \( P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]} \)

\[
\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \quad \Delta O \propto \frac{1}{\sqrt{n}} \quad n \to \infty \to 0
\]
Input: discretized $\mathcal{L}_{QCD} = \frac{1}{16\pi\alpha_L(a)} FF + \bar{q}_f(D + m_f(a))q_f$

\begin{align*}
m_{\text{latt}}^N &= m_{\text{phys}}^N \\
m_{\pi}^\text{latt} / m_{\pi}^\text{latt} &= m_{\pi}^\text{phys} / m_{\pi}^\text{phys} \
&\to m_u(a) \approx m_d(a) \\
\end{align*}

Output: hadron masses, matrix elements, decay constants, etc...

Required:

1. $L = Na \to \infty$: FSE suppressed with $\exp(-Lm_\pi) \Rightarrow Lm_\pi \gtrsim 4.$
2. $m_q^\text{latt} \to m_q^\text{phys}$: chiral perturbation theory ($\chi$PT) helps for $m_{ud}$ but $m_{ud}^\text{latt}$ must be sufficiently small to start with ($m_\pi \lesssim 200$ MeV?).
3. $a \to 0$: functional form known: $\mathcal{O}(a^2), \mathcal{O}(\alpha_s a) \Rightarrow \approx 4$ lattice spacings.
Landscape of recent lattice simulations

Figures taken from
C Hoelbling, arXiv:1410.3403
Computational challenges

Cost of simulation is proportional to

- number of points: \((L/a)^4\)
- condition number of linear system: \(1/m_\pi^2\)
- \(L^{1/2}/m_\pi\) in (Omelyan) time integration within hybrid Monte Carlo
- \(1/a^{\geq 2}\) critical slowing down (autocorrelations)

Adjusting \(L \propto 1/m_\pi\) this means:

\[
\text{cost} \propto \frac{1}{a^{\geq 6} m_\pi^{7.5}}
\]

In addition: for baryonic observables at small \(m_\pi\) serious signal/noise problem.

State of the art: \(64^3 \times 128\) sites, corresponding to \(\approx (4 \times 10^9)^2\) (sparse) complex matrices.

Tremendous progress in Hybrid Monte Carlo, solver, noise reduction.
HW Hamber, E Marinari, G Parisi, C Rebbi, NPB225 (83) 475
(Appendix B)
GP Lepage 89, http://inspirehep.net/record/287173

\[ C_N(t) \sim \exp(-m_N t) \]
\[ [\Delta C_N(t)]^2 \sim \exp(-3m_\pi t) \]
\[ \frac{\Delta C_N(t)}{C_N(t)} \sim \exp \left[ \left( m_N - \frac{3}{2} m_\pi \right) t \right] \]

“Self-averaging” over many source points increases statistics. Becomes increasingly important towards small \( m_\pi \).
Three point functions

Evaluate $\langle N | \bar{q} \Gamma q | N \rangle$ (Lines: quark “propagators” $M_{xy}^{-1}$, $M = \not{D} + m_q$)

$q \in \{u, d\}$: both quark-line connected and disconnected terms.
$q = s$: only the disconnected term.

“Connected” requires only 12 rows (spin $\times$ colour) of $M^{-1}$.
“Disconnected” $12 N^3$ rows (timeslice): stochastic “all-to-all” methods.

“Disconnected” cancels ($m_u = m_d$, $Q^2 E \not{D}$) from isovector combinations: “proton minus neutron”, i.e. $\langle p | (\bar{u} \Gamma u - \bar{d} \Gamma d) | p \rangle = \langle p | \bar{u} \Gamma d | n \rangle$. 
Action and ensembles

- \( N_f = 2 \) NP improved Sheikholeslami-Wilson fermions, Wilson glue.
- \( Lm_\pi \) up to 6.7, \( a \) down to 0.06 fm, \( m_\pi \) down to 150 MeV.
- Two lattice spacings around \( m_\pi \approx 290 \) MeV, three around 425 MeV.
- 300–600 Wuppertal=Gauss smearing iterations on top of APE smearing.

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<th>( \beta )</th>
<th>( a/\text{fm} )</th>
<th>( \kappa )</th>
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Ensembles II

\[ Lm_\pi \approx 6.7: \] ▶
\[ Lm_\pi > 4.1: \] ▶▶
\[ Lm_\pi > 3.4: \] ★★★
\[ Lm_\pi \approx 2.8: \] △

\[ m_\pi \] [GeV]
Decomposition of the proton (and pion) mass 1

\[ m_N = \sum_{q \in \{u,d,s,\ldots\}} m_q \langle N| \bar{q} 1 q | N \rangle + \left\langle N \left| \frac{1}{8\pi\alpha_L} (E^2 - B^2) + \sum_q \bar{q} D \cdot \gamma q \right| N \right\rangle \]

\[ + \frac{1}{4} \left( m_N - \sum_q m_q \langle N| \bar{q} 1 q | N \rangle \right) \]

VEV \[ \langle 0| \bar{q} q | 0 \rangle \] is understood to be subtracted from \[ \langle N| \bar{q} q | N \rangle \].

Pion-nucleon \( \sigma \)-term: \( \sigma_{\pi N} = m_u \langle N| \bar{u} u | N \rangle + m_d \langle N| \bar{d} d | N \rangle = \sigma_u + \sigma_d \).

Scalar particles (Higgs, neutralino etc.) couple \( \propto \) quark matrix elements.
Decomposition of the proton (and pion) mass II

\[ \sigma_\pi = m_{ud} \langle \pi | \bar{u}u + \bar{d}d | \pi \rangle = m_{ud} \frac{\partial m_\pi}{\partial m_{ud}} = \frac{m_\pi}{2} + O(m_\pi^3). \]

Therefore:

\[ m_\pi \approx \frac{1}{2} m_\pi + \frac{3}{8} m_\pi + \frac{1}{8} m_\pi \]

\( \sigma_\pi \) can be further decomposed into valence and sea quark contributions.

Wilson fermions: singlet and non-singlet mass renormalization constants differ by \( r_m > 1 \Rightarrow \) “valence” > “connected”:

\[ r := \frac{\langle \pi | \bar{u}u + \bar{d}d | \pi \rangle_{\text{sea}}}{\langle \pi | \bar{u}u + \bar{d}d | \pi \rangle} = r_m \left( \frac{\langle \pi | \bar{u}u + \bar{d}d | \pi \rangle_{\text{dis}}}{\langle \pi | \bar{u}u + \bar{d}d | \pi \rangle_{\text{lat}}} - 1 \right) + 1 \]
Pion mass: $\sigma_\pi$ compared to $m_\pi/2$

The theoretical expectation $\sigma_\pi \approx m_\pi/2$ is confirmed.

[S Collins, D Richtmann]
Pion mass: light sea quark and strange quark contribs.

Less than $\sim 10\%$ of $\sigma_\pi$ (or $\sim 5\%$ of the mass) is due to sea quarks.
Strange quarks are negligible too.

Nevertheless, $r_m = \frac{Z_m^{\text{singlet}}}{Z_m^{\text{nonsinglet}}} > 1$ means at $a \approx 0.071$ fm about 30% of the signal originates from the disconnected contribution. So this needs to be computed even for the valence quark contribution.
The non-vanishing light quark masses are directly responsible for only $\approx 35 \text{ MeV}$ of the nucleon mass but for $68 \text{ MeV}$ of the pion mass!

This may not be too surprising since $m_N \not\to 0$ as $m_{ud} \to 0$. 

$[S \text{ Collins}]$
Chiral extrapolation of the nucleon mass

\[
(r_0 m_\pi)^2 \quad \text{versus} \quad r_0 M_N
\]

- Preliminary fit (unconstrained)
- Combined fit

Data points:
- \( r_0 \sigma \)
- \( r_0 M_N \)
- \( r_0 \equiv 0.501 \text{ fm} \)

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The scalar matrix elements $m_q \langle N | \bar{q} q | N \rangle$ determine the coupling of the nucleon to scalar particles at zero recoil:

$$\frac{f_N}{m_N} \approx \sum_{q \in \{u, d, s\}} f_{Tq} \frac{\alpha_q}{m_q} + \frac{2}{33 - 6} f_{Tg} \sum_{q \in \{c, b, t, \ldots\}} \frac{\alpha_q}{m_q}.$$ 

Cross section $\propto |f_N|^2$. Higgs example: $\alpha_q \propto m_q / m_W$.

$$f_{Tq} \equiv \frac{m_q \langle N | \bar{q} q | N \rangle}{m_N}$$

are the contributions of the light quark masses to the proton mass and

$$f_{Tg} \approx 1 - \sum_{q \in \{u, d, s\}} f_{Tq}.$$ 

Little about $f_{Tq}$ is known experimentally.
Scalar strangeness content

\[ f_T^s \]

\[ m_\pi^2 \text{ [GeV}^2\text{]} \]

\[ a \approx 0.08 \text{ fm} \]
\[ a \approx 0.07 \text{ fm} \]
\[ a \approx 0.06 \text{ fm} \]

Engelhardt \( N_f = 2 + 1 \)
ETMC \( N_f = 2 + 1 + 1 \)

[M Engelhardt, arXiv:1210.0025]: domain wall on staggered
[ETMC, C Alexandrou et al, arXiv:1309.7768]: twisted mass
Spin of the nucleon

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \sum_{q, \bar{q}} L_q + J_g : \]

Ji decomposition into the contributions of the (longitudinal) quark spins

\[ \Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \cdots, \]

the (longitudinal) quark and antiquark orbital angular momenta \( L_q = J_q - \frac{1}{2} \Delta q \) and the (longitudinal) gluon total angular momentum \( J_g \).

Naïve non-relativistic SU(6) quark model: \( \Delta \Sigma = 1, L_q = J_g = \Delta s = 0 \).

Relativistic quark models: \( \Delta \Sigma \sim 0.6, L_{\text{quarks}} \sim 0.2 \).

I will say nothing about the Jaffe and Manohar decomposition:

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_{\text{quarks}} + \Delta G + L_g \quad \left( J_g \neq \Delta G + L_g, J_q \neq \frac{1}{2} \Delta q + L_q \right). \]
The total quark angular momenta \( J_q = \frac{1}{2} \Delta q + L_q \) can be extracted from generalized form factors at \( q^2 = 0 \):

\[
J_q + J_{\bar{q}} = \frac{1}{2} \left[ A_{20}^q(0) + B_{20}^q(0) \right],
\]

where \( A_{20}^q(q^2) \) and \( B_{20}^q(q^2) \) are obtained from matrix elements of local quark bilinears of the form

\[
\left< N, s', p + q | \bar{q} \gamma \{ \mu_1 \overleftrightarrow{D} \mu_2 \} q | N, s, p \right>.
\]

Then

\[
L_q = J_q - \frac{1}{2} \Delta q, \quad J_g = \frac{1}{2} - \sum_{q, \bar{q}} J_q .
\]
Individual quark spin contributions ($q \in \{u, d, s\}$)

$$(\Delta q + \Delta \bar{q}) \, s_\mu = \frac{1}{m_N} \langle N, s \mid \bar{q} \gamma_\mu \gamma_5 q \mid N, s \rangle = F_A^q(0) = \tilde{A}_q(0)$$

Axial charges:

$$a_3 = -s_\mu \frac{1}{m_N} \langle N, s \mid \bar{\psi} \gamma_\mu \gamma_5 \lambda_3 \psi \mid N, s \rangle = \Delta u - \Delta d = g_A$$

$$a_8 = -s_\mu \frac{\sqrt{3}}{m_N} \langle N, s \mid \bar{\psi} \gamma_\mu \gamma_5 \lambda_8 \psi \mid N, s \rangle$$

$$= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2\Delta s - 2\Delta \bar{s}$$

$$a_0(Q^2) = -s_\mu \frac{1}{m_N} \langle N, s \mid \bar{\psi} \gamma_\mu \gamma_5 1 \psi \mid N, s \rangle$$

$$= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} = \Delta \Sigma(Q^2).$$

$\psi = (u, d, s)^t$, $\lambda_j$ are Gell-Mann flavour matrices.

$a_3 = g_A$ known from neutron $\beta$ decay, assuming isospin symmetry.

$a_8$ usually estimated from hyperon $\beta$ decay, assuming $SU(3)_F$ symmetry.
Extraction of the $\Delta q$'s from experiment

DIS gives spin structure functions of proton and neutron $g_1^{p,n}(x, Q^2)$.

First moment related to $a_i$'s via OPE (leading twist):

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 dx \ g_1^{p,n}(x, Q^2) = \frac{1}{36} [(a_8 \pm 3a_3)C_{NS} + 4a_0C_S]$$

Use models to extrapolate $g_1$ from experimental $x_{\text{min}}$ to $x = 0$!

$C_{S/NS} = C_{S/NS}(\alpha_s(Q^2))$. 

Combinations of $a_i$ give $\Delta q$'s, e.g., $(\Delta s + \Delta \bar{s})(Q^2) = \frac{1}{3}[a_0(Q^2) - a_8]$

SIDIS allows for direct measurements of the $\Delta q(x)$ but requires fragmentation functions.

[COMPASS, arXiv 1001.4654]

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<th>Naive Extrap.</th>
<th>combined with DSSV</th>
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<td>$(\Delta s + \Delta \bar{s})(5 \text{GeV}^2)$</td>
<td>$-0.02 \pm 0.02 \pm 0.02$</td>
<td>$-0.10 \pm 0.02 \pm 0.02$</td>
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DSSV: [de Florian et al, arXiv:0904.3821]
No continuum limit, $m_\pi \approx 290$ MeV $\Rightarrow$ add 20% systematic error.

Result in the $\overline{\text{MS}}$ scheme at $\mu^2 = 7.4$ GeV$^2$:

\[
\Delta \Sigma = \Delta u + \Delta d + \Delta s = 0.45(4)(9)
\]

\[
\Delta s = -0.020(10)(4)
\]
Comparison of recent lattice calculations

Consistency between different determinations: small $\Delta s + \Delta \bar{s}$.

ETMC result shows statistical accuracy that is possible. Systematics!

[QCDSF: GB et al, 1112.3354; M Engelhardt, 1210.0025; ETMC: A Abdel-Rehim et al, 1310.6339; $\chi$QCD: Y Yang et al, unpublished.]
\[ J_q + J_{\bar{q}} = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \]

From Lattice 14 review [M Constantinou, 1411.0078]

[\textit{LHPC: S Syritsyn et al, 1111.0718 (}N_f = 2 + 1\text{);} \textit{QCDSF/UKQCD: A Sternbeck et al, 1203.6579 (}N_f = 2\text{);} \textit{ETMC: C Alexandrou et al, 1104.1600, unpublished (}N_f = 2\text{);} \textit{ETMC: C Alexandrou et al, 1303.5979 (}N_f = 2 + 1 + 1\text{).}]
\[ g_A = \Delta u - \Delta d \]

Comparing similar volumes: no significant discretization effects.

- \( m_\pi \approx 425 \text{ MeV} \): \( g_A \) increases by \( \approx 5\% \) with \( Lm_\pi \approx 3.7 \rightarrow 4.9 \)
- \( m_\pi \approx 290 \text{ MeV} \): \( g_A \) up by \( \approx 6\% \) with \( Lm_\pi \approx 3.4 \rightarrow 4.2 \), then constant.
- \( m_\pi \approx 150 \text{ MeV} \): No difference between \( Lm_\pi \approx 2.8 \) and \( Lm_\pi \approx 3.5 \).

[S Collins, R Rödl]
Finite volume effects predicted by $\chi$PT similar for $g_A$ and $F_\pi$ $
\implies$ follow QCDSF: R Horsley et al, arXiv:1302.2233 and plot ratio

Extrapolation to physical point: $g_A/F_\pi = 13.88(29) \text{ GeV}^{-1}$

Expt: $g_A/F_\pi = 13.797(34) \text{ GeV}^{-1}$.

Using $F_\pi(\text{expt}) = 92.21 \text{ MeV}$ we obtain $g_A = 1.280(27)(35)$

Expt: $g_A = 1.2670(35)$. 
\( g_A \): summary of recent lattice results

QCDSF: 1302.2233  
Mainz: 1311.5804  
ETMC 2: 1312.2874  
LHPC: 1209.1687  
RBC/UKQCD: 1309.7942  
ETMC 2+1+1: 1303.5979  
PNDME: 1306.5435  
RQCD: 1412.7336
Isovector scalar charge

LHPC: 1206.4527, PNDME: 1306.5435, ETMC: 1411.3494, RQCD: 1412.7336
Isovector tensor charge

ETMC 2: 1311.4670, RBC: 1003.3387, LHPC: 1206.4527,
PNDME: 1306.5435, ETMC 2+1+1: 1311.4670, RQCD: 1412.7336

General remark: we vary $a^2$ only by a factor 1.8 ⇒ we cannot exclude lattice spacing effects of up to $0.071^2/(0.081^2 - 0.060^2) \cdot \Delta g \approx 1.7 \cdot \Delta g$. 
Isovector electromagnetic formfactors

\[ \langle p|\bar{u}\gamma_\mu d|n\rangle = \bar{u}_p(p_f) \left[ g_V(q^2)\gamma_\mu + \frac{\tilde{g}_T(q^2)}{2m_N}i\sigma_\mu\nu q^\nu \right] u_n(p_i) \]

Dirac FF: \( g_V(q^2) = F_1^p(q^2) - F_1^n(q^2) \xrightarrow{q^2 \to 0} 1 \)

Pauli FF: \( \tilde{g}_T(q^2) = F_2^p(q^2) - F_2^n(q^2) \xrightarrow{q^2 \to 0} \kappa_p - \kappa_n \approx 3.7058901(5) \)

\[ g_V(Q^2) = 1 - \frac{r_1^2}{6} Q^2 + \mathcal{O}(Q^4), \quad \tilde{g}_T(Q^2) = \tilde{g}_T(0) \left[ 1 - \frac{r_2^2}{6} Q^2 + \mathcal{O}(Q^4) \right] \]

Proton radius:

\[ r_p^2 \approx r_1^2 + \frac{3\tilde{g}_T(0)}{2m_N^2} \cdot \]

Dipole fit to determine the induced tensor charge \( \tilde{g}_T = \tilde{g}_T(0) \):

\[ \tilde{g}_T(Q^2) = \frac{\tilde{g}_T(0)}{(1 + r_2^2/Q^2/12)^2} \cdot \]
Extrapolation of the Pauli formfactor at $m_\pi = 290$ MeV

Difference between magnetic moment anomalies $\tilde{g}_T(0) = \kappa_p - \kappa_n$.

Extrapolation error decreases with smaller $Q^2_{\text{min}} = \pi^2/L^2$. Therefore, invisible FSE for $Lm_\pi > 3.4$ at $m_\pi = 290$ MeV ($L > 2.3$ fm) do not necessarily imply they are irrelevant within the smaller statistical errors at $m_\pi = 150$ MeV ($L > 4.5$ fm).
Induced isovector tensor charge

Extrapolating in the usual way... however, FSE are unquantifiable at the lightest mass point and $O(a)$ improvement is not yet included.

QCDSF: 1106.3580, Mainz: 1311.5804 + 1411.4804,
ETMC 2: 1102.2208, LHPC: 1404.4029, RBC: 0904.2039,
ETMC 2+1+1: 1303.5979, PNDME: 1306.5435, RQCD: 1412.7336
Isovector quark momentum fraction: $\langle x \rangle_{u-d}^{\overline{MS}}(2 \text{ GeV})$


\begin{align*}
L m_\pi &\approx 6.7: \Box \\
L m_\pi &> 4.1: \circ \\
L m_\pi &> 3.4: \ast \ast \ast \\
L m_\pi &\approx 2.8: \triangle
\end{align*}

Mild dependence on $V, m_\pi$.
Renormalised non-perturbatively.
$O(a)$ leading errors, $a$ varied from 0.08 to 0.06 fm.

Improvement on earlier calculations which suffered from excited state contamination $\langle x \rangle_{u-d}^{\overline{MS}}(2 \text{ GeV}) \sim 0.25$.

Near physical point but more work needs to be done — lattice spacing dependence?
\[ \langle x \rangle_{\overline{\text{MS}}}^{\text{u} - \text{d}}(2 \text{ GeV}) \]: summary of recent lattice results

RQCD: GB et al, 1408.6850;
LHPC: J Green et al, arXiv:1209.1687;
ETMC 2+1+1: C Alexandrou et al, arXiv:1312.2874;

PDFs from
S Alekhin et al, 1310.3059; CT10: J Gao et al, 1302.6246;
NNPDF: R Ball et al 1207.1303; A Martin et al 0905.3531.

[ETMC, arXiv:1410.8761]: disconnected contributions small \( \Rightarrow \)
predictions for \( \langle x \rangle_{\overline{\text{MS}}}^{\text{q}}(2 \text{ GeV}) \) soon. Mixing between quarks and glue!
Summary

- Lattice can contribute to many quantities that are hard to constrain by experiment, e.g., $\sigma_{\pi N}$, $f_{Ts}$, $g_S$, $g_T$.
- Lattice calculations are important to determine the spin content of the nucleon: $\Delta q$, $\Delta \Sigma$, $J_q$, $\langle x \rangle \Delta q$, ....
- In the past disconnected quark line diagrams were often omitted and differences quoted: $g_A$, $\langle x \rangle_{u-d}$, ..., but no $\Delta s$, $\Delta \Sigma$, $J_q$, $\langle x \rangle_q$ etc.
- Improved methods now allow for the calculation of these contributions.
- $g_A$ seems to approach the physical value, once $Lm_\pi > 4$.
- $\langle x \rangle_{u-d}$ comes out 20% bigger than expected. lattice spacing effects? Renormalization?
- Precision physics requires an extrapolation $a \rightarrow 0$. For quite a few quantities however errors of 20% are acceptable.
- High Mellin moments almost impossible to compute $\Rightarrow$ recent interest also in “quasi” parton distribution functions.