Charm & Bottom Baryon spectrum

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with

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Summary

• Charmed and Baryon spectrum
  • observations

• Lattice QCD calculation
  • Methodology
  • Heavy quarks on the lattice
  • Results and comparisons with models and observations
The flavor symmetries shown in Fig. 2 are of course badly broken, but the figure is the simplest way to see what charmed baryons should exist. For example, from Fig. 2(b), we expect to find, in the same $J^P = 1/2^+ \, 20$-plet as the nucleon, a $\Lambda_c$, a $\Sigma_c$, two $\Xi_c$'s, and an $\Omega_c$. Note that this $\Omega_c$ has $J^P = 1/2^+$ and is not in the same SU(4) multiplet as the famous $J^P = 3/2^- \, \Omega^-$.

Figure 2: SU(4) multiplets of baryons made of $u$, $d$, $s$, and $c$ quarks. (a) The 20-plet with an SU(3) decuplet on the lowest level. (b) The $20^\prime$-plet with an SU(3) octet on the lowest level. (c) The 4-plet. Note that here and in Fig. 3, but not in Fig. 1, each charge state is shown separately.

Figure 3 shows in more detail the middle level of the $20^\prime$-plet of Fig. 2(b); it splits apart into two SU(3) multiplets, $\bar{3}$ and $6$. The states of the $\bar{3}$ are antisymmetric under the interchange of the two light quarks (the $u$, $d$, and $s$ quarks), whereas the states of the $6$ are symmetric under this interchange. We use a prime to distinguish the $\Xi_c$ in the $6$ from the one in the $\bar{3}$.
(a) Charmed baryons

Charmed Baryon spectrum
Bottom Baryon spectrum

Mass (MeV)

$\Lambda_b^-$  $\Sigma_b^+$  $\Xi_b^-$  $\Omega_b^+$

5619.4(6)  1/2
5813.5(1.3)  1/2
5833.4(1.3)  3/2
5945.5(2.3)  3/2
6046.8(2.1)  1/2
5790.6(2.0)  1/2

PDG 2014
Unobserved states

\[ \Xi_{cc} \quad \Xi_{bb} \quad \Xi_{bc} \]

\[ \Omega_{cc} \quad \Omega_{bb} \quad \Omega_{bc} \]

\[ \Omega_{ccc} \quad \Omega_{bbb} \quad \Omega_{bbc} \quad \Omega_{bcc} \]

Multi-charm  Multi-bottom  Mixed flavor
Controversies

$\Xi_{cc}^+$ Reported by SELEX


Not found by FOCUS, BaBar, Belle LHCb

• First principles calculations can predict masses of unobserved states

• Can help resolve controversies

• Complete Lattice QCD calculations of the ground state masses are now possible
Figure 15.7: Spectroscopy for mesonic systems containing one or more heavy quarks (adapted from Ref. 50). Particles whose masses are used to fix lattice parameters are shown with crosses; the authors distinguish between "predictions" and "postdictions" of their calculation. Lines represent experiment.

As we move away from hadrons which can be created by the simplest quark model operators (appropriate to the lightest meson and baryon multiplets) we encounter a host of new problems: either no good interpolating fields, or too many possible interpolating fields, and many states with the same quantum numbers. Techniques for dealing with these interrelated problems vary from collaboration to collaboration, but all share common features: typically, correlation functions from many different interpolating fields are used, and these signal what amounts to a variational calculation using the chosen operator basis. In addition to mass spectra, wave function information can be garnered from the form of the best variational wave function.

Of course, the same problems which are present in the spectroscopy of the lightest hadrons (the need to extrapolate to infinite volume, physical values of the light quark masses, and zero lattice spacing) are also present. We briefly touch on three different kinds of hadrons: excited states of baryons, glueballs, and hybrid mesons. The quality of the data is not as good as for the ground states, and so the results continue to evolve.

Ref. 60 is a good recent review of excited baryon spectroscopy. The interesting physics questions to be addressed are precisely those enumerated in the last section. An example of a recent calculation, due to Ref. 61 is shown in Fig. 15.10. Notice that the pion is not yet at its physical value. The lightest positive parity state is the nucleon, and the Roper resonance has not yet appeared as a light state.
Flavored baryons (LQCD vs Exp)

Outline of LQCD calculations

- Include the vacuum polarization effects
  - 2 light (up down) 1 heavy (strange)
  - ... and ... 1 very heavy (charm)

- Finite Volume
  - Compute in multiple and large volumes

- Continuum Limit
  - Compute with several lattice spacings

- Quark masses
  - Compute with several values for the quark masses
  - Study quark mass dependence of QCD
  - Physical light (up down) quark masses
Light quark actions

- Light quarks (up, down, strange) \( m_q \ll \Lambda_{QCD} \sim 250\, MeV \)

- Straight forward to put on the lattice

- Fermion doubling problem
  - Wilson action, Kogut-Susskind action
    - Chiral symmetry breaking
    - Flavor symmetry breaking (KS)

- Domain Wall fermions or Overlap fermions
  - Chiral symmetry
  - \( O(a^2) \) lattice spacing errors
The heavy quark action

- Light quarks (up, down, strange) \( m_q \ll \Lambda_{QCD} \sim 250 \text{MeV} \)
- Heavy quarks (charm, bottom, top) \( m_q \gg \Lambda_{QCD} \)

\[
\begin{align*}
m_{\text{charm}} & \sim 1300 \text{ MeV} \\
m_{\text{bottom}} & \sim 4200 \text{ MeV} \\
m_{\text{top}} & \sim 174200 \text{ MeV}
\end{align*}
\]

- Lattice fermion Lagrangians assume for light quirks \( a \, m_q \ll 1 \)

\[
\mathcal{L} = \mathcal{L}_{\text{wilson}} + \mathcal{O}(a)
\]

- For typical lattice QCD calculations (\( a=0.125\text{fm}, 0.09\text{fm}, 0.06\text{fm} \))

\[
\begin{align*}
0.4 < a m_{\text{charm}} & < 0.8 \\
1.3 < a m_{\text{bottom}} & < 2.7
\end{align*}
\]

- Special care is needed in treating heavy quarks on the lattice
  - Or use very small lattice spacing (\( a < 0.01\text{fm} \))....
The heavy quark action (charm)

- Fermilab action: Symanzik improvement taking into account $a m_q \sim 1$
  
  $$S = S_0 + S_B + S_E$$
  
  $$S_0 = \sum_x \bar{q}(x)[m_0 + (\gamma_0 \nabla_0 - \frac{1}{2}\triangle_0) + \nu \sum_i (\gamma_i \nabla_i - \frac{1}{2}\triangle_i)]q(x)$$

- For Wilson action $\nu = 1$ for light quarks. For heavy quarks $\nu$ needs to be adjusted to remove lattice artifacts $O(m a)$.

- The $O(a)$ lattice artifacts are removed by the terms
  
  $$S_B = -\frac{1}{2} c_B \sum_x \bar{q}(x) (\sum_{i<j} \sigma_{ij} F_{ij}) q(x)$$
  
  $$S_E = -\frac{1}{2} c_E \sum_x \bar{q}(x) (\sum_i \sigma_{0i} F_{0i}) q(x)$$

- These coefficients can be computed perturbatively. At tree-level (with tadpole improvement)
  
  $$c_B = \frac{\nu}{u_0^3}, \quad c_E = \frac{1}{2} (1 + \nu) \frac{1}{u_0^3}$$


P. Chen, Phys. Rev. D64, 034509 (2001)
Heavy quark action (cont.)

Tune $v$ and $m_0$ so that the speed of light is 1 and the spin averaged meson mass matches experiment

\[
\overline{M} = \frac{3}{4} E_{J/\psi}(0) + \frac{1}{4} E_{\eta_c}(0).
\]

\[
c^2(p) = \frac{E_{J/\psi}^2(p) - E_{J/\psi}^2(0)}{p^2}.
\]
Heavy quark action (Bottom quark)

Typical momenta of the bottom quark in a baryon is between 0.5 and 1.5 GeV resulting velocities of \( v = 0.1c \)

Non-relativistic approximation is applicable NRQCD

\[
S_\psi = a^3 \sum \psi^\dagger(x, t) \left[ \psi(x, t) - K(t) \psi(x, t - a) \right]
\]

\[
H_0 = -\frac{\Delta^{(2)}}{2m_b},
\]

\[
\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8m_b^3} + c_2 \frac{ig}{8m_b^2} \left( \nabla \cdot \vec{E} - \vec{E} \cdot \nabla \right)
- c_3 \frac{g}{8m_b^2} \sigma \cdot \left( \nabla \times \vec{E} - \vec{E} \times \nabla \right) - c_4 \frac{g}{2m_b} \sigma \cdot \vec{B}
+ c_5 \frac{a^2 \Delta^{(4)}}{24m_b} - c_6 \frac{a (\Delta^{(2)})^2}{16n m_b^2}.
\]

\( O(v^2) \)

\( O(v^4) \)

Tadpole improved tree level matching
Lattice calculation set up

• Domain wall fermions for the light quarks (RBC/UKQCD)

• Relativistic heavy quark action for charm quark

• NRQCD for bottom quark

• Two lattice spacings (0.11fm and 0.085fm determined from bottomonium spectroscopy)

• Pion mass range 220MeV - 420MeV
  • Extrapolate to the physical pion mass point

• Single volume of about 2.7fm

• Combined chiral and continuum extrapolations based on HBchiPT
Mass computation

- Use simplest non-relativistic interpolating fields for spin 1/2, 3/2 baryons
- Smeared quark sources (with several smearing widths)
- Masses where estimated from fits to exponentials of the euclidean time dependence of two point functions
FIG. 1. Matrix fit of the two-point functions using Eq. (37). The data shown here are from the C54 set. The correlator, which equals $\langle \bar{O}[c, c, b] O[c, c, b] \rangle$ in the limit of infinite statistics, is not shown for clarity.

To illustrate how the results from methods 1 through 4 compare with each other, we show the $\Delta \sigma_{cc}$ energies in Fig. 3. The different methods generally give quite consistent results, and we use the correlated weighted average for the further analysis. The correlations between the energies from the different methods are taken into account using statistical bootstrap; we perform the weighted averages for each bootstrap sample to obtain a new bootstrap distribution for the average energy. The statistical uncertainty of the average energy is then obtained from the width of this distribution. In some cases, the energies obtained using the different fit methods are not consistent with each other (as can be seen for the C14 data set in Fig. 3), and we inflate the uncertainty of the average using a scale factor.

To this end, we compute the value of $\delta^2$ for a constant fit to the four energies. If $\delta^2 / (N - 1) > 1$ (where $N = 4$), we inflate the uncertainty of the weighted average by a factor of $S = \sqrt{\frac{\delta^2}{N - 1}}$.

The averaged baryon and quarkonium energies from all data sets are given in Tables VII and VIII, respectively.
FIG. 2. Effective-energy plot for the $2 \times 2$ matrix of two-point functions from the $C54$ set. The effective energy for a correlator $C(t)_{ij}$ is computed as $aE_{\text{eff}}(t_a) = \ln \frac{C(t)}{C(t_a)}$. The lines indicate the time ranges and the energy obtained from the fit shown in Fig. 1.

FIG. 3. $\langle O[c, c, b] \overline{O}[c, c, b] \rangle$ energies obtained using the four different fit methods for each data set as explained in the main text. Also shown are the method-averaged energies (correlations are taken into account). For the method-averaged energies, the outer error bars include a scale factor in the cases where the average has $\frac{\sigma}{\sigma_f} > 1$ (here, for the $C14$ and $C54$ data sets).
Chiral and continuum extrapolations

\[ E_{\Lambda_Q}^{(sub)} = E_{(sub,0)} + d^{(vv)}_\pi \frac{[m^{(vv)}_\pi]^2}{4\pi f} + d^{(ss)}_\pi \frac{[m^{(ss)}_\pi]^2}{4\pi f} + \mathcal{M}_{\Lambda_Q} + d_a a^2 \Lambda^3, \]

\[ E_{\Sigma_Q}^{(sub)} = E_{(sub,0)} + \Delta^{(0)} + c^{(vv)}_\pi \frac{[m^{(vv)}_\pi]^2}{4\pi f} + c^{(ss)}_\pi \frac{[m^{(ss)}_\pi]^2}{4\pi f} + \mathcal{M}_{\Sigma_Q} + c_a a^2 \Lambda^3, \]

\[ E_{\Sigma_Q^*}^{(sub)} = E_{(sub,0)} + \Delta^{(0)} + \Delta^{(0)}_* + c^{(vv)}_\pi \frac{[m^{(vv)}_\pi]^2}{4\pi f} + c^{(ss)}_\pi \frac{[m^{(ss)}_\pi]^2}{4\pi f} + \mathcal{M}_{\Sigma_Q^*} + c_a a^2 \Lambda^3, \]

The data sets with the lowest two pion masses (Table XI for the values of the shifts); data points at the coarse lattice spacing are plotted with circles, and data points at the fine lattice spacing are plotted with squares. The partially quenched data points, which have...
FIG. 15. Our results for the masses of charmed and/or bottom baryons, compared to the experimental results where available [8, 10, 12]. The masses of baryons containing $n_b$ bottom quarks have been set by $n_b \cdot (3000 \text{ MeV})$ to fit them into this plot. Note that the uncertainties of our results for nearby states are highly correlated, and hyperfine splittings such as $M_\pi^\star$ and $M_\Omega$ can in fact be resolved with much smaller uncertainties than apparent from this figure (see Table XIX).
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of a quenched lattice calculation can be found in Ref. [31]). We therefore compare our lattice QCD results for the
this value than the ratios in the charm sector.

\[ \frac{3}{4} \] in the heavy-quark limit [4]. We do indeed see some evidence that the ratios in the bottom sector are closer to
results). Heavy quark-diquark symmetry [3] predicts that the ratio of these hyperfine splittings approaches the value
meson hyperfine splittings from the same data sets as used for the baryons are consistent with the experimental
the experimental results of the heavy-light meson hyperfine splittings (preliminary lattice results for the heavy-light
[42], and we refer the reader to Ref. [42] for further discussions.

\[ \text{TABLE XX. Hyperfine splittings of doubly heavy baryons calculated in this work, compared to experimental results [8] for the} \]

\[ \text{Namekawa et al., 2010 (} a \approx 0.12 \text{ fm)} \]
\[ \text{Briceno et al., 2012 (} a = 0 \text{)} \]
\[ \text{Liu et al., 2010 (} a \approx 0.12 \text{ fm)} \]
\[ \text{Brown et al., 2014 (} a = 0 \text{)} \]
\[ \text{Padmanath et al., 2013 (} a_s \approx 0.12 \text{ fm; stat. only)} \]
\[ \text{Alexandrou et al., 2014 (} a = 0 \text{)} \]

\[ M (\text{MeV}) \]
\[ \Xi_{cc} \]
\[ \Xi^*_{cc} \]
\[ \Omega_{cc} \]
\[ \Omega^*_{cc} \]
\[ \Omega_{ccc} \]
FIG. 17. Comparison of lattice QCD results for the doubly bottom baryon masses. The only other published unquenched calculation is the one of Ref. [52]. Our results have larger statistical uncertainties, but our calculation was performed with closer-to-physical pion masses and included a combined chiral and continuum extrapolation.

We hope that our lattice QCD results provide a useful benchmark for future studies of these interesting systems.
FIG. 18. Comparison of our lattice QCD results for the $\Lambda_c$, $\Lambda_c'$, $\Lambda_c^*$, $\Omega_c$, $\Omega_c'$, and $\Omega_c^*$ baryon masses with estimates based on continuum methods, including quark models and QCD sum rules [99–112]. From Refs. [100] (Silvestre-Brac, 1996) and [111] (Ghalenovi et al., 2014), we show results for multiple different choices of the interquark potentials. Note that the bag-model calculation of Ref. [103] (He et al., 2004) predicts $m^{\Lambda_c^*}_c < m^{\Lambda_c}_c$ and $m^{\Omega_c'}_c < m^{\Omega_c}_c$, both rather unusual. The sum-rule calculation of Ref. [108] (Zhang et al., 2008) gives extremely large hyperfine splittings $m^{\Lambda_c^*}_c - m^{\Lambda_c}_c \approx 1$ GeV and $m^{\Omega_c'}_c - m^{\Omega_c}_c \approx 0$. Our results for the hyperfine splittings are $m^{\Lambda_c^*}_c - m^{\Lambda_c}_c = 26.7(3.6)(8.4)$ MeV, $m^{\Omega_c'}_c - m^{\Omega_c}_c = 27.4(2.6)(6.7)$ MeV; the $\Lambda_c^*$ mass from Ref. [108] is beyond the upper limit of the plot.
Conclusions

- We have presented a comprehensive study of the baryon spectrum with charm and bottom quarks for both spin 1/2 and spin 3/2 baryons.
- Our results are in good agreement with experiment.
- We make a large number of predictions states yet to be observed.
- LHCb has already confirmed one of our predictions.
- Hopefully more to come...