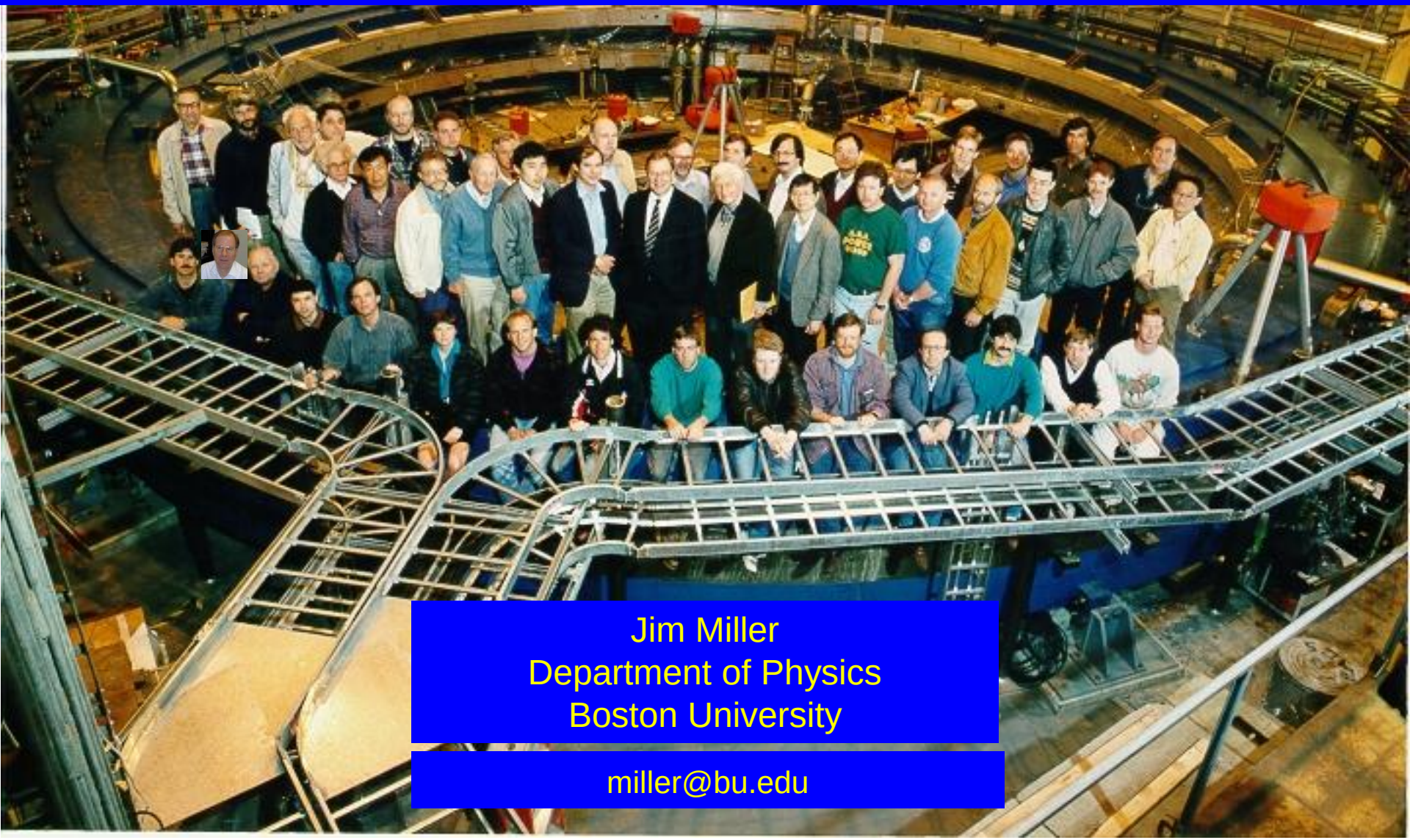
Muon (g-2): Inside or outside of the Standard Model?



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Muon (g-2)

For the muon magnetic anomaly, $a_\mu = (g-2)/2$,
SM expected value (2009): 116 591 802(49)x10^{-11}
BNL-E821 (2004) measured value: 116 592 089(63) x10^{-11}
Δa_μ=287(80) x10^{-11}=(expt) – (SM)
About 3.6 s.d. difference
A new experiment at Fermilab is being developed to reduce the experimental uncertainty and to sharpen comparison between theory and experiment

Magnetic Dipole Moments

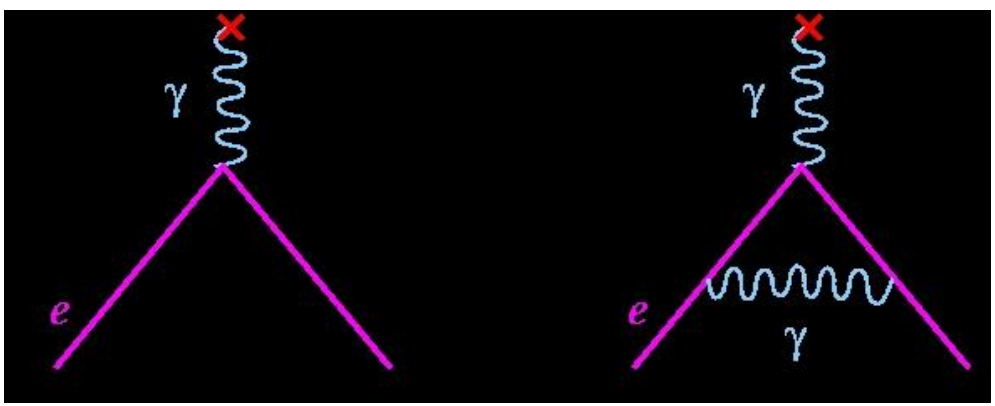
$$\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}$$

Dirac Equation Predicts g=2

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}]\psi$$

Radiative corrections change g

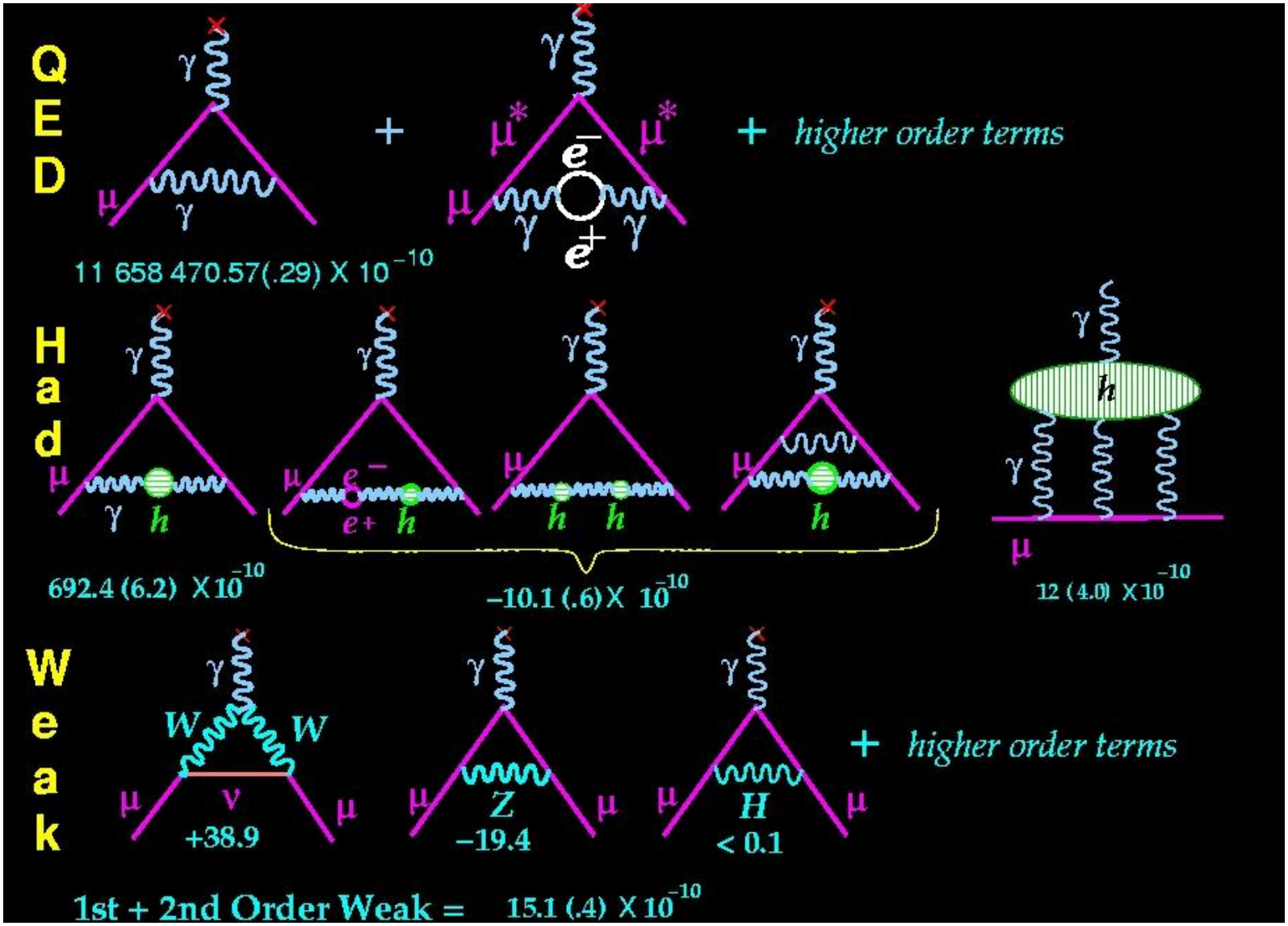
$$a = \frac{(g-2)}{2}$$



$$a = \frac{\alpha}{2\pi} = 0.001161$$

New virtual particles that couple to muons also add to *a*→ sensitive to new physics!

Standard Model Value for (g-2)



relative contribution of heavier things $\left(\frac{m_\mu}{m_e}\right)^2 \simeq 40,000$

QED now calculated to 5-loops!

Complete Tenth-Order QED Contribution to the Muon $g-2$

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

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[4] *Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York, 14853, U.S.A*

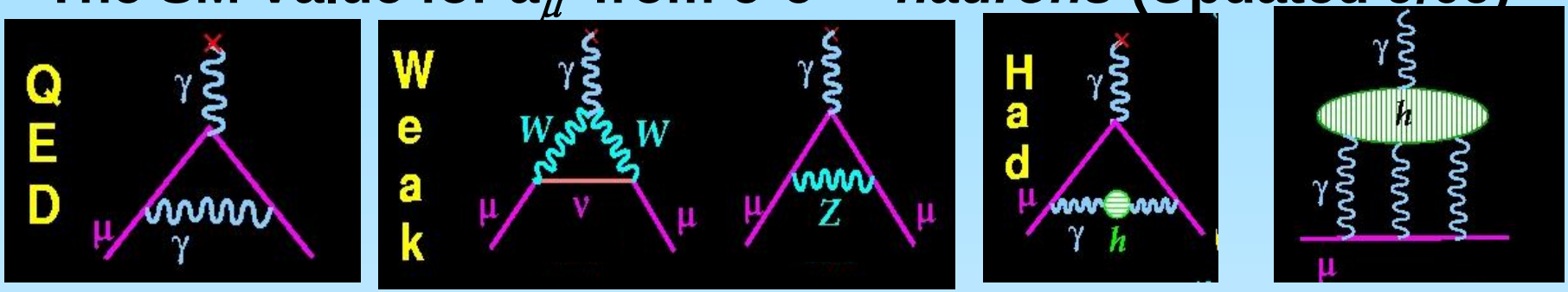
(Dated: May 29, 2012)

We report the result of our calculation of the complete tenth-order QED terms of the muon $g-2$. Our result is $a_\mu^{(10)} = 753.29\ (1.04)$ in units of $(\alpha/\pi)^5$, which is about 4.5 s.d. larger than the leading-logarithmic estimate 663 (20). We also improved the precision of the eighth-order QED term of a_μ, obtaining $a_\mu^{(8)} = 130.8794\ (63)$ in units of $(\alpha/\pi)^4$. Using the best non-QED value of α, we obtain the standard model prediction $a_\mu(\mathrm{SM}) = 116\ 591\ 840\ (59) \times 10^{-11}$, to be compared with the measured value $a_\mu(\mathrm{exp}) = 116\ 592\ 089\ (63) \times 10^{-11}$. The difference $a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = 249\ (87) \times 10^{-11}$ is about 2.9 s.d.

PACS numbers: 13.40.Em,14.60.Ef,12.20.Ds

- arXiv:1205.5370v2 [hep-ph] 27 May 2012
- 12,672 diagrams

The SM Value for a_μ from $e^+e^- \to$ *hadrons* (Updated 9/09)



well known

significant work ongoing

	VALUE ($\times 10^{-11}$) UNITS
QED ($\gamma + \ell$)	$116\,584\,718.853 \pm 0.022 \pm 0.029_\alpha$
HVP(lo)*	$6\,923 \pm 42$
HVP(ho)	-98.4 ± 0.7
H-LBL	105 ± 26
EW	$154 \pm 1 \pm 2$
Total SM	$116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}}\ (\pm 49_{\text{tot}})$

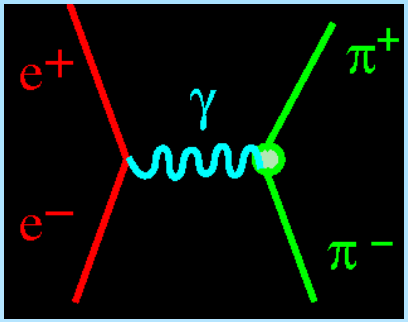
*Davier et al, Eur. Phys. J. C (2011) 71:1515

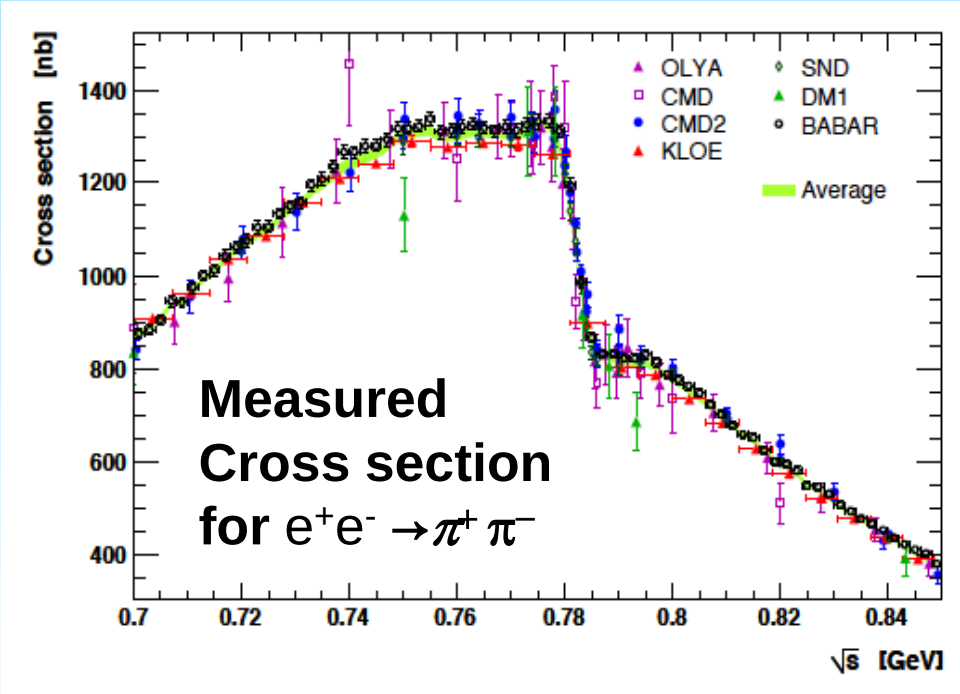
$\sigma_{\text{exp}} = \pm 63$

$\Delta a_\mu = 287(80) \times 10^{-11}$

a_μ^{had}(LO) Analyticity + Optical Theorem

$$a_\mu(\text{had}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) \left(\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\right)$$



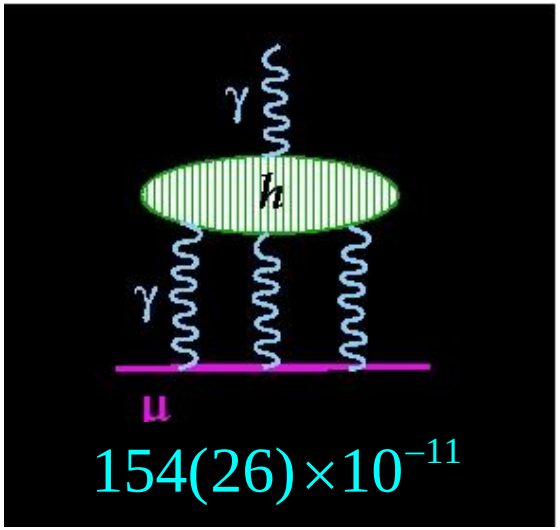


- Future efforts will reduce errors
  - CMD3 at VEPP2000, up to 2.0 GeV (next 5 years)
  - perhaps Belle

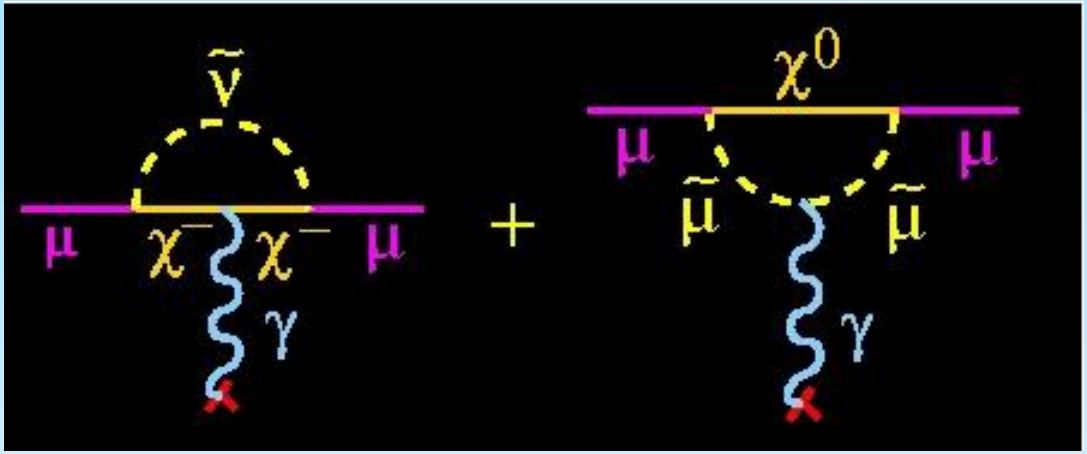
Hadronic light-by-light

- This contribution must be determined by calculation.

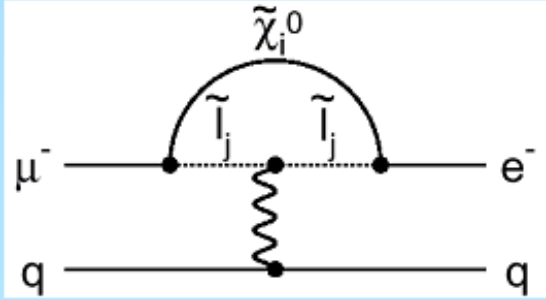
- the knowledge of this contribution limits knowledge of theory value.



a_μ is sensitive to a wide range of new physics, e.g.SUSY



Compare $\mu \rightarrow e$ conversion



$$a_\mu(\text{SUSY}) \simeq (\text{sgn}\mu)\, 130 \times 10^{-11} \tan\beta \left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^2$$

difficult to measure at LHC

Related processes in SUSY

$$\mu^+ \rightarrow e^+\gamma; \quad \mu^- + \mathcal{N} \rightarrow e^- + \mathcal{N}$$

Other Models

- Technicolor
  - small Δa_μ
- Littlest Higgs with T-parity
  - small Δa_μ
- Universal Extra Dimensions
  - small Δa_μ
- Randall Sundrum
  - could accommodate large Δa_μ
- Two Higgs doublets, shadow Higgs
  - small Δa_μ
- Additional light bosons that can affect EM interactions (difficult to study at LHC)
  - secluded U(1),etc., could have significant Δa_μ

Experimental Determination of a_μ (BNL)

Measure muon spin precession rate in a magnetic storage ring

The magnetic, electric dipole moments $\vec{\mu} = \frac{ge}{2m}\vec{s}, \quad \vec{d} = \frac{\eta e}{2mc}\vec{s}; \quad a_\mu = \frac{g-2}{2}$

Start with $\frac{d\vec{s}}{dt'} = \vec{\mu}\times\vec{B}' + \vec{d}\times\vec{E}'$ in muon rest frame, transform t',E',B' to lab frame,

include Thomas precession, subtract the cyclotron frequency,

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

≈ 0 at magic momentum

For E=0 and zero EDM $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}a_\mu\vec{B}$

- Precession does not depend on momentum
- Uniform B highly desirable
  - all particles precess at same rate regardless of trajectory or momentum- but then no focusing to hold muons in storage ring
- BNL Solution: uniform B with quadrupole E fields, choose 'magic gamma':

Choose p such that: $a_\mu - \frac{m^2}{p^2} = 0 \rightarrow$ p= $\frac{m}{\sqrt{a_\mu}}$ =3.096 GeV/c

- Correlation between electron and spin directions causes the number of high energy decay electrons to oscillate at the precession frequency

The miracles that make the experiment possible:

- Parity violation
  - produces polarized muons

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

  - analyzes the spin orientation at the decay time

$$\mu^- \rightarrow e^- + \bar{\nu}_e \nu_\mu$$

- The 2.2μs lifetime permits precision measurements
- The mass ~107 m_e gives sensitivity to the TeV scale
- The rate at which the spin turns relative to the momentum only depends on the anomaly and B field.

$$\vec{\omega}_a = \omega_S - \omega_C = -\frac{Qe}{m} a_\mu \vec{B}$$

- There is a "magic" value of γ such that an applied E field doesn't mess up this dependence

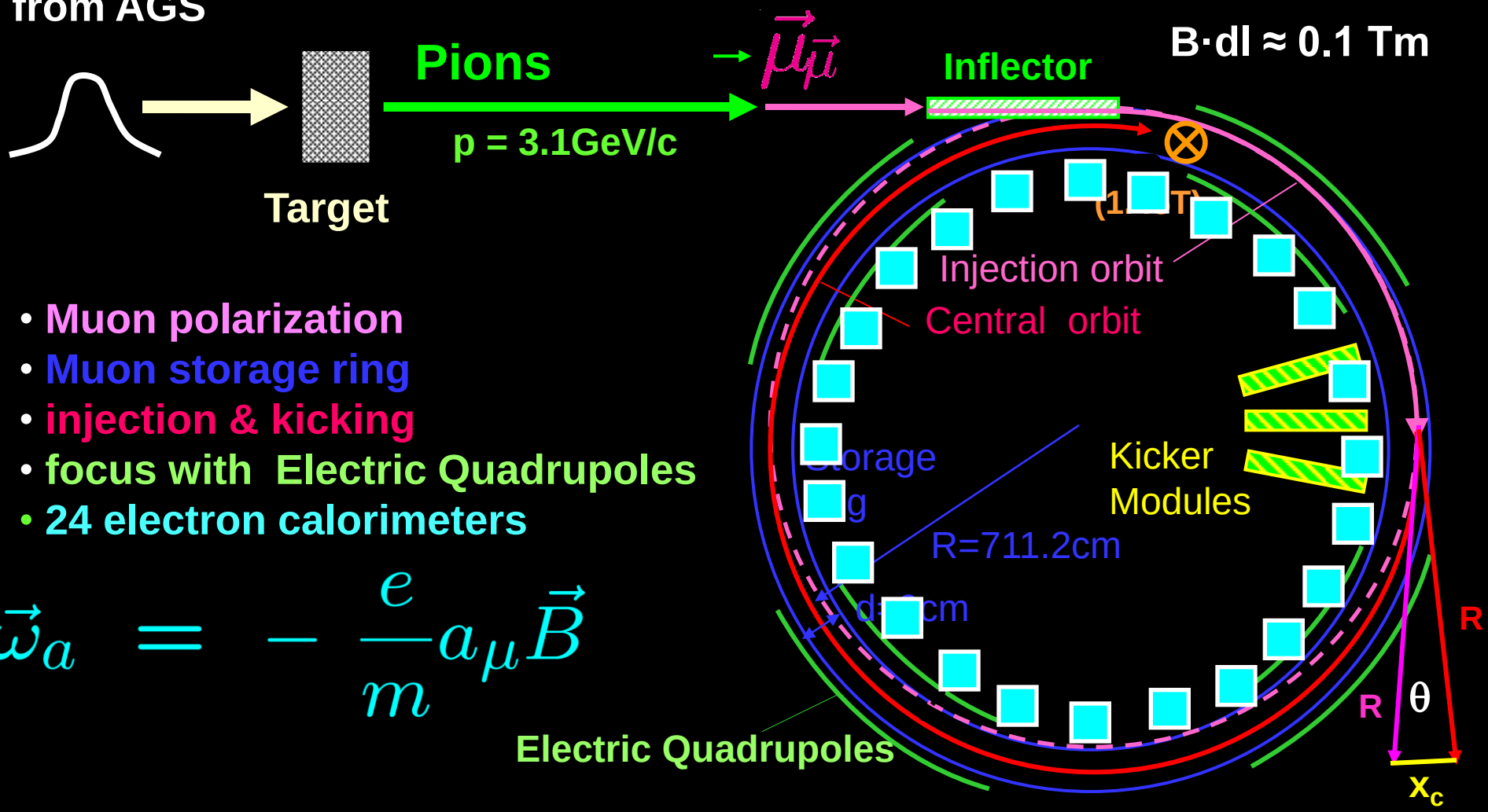
BNL and FNAL Experimental Technique

25ns bunch of
5 X 10^{12} protons
from AGS

$x_c \approx 77$ mm

$\theta \approx 10$ mrad

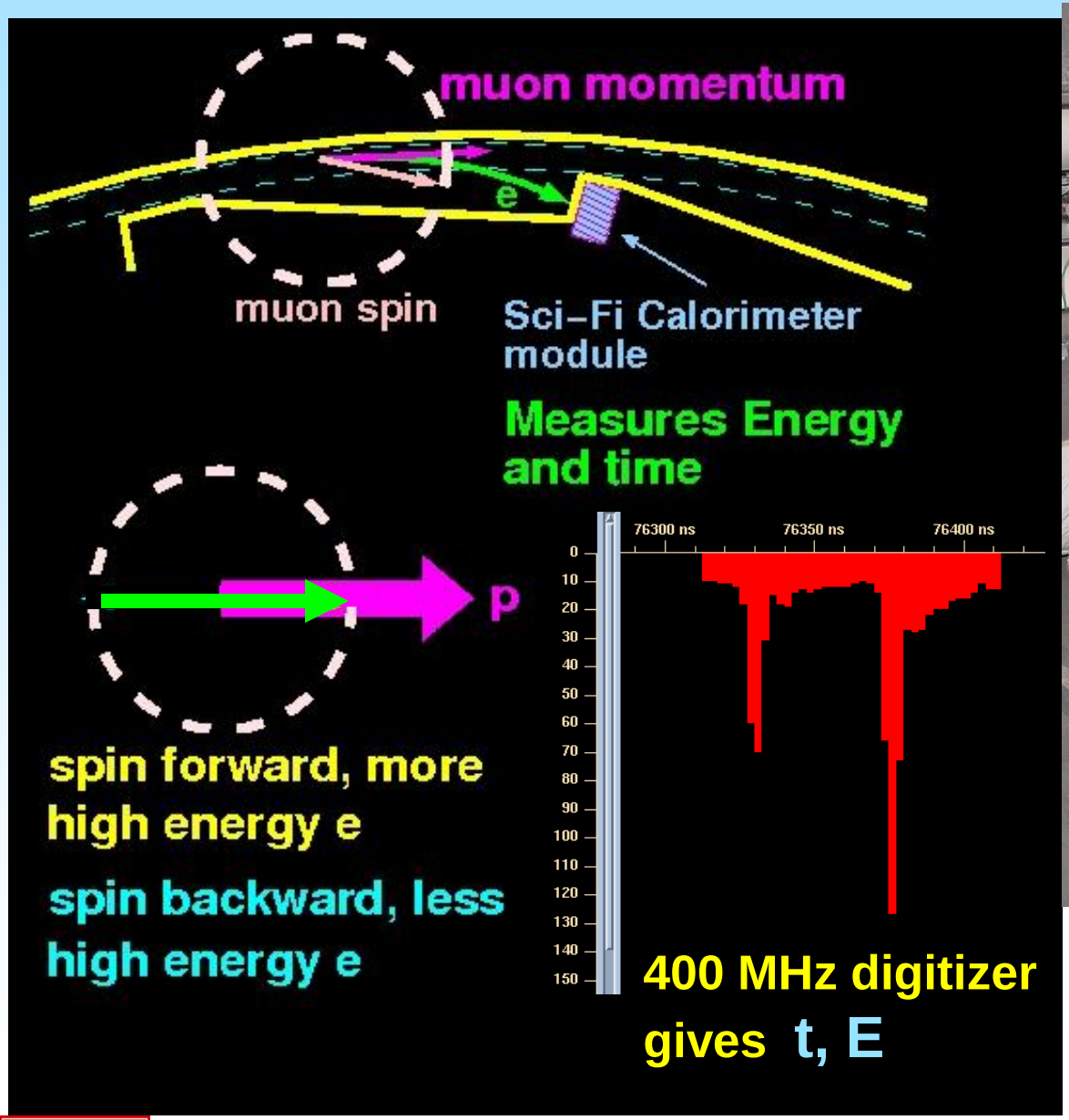
$B \cdot dl \approx 0.1$ Tm



- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

To measure ω_a, we used Pb-scintillating fiber calorimeters.

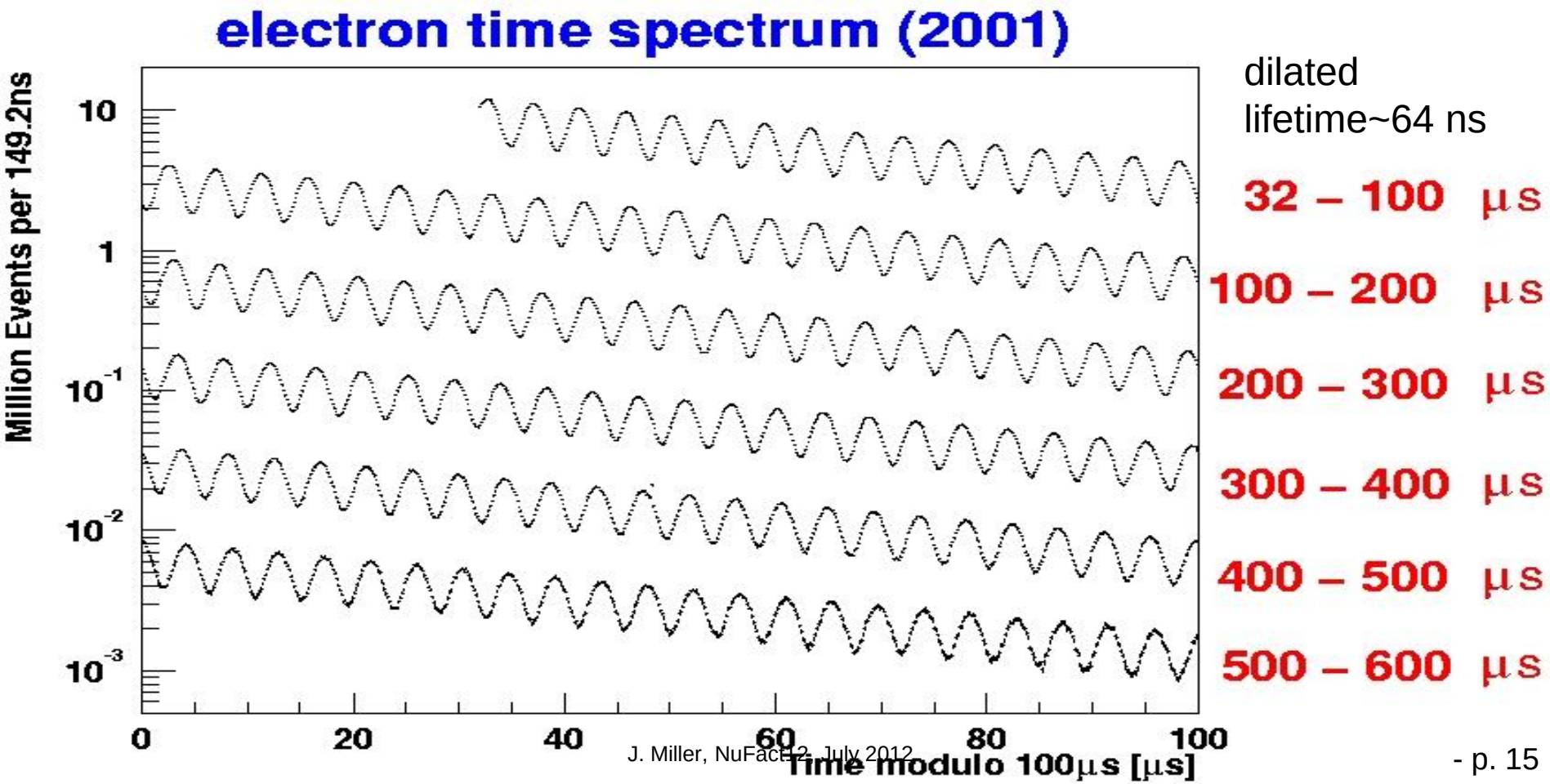


Count number of e^- with $E_e \geq 1.8$ GeV

Arrival time spectrum

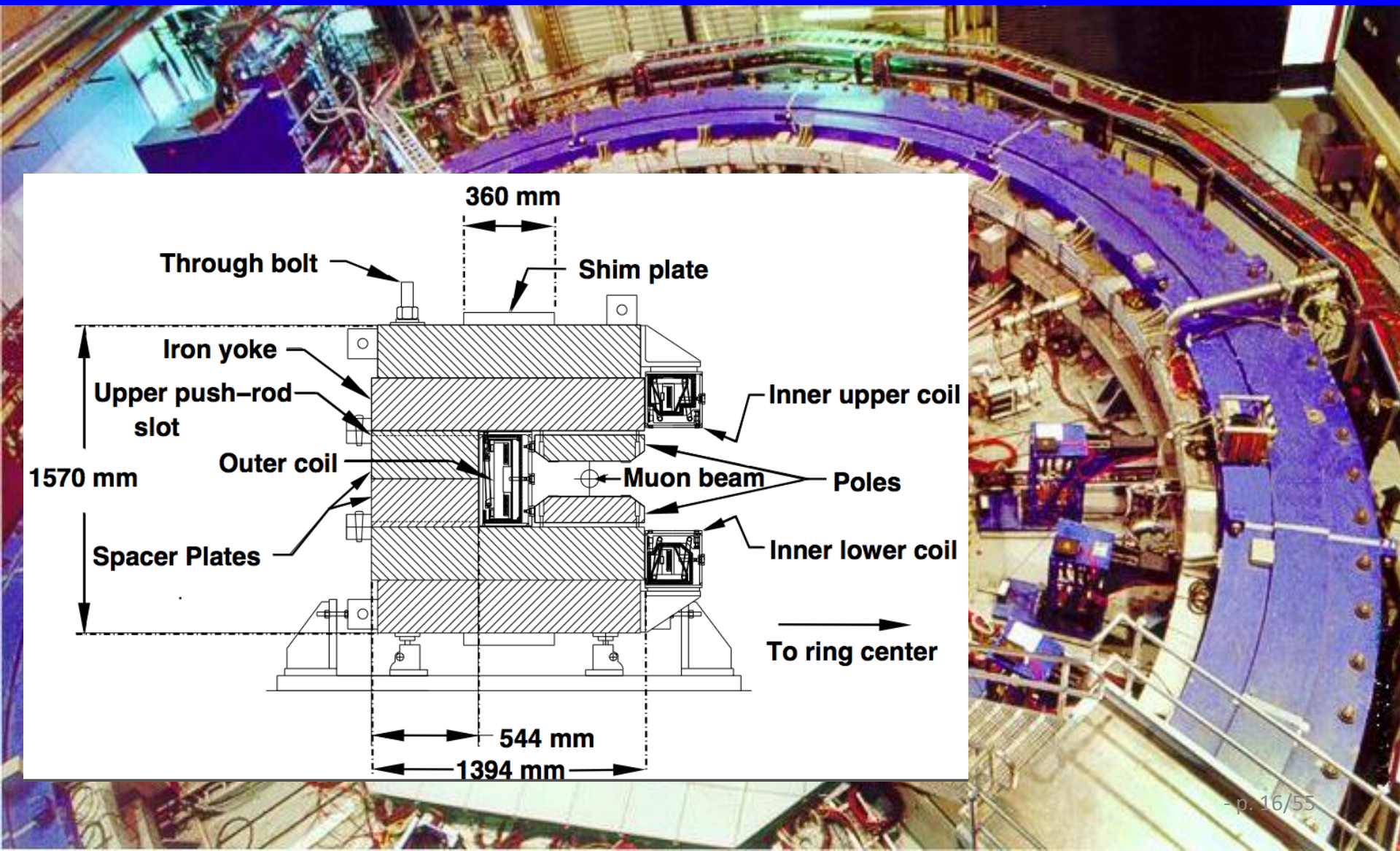
$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos\omega_a t + \phi)]$$

$$4 \times 10^9\ e^-,\ E_{e^-} \geq 1.8\ \text{GeV}$$



dilated lifetime~64 ns

The Experiment



The Storage Ring Magnet

$r = 7112$ mm

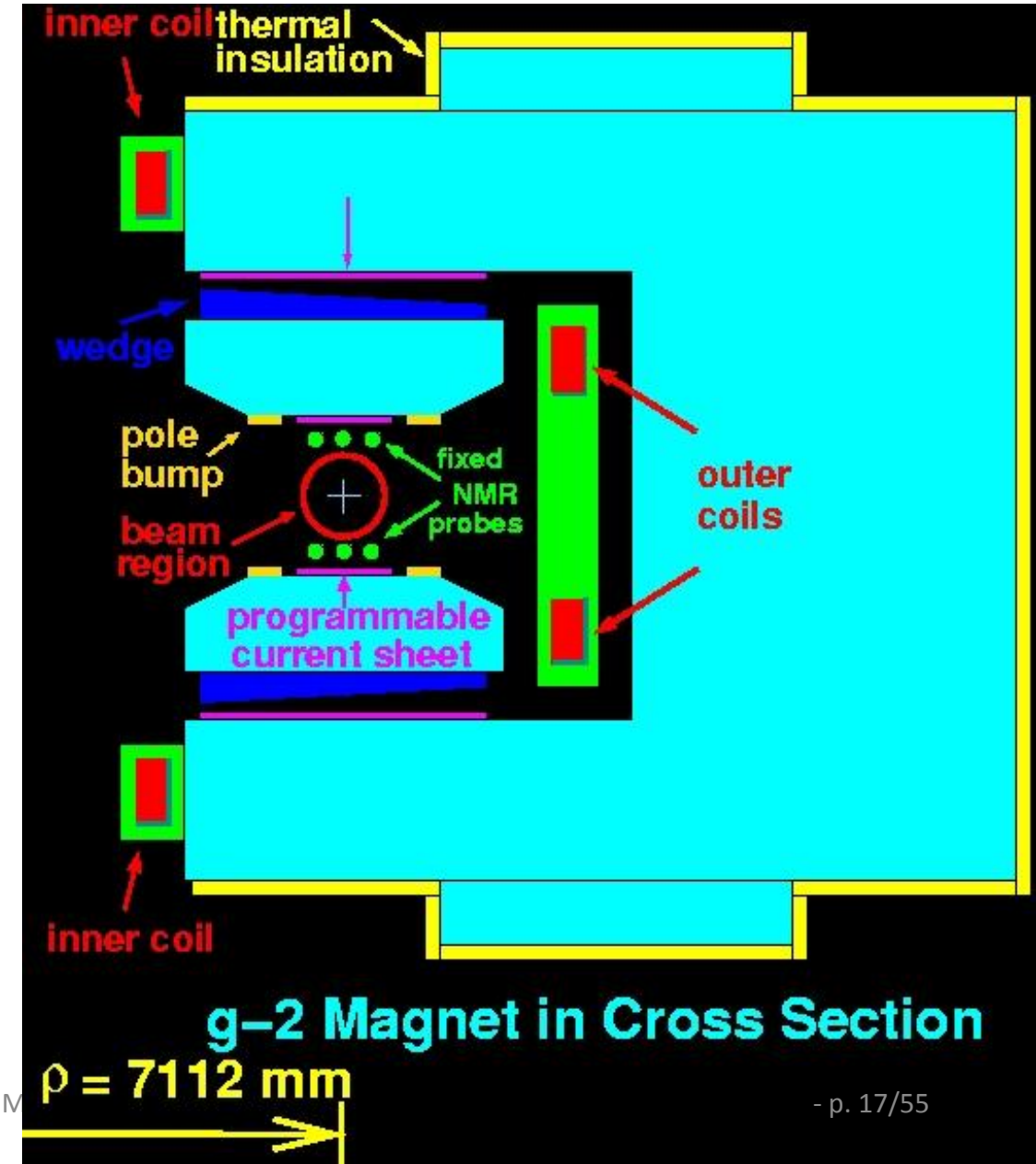
$B_0 = 1.45$ T

$\tau_{cyc} = 149$ ns

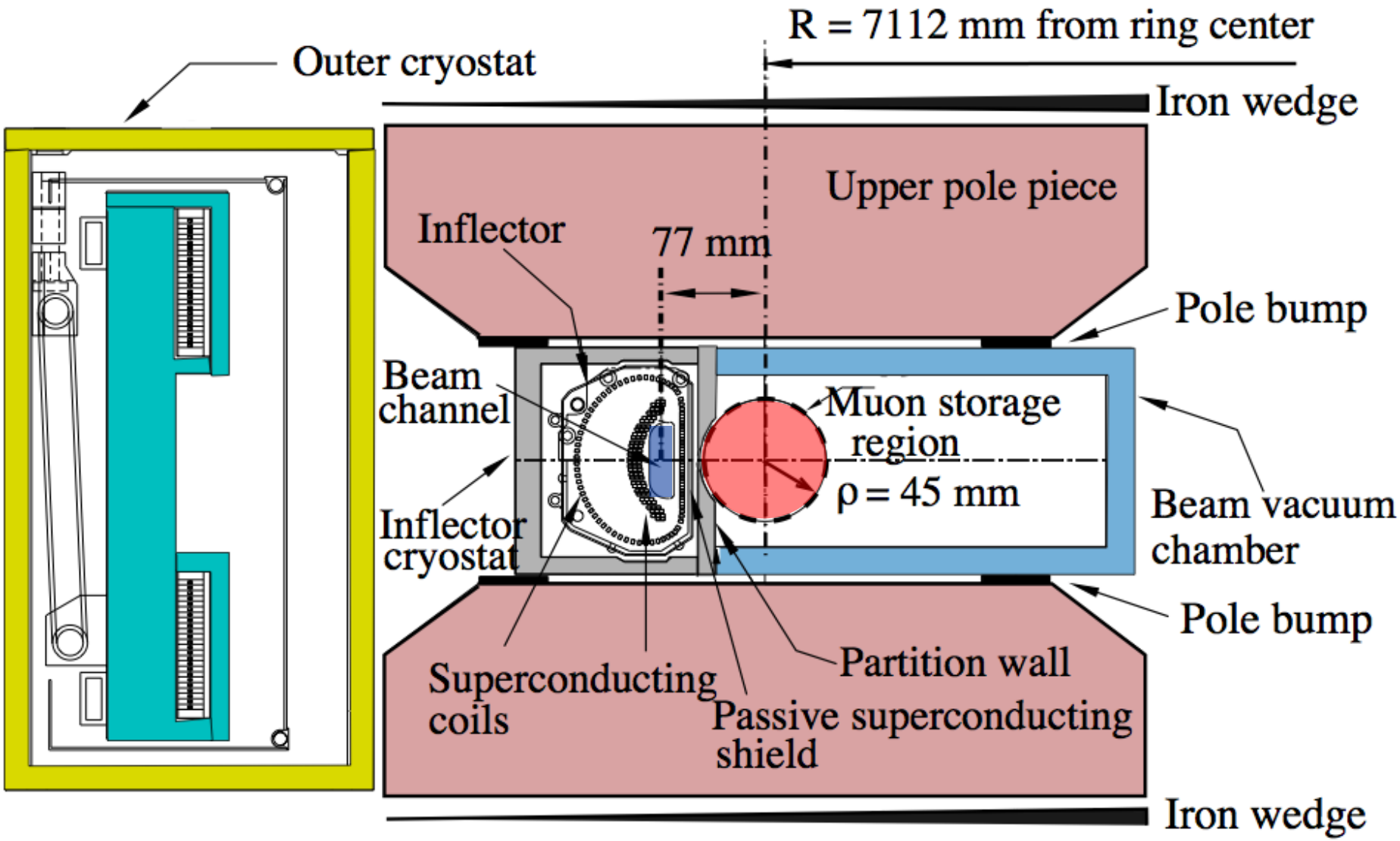
$\tau_{(g-2)} = 4.37\ \mu s$

$\tau_\mu = 64.4\ \mu s$

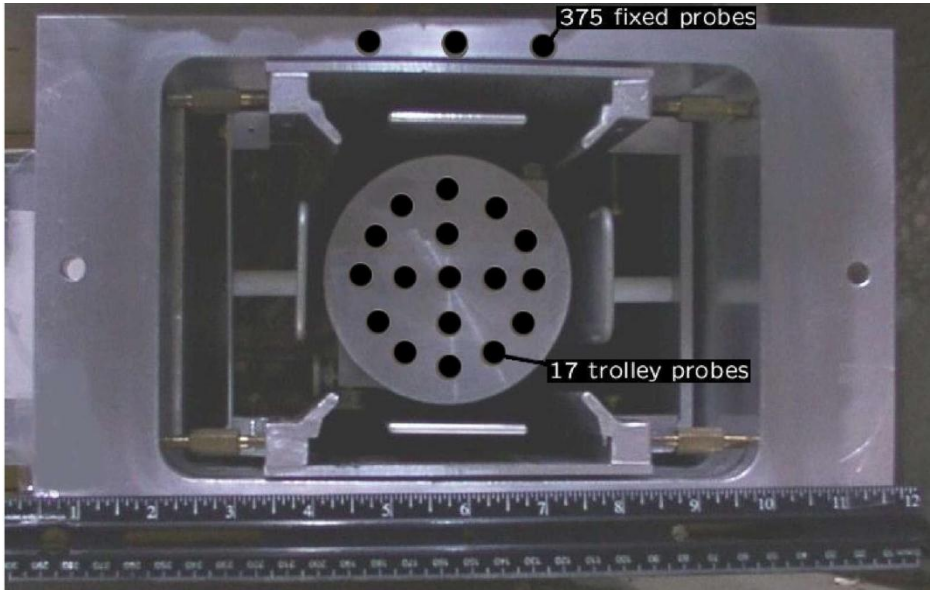
$p_\mu = 3.094$ GeV/c

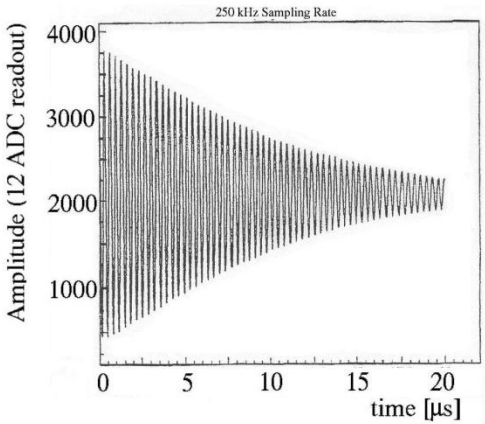


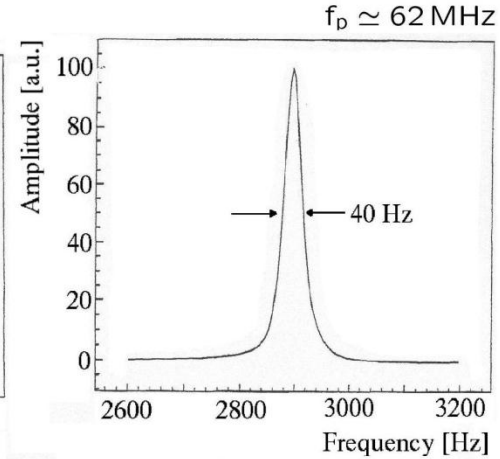
Magnet shimming tools (Gordon Danby's design):
wedge angle – quadrupole
pole bumps - sextupole

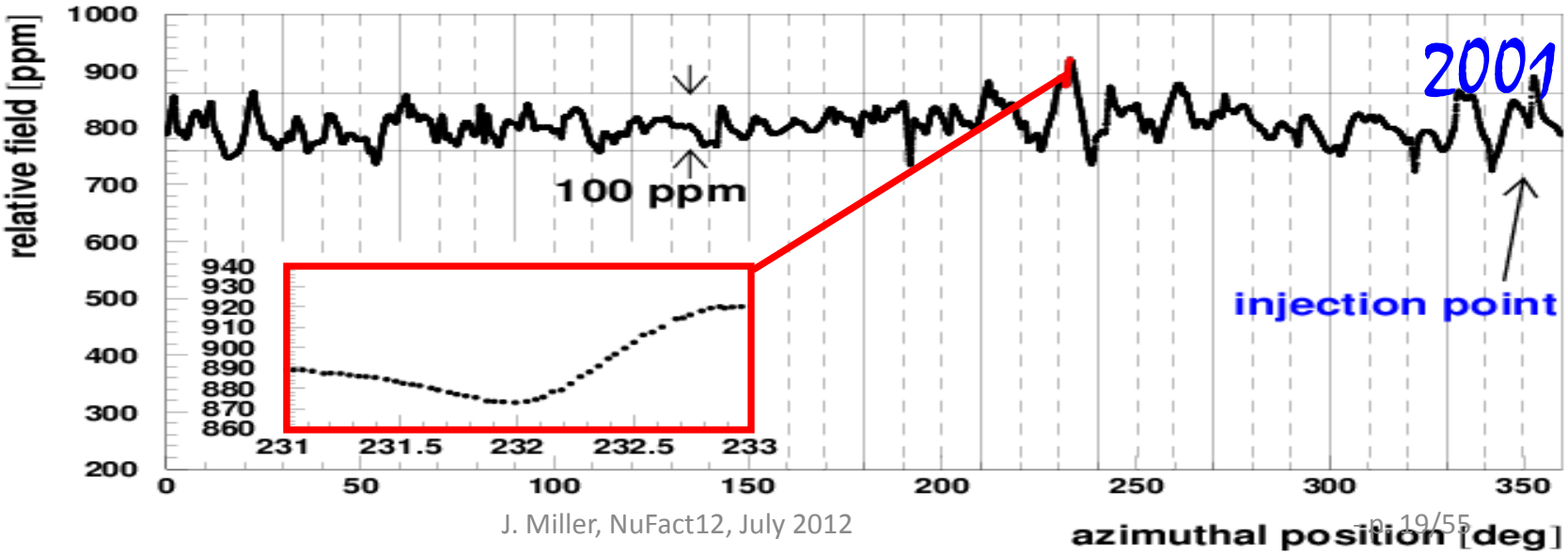


B Field Measurement





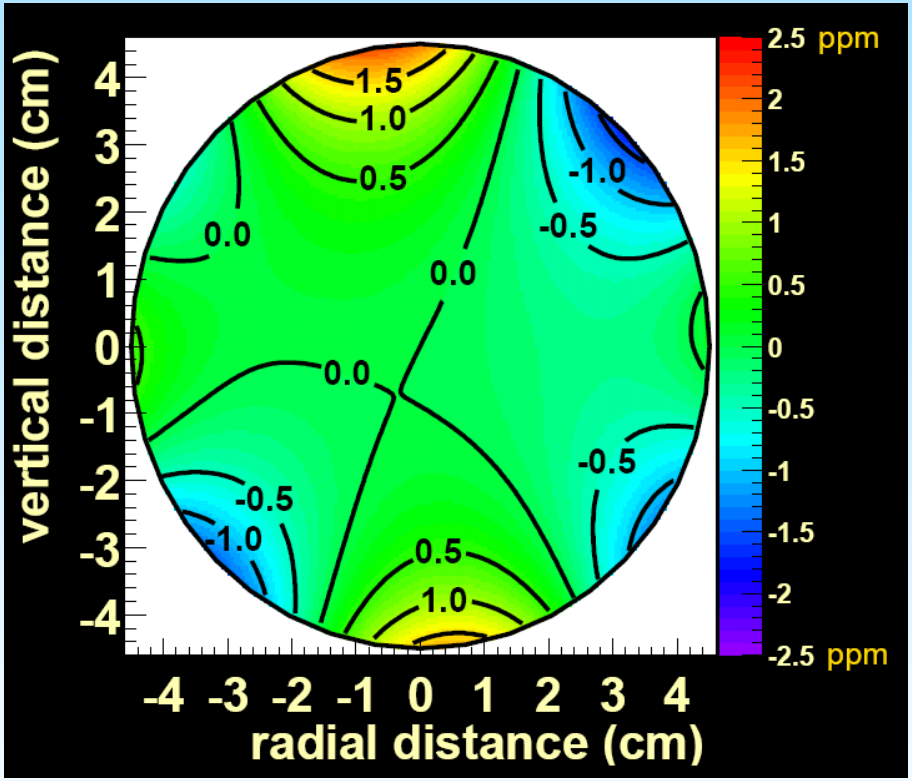




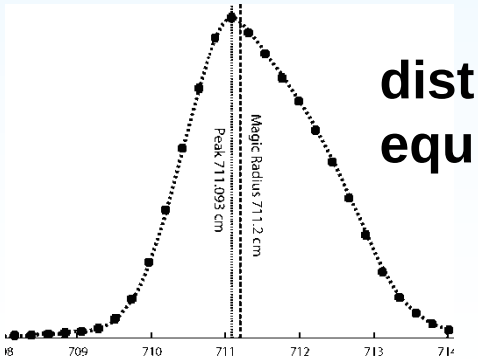
Average field uniformity ± 1 ppm

$$\langle B \rangle_{\text{azimuth}}$$

0.5 ppm contours



$$\sigma_{\text{syst}} \langle B \rangle_{\mu-\text{dist}} = \pm 0.03 \text{ ppm}$$



dist. of μ equil. radii

To determine a_μ we measure two numbers:

The muon spin frequency: ω_a

$$\omega_a = a\frac{eB}{mc}$$

The magnetic field normalized to the Larmor frequency of the free proton: ω_p

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

Best value of ratio from muonium hfs experiments

Muon EDM

- A non-vanishing EDM signals violations of T and P symmetries
- Assuming CPT this implies CP violation
- New sources of CPV sought to help explain BAU
- Muon EDM is the only measureable EDM outside the first generation
- SM predicts scaling w/r electron proportional to mass, $<10^{-38}$ e-cm
- Lepton universality, current e limit→$d_\mu<5\times10^{-25}$ e-cm
- Some new physics models predict muon EDM: few x 10^{-22} while at the same time keeping the electron EDM small (Babu, Barr, Dorsner, 2001)

Present EDM Limits

Particle	*Present EDM limit* (e-cm)	*SM value* (e-cm)
n	$<2.9\times10^{-26}$	$10^{-32} - 10^{-31}$
e^-	$<1.05\times10^{-27}$	$< 10^{-41}$
μ	$<10^{-18}$ (CERN g-2) $<1.8\times10^{-19}$ (BNL g-2)	$< 10^{-38}$
future exp μ	**10^{-20} to 10^{-25} ?** **x10-100 (g2@FNAL)**	

BNL and FNAL g-2 Expt Method to extract EDM

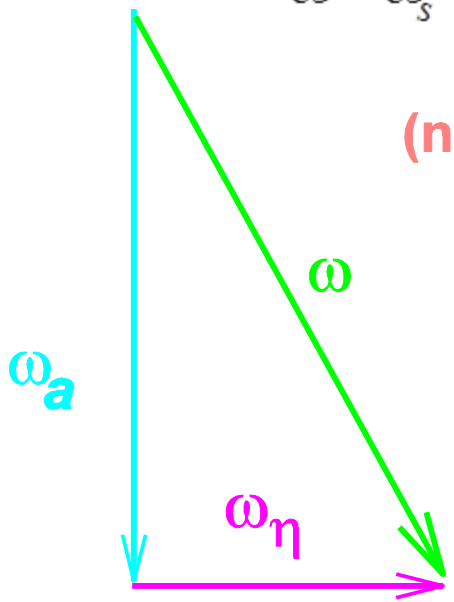
Recall:

The magnetic, electric dipole moments $\vec{\mu} = \frac{ge}{2m}\vec{s}, \quad \vec{d} = \frac{\eta e}{2mc}\vec{s}; \quad a_\mu = \frac{g-2}{2}$

$$\vec{\omega} = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

≈ 0 at magic momentum

(not to scale)



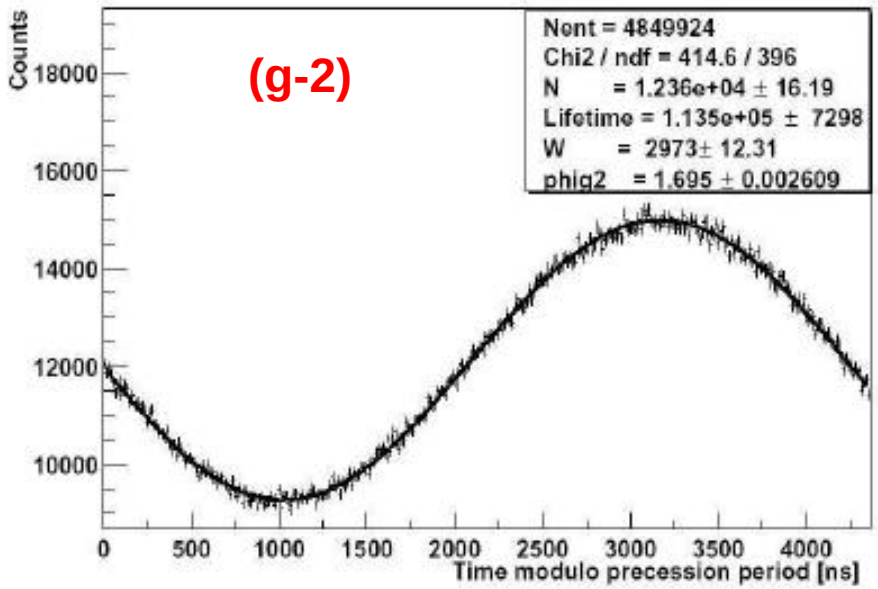
The EDM causes the spin to precess out of plane.

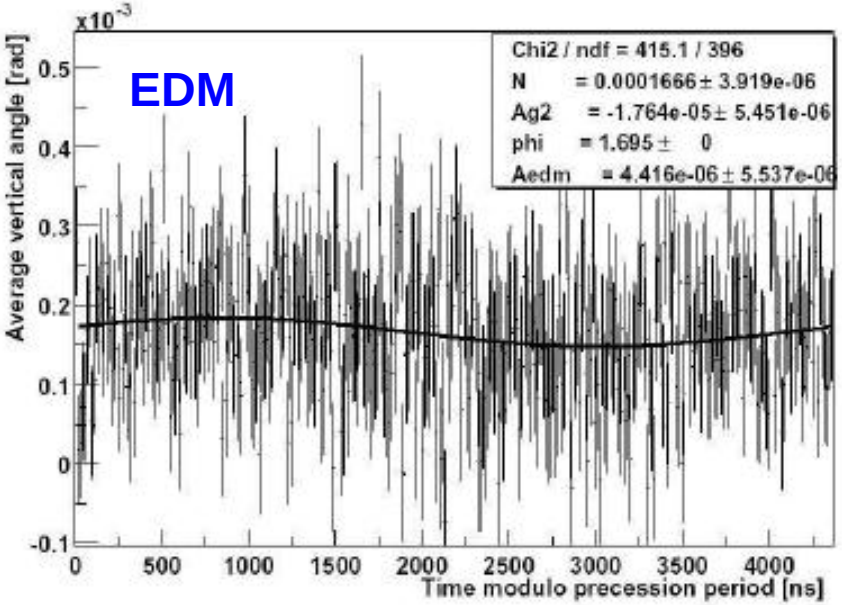
Note: $\omega = \sqrt{\omega_a^2 + \omega_\eta^2}$

BNL result, $d_\mu < 1.8 \times 10^{-19}$ e-cm (90% c.l.)

Signal: up-down oscillation $\pi/2$ out of phase with ω_a using upward-going and downward-going tracks

E821 Data





Vertical Oscillation out of phase with ω_a

$$N^{\pm}(t) \propto 1 + A_\mu \cos(\omega t + \phi) \mp A_{EDM} \sin(\omega t + \phi)$$

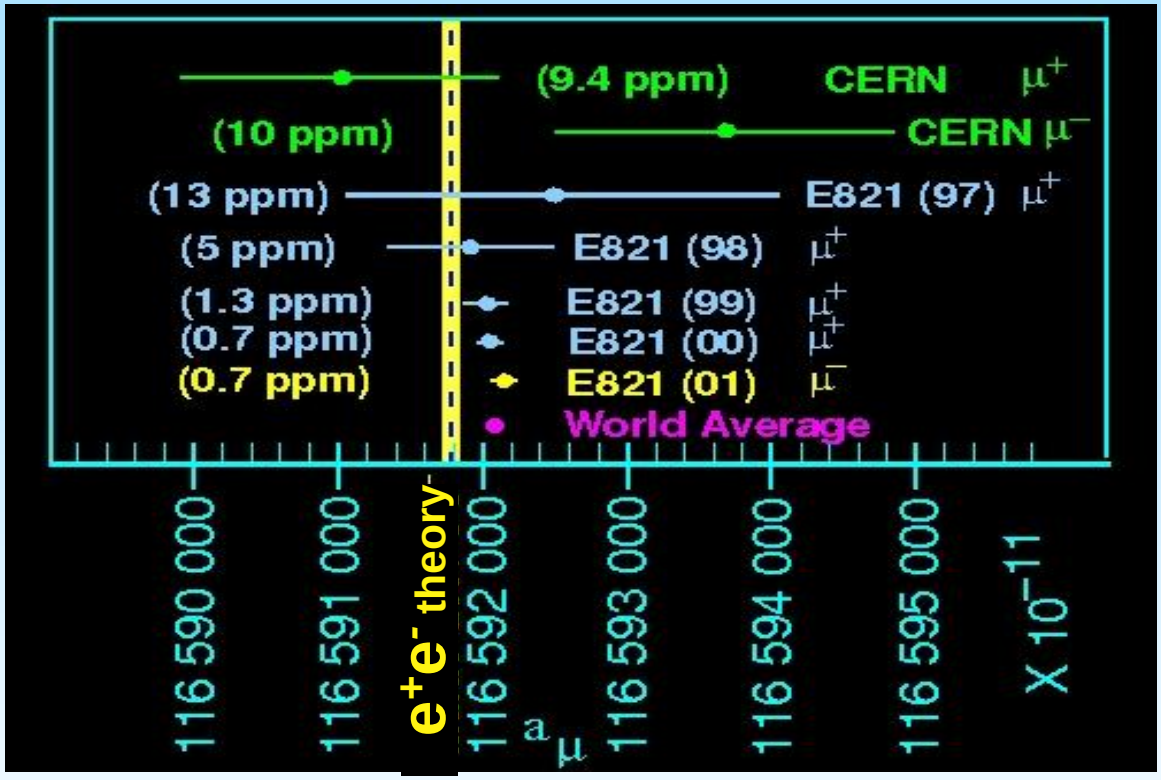
$$d_\mu < 1.8 \times 10^{-19} \ (95\% \ \mathrm{CL})$$

This EDM would shift a_μ by $(0.0 \pm 42) \times 10^{-11}$

E821 achieved ± 0.54 ppm. The e^+e^- based theory is at the ~0.42 ppm level. Difference is >3σ

SM: Davier et al, , Eur. Phys. J. C (2011) 71:1515

Hagiwara, et al., J.Phys. G **38** (2011) 085003



$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \ (0.54\,\text{ppm})$$

$$\Delta a_\mu^{(\text{today})} = (288 \pm 80) \times 10^{-11}$$

$$a_\mu^{EW} = 154(1)(2) \times 10^{-11}$$

3.6 σ: Theory & Experiment must do better

The New $g-2$ Experiment:

An experiment to Measure the Muon Anomalous Magnetic Moment

to ± 0.14 ppm Precision

- Experiment: E989 at Fermilab ≥ X4 better
  - relocate the storage ring to Fermilab (operations $)
  - use the p-bar beam line and debuncher storage ring (now called the delivery ring) as a long decay line.
- CD0 expected very soon.
- Building construction will begin in November 2012

The error budget for a new experiment represents a continuation of improvements already made during E821

Systematic uncertainty (ppm)	1998	1999	2000	2001	E821 final	P989 Goal
Magnetic field – ω_p	0.5	0.4	0.24	0.17		0.07
Anomalous precession – ω_a	0.8	0.3	0.31	0.21		0.07

Statistical uncertainty (ppm)	4.9	1.3	0.62	0.66	0.46	0.1
Systematic uncertainty (ppm)	0.9	0.5	0.39	0.28	0.28	0.1
Total Uncertainty (ppm)	5.0	1.3	0.73	0.72	0.54	0.14

Proposed Improvements

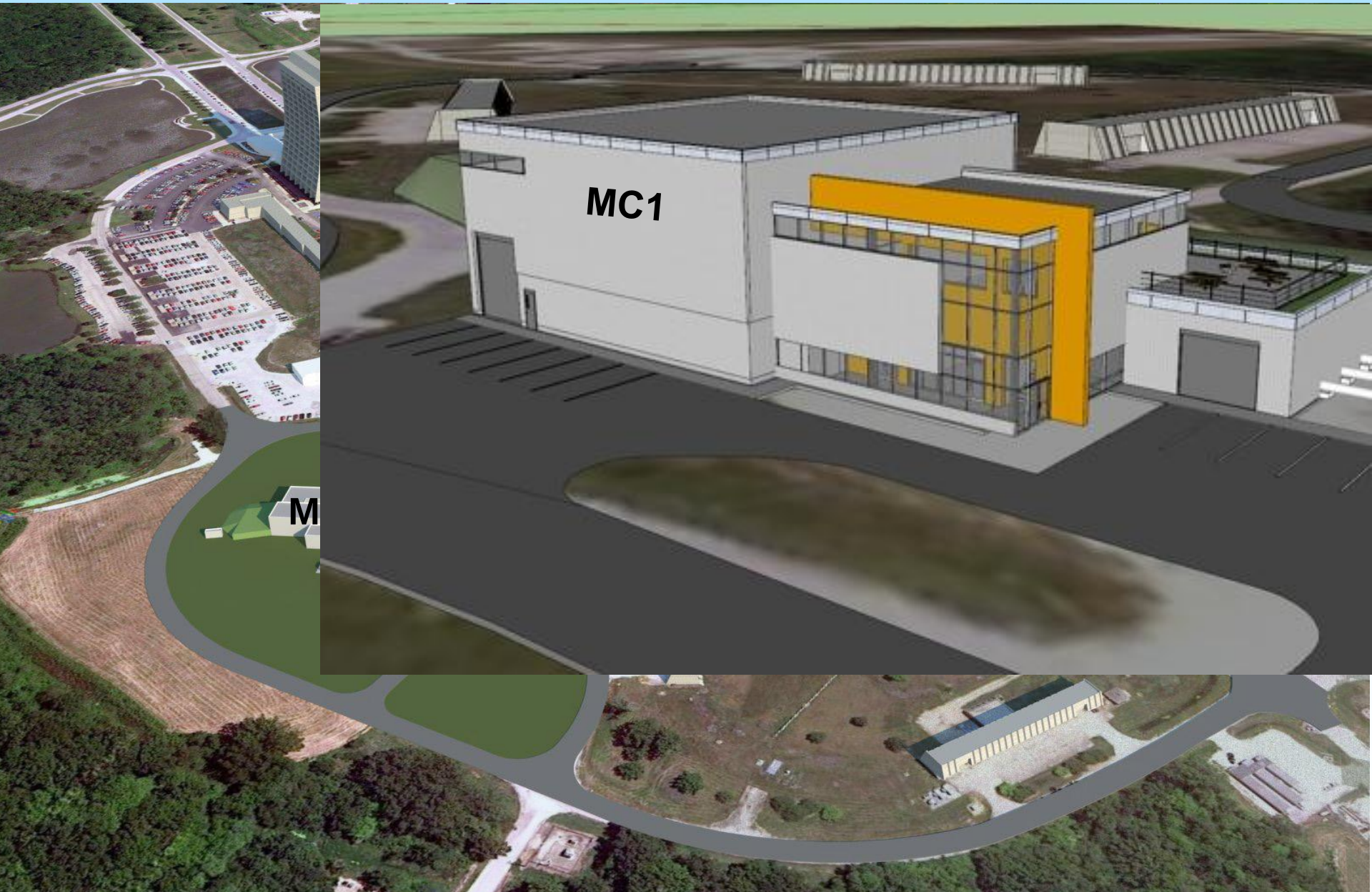
- Improve magnet shimming to improve magnet uniformity
- Improve magnetic field-measuring trolley, positioning of fixed NMR probes
- Improve calorimeter
  - increase segmentation to reduce pileup systematic
- Improve particle tracking with trackers upstream of several detector stations to improve EDM and understanding of muon distribution in ring
- Increase number of injection pulses per second- less intensity per injection hence less pileup
- Longer beamline means most pions decay
  - much less beam 'flash' in detectors
  - can tune muon momentum to that of pion beam, giving many more injected muons from forward muon decays compared to BNL
  - can turn on calorimeters much sooner after injection, gives much less effect on gains of photo-sensitive devices, better data efficiency
- Replace inflector (BNL version has closed end causing multiple scattering and reduced muon storage)
- Improvements in kicker, electrodes, ...

Sikorsky S64F 12.5 T hook weight (Outer coil 8T)

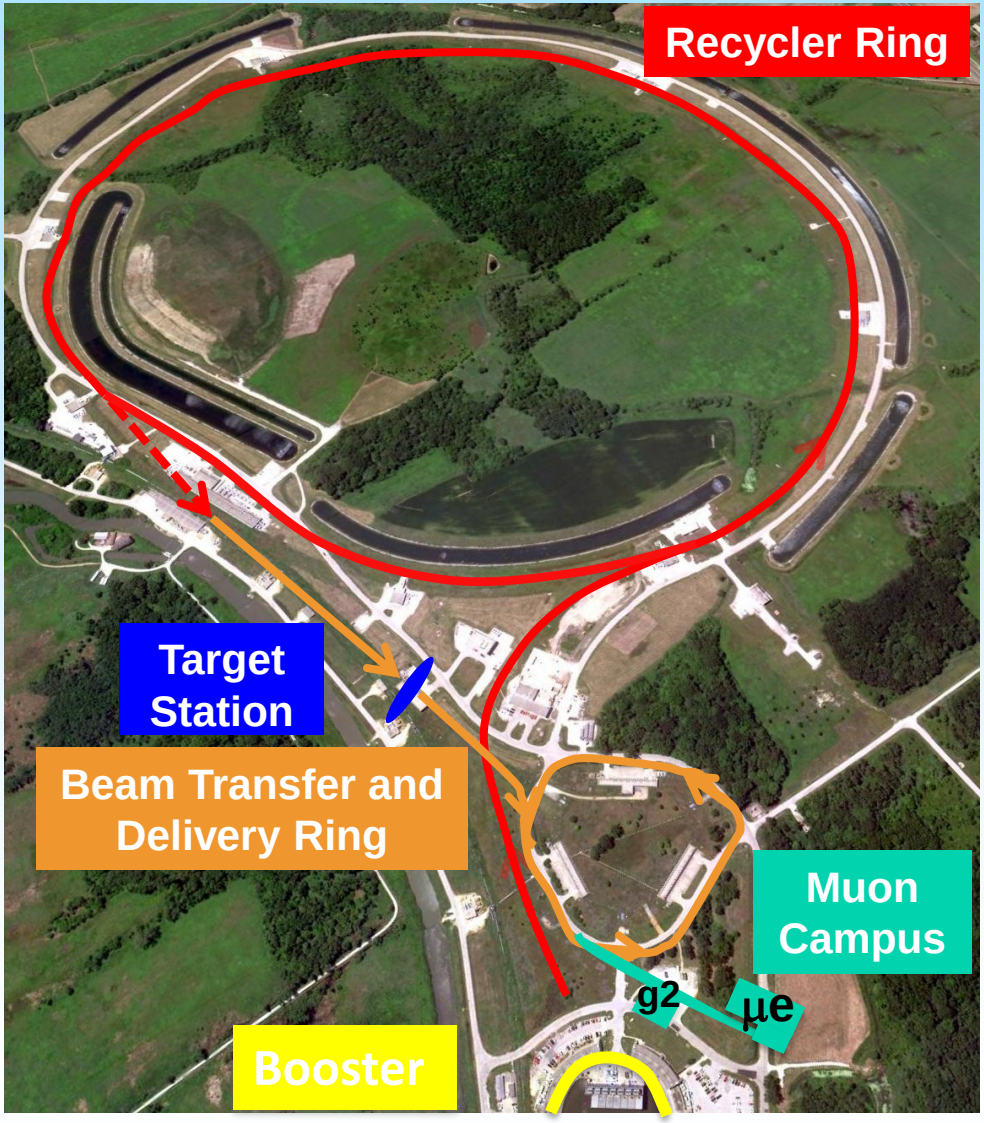
- Transport coils to and from barge via Sikorsky aircrane
- Ship through St Lawrence –> Great Lakes –> Calumet SAG
- Subsystems can be transported overland, but probably more cost effective to ship steel on barge as well.

Fermilab Muon Campus

Multipurpose Building designed for future experiments as well



Fermilab Muon Beam



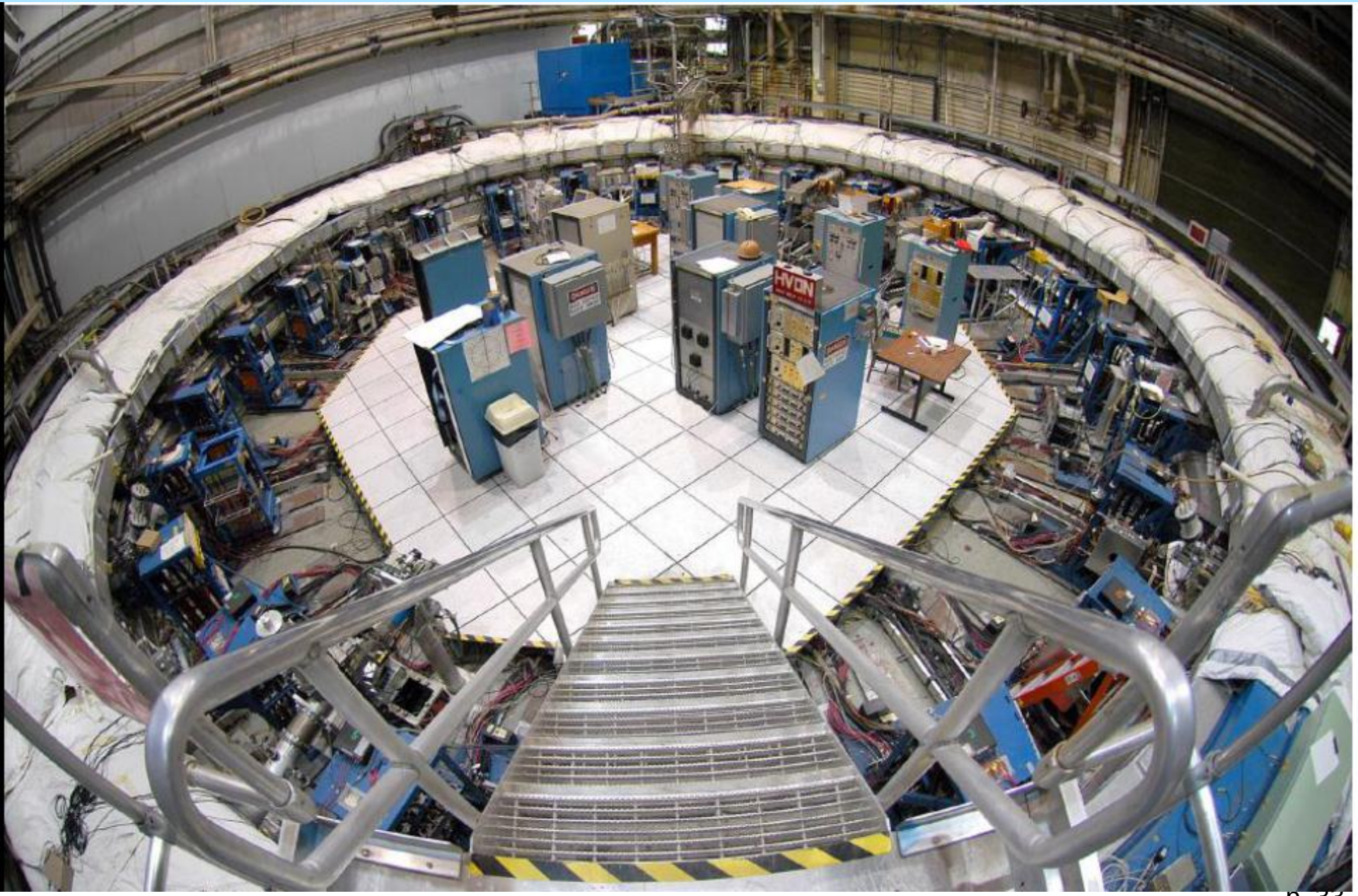
One 4×10^{12} p bunch to recycler

Re-bunch to 4 bunches

Extract one bunch at a time to target

Use the injection line and delivery ring as a 1,900 m decay line

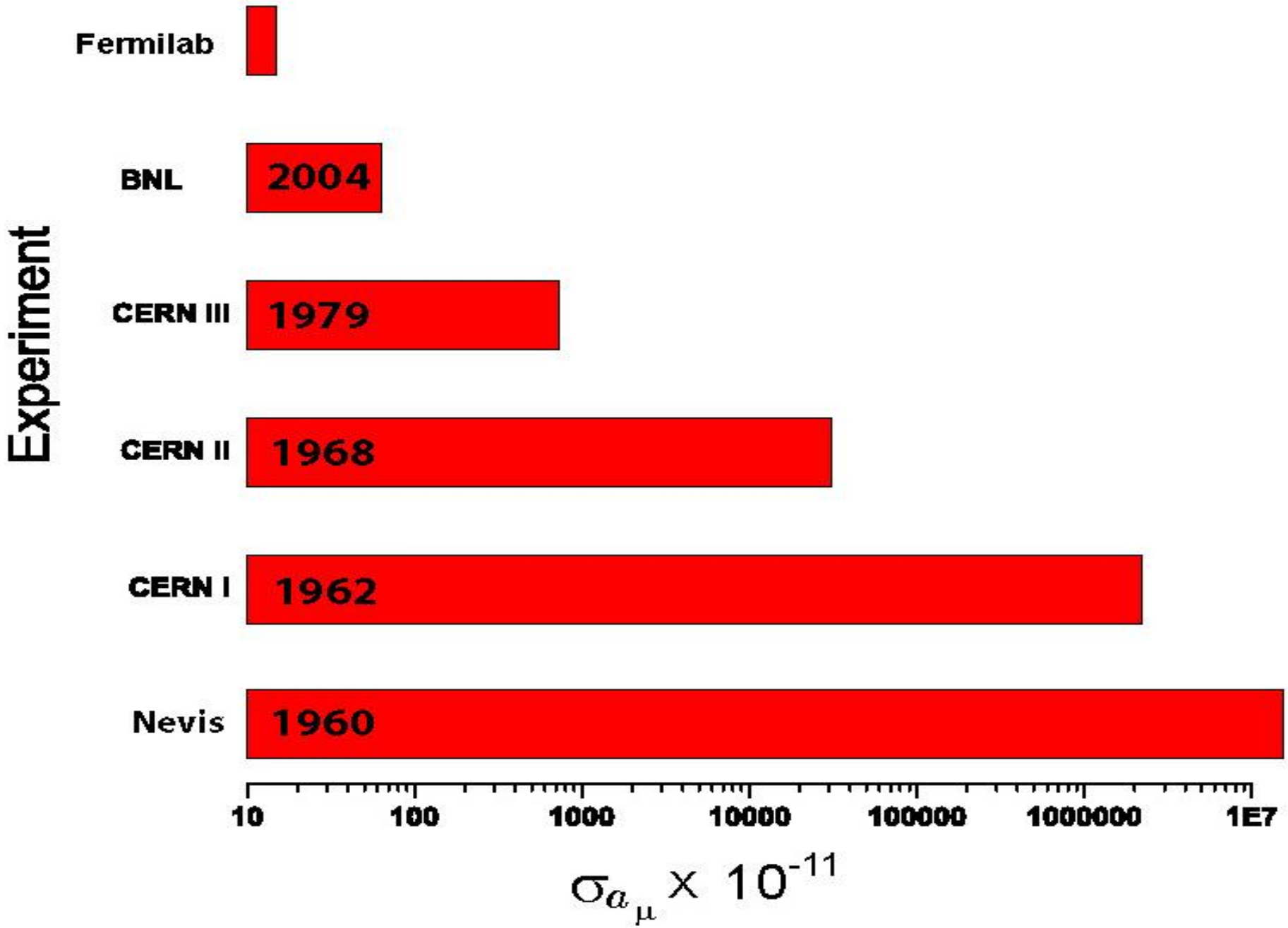
Storage Ring at BNL in 2011



May 2012

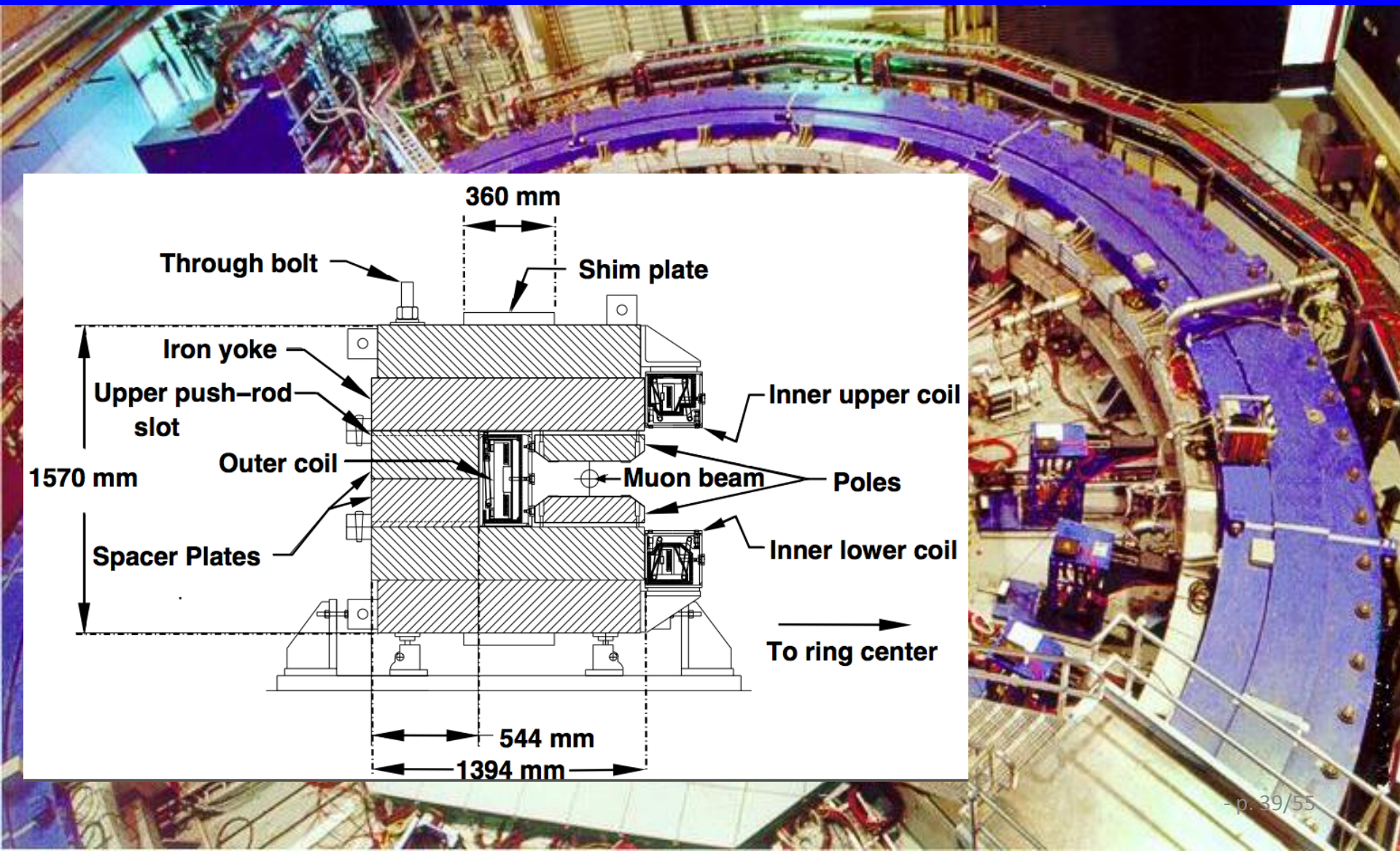
Final words:

- There appears to be a 3.6 s.d. difference between experiment and theory
- E989 is at an exciting point
  - will improve on E821 by at least X4
  - CD0 soon, building construction starting November 2012
  - plan to relocate the storage ring in 2013
- Worldwide effort on all aspects of the theory continues
  - more e^+e^- data from BES, KLOE, Mainz, Novosibirsk
  - H-LBL work continues on several fronts (including KLOE)
  - The lattice is becoming relevant, both for H-LO and H-LBL


Experiment
Fermilab
BNL
2004
CERN III
1979
CERN II
1968
CERN I
1962
Nevis
1960
10
100
1000
10000
100000
1000000
1E7
$\sigma_{a_\mu} \times 10^{-11}$

END

The Experiment



The miracles that make the muon g-2 experiment possible:

- Parity violation
  - produces polarized muons

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

  - analyzes the spin orientation at the decay time

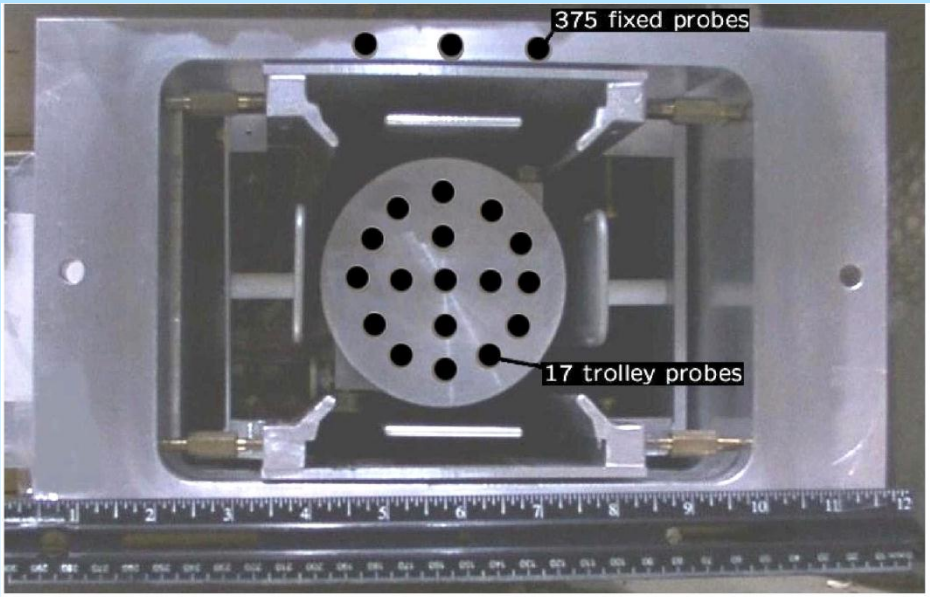
$$\mu^- \rightarrow e^- + \bar{\nu}_e \nu_\mu$$

- The 2.2μs lifetime long to permit precision measurements
- The mass ~107 m_e gives sensitivity to the TeV scale
- The rate at which the spin turns relative to the momentum only depends on the anomaly and B field.

$$\vec{\omega}_a = \omega_S - \omega_C = -\frac{Qe}{m} a_\mu \vec{B}$$

- There is a "magic" value of γ such that an E field doesn't mess up this dependence

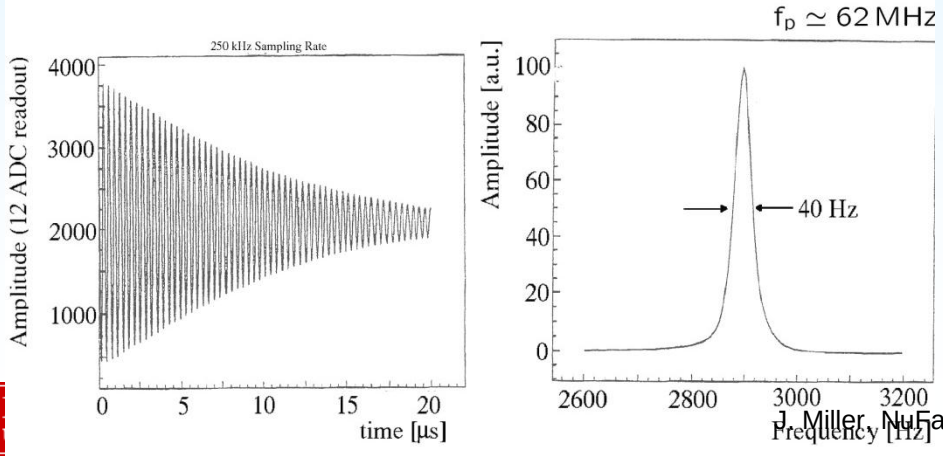
The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



- **Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.**

$$a_\mu = \frac{\dfrac{\omega_a}{\omega_p}}{\dfrac{\mu_\mu}{\mu_p} - \dfrac{\omega_a}{\omega_p}}$$

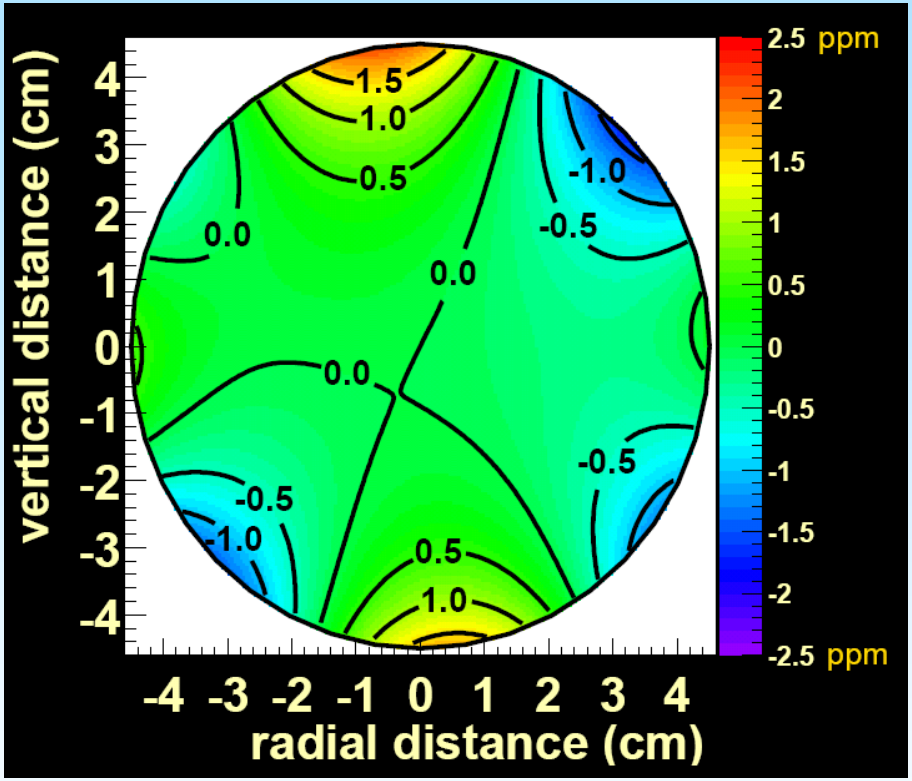
Free induction decay signals:



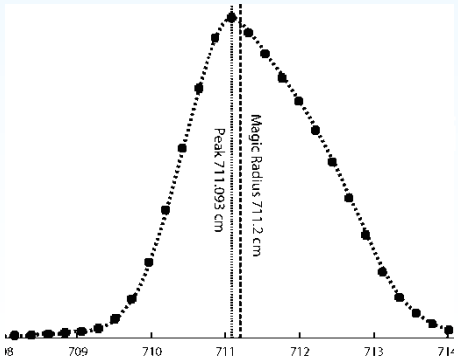
Average field uniformity ± 1 ppm

$\langle B \rangle_{\text{azimuth}}$

0.5 ppm contours



$$\sigma_{\text{syst}} \langle B \rangle_{\mu-\text{dist}} = \pm 0.03 \text{ ppm}$$

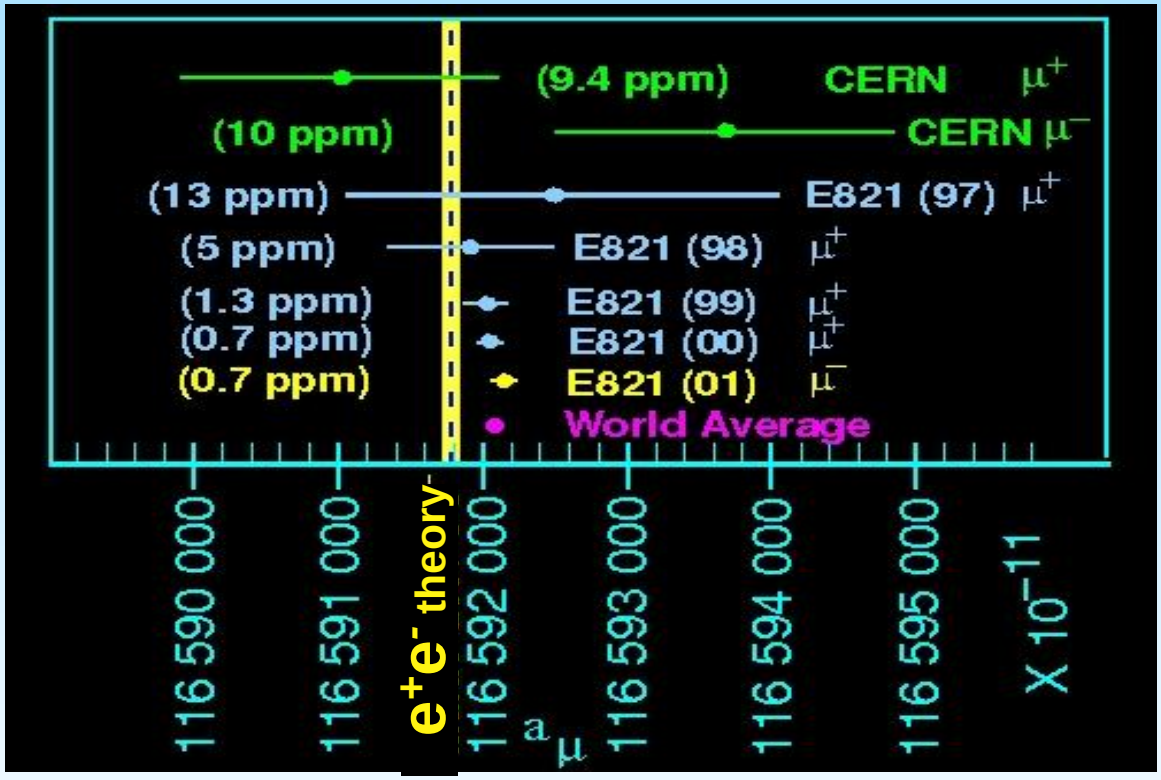


of μ equil.

E821 achieved ± 0.54 ppm. The e^+e^- based theory is at the ~0.49 ppm level. Difference is >3σ

SM: Davier et al, , Eur. Phys. J. C (2011) 71:1515

Hagiwara, et al., J.Phys. G **38** (2011) 085003



$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \ (0.54\,\text{ppm})$$

$$\Delta a_\mu^{(\text{today})} = (288 \pm 80) \times 10^{-11}$$

$$a_\mu^{EW} = 154(1)(2) \times 10^{-11}$$

Dedicated Muon EDM Measurements

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

- *Frozen spin method:*
- In a storage ring, apply radial E-field so that terms in red add to zero
- Effect of EDM accumulates with time- much enhanced signal
  - Spin vector, initially aligned with the momentum, acquires a
  - vertical component proportional to the EDM
- Proposal to J-PARC (2003), Ring: r=7 m, B=0.25 T, E=2.2 MV/m, p=125MeV/c → d_μ<1x10^{-24} e-cm
- See also Adelman, et al., hep-ex/0606034v3 (2009): Ring: r=0.42 m, B=1 T, E=0.64 MV/m, p=125MeV/c → d_μ<5x10^{-25} e-cm
- Likely limited by number of muons- A natural for future high intensity machines.

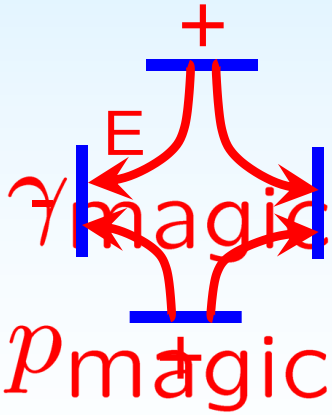
Need vertical focusing:

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{QeB}{m} \qquad B \Rightarrow \langle B \rangle_{\mu-\text{dist}}$$

With an electric quadrupole field for vertical focusing

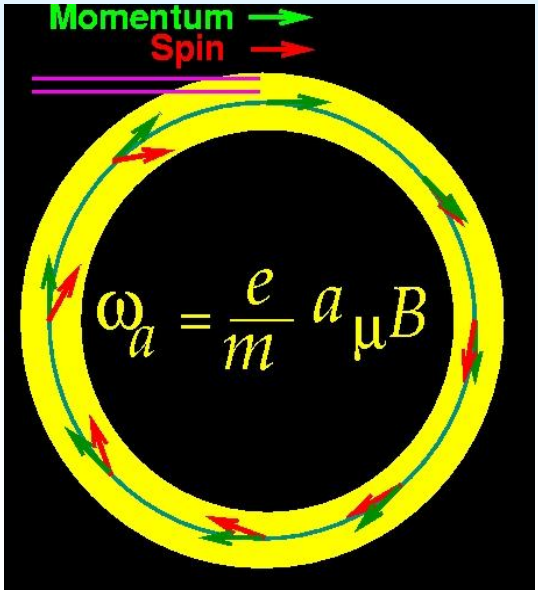
without vertical focussing

$$\vec{\omega}_a = -\frac{Qe}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] \rightarrow 0$$



$$\gamma_{\text{magic}} = 29.3$$

$$p_{\text{magic}} = 3.09 \text{ GeV/c}$$



Experimental Determination of a_μ

Measure muon spin precession rate in a magnetic storage ring

In the muon rest frame $\frac{d\vec{s}}{dt'} = \vec{\mu} \times \vec{B}' + \vec{d} \times \vec{E}'$

where the magnetic and electric dipole moments are $\vec{\mu} = \frac{ge}{2m}\vec{s}$ and $\vec{d} = \frac{\eta e}{2mc}\vec{s}$

Transforming E, B, t to the lab frame, include Thomas precession, subtract the cyclotron frequency, we get

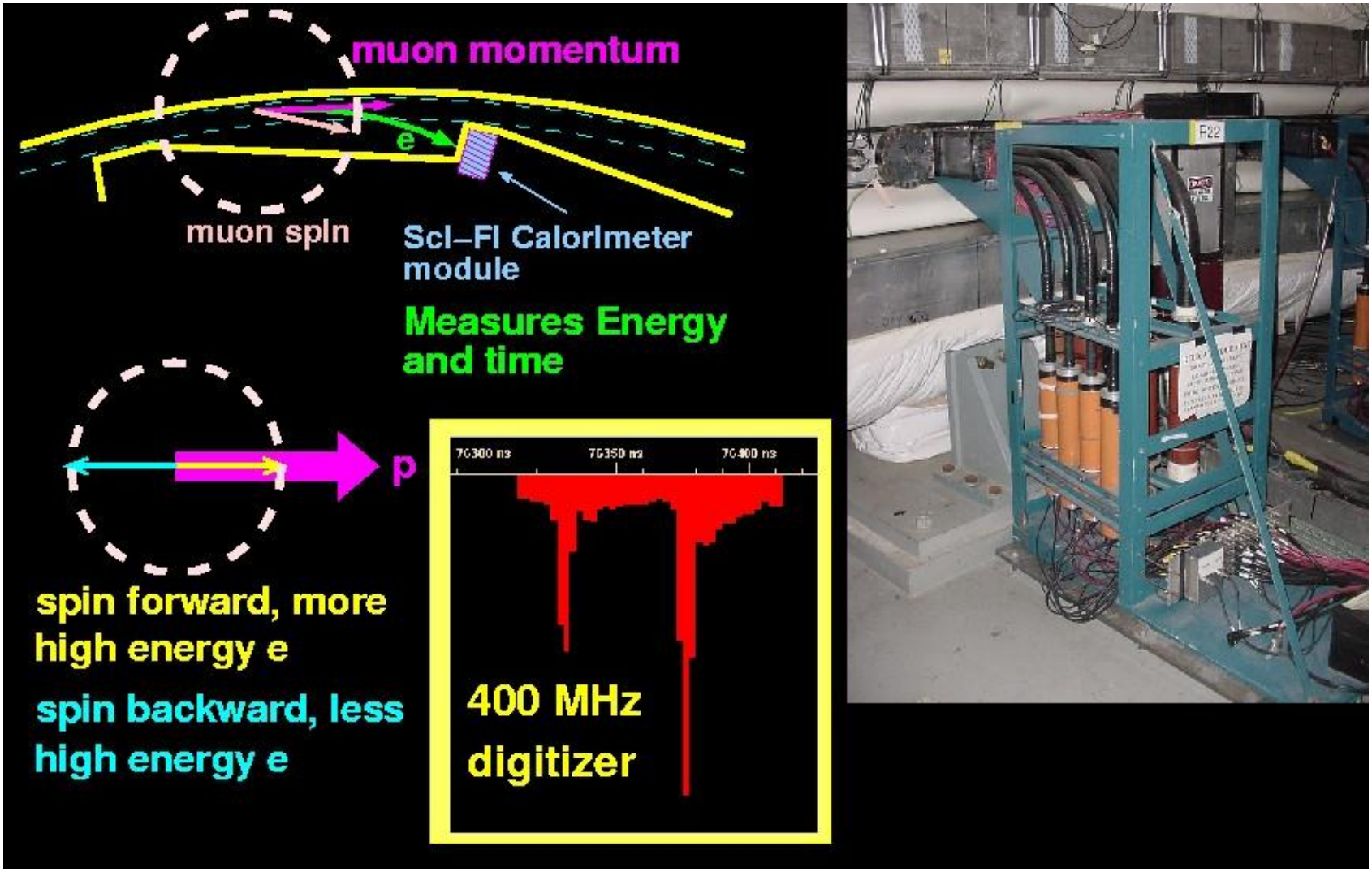
$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{m^2}{p^2}\right)\frac{\vec{\beta} \times \vec{E}}{c}} + \frac{\eta}{2}\left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c}\right)\right] \quad (\vec{\beta} \bullet \vec{B} \approx \vec{\beta} \bullet \vec{E} \approx 0)$$

In absence of E-field and with zero EDM $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m} a_\mu \vec{B}$

- Precession does not depend on momentum
- Uniform B highly desirable- all particles precess at same rate regardless of trajectory or momentum- but then no focusing to hold muons in storage ring
- Solution: focus with quadrupole E fields, choose ‘magic gamma’:

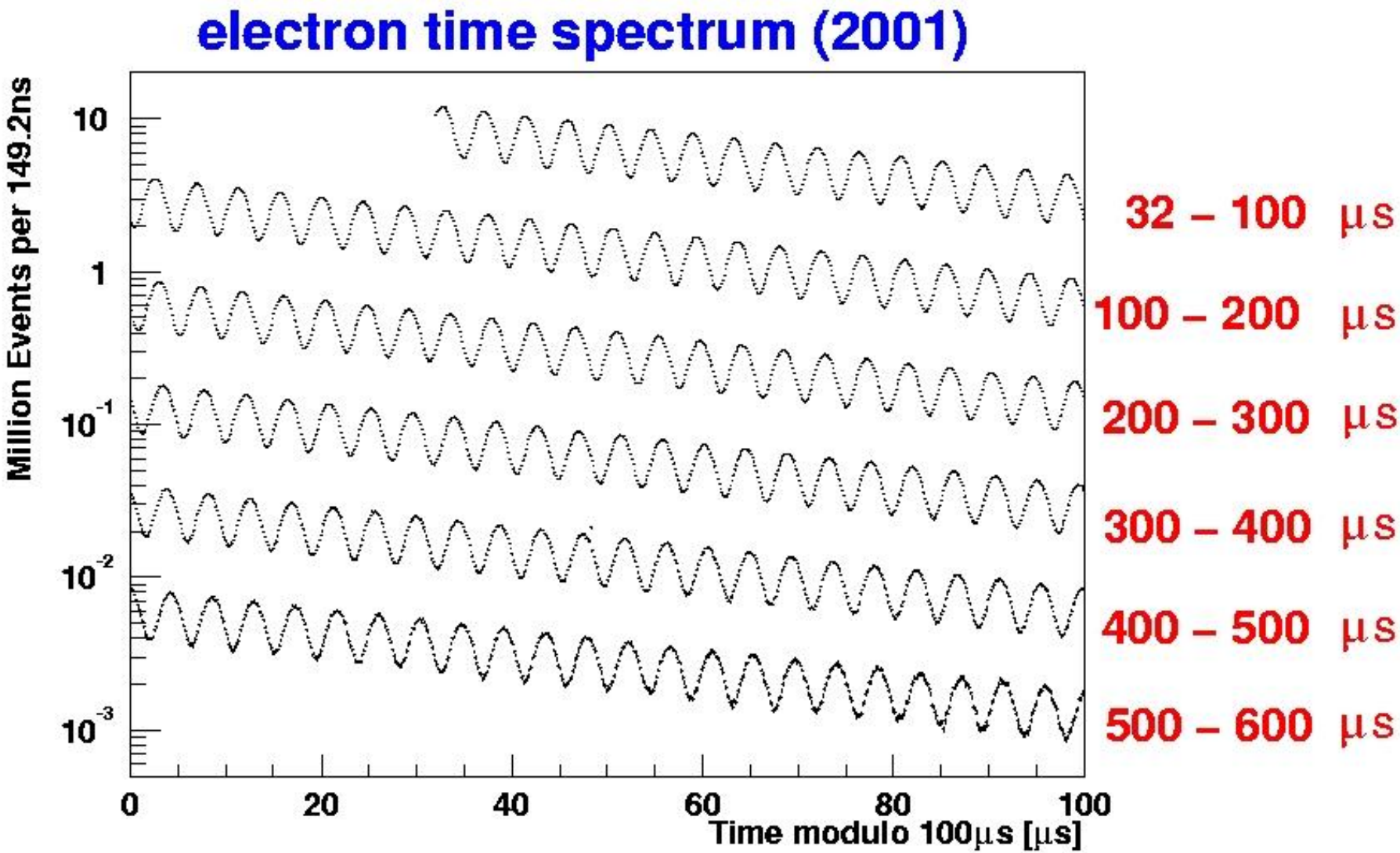
Choose p such that: $a_\mu - \frac{m^2}{p^2} = 0 \rightarrow$ p= $\frac{m}{\sqrt{a_\mu}}$ =3.096 GeV/c

Detectors and vacuum chamber



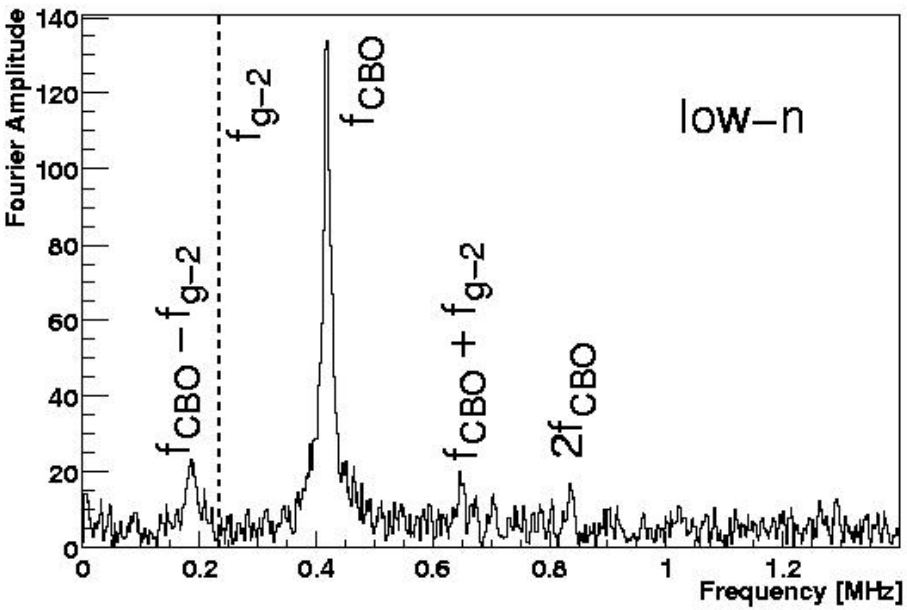
Detector acceptance depends on radial position of the μ when it decays.

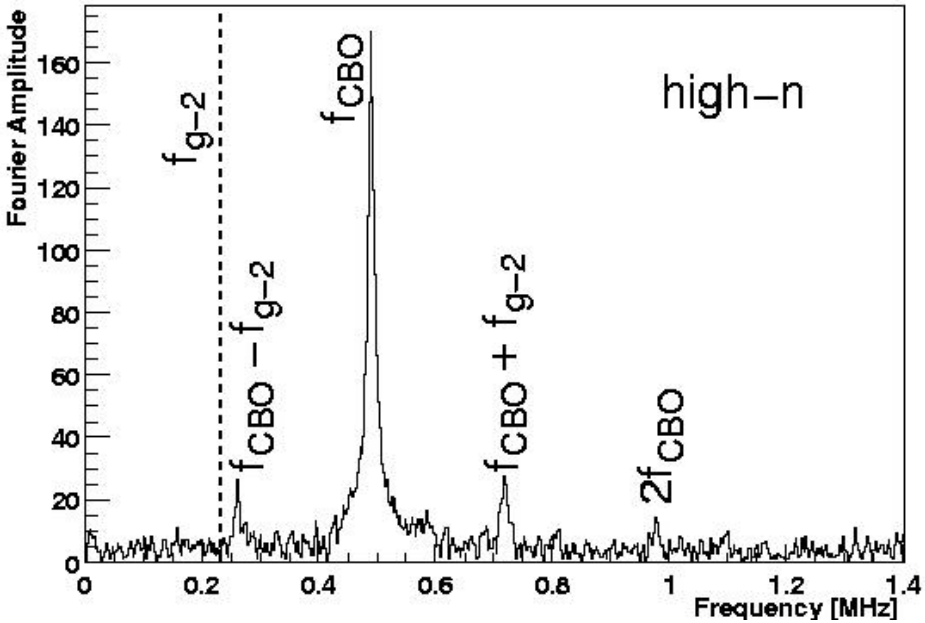
$4 \times 10^9 \quad e^-, \; E_{e^-} \geq 1.8$ GeV



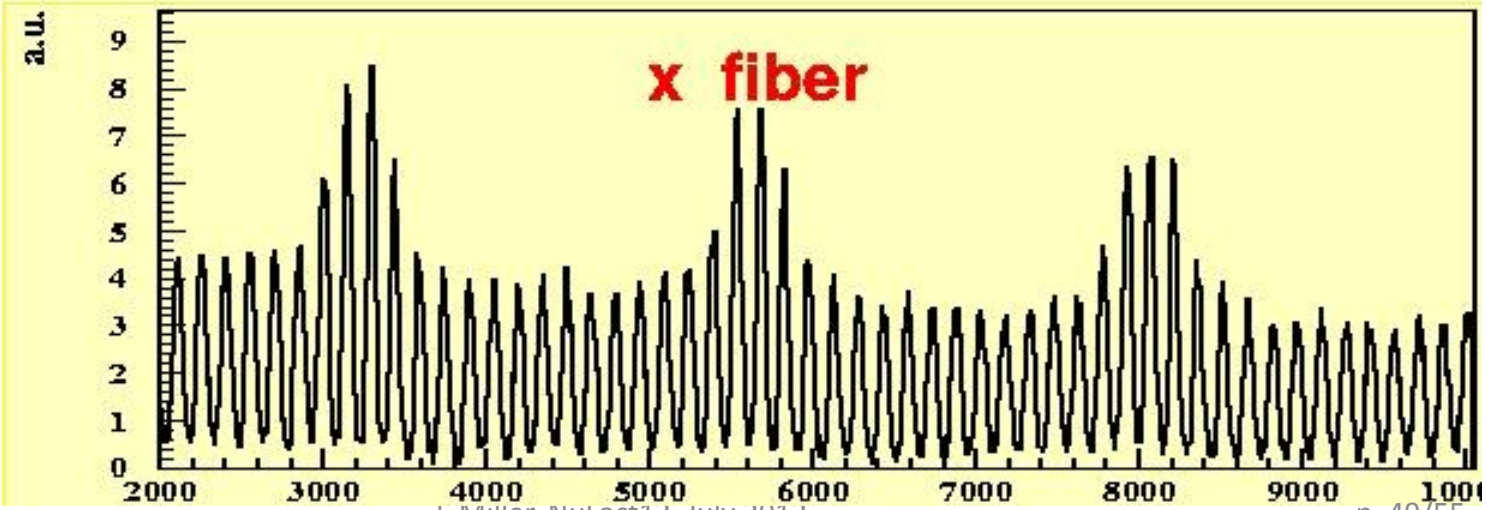
$$f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

Fourier Transform: residuals to 5-parameter fit

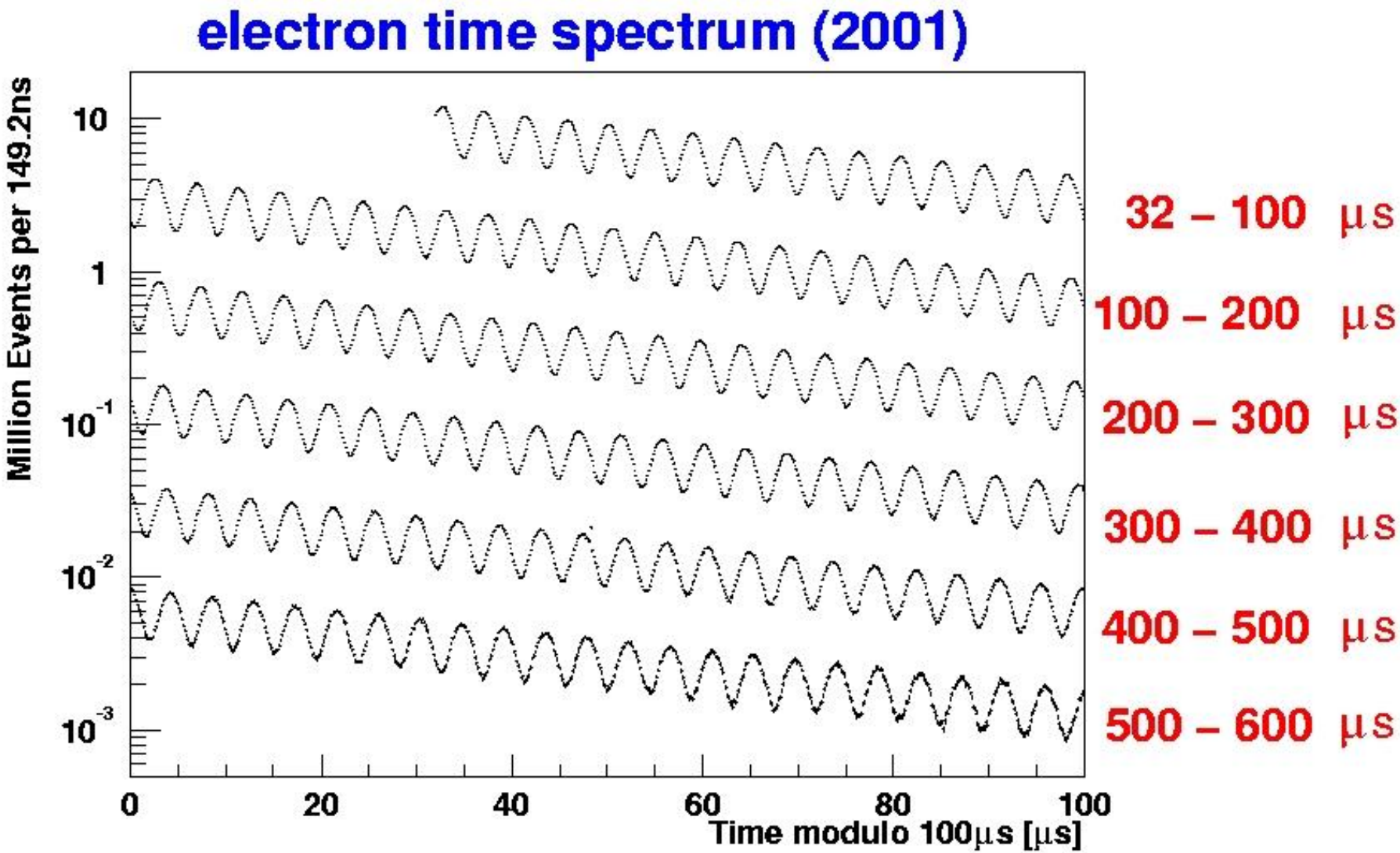




beam motion across a scintillating fiber – ~15 turn period



$4 \times 10^9 \quad e^-, \; E_{e^-} \geq 1.8$ GeV



$$f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

E821 ω_p systematic errors (ppm)

Source of Uncertainty	1998	1999	2000	2001	
Absolute Calibration	0.05	0.05	0.05	0.05	(i)
Calibration of Trolley	0.3	0.20	0.15	0.09	
Trolley Measurements of B0	0.1	0.10	0.10	0.05	(II)
Interpolation with the fixed probes	0.3	0.15	0.10	0.07	(III)
Inflector fringe field	0.2	0.20	-	-	
uncertainty from muon distribution	0.1	0.12	0.03	0.03	
Other*		0.15	0.10	0.10	(iv)
Total	0.5	0.4	0.24	0.17	

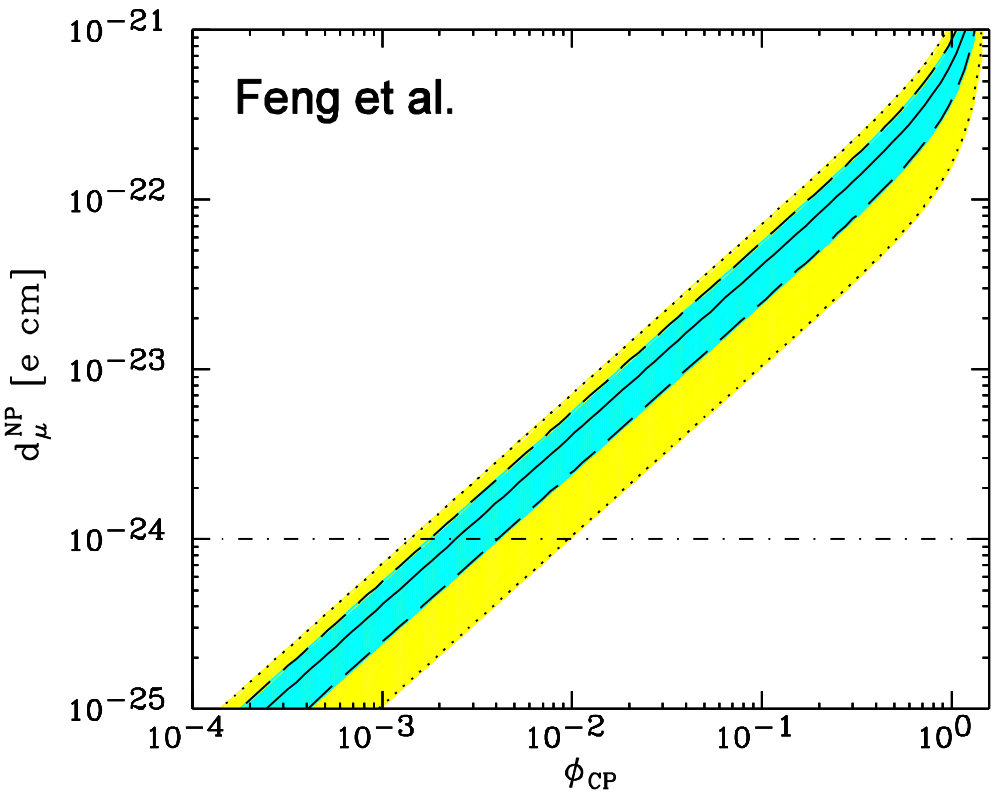
*higher multipoles, trolley voltage and temperature response, kicker eddy currents, and time-varying stray fields.

Systematic errors on ω_a (ppm)

$\sigma_{systematic}$	1999	2000	2001
Pile-up	0.13	0.13	0.08
AGS Background	0.10	0.10	*
Lost Muons	0.10	0.10	0.09
Timing Shifts	0.10	0.02	0.02
E-Field, Pitch	0.08	0.03	*
Fitting/Binning	0.07	0.06	*
CBO	0.05	0.21	0.07
Beam Debunching	0.04	0.04	*
Gain Change	0.02	0.13	0.13
total	0.3	0.31	0.21

$\Sigma^* = 0.11$

a_μ implications for the muon EDM



assuming that

$$a_\mu^{\rm NP} = 3(1) \times 10^{-9}$$

± 2 σ

± 1 σ

$$d_\mu^{\rm NP} \simeq 3 \times 10^{-22} \left(\frac{a_\mu^{\rm NP}}{3 \times 10^{-9}} \right) \tan \phi_{CP} \quad {\rm e} \cdot {\rm cm}$$

where ϕ_{CP} is a CP violating phase.

muon (g-2) storage ring at BNL→Fermilab

Electric and Magnetic Dipole Moments

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

Transformation properties:

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	$-$	$+$	$+$
C	$-$	$-$	$-$
T	$+$	$-$	$-$

An EDM implies both *P* <u>and</u> *T* are violated. Assuming CPT invariance, an EDM at a measureable level would imply non-standard model $\not{CP}$. The baryon/antibaryon asymmetry in the universe, needs new sources of $\not{CP}$.