Dipole Moments (Theory)

an overview with emphasis on EDMs

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Overview

Why are particle dipole moments special?

The SM treats *L*- and *R*- handed fermions *separately*, and their interactions are *distinct*.

The *anomalous* magnetic moment μ_a and electric dipole moment *d* are generated by chirality-changing operators, i.e.,
$\psi_R \sigma^{\mu\nu} \psi_L F_{\mu\nu}$
and
$\psi_R \sigma^{\mu\nu} \gamma_5 \psi_L F_{\mu\nu}$

An anomalous magnetic moment or a permanent EDM do not appear at tree level. They are *sensitive* to new (scalar!!) particles and interactions which can appear in loops — a distinct window on new physics.

If $d \neq 0$ then both parity *P* and time-reversal *T* symmetries are **broken**. We might think that $d = 0$, but this is not true even in the SM.

An Insight from the GDH Sum Rule

A particle of charge e, spin S, and mass M has $\mu = egS/2M$.
For any spin S, an unsubtracted dispersion relation for the forward spin-flip Compton amplitude with the low-energy theorem, yields

$$\mu_a^2 = \frac{4\pi\alpha S^2}{M^2}(g-2)^2 = \frac{S}{\pi}\int_{\nu_{\rm th}}^{\infty}\frac{d\nu}{\nu}[\sigma_P(\nu) - \sigma_A(\nu)]$$

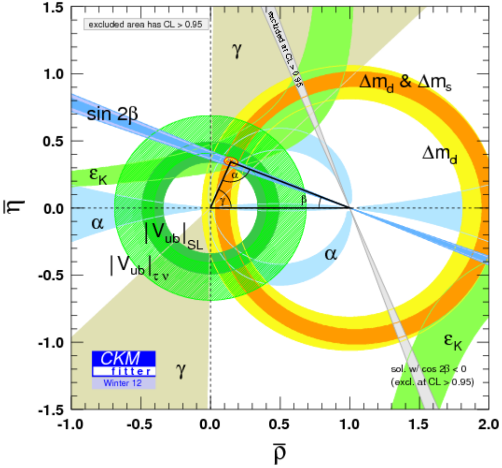
[Gerasimov, Yad. Fiz., 1965; Drell and Hearn, PRL, 1966]

$\sigma_{P(A)}$ is the photoabsorption cross section for the photon spin *parallel* or *antiparallel* to the target spin.
Expanding in powers of α...

- $g = 2$ at tree level for particles of any S.
  [Weinberg, 1970]
- Also yields the Schwinger result $(g-2)/2 = \alpha/2\pi$ in QED.
  [Altarelli, Cabibbo, Maiani, PLB, 1972]

$g - 2$ is clearly complementary to collider studies of new physics.
Why study EDMs?

Context: CP violation in 2012



Flavor and CP violation in flavor-changing processes is dominated by the CKM mechanism.

[CKMfitter: hep-ph/0104062, hep-ph/0406184 ; http://ckmfitter.in2p3.fr – Winter, 2012 update]

But the CKM mechanism cannot explain the Baryon asymmetry of the Universe....

What's Next?

We can

i) continue to test the **relationships** that a single CP-violating parameter entails to higher precision

– as well as –

ii) continue to make "null" tests.
Enter EDMs, as they are inaccessibly small in the CKM model.

QCD can possess a CP-violating parameter: $\bar{\theta}$. Namely,
$\mathcal{L}_{\rm CP} = \frac{\alpha_s \bar{\theta}}{8\pi} \epsilon^{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu}$
Experiment tells us $\bar{\theta} \ll 1$, but the SM offers no explanation – a strong CP "problem".
We will want to distinguish $\bar{\theta}$ from other sources of low-energy CP violation....

The electric dipole moment d and magnetic moment μ of a nonrelativistic particle with spin S is defined via $\mathcal{H} = -d\frac{\mathbf{S}}{S} \cdot \mathbf{E} - \mu\frac{\mathbf{S}}{S} \cdot \mathbf{B}$

This in itself suggests an experimental strategy.

Applied electric fields can be greatly enhanced within atoms and molecules...

[Purcell and Ramsey, 1950]

Assuming CPT invariance the relativistic generalization is:

$\mathcal{L} = -d\frac{i}{2}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu} - \mu_a\frac{1}{2}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$

Through the EDM d, a P-odd, T-odd observable, we probe new sources of the CP-violation.

On dimensional grounds, under SU(2)$_L \times$ U(1) gauge invariance, with $\sin\phi_{\rm CP} \sim 1$, [de Rujula et al., Nucl Phys B 357, 311 (1991)]

$d_d \sim 10^{-3} e \frac{m_d(\text{MeV})}{\Lambda(\text{TeV})^2} \sim 10^{-25}/\Lambda(\text{TeV})^2$ e-cm.

Λ is the scale CP is broken.

Thus $|d_n^{\rm expt}| < 2.9 \times 10^{-26}$ e-cm at 90%CL [Baker et al., PRL 97, 131801 (2006)]

implies that $\Lambda \sim 1$ TeV.

The n EDM probes TeV-scale physics (at least).

SU(2)$_L \times$U(1) gauge invariance brings an EDM probe *down* to the weak scale.

Comparing EDM limits

This simple scaling argument can be used broadly...

We can use it to crudely (!) compare EDM limits in different systems, though they probe *different physics*.

Note $(m_e/m_d) \sim 0.1$ and $(m_\mu/m_d) \sim 20$ to yield “neutron equivalent” limits of $3 \cdot 10^{-27}$ e-cm and $6 \cdot 10^{-25}$ e-cm, respectively, for the electron and muon.

cf. the experiment limits...

Tl atom: $|d_e| \leq 1.6 \cdot 10^{-27}$ e-cm [Regan et al., PRL 88, 071805 (2002)]

YbF: $|d_e| < 1.05 \cdot 10^{-27}$ e-cm [Hudson et al., Nature (2011)]

μ: $|d_\mu| < 1.9 \cdot 10^{-19}$ e-cm [Bennett et al., 2009]

The μ EDM is relatively poorly known. Interestingly if $a_\mu^{\rm NP} \sim 10^{-}9$, then one might “expect” $d_\mu^{\rm NP} \sim 10^{-}22$ e-cm. A limit of 10^{-24} e-cm with a dedicated experiment may prove possible.

EDMs in extensions of the SM

Many models of electroweak-symmetry breaking also give rise to substantial EDMs.

Models with weak-scale supersymmetry are a particular stand-out.

Such can resolve the hierarchy problem and are sufficiently weakly coupled to be consistent with precision electroweak constraints.

Moreover, SUSY models generically have many new sources of CP violation, and can produce a BAU in the electroweak phase transition more efficiently than in the SM.

This in itself is indirect support for supersymmetry.

However, the non-observation of additional *chirality-changing* effects constrain new scalar masses and mixings, both their Re and Im parts.

Note too recent CMS limits [1205.0272] on pair-produced new particles at 95% CL:

$m_{\tilde{g}} > 1098$ GeV $\qquad m_{\tilde{t}} > 737$ GeV $\qquad m_{\tilde{\tau}} > 223$ GeV

In the absence of signals from the LHC, EDMs can give crucial insight.

The SUSY CP Problem

The MSSM generically has many additional sources of CP violation because all the soft breaking terms can be complex.
Many constraints come from the non-observation of flavor-changing CP-violating effects beyond those of the SM:

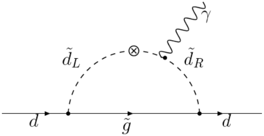
in K's:
$\Gamma(K_L \to 2\pi)$ (ϵ_K) and from the pattern of isospin-breaking in $\Gamma(K_L, K_S \to \pi^+\pi^-, \pi^0\pi^0)$ (ϵ').

in B's:
$A_{CP}(b \to s\gamma)$, $\Gamma(B, \bar{B}(t)) \to \psi K_S, \ldots$,

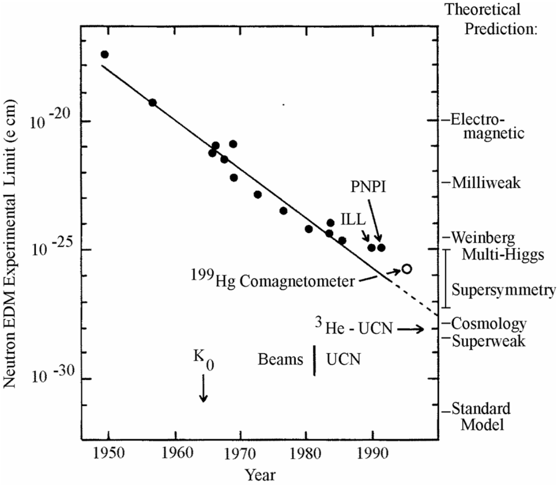
We can study flavor-conserving, CP-violating processes also.
$\Longrightarrow$ Enter the EDM of the neutron, electron,....
In the case of the μ, this is the "complex", i.e., CP-violating, analogue of the study of $\kappa_\mu \equiv (g-2)_\mu$. We can compute $(g-2)_\mu$ in the SM.
Such studies are highly complementary.

[Graesser and Thomas, 2001; Feng, Matchev, Shadmi, 2001]

The leading contribution to the neutron EDM in the MSSM:



Neutron EDM Timeline – The "Model Killer"



[S. Lamoreaux (Yale)]

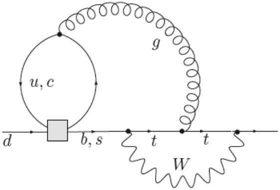
EDMs in the CKM Model

Why are they so small?

The structure of the CKM matrix guarantees that d_q is zero at two-loop order.

[Shabilin, Sov. J. Nucl. Phys. 28, 75 (1978)]

The first non-trivial contributions come at 3 loops, the largest involving a gluon [Khriplovich, PLB 173, 193 (1986)]



In leading logarithmic order at three loops $d_d^{\mathrm{CKM}} \simeq 10^{-34}$ e-cm.

[Czarnecki and Krause, PRL 78, 4339 (1997)]

d_e for *massless neutrinos* first appears at 4-loop order:
$d_e^{\mathrm{CKM}} \leq 10^{-38}$ e-cm

[Khriplovich and Pospelov, Sov. J. Nucl. Phys. 53 (1991) 638.]

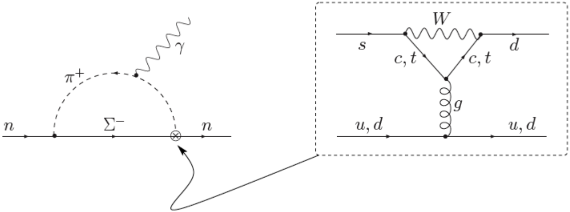
N.B. ν mixing gives enhancement

[Ng and Ng, 1996]

There is a huge mismatch between the SM predictions and the current experimental reach; we will explore ways to fill the gap!

EDM Enhancements in the CKM Model

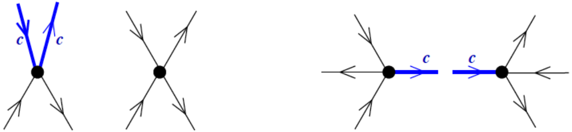
Chiral dynamics can give an enhancement



Namely, $d_n^{\mathrm{CKM}} \simeq 10^{-32}$ e-cm.

[Gavela et al., PLB 109, 215 (1982); Khriplovich and Zhitnitsky, PLB 109, 490 (1982).]

Bound-state effects involving charm may also yield an enhancement:



a b

Namely, $d_n^{\mathrm{CKM}} \simeq 10^{-31}$ e-cm. [Mannel and Uraltsev, 1202.6270]

The Effective CP-Violating Lagrangian at $\Lambda \sim 1$ GeV

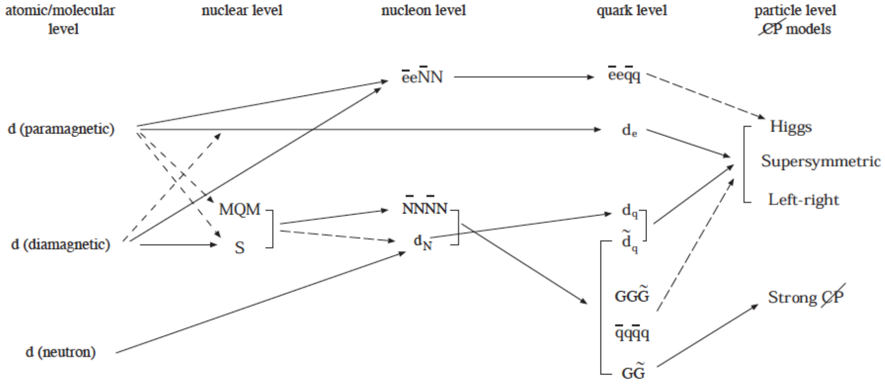
We now consider sources of CP violation beyond the SM. Thinking broadly and systematically...

$$
\begin{aligned}
\mathcal{L}_\Lambda &= \frac{\alpha_s \bar{\theta}}{8\pi} \epsilon^{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu} \\
&\quad - \frac{i}{2} \sum_i d_i \bar{\psi}_i F_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \psi_i - \frac{i}{2} \sum_i \tilde{d}_i \bar{\psi}_i F^a_{\mu\nu} t^a \sigma^{\mu\nu} \gamma_5 \psi_i \\
&\quad + \frac{1}{3} w f^{abc} F^a_{\mu\nu} \epsilon^{\nu\beta\rho\delta} F^b_{\rho\delta} F_\beta^{\ \mu,c} + \sum_{i,j} C_{ij} (\bar{\psi}_i \psi_i)(\bar{\psi}_j i \gamma_5 \psi_j) + \ldots
\end{aligned}
$$

with $i, j \in u, d, s, e, \mu$. [Pospelov and Ritz]
We neglect terms higher than dimension 6.
Models with new scalar degrees of freedom (MSSM!) generate d_i and $\tilde{d}_i$, e.g.
The EDMs of different systems can be helpful in disentangling the various sources.

EDMs of Complex Systems

There is a hierarchy of scales to consider:



[Ginges and Flambaum, 2004]

EDMs in neutrons, nuclei, atoms, and molecules are broadly complementary.

Atomic Scale Enhancements of EDMs

Limits on the electron EDM d_e come from paramagnetic ($J_e \neq 0$) and diamagnetic (e.g. $J_e = 0$) atoms — and molecules.

These systems can evince various long-distance enhancements.

Schiff theorem (1963):

in the nonrelativistic limit a neutral, point-like atom will shield an applied electric field, so that there is no atomic EDM even if $d_e \neq 0$.

Schiff's theorem can be strongly violated by relativistic and finite-size effects. Thus the "effective" electric field is much enhanced and better EDM limits emerge.

For paramagnetic atoms, relativistic effects rule. For alkali atoms

$$d_{\rm para}(d_e) \sim 10 \frac{Z^3\alpha^2}{J(J+1/2)(J+1)^2} d_e$$

[Flambaum, 1976; Sandars, 1965]

Such effects can be larger still for polar molecules [e.g., YbF].

Studies in paramagnetic atoms/molecules are most sensitive to d_e.

Atomic/Nuclear Scale Enhancements of EDMs

[Ginges and Flambaum, 2004]

Enhancement factors of the electron EDM for atoms of interest

	Atom	Enhancement factor $K = d_{\mathrm{atom}}/d_e$ Semi-empirical	Ab initio
Paramagnetic	Rb	24[a], 24.6[b], 16.1[c], 23.7[c], 22.0[c]	24.6[c]
	Cs	119[a], 131[d], 138[b], 138[e], 80.3[c], 106.0[c], 100.4[c]	114.9[c], 114[f], 115[g]
	Fr	1150[a]	910(50)[h]
	Tl	−716[i], −500[e], −502[c], −607[c], −562[c]	−1041[c], −301[j], −179[f], −585[k]
	Xe $^3P_2$	130[l], 120[e]	
Diamagnetic	Xe		−0.0008[m], −0.0008[g]
	Hg		−0.014[m], 0.0116[g]

Atomic EDMs can also be induced by P-odd, T-odd nuclear moments – here finite-size effects rule

Ground state EDMs of diamagnetic atoms induced by nuclear Schiff moments. Units: $10^{-17}(S/(e\ \mathrm{fm}^3))e$ cm

^{129}Xe	^{223}Rn	^{199}Hg	^{225}Ra
0.27[a], 0.38[b]	2.0[c], 3.3[b]	−4.0[d], −2.8[b]	−7.0[c], −8.5[b]

[a] Ref. [267]. Relativistic Hartree–Fock calculation.
[b] Ref. [295]. Average of two ab initio many-body calculations; core polarization and correlation corrections included.
[c] Ref. [284]. Estimate found by scaling (with Z) calculations for lighter analogous atoms. Radon result scaled from xenon calculation [267]; radium result scaled from mercury calculation [290,289].
[d] Ref. [290,289]. Estimated from the calculation of $d_{\mathrm{atom}}(C^T)$ for mercury performed in Ref. [266].

Nuclear deformation and atomic state mixing can also play a role [Ra, Rn].
Different enhancement mechanisms can operate simultaneously.

To interpret an EDM limit in ^{199}Hg, say, one must not only compute *d*(atom) in terms of *S*, the Schiff moment, but also compute the Schiff moment in terms of T-odd, P-odd πNN coupling – and connect the latter to fundamental sources of CP violation!

When all is said and done, the uncertainty is dominated by our uncertainty in the effective interaction. Our preferred interaction is SkO′, which (to repeat) gives the result

$$S^{\mathrm{SkO}'}_{^{199}\mathrm{Hg}} = 0.010 g\bar{g}_0 + 0.074 g\bar{g}_1 + 0.018 g\bar{g}_2 \, [e\,\mathrm{fm}^3]. \quad (14)$$

If instead we average the results from the five interactions, we get

$$S^{\mathrm{ave}}_{^{199}\mathrm{Hg}} = 0.007 g\bar{g}_0 + 0.071 g\bar{g}_1 + 0.018 g\bar{g}_2 \, [e\,\mathrm{fm}^3]. \quad (15)$$

The range of results in Table III is a measure of the uncertainty.

As noted in the introduction, Refs. [8,9] contain a similar calculation. They report

$$S^{\mathrm{Ref.[8]}}_{^{199}\mathrm{Hg}} = 0.0004 g\bar{g}_0 + 0.055 g\bar{g}_1 + 0.009 g\bar{g}_2 \, [e\,\mathrm{fm}^3], \quad (16)$$

[de Jesus and Engel, 2005]

What is the road *from* discovery? How do we interpret what we have found?

EDMs of the neutron, proton, and simple nuclei?

QCD matrix elements

How well can we interpret an EDM limit?

Let compare matrix element calculations for $\bar{\theta}$-QCD:
chiral: $d_n(\bar{\theta}) = 5.2 \cdot 10^{-16}\bar{\theta}$ e-cm [Crewther et al., 1979]
QCD sum rules: [Pospelov and Ritz, PRL 1999]

$$d_n(\bar{\theta}) = (1 \pm 0.5)\frac{\langle \bar{q}q \rangle}{(225\,\mathrm{MeV})^3} 2.5 \cdot 10^{-16}\bar{\theta}\ \text{e-cm}$$

They are crudely comparable, but... [Narison, PL B666, 455 (2008)]

$D_N|_{\rm exp} \leq 6.3 \times 10^{-26}$ cm ,

one can deduce in units of 10^{-10}:

$$\begin{aligned} \theta &\leq (1.6 \pm 0.4)\ [\text{Chiral}]\ :\ \nu = M_N \\ &\leq (1.3 \sim 11.7)\ [\text{ChPT}]\ :\ M_N/3 \leq \nu \leq M_N \\ &\leq (6.9 \pm 3.5)\ \ [\text{Constituent quark}] \\ &\leq (14.9 \pm 4.9)\ \ [\text{QSSR}]\ . \end{aligned}$$

cf. claimed 50% error in QSR method for CP-violating ops. w/ dimension ≤ 5
[Pospelov & Ritz, PRL 1999]

N.B. Test germane nucleon matrix element computations by confronting the empirical anomalous moments [Brodsky, SG, Hwang, 2006]

Summary and Outlook

Dipole Moments provide a unique window on new physics and are particularly sensitive to new scalar degrees of freedom.

EDMs are powerful probes of new sources of CP violation.

EDM studies in a range of systems are complementary.

Various enhancement factors can act in the EDMs of atomic nuclei (and can yield d_e as well!); these experiments could greatly benefit from a Project X.

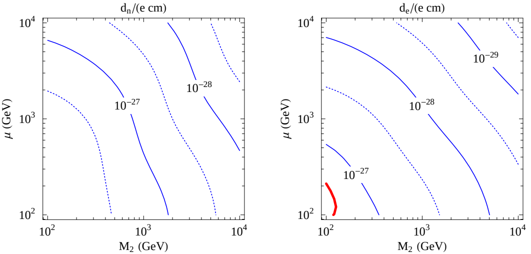
Improving EDM limits give us key information on the energy scale of new physics.

Backup Slides

Electric Dipole Moments in Split Supersymmetry

Can resolve SUSY CP problem by making superpartners heavy or CP phases small....

Models with "split" supersymmetry (heavy scalars!) can still produce significant EDMs at two-loop order: [Chang, Chang, Keung, 2005; Giudice and Romanino, 2006]



n and "e" EDMs are complementary! [see also Pospelov and Ritz]

Both d_e and d_n are expected to improve.

$|d_n| \leq 2.9 \cdot 10^{-26}$ e-cm (90% CL) [Baker et al., PRL 97, 131801 (2006)]

$|d_e| \leq 1.6 \cdot 10^{-27}$ e-cm [Regan et al., PRL 88, 071805 (2002)]

Some supersymmetric models (from "M Theory") realize CP violation only in the quark and lepton Yukawas $\Longrightarrow$ EDMs are SM-like [Kane, Kumar, Shao, arXiv:0905.2986]

Other Possibilities

Suppose the weak scale is simply fine-tuned and/or that no future LHC discoveries are compatible with supersymmetry.

Are EDM measurements still important? Yes!

	B hints	BAU	EDM
extra generation (SM)	yes	yes	yes
other (walking technicolor; asymmetric dark matter)	?	?, yes	?
supersymmetry	-	yes	yes

Note anomaly cancellation demands that SU(2) doublets appear in twos.

[Witten, 1982]

Some models will also have interesting magnetic moment signals.

If the fourth generation of quarks are sufficiently massive, they can **significantly enhance** the produced BAU as well as the expected EDM using SM mechanisms.

Precision electroweak measurements tolerate this addition.

[Erler and Langacker, 2010 (1003.3211)]

Heavy quarks dramatically enhance the Jarlskog invariant.

[Hou, 2008 (0810.3396v2)]

In the SM

$$J = (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)A$$

where $A \simeq 3 \times 10^{-5}$. With $(t', b') \sim 300\,\mathrm{GeV}$ we have in effect

$$J = (m_{t'}^2 - m_u^2)(m_{t'}^2 - m_c^2)(m_t^2 - m_c^2)(m_{b'}^2 - m_s^2)(m_{b'}^2 - m_s^2)(m_b^2 - m_s^2)A_{234}^{sb}$$

where $A_{234}^{sb}/A \sim 30$ (if B hints hold up) and the mass factors yield an enhancement of 10^{13}.

This may well suffice to make the SM mechanism of BAU viable.

This scenario has also been explored for EDMs. Here 3-loop electroweak effects dominate. If $m_{t'} \sim 500 - 600\,\mathrm{GeV}$, then $d_d \sim 10^{-29}$ e-com. [Hamzaoui and Pospelov, 1995 (hep-ph/9508222v1)]