Search for the Origin of Duality

Nathan Isgur, Sabine Jeschonnek, Wally Melnitchouk, Wally Van Orden

Jefferson Lab

Goal: understand the **qualitative origin** of duality

- The Model
- Duality in the Excitation Form Factors
- Duality in the Structure Function

Work in progress - all results are preliminary!

The Model (I)

Properties of the Model:

- confinement
- relativity
- valence quark model
- no spin

choose relativistic harmonic oscillator potential

The Model (II)

1st step: Simplifications

- replace qq by $\bar{q}$, as $3 \otimes 3 = \bar{3} \oplus 6$:

  "meson" instead of baryon target

- take $m_{\bar{q}} \rightarrow \infty$

  Bethe-Salpeter equation reduces to a Klein-Gordon equation

Wave Function & Energy Eigenvalues (I)

Klein-Gordon equation:

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + m^2 + V^2\right)\Phi(x) = 0$$

and $\Phi(x) = \Phi(\vec{r})\exp(iEt)$

confining scalar potential:
$V^2(\vec{r}) = \alpha\, r^2, [\alpha] = [m^4]$

$$\Rightarrow (-\vec{\nabla}^2 + \alpha\, r^2 - E^2 + m^2)\Phi(\vec{r}) = 0$$

compare to the Schrödinger equation for the harmonic oscillator:

$$\left(-\frac{\vec{\nabla}^2}{2m} + \frac{1}{2}\kappa\, r^2 - E\right)\Psi(\vec{r}) = 0$$

with the solution:

$E_N = \sqrt{\frac{\kappa}{m}}\,(N + \frac{3}{2})$ equidistant spacing of states!

$$\Psi(x) = \frac{\sqrt{\beta}}{\sqrt{2^{n_x} n_x! \sqrt{\pi}}} H_{n_x}(\beta x)\exp\left(-\frac{1}{2}x^2\beta^2\right)$$

with $\beta = (\kappa m)^{1/4}$.

Wave Function & Energy Eigenvalues (II)

Klein-Gordon equation:

$$\Rightarrow \left(-\frac{\vec{\nabla}^2}{2m} + \frac{1}{2}\underbrace{\frac{\alpha}{m}}_{\tilde{\kappa}} r^2 - \underbrace{\left(\frac{1}{2}\frac{E^2}{m} - \frac{1}{2}m\right)}_{\tilde{E}}\right)\Phi(\vec{r}) = 0$$

energy eigenvalues:

$$\tilde{E}_N = \sqrt{\frac{\tilde{\kappa}}{m}}\,(N + \tfrac{3}{2})$$

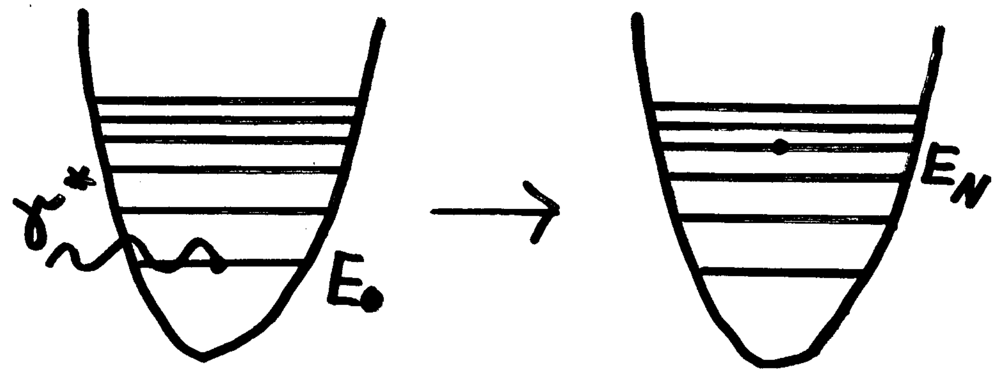
$$\Rightarrow \boxed{E_N = \sqrt{2\sqrt{\alpha}(N + \tfrac{3}{2}) + m^2}}$$

Relativity introduces a much higher density of states!

wave functions: the same as for the non-relativistic oscillator, with $\beta \leftrightarrow \alpha^{\frac{1}{4}}$.

choose parameters: m = 330 MeV, $\alpha^{\frac{1}{4}} = 400$ MeV

Exciting the Resonances



No decays yet!

We can calculate:

- Excitation Form Factors
- Structure Functions
- Cross Sections

Do we see local duality, global duality, scaling?

Questions: What is the correct scaling variable? Which quantity is supposed to scale?

Duality in the Form Factor

Free quark form factor: 1

Meson excitation form factor:

$$F_{0N}(\vec{q}) = \int d\vec{r} \exp(i\vec{q}\cdot\vec{r})\Phi_N^*(\vec{r})\Phi_0(\vec{r})$$

$$\Rightarrow |F_{0N}(\vec{q})|^2 = \frac{1}{N!}\left(\frac{\vec{q}^2}{2\beta^2}\right)^N \exp(-\frac{\vec{q}^2}{2\beta^2})$$

Sum over all possible hadronic excitations:

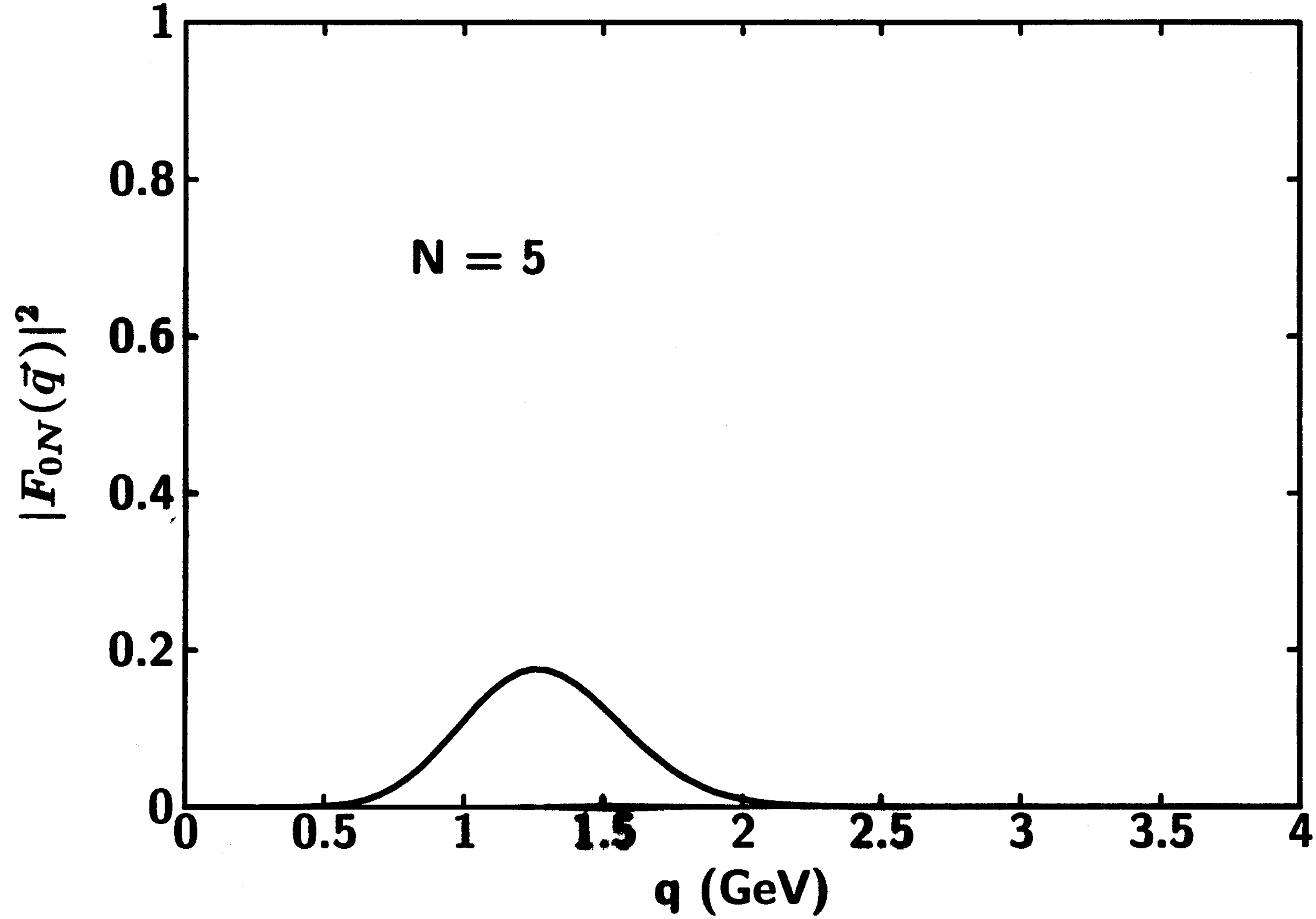
$$\sum_{N=0}^{N_{max}} |F_{0N}(\vec{q})|^2 = \exp(-\frac{\vec{q}^2}{2\beta^2}) \sum_{N=0}^{N_{max}} \frac{1}{N!}\left(\frac{\vec{q}^2}{2\beta^2}\right)^N$$

$$= 1 \text{ if } N_{max} \rightarrow \infty$$

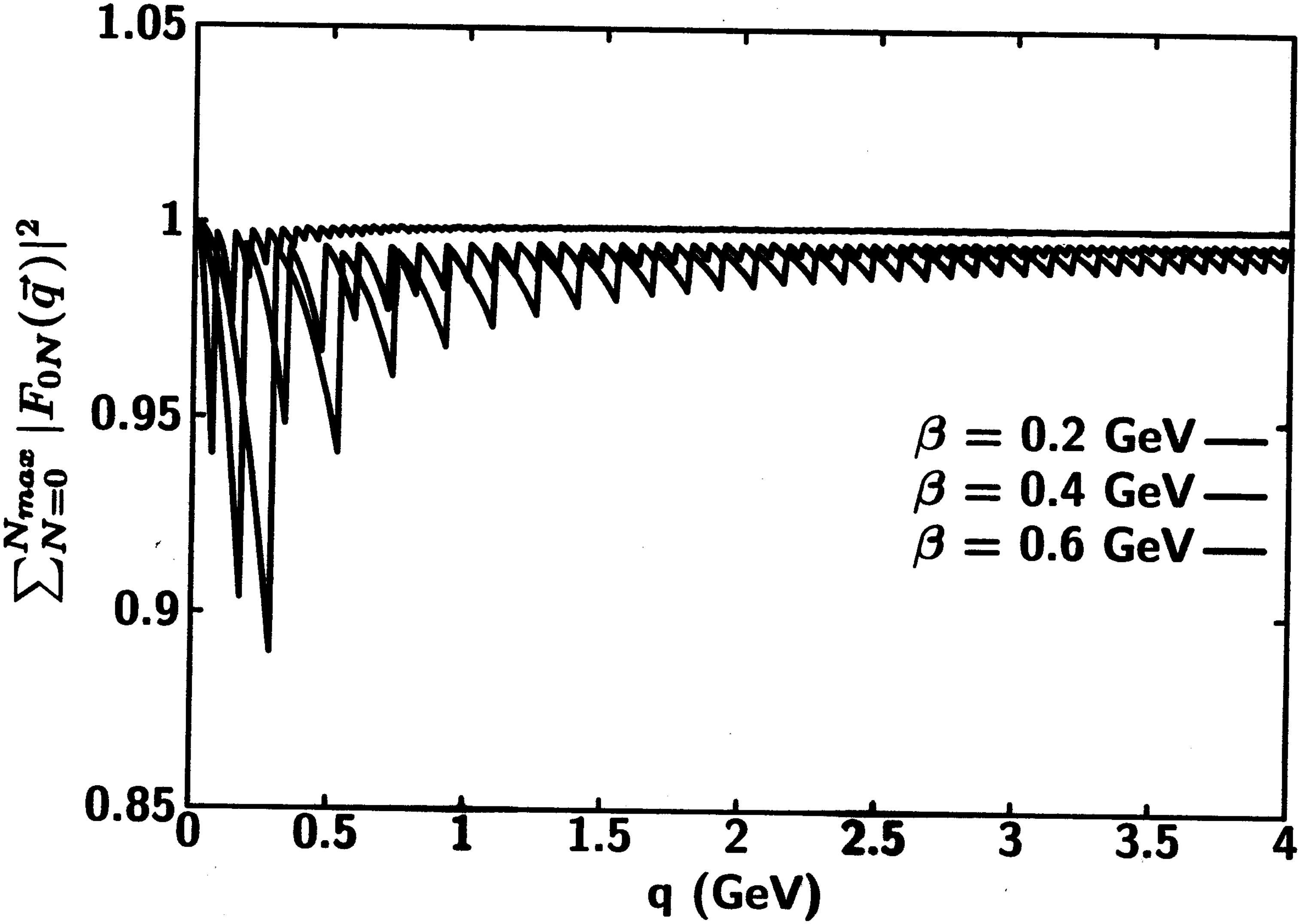
Kinematic constraint: space-like region:

$$|\vec{q}| > \nu = E_N - E_0 \Leftrightarrow Q^2 > 0$$

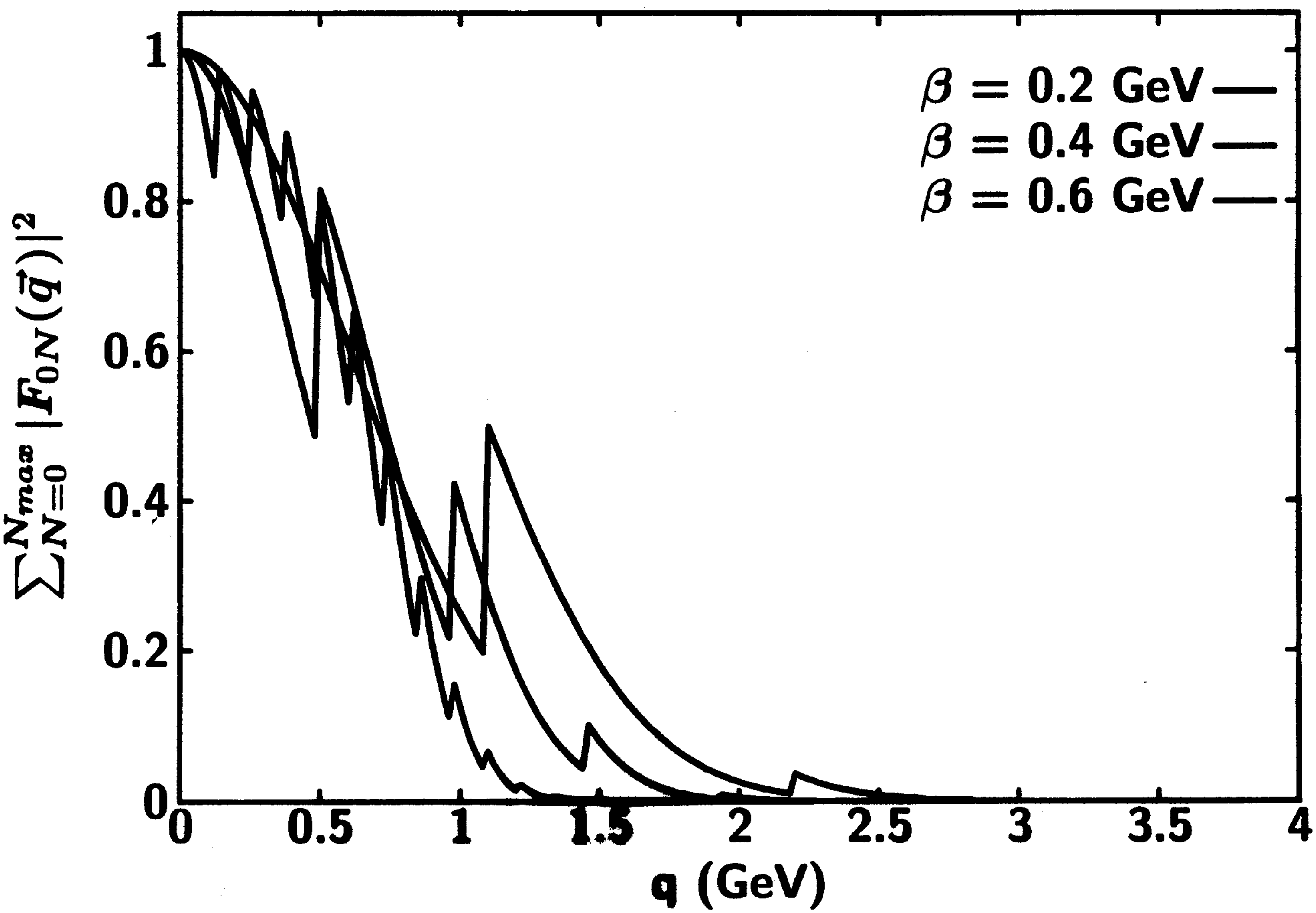
Form Factor for a Single Resonance



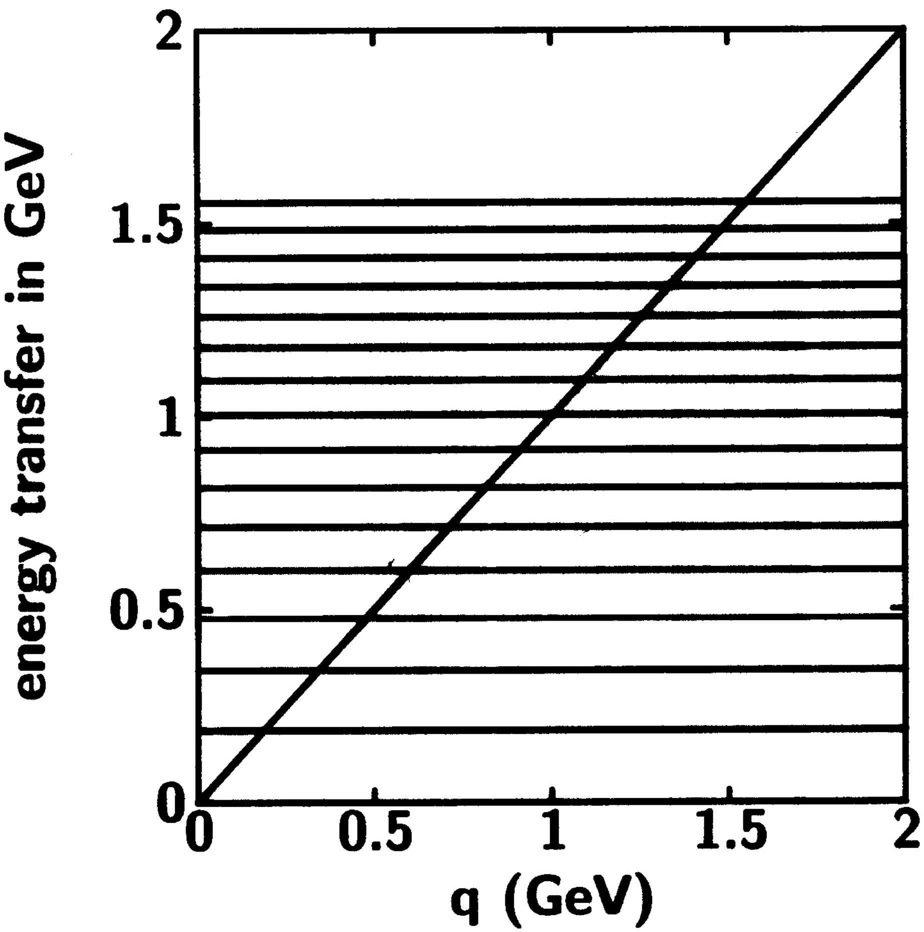
Duality in the Form Factor: Results

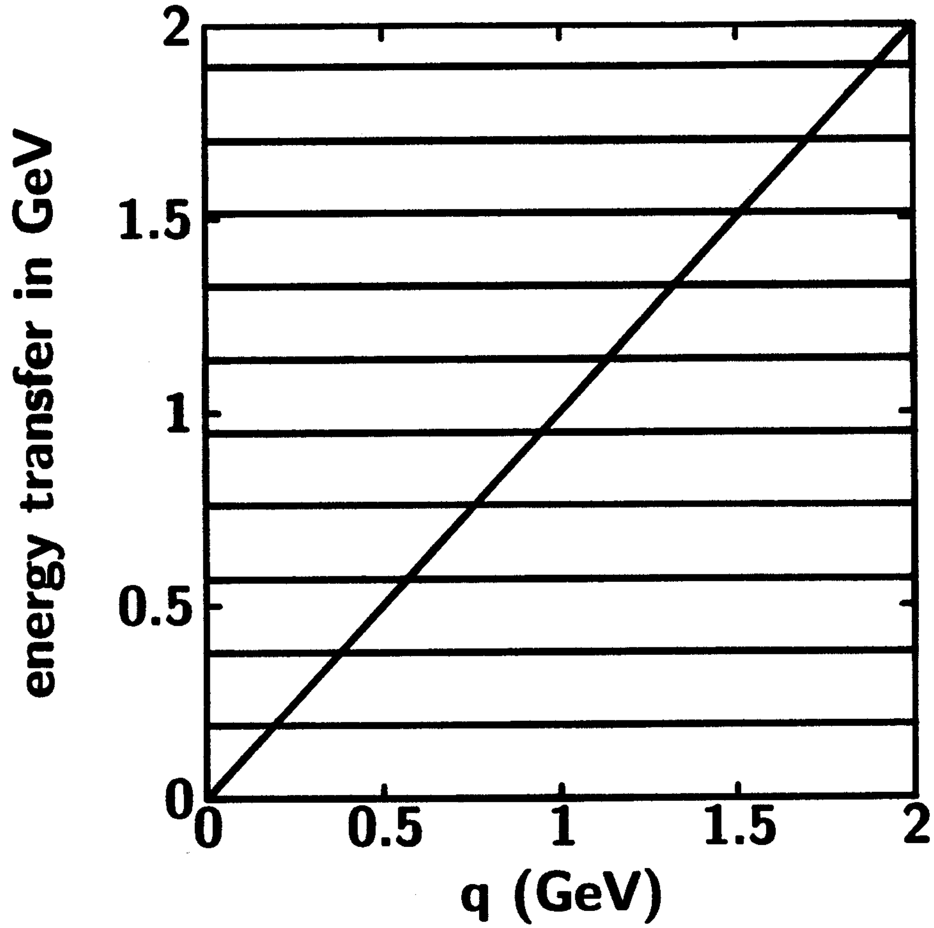


Duality in the Form Factor: Non-Relativistic



Energy Levels: Relativistic and Non-Relativistic





Note: only the first 15 levels are shown in the relativistic case.

What have we learned from the Form Factor Calculation?

In our model, the form factors for the quark process and the hadronic process are almost the same: duality is fulfilled at the 2 % level.

- duality in this case is closely related to completeness
- phase space is very important → relativistic description is necessary
- violation of duality is proportional to the strength of binding, i.e. β

Duality in the Structure Function

What we expect to see:

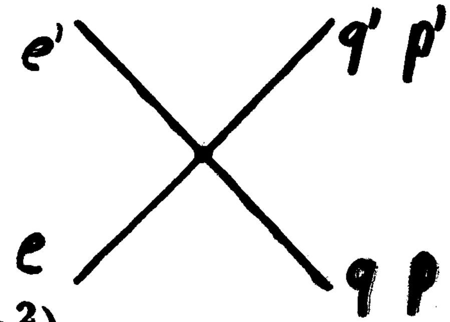
- Scaling for $Q^2 \rightarrow \infty$
- Local Duality: the resonances average to the scaling curve
- Global Duality: the moments will be constant if Q^2 is large enough

Questions: What is the correct scaling variable? Which quantity is supposed to scale?

Consider the all scalar case to answer these questions!

Scaling Variable & Scaling Function (I)

Scalar electron scatters from free scalar quark, pointlike interaction $g(e^\dagger e)(q^\dagger q)$:



structure function $W \propto g^2 \delta(p'^2 - m^2)$

IMF: $p = xP$, $P = (\sqrt{P^2 + M^2}, 0, 0, P)$, so:

$$p'^2 - m^2 = (p+q)^2 - m^2 =$$
$$x^2 M^2 - Q^2 + 2xM\nu - m^2$$

Bjorken limit: $p'^2 - m^2 \longrightarrow 2xM\nu - Q^2$

$$W_{Bj} = g^2\, \delta(2xM\nu - Q^2) = g^2 \frac{1}{2M\nu} \delta(x - \frac{Q^2}{2M\nu})$$

conclusion: $\boxed{\nu W_{Bj} \text{ scales in } x_{Bj}}$

Scaling Variable & Scaling Function (II)

General case - arbitrary value of Q^2: keep target mass M and quark mass m. Assume a free quark with a momentum distribution.

The appropriate scaling variable for this case is

$$x = \frac{1}{2M}\left(\sqrt{\nu^2 + Q^2} - \nu\right)\left(1 + \sqrt{1 + \frac{4m^2}{Q^2}}\right)$$

$$\boxed{\sqrt{\nu^2 + Q^2}\, W = |\vec{q}|\, W \text{ scales in } x}$$

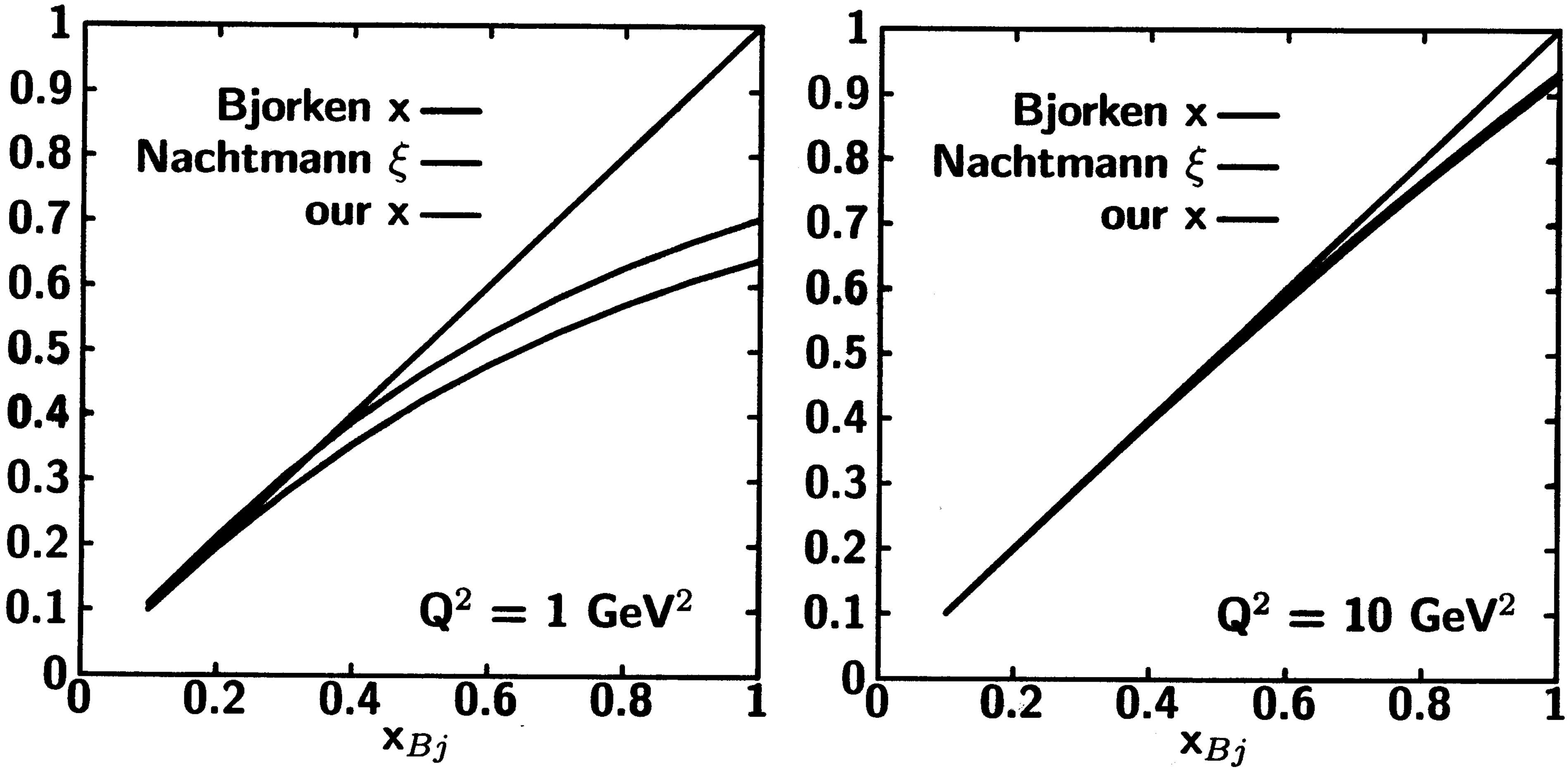
rescale x:

$$u = x\frac{M}{m} = \frac{1}{2m}\left(\sqrt{\nu^2 + Q^2} - \nu\right)\left(1 + \sqrt{1 + \frac{4m^2}{Q^2}}\right)$$

note: for a given Q^2, the maximum attainable value of u is:

$$u_{max} = \frac{1}{2m} Q \left(1 + \sqrt{1 + \frac{4m^2}{Q^2}}\right)$$

Different Scaling Variables



Scaling Variable & Scaling Function (III)

General case - arbitrary value of Q^2: Assume a bound quark. Keep target mass M, heavy quark mass μ and quark mass m. In our case, we take the limit of $M, \mu \to \infty$, $M - \mu = E_0$.

The appropriate scaling variable for this case is

$$x_b = \frac{E_o + \sqrt{Q^2 + \nu^2} - \sqrt{(E_0 + \nu)^2 - m^2}}{M}$$

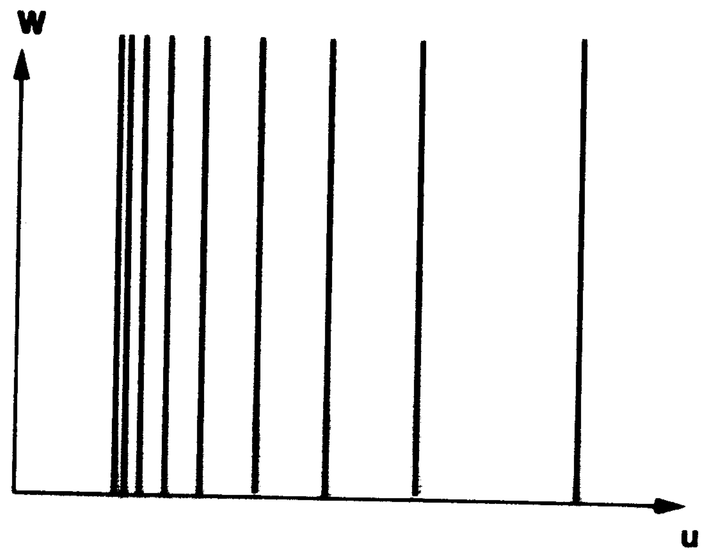
$\sqrt{\nu^2 + Q^2}\, W = |\vec{q}|\, W$ scales in x_b

rescale x: $u_b = x_b \frac{M}{m}$

note: for a given Q^2, the maximum attainable value of u_b is:

$$u_{b,max} = \frac{E_o + Q - \sqrt{E_0^2 - m^2}}{m}$$

Smoothing Procedures



- integrate over a small interval in energy transfer $\rightarrow$ histogram-type plot

- continuize the sum over N to an integral

- replace the δ-function by a Breit - Wigner

$$\delta(\nu - E_N + E_0) \rightarrow \frac{\Gamma}{2\pi} \frac{1}{(\nu - E_N + E_0)^2 + 0.25\Gamma^2}$$

The Scalar Structure Function

Cross section: $\frac{d\sigma}{d\Omega dE_f} = \frac{g^2}{(2\pi)^2} \frac{E_f}{4E_i} \frac{1}{Q^4} W_{scalar}$

Structure Function:

$$W^{h.o.}_{scalar}(\nu, Q^2) =$$

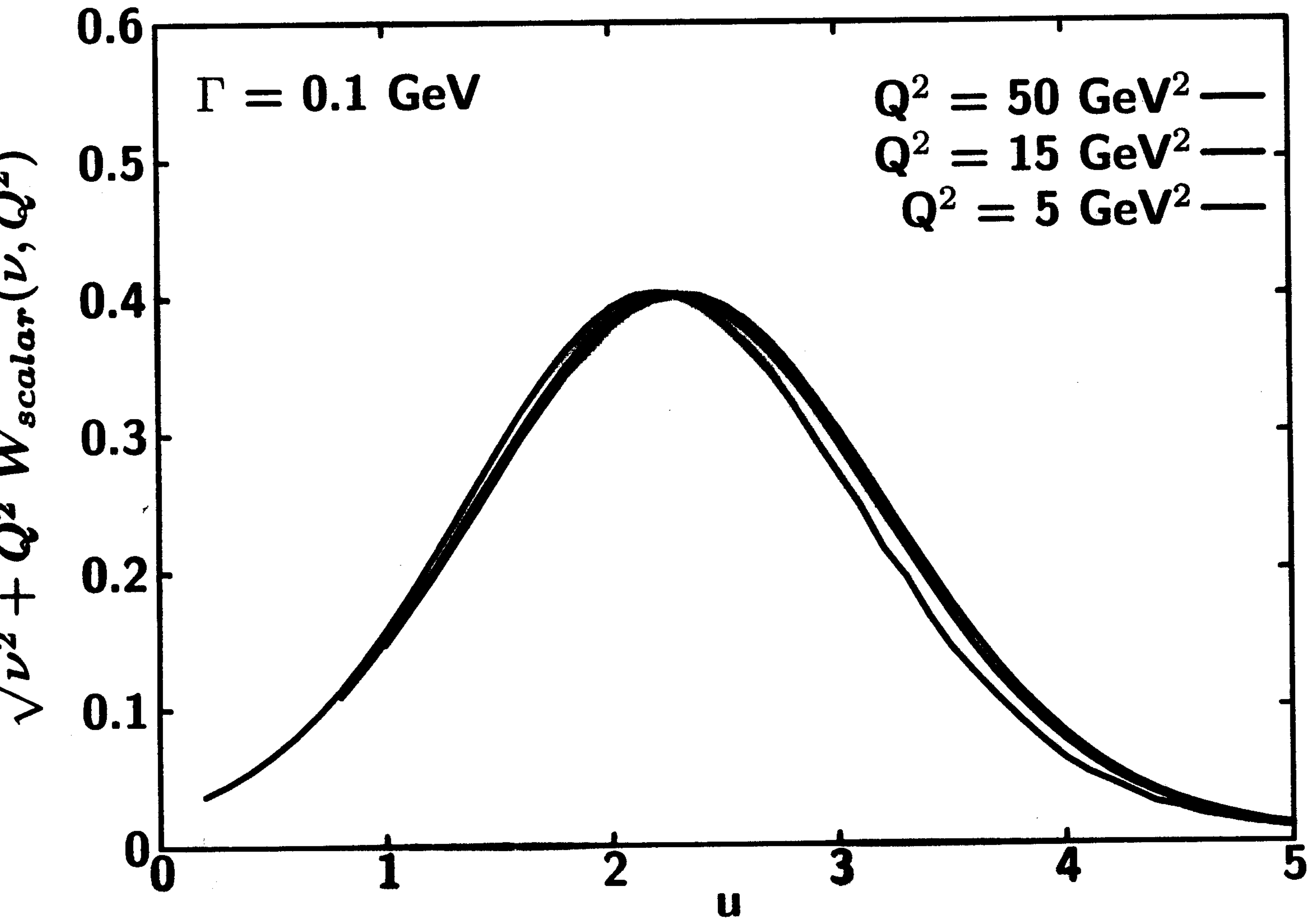
$$\sum_N \frac{1}{4E_0 E_N} \, |F_{0N}(\sqrt{Q^2 + \nu^2})|^2 \, \delta(\nu - E_N - E_0)$$

no decays yet:

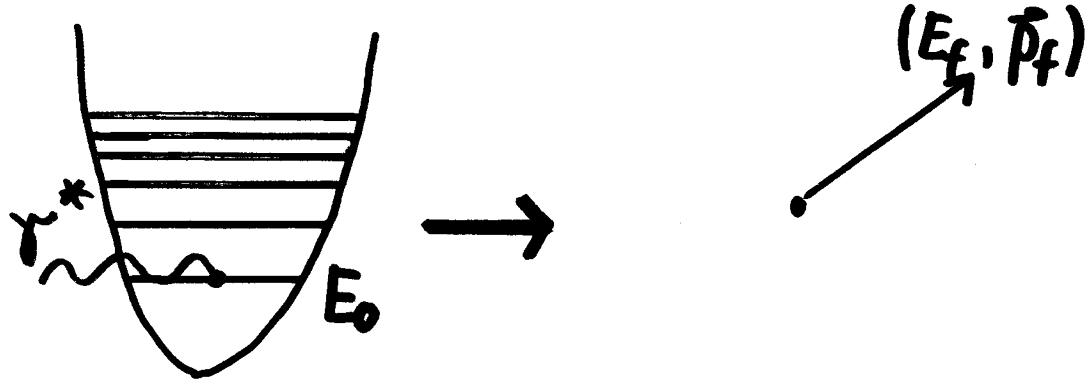
$$\nu = E_N - E_0 \Rightarrow \delta(\nu - E_N - E_0)$$

Need a smoothing procedure!

Scaling for the All Scalar Case



The Bound-Free Case



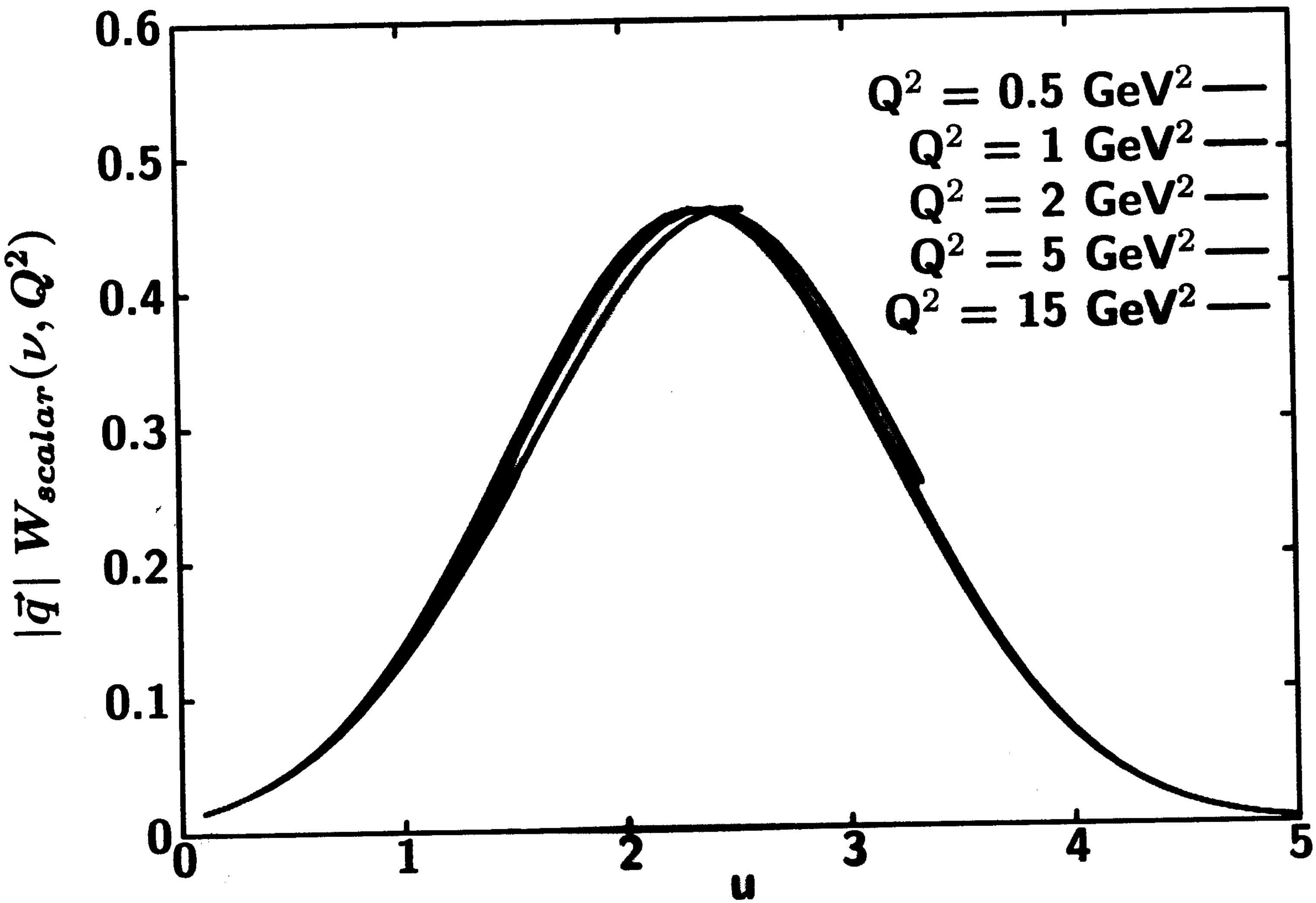
Structure Function:

$$W^{b.f.}_{scalar}(\nu, Q^2) = \frac{1}{4\sqrt{\pi}\, E_0\, \beta\, |\vec{q}\,|}.$$

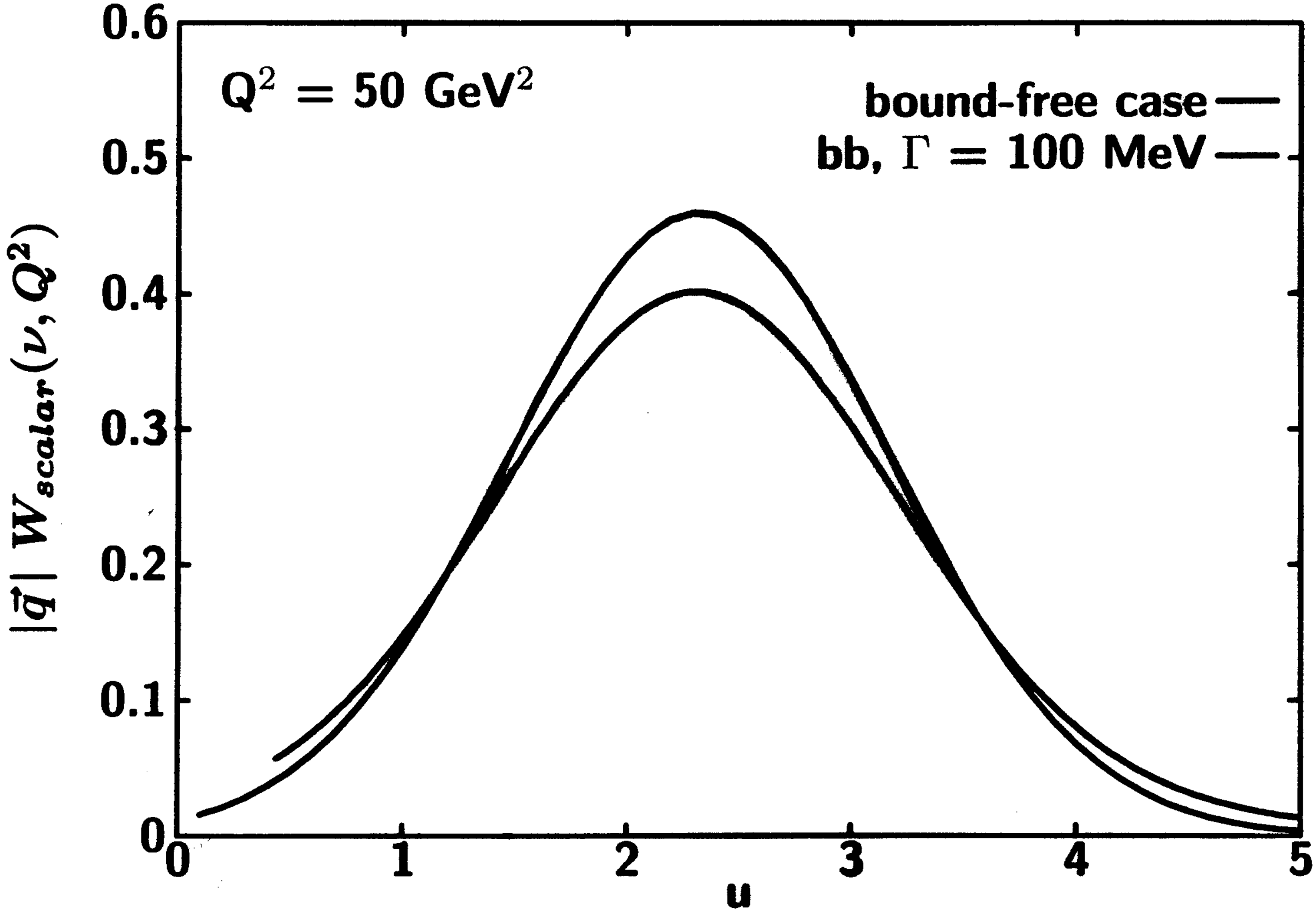
$$\left(\exp\left[-\tfrac{1}{\beta^2}(|\vec{q}\,| - |\vec{p}_f\,|)^2\right] - \exp\left[-\tfrac{1}{\beta^2}(|\vec{q}\,| + |\vec{p}_f\,|)^2\right]\right)$$

with $|\vec{p}_f\,| = \sqrt{(\nu + E_0)^2 - m^2}$

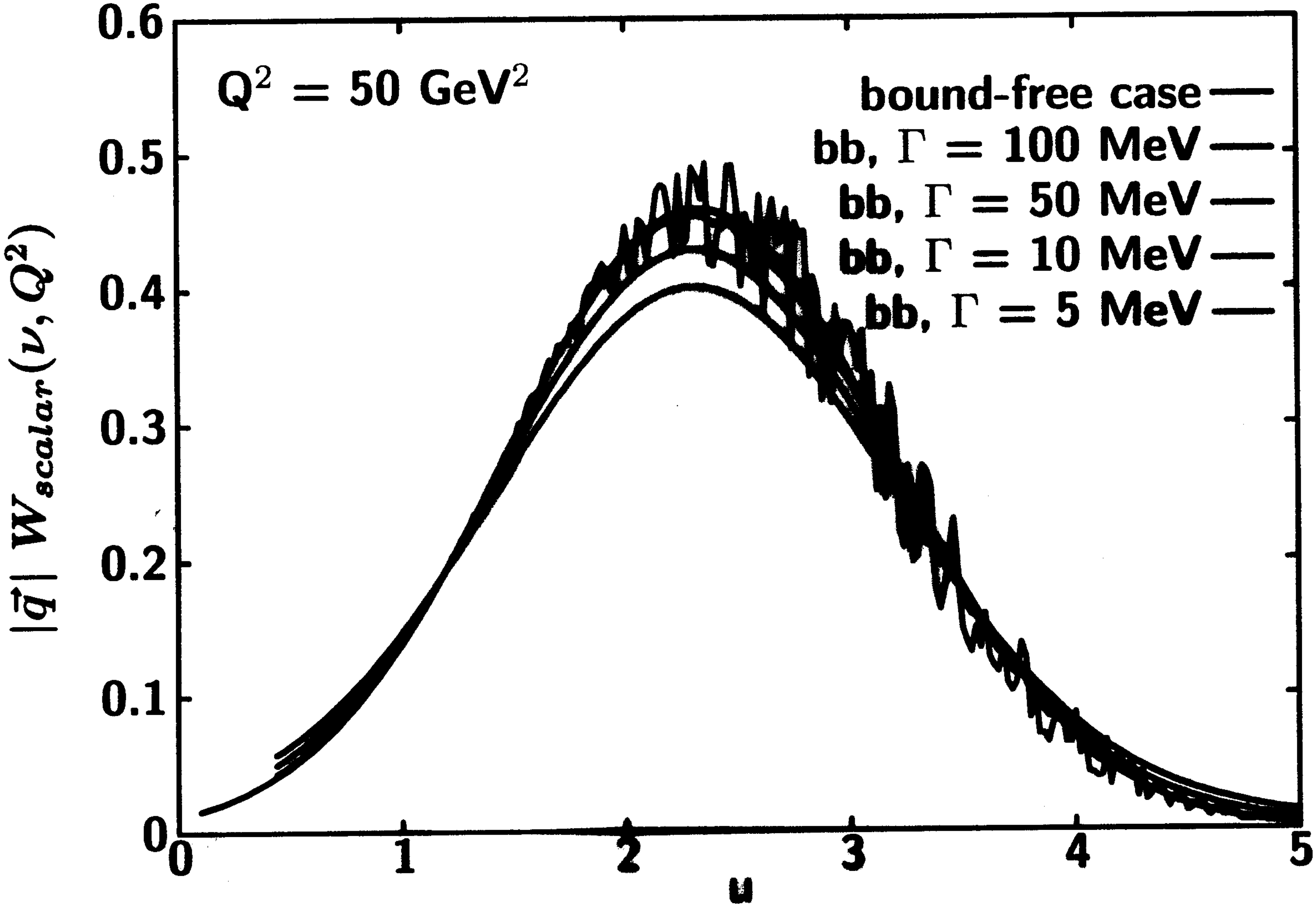
Scaling for the All Scalar Case: Bound-Free



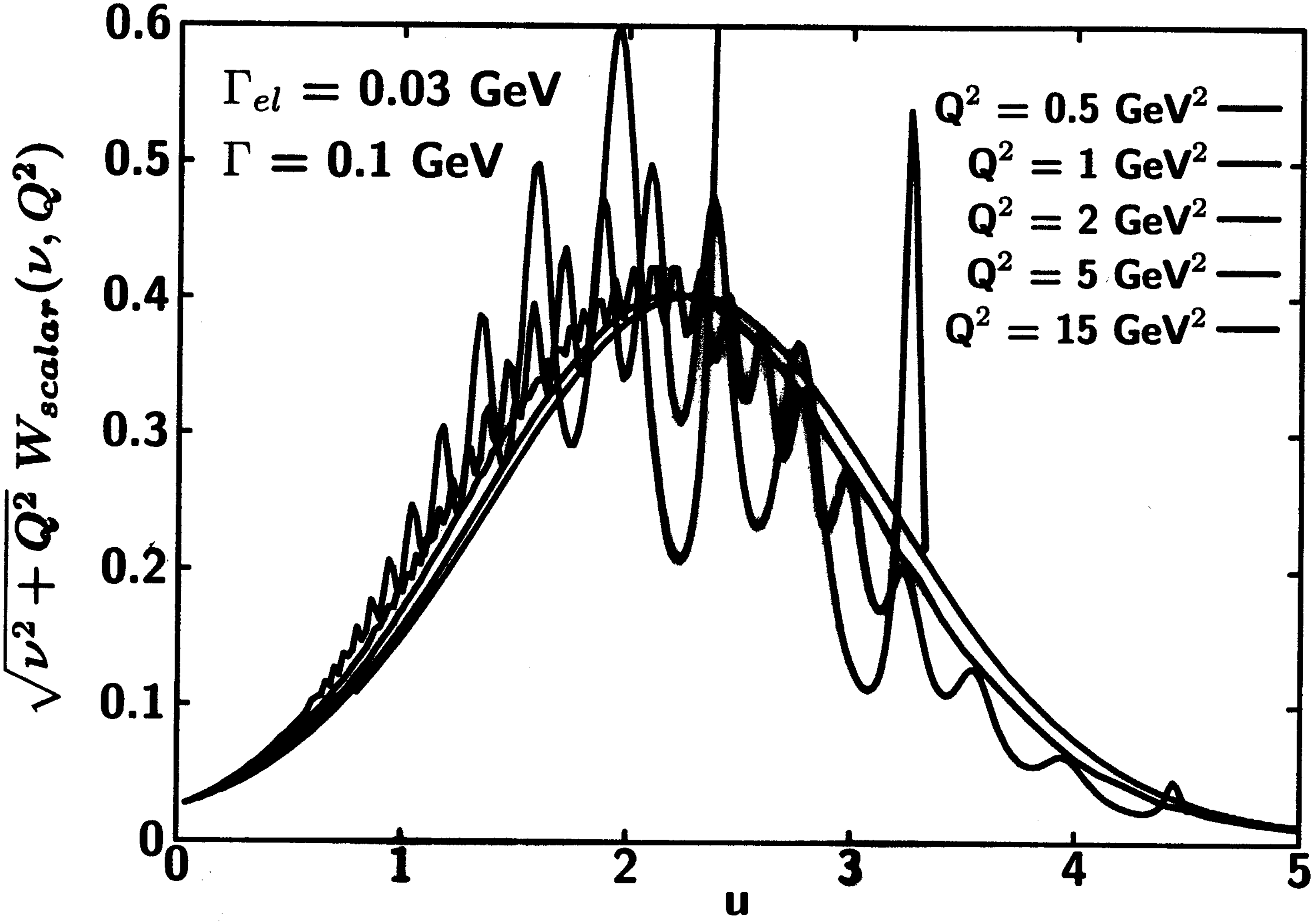
Scaling: Bound-Bound and Bound-Free (I)



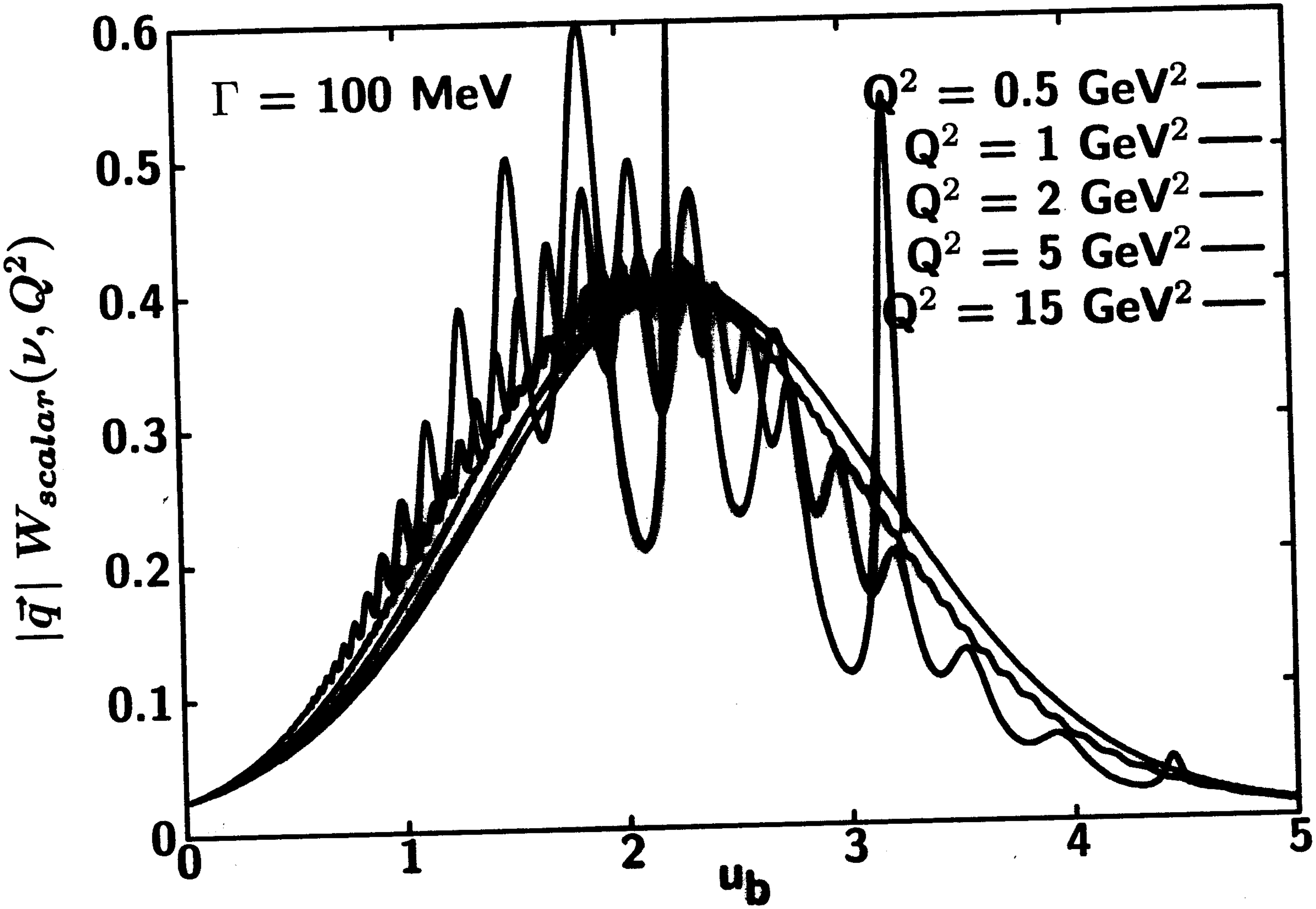
Scaling: Bound-Bound and Bound-Free (II)



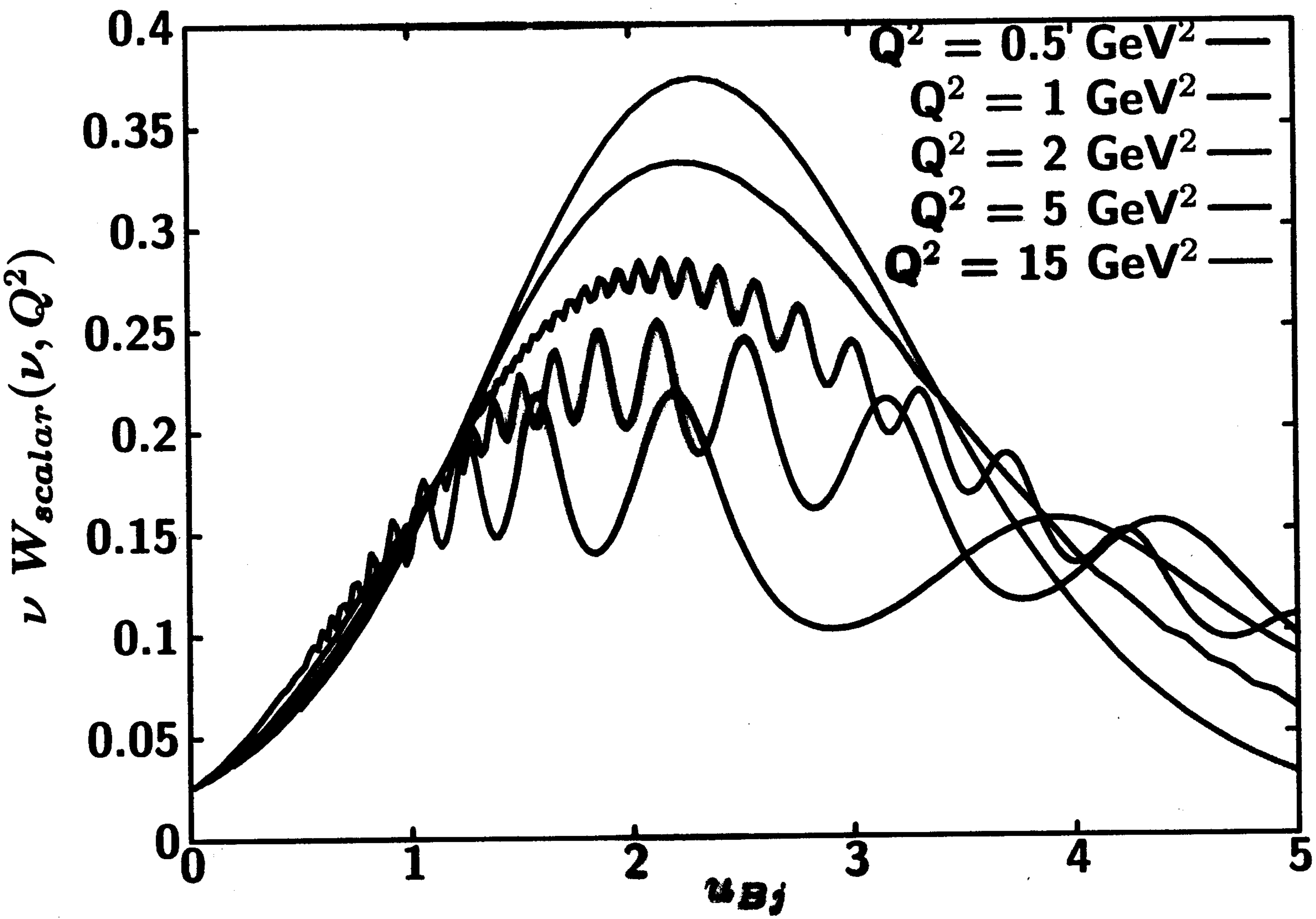
Local Duality for the **All** Scalar Case



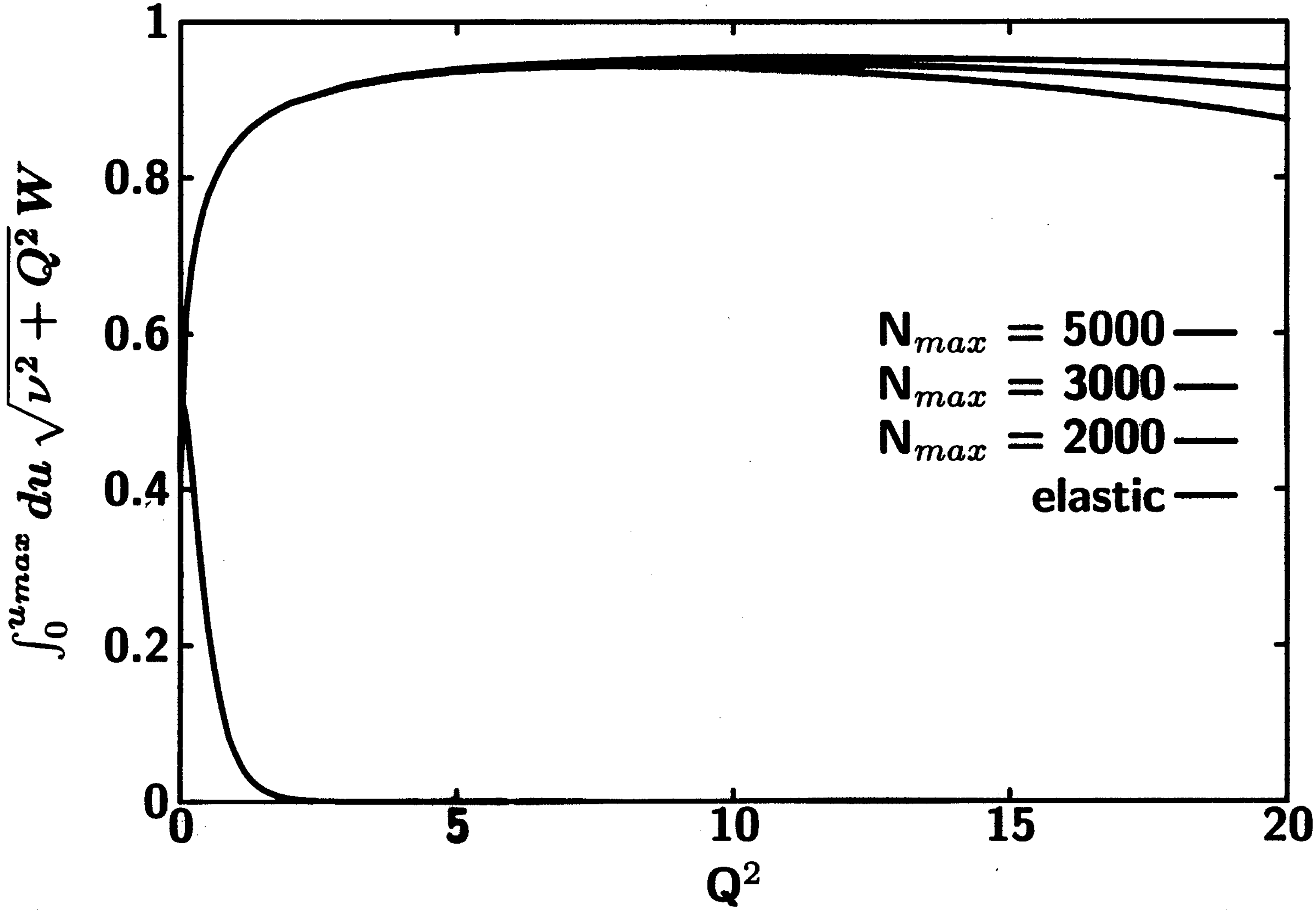
Local Duality for the All Scalar Case (II)



Local Duality with Bjorken's variable



Global Duality: First Moment



Summary and Outlook

- confining, relativistic model
- duality is realized in the form factors
- nice qualitative reproduction of scaling, global and local duality, which are observed in Bloom – Gilman duality
- bound-bound results converge to the bound-free result

What's next?

- decay of resonances, consistent calculation of the widths
- introduce spin