Jefferson Lab
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Duality and Quark-Gluon Correlations in Electron Scattering

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An illustration of the interplay
between

structure functions,
resonances,
and high twists $\cdots$

N$^{\text{TH}}$ MOMENT OF STRUCTURE FUNCTION

$$\int_0^1 dx\, x^{N-2} F_2(x,Q^2) = \mathcal{A}^N \, C_2^N(Q^2) + \cdots$$

TARGET-DEPENDENT
Q^2-INDEPENDENT

TARGET-INDEPENDENT
CALCULABLE FROM
PERTURBATIVE QCD

TWIST EXPANSION FOR ξ – MOMENTS OF STRUCTURE FUNCTION

$$\int d\xi \; \xi^{n-2} F_2(\xi, Q^2)$$

$$= \underline{\underline{A_n(Q^2)}} + \sum_{k=1}^{\infty} \left(\frac{n M_0^2}{Q^2}\right)^k \underline{\underline{B_n^k(Q^2)}}$$

$$A_n(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d_n} \left(1 + \frac{\alpha_s(Q^2)}{3\pi} f_n + \cdots\right) A_n$$

$$B_n^k / A_n = \mathcal{O}(1)$$

DE RUJULA ET AL. 1977
JI ET AL. 1995

DUALITY $\Rightarrow$ HIGHER TWIST ($1/Q^{2k}$) TERMS SMALL

TWIST OF AN OPERATOR

= DIMENSION − SPIN

$\bar{\psi}\gamma_\mu\gamma_5\psi$ TWIST − 2

$\bar{\psi}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\psi$ TWIST − 2

$\bar{\psi}\tilde{F}_{\mu\nu}\gamma^\nu\psi$ TWIST − 4

$\boxed{\textbf{Bloom-Gilman Duality}}$

"Resonances average to a universal curve"

Lowest moment ($n = 2$) of structure function:

$$M_2(Q^2) \;=\; \int_0^1 d\xi\ F_2(\xi, Q^2)$$

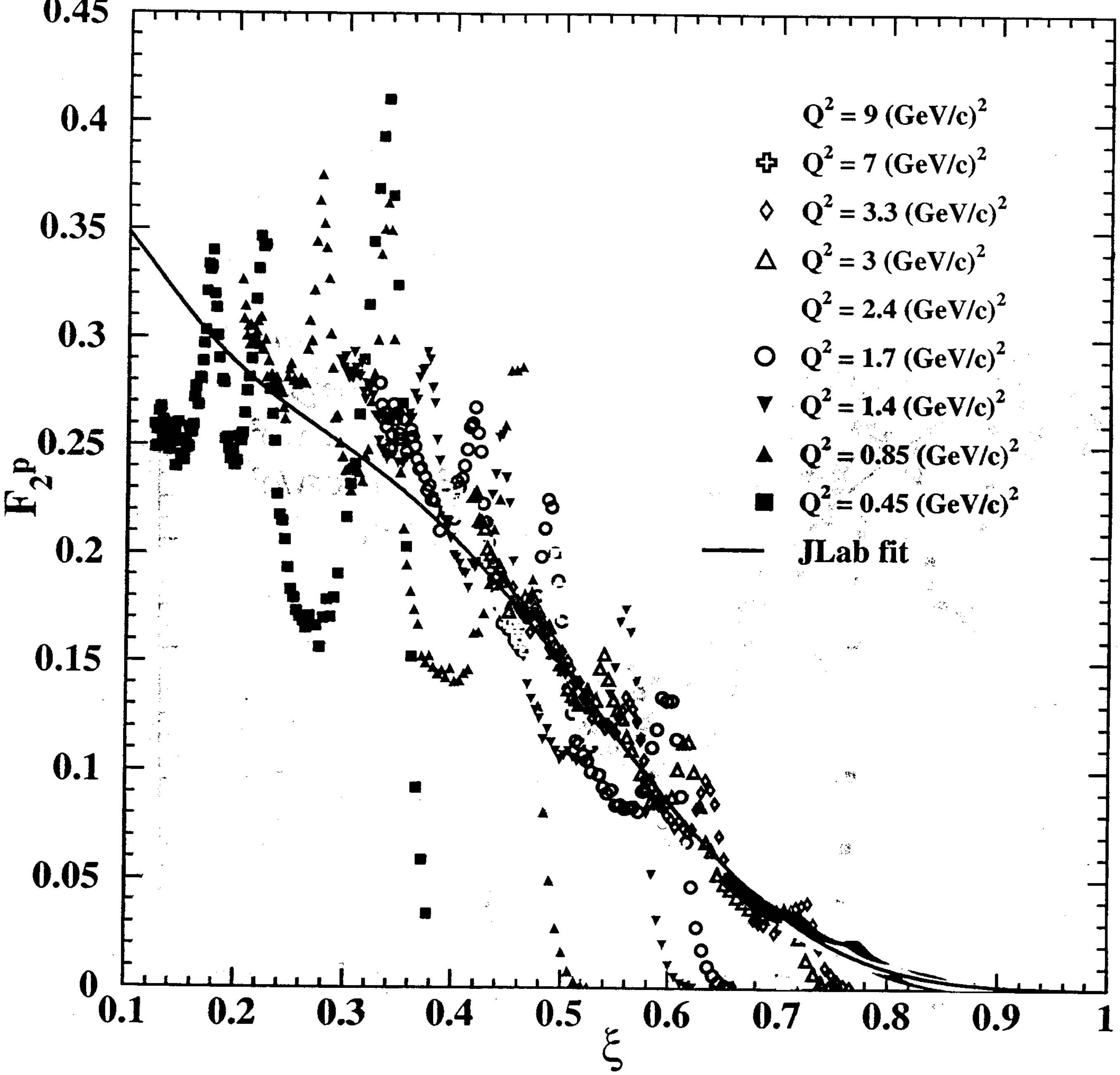
$$\frac{dM_2(Q^2)}{dQ^2} \approx 0$$

$$\Updownarrow$$

high twists B_2^k are small or cancel

F_2^p IN THE RESONANCE REGION

KEPPEL ET AL, JEFF. LAB



Three regions (de Rujula, Georgi, Politzer 1976):

- A: $n \ll M_0^2/Q^2$

  $\longrightarrow$ scaling

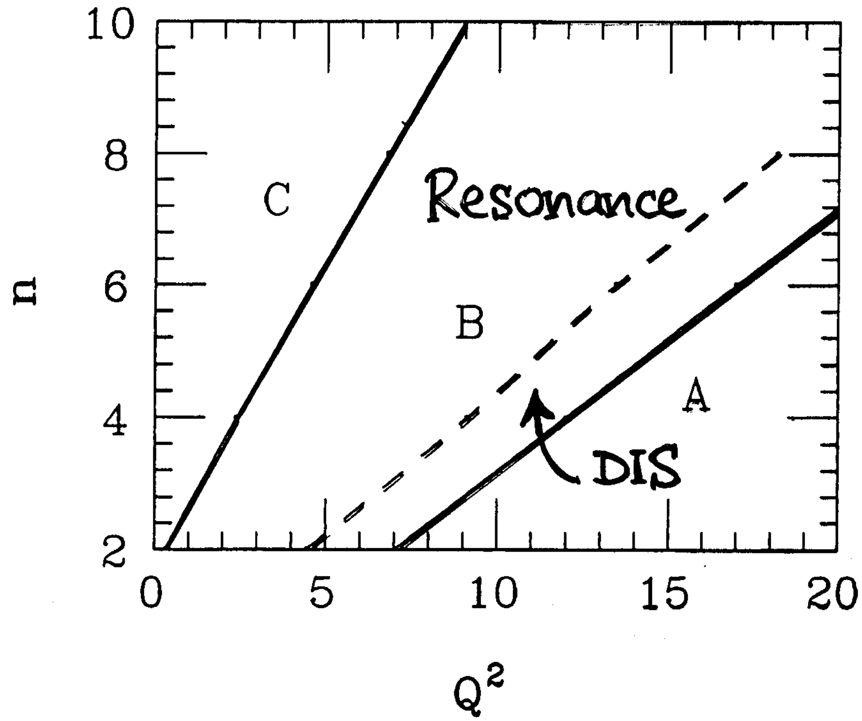
- B: $n \leq M_0^2/Q^2$

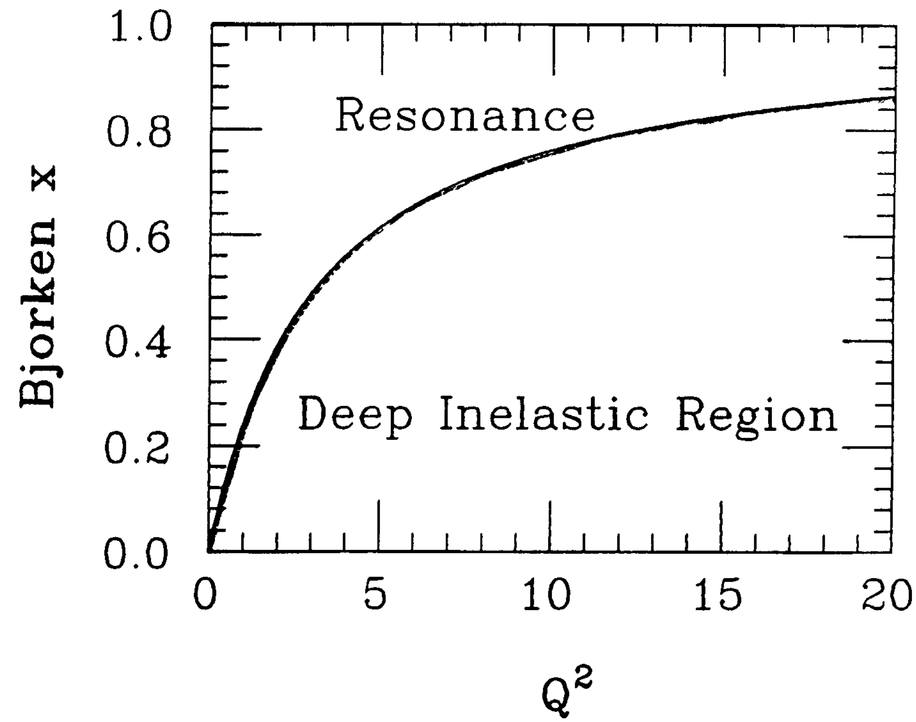
  $\longrightarrow$ resonances/high twists

- C: $n \geq M_0^2/Q^2$

  $\longrightarrow$ OPE breaks down
  $\longrightarrow$ intractable?

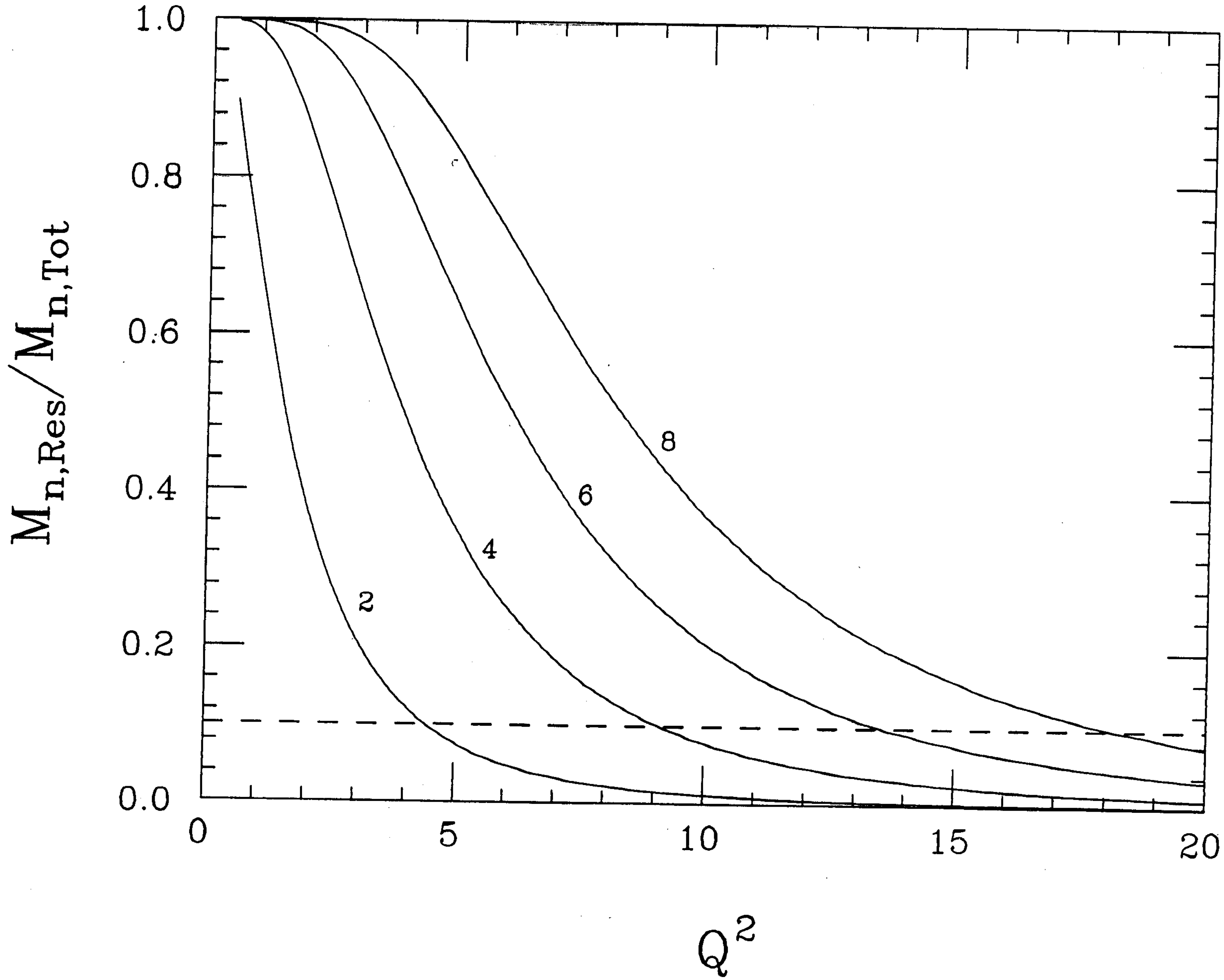
JI, UNRAU 1995



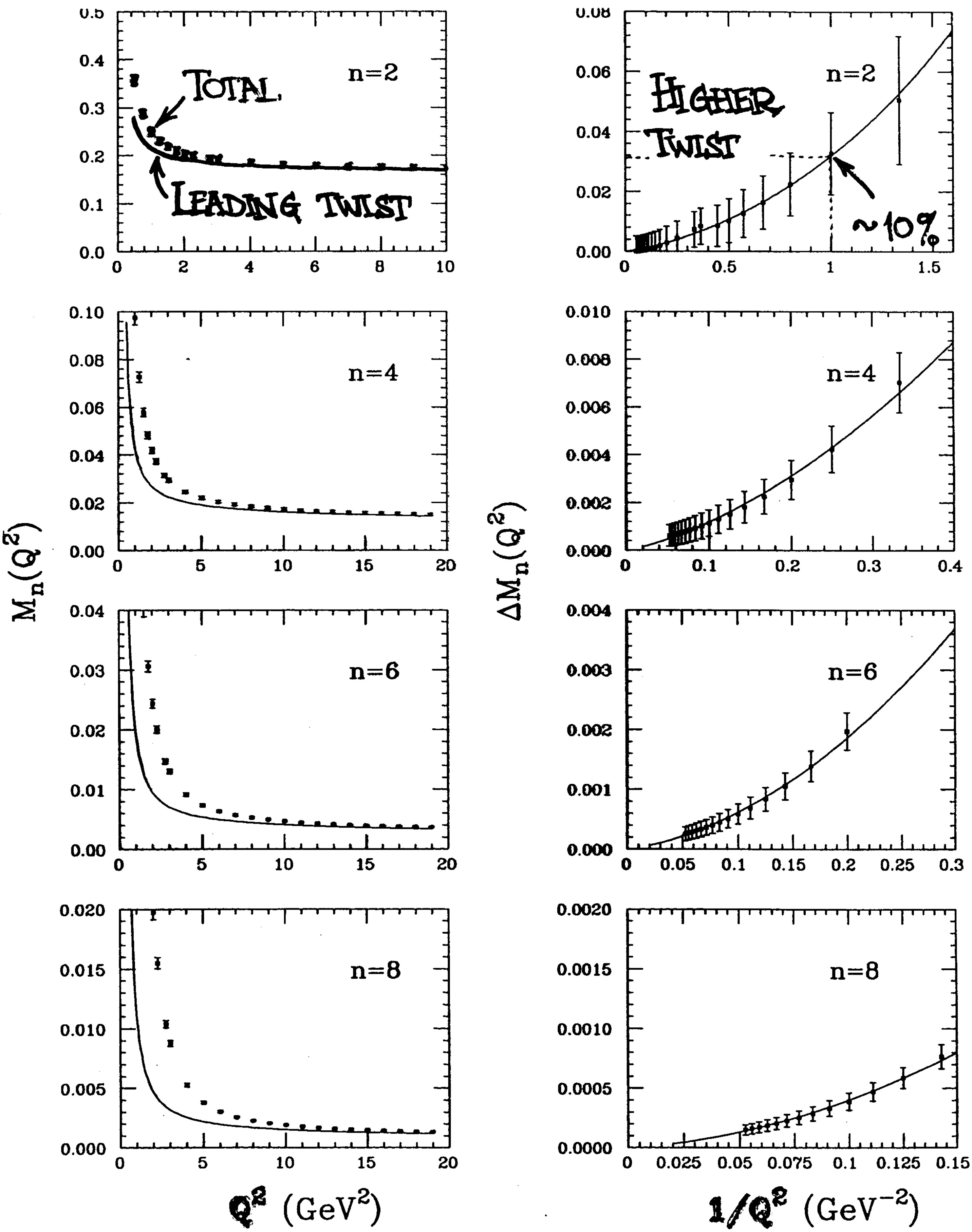


$$W^2 = M^2 + \frac{Q^2}{x}(1-x) = (2\ \mathrm{GeV})^2$$

JI, UNRAU 1995



Ji, Unrau '95



TOTAL
LEADING TWIST
HIGHER TWIST
~10%
n=2
n=4
n=6
n=8
$M_n(Q^2)$
$\Delta M_n(Q^2)$
Q^2 (GeV^2)
$1/Q^2$ (GeV^{-2})

WHY DOES F_2 APPEAR TO BE VALENCE-LIKE AT LOW ξ (LOW Q^2)?

$$F_2 = F_2^{VAL} + F_2^{SEA}$$

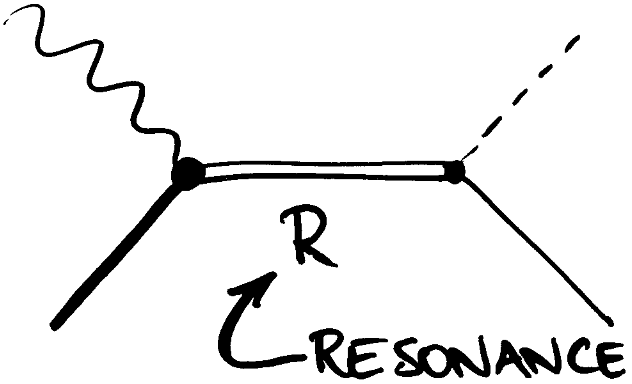
$$F_2 = F_2^{NON-DIFF} + F_2^{DIFF}$$

REGGE THEORY :

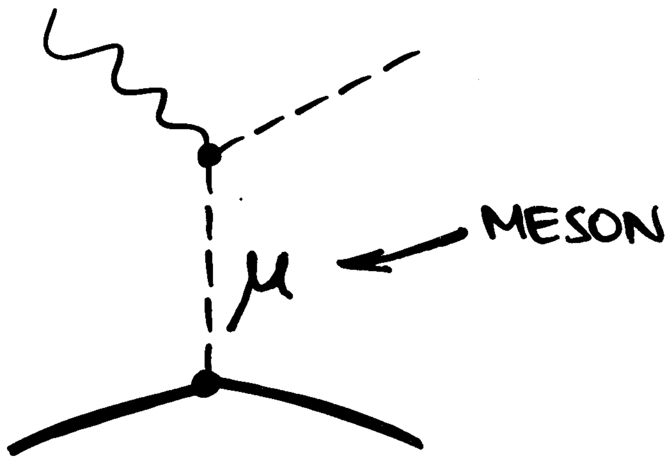
NON-DIFF. $\sim$ VALENCE

DIFFRACTIVE $\sim$ SEA

$$\sum_R$$



$$\sim \sum_\mu$$



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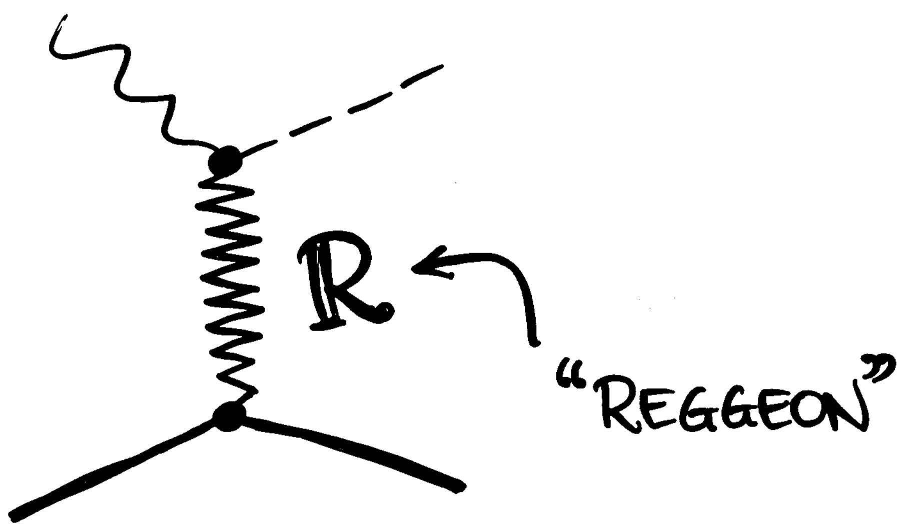


RESONANCES

$\Downarrow$

NON-DIFFRACTIVE

$\Downarrow$

REGGEON EXCHANGE

$\Downarrow$

VALENCE-LIKE

$$F_2^{VAL} \sim x^{1/2} \xrightarrow[x \to 0]{} 0$$

NON-RESON. BCKGND.

$\Downarrow$

DIFFRACTIVE

$\Downarrow$

POMERON EXCHANGE

$\Downarrow$

SEA-LIKE

$$F_2^{SEA} \sim x^{\alpha \sim 0} \xrightarrow[x \to 0]{} \text{const.}$$

DEEP-INELASTIC LEPTON-NUCLEON CROSS SECTION

$$\frac{d\sigma}{dx\,dQ^2}^{\uparrow\downarrow-\uparrow\uparrow} \propto (E+E'\cos\theta)\,g_1 - 2Mx\,g_2$$

$$\frac{d\sigma}{dx\,dQ^2}^{\uparrow\rightarrow-\uparrow\leftarrow} \propto g_1 + \frac{2E}{\nu}\,g_2$$

$$x = \frac{Q^2}{2M\nu}$$ LIGHT-CONE MOMENTUM FRACTION

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}$$ PHOTON VIRTUALITY

FIRST MOMENT

OF SPIN-DEPENDENT

LONGITUDINAL S.F.

$$\Gamma^N(Q^2) = \int_0^1 dx\, g_1^N(x, Q^2)$$

INCLUDES ELASTIC

(X = 1) CONTRIBUTION

★★★

OPERATOR PRODUCT EXPANSION FOR Γ

$$\Gamma(Q^2) = \sum_{\tau=2,4\cdots} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$

GERASIMOV-DRELL-HEARN SUM RULE

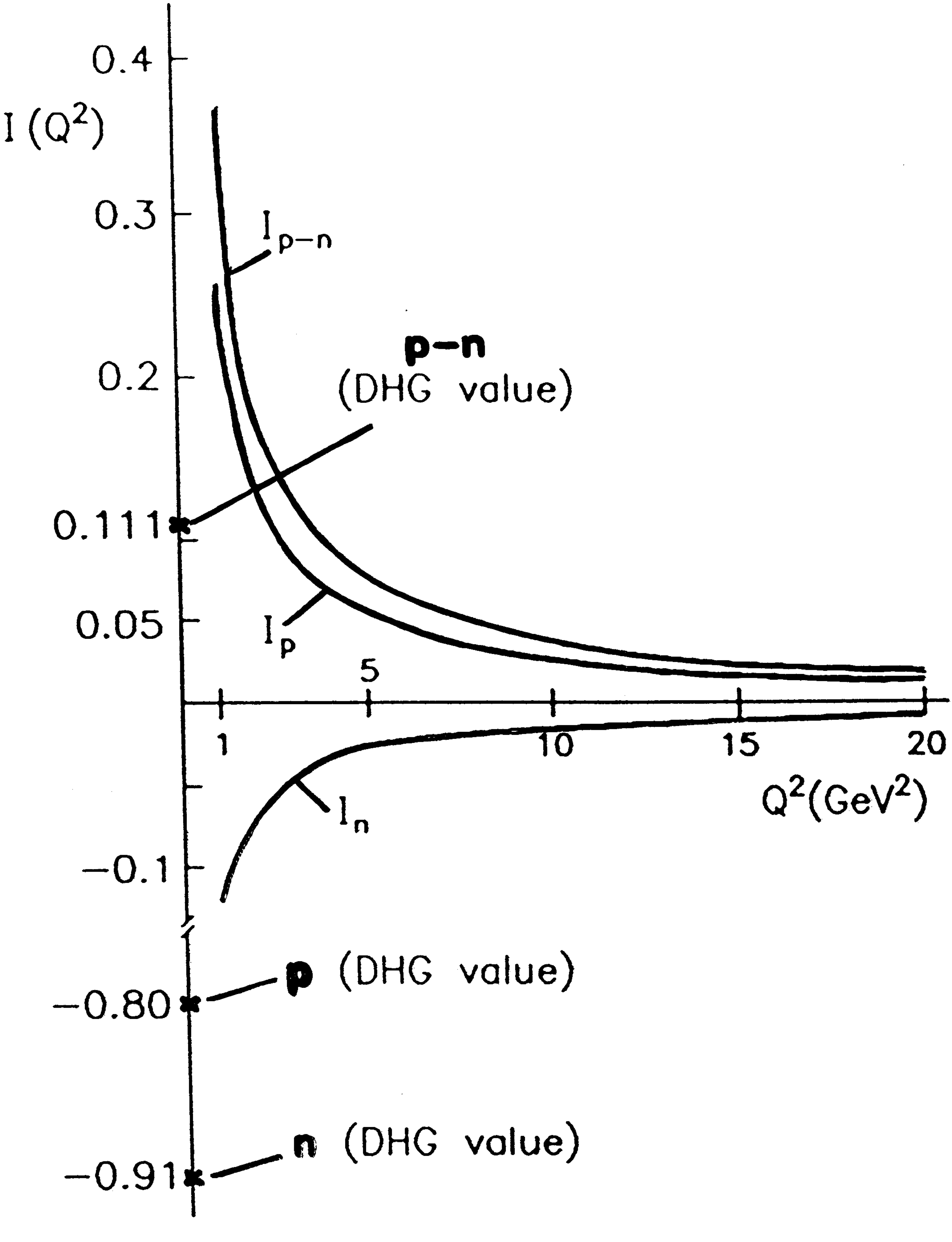
$$I(Q^2) = \frac{2M^2}{Q^2}\,\Gamma(Q^2)$$

REAL PHOTON LIMIT:

$$I(0) = -\frac{1}{4}\kappa^2$$

NON-TRIVIAL Q^2 DEPENDENCE!

→ ANSELMINO, IOFFE, LEADER 1989



AXIAL CHARGES

$$\Gamma^{p,n} = \pm \frac{1}{12} g_3 + \frac{1}{36} g_8 + \frac{1}{9} \Delta\Sigma$$

$$g_3 = \Delta u - \Delta d$$

$$= F + D = g_A / g_V$$

$$g_8 = \Delta u + \Delta d - 2\Delta s$$

$$= 3F - D$$

WEAK HYPERON DECAYS

$$F = 0.459\,(8)$$

$$D = 0.798\,(8)$$

pQCD CORRECTIONS

$$\Delta q_{NS} \longrightarrow C_{NS}(Q^2)\ \Delta q_{NS}$$

$$\Delta\Sigma \longrightarrow C_S(Q^2)\ \Delta\Sigma$$

$$C_{NS}(Q^2) = 1 - a_s(Q^2) - 3.58\, a_s^2(Q^2) - 20.22\, a_s^3(Q^2)$$

$$C_S(Q^2) = 1 - \frac{1}{3} a_s(Q^2) - 0.55\, a_s^2(Q^2) - 4.45\, a_s^3(Q^2)$$

$$\left[a_s(Q^2) \equiv \alpha_s(Q^2)/\pi \right]$$

$$\Gamma^{p,n}(Q^2) = C_{NS}(Q^2)\left(\pm\frac{1}{12} g_3 + \frac{1}{36} g_8\right) + C_S(Q^2)\,\frac{1}{9}\Delta\Sigma$$

Duality, High Twist & Color

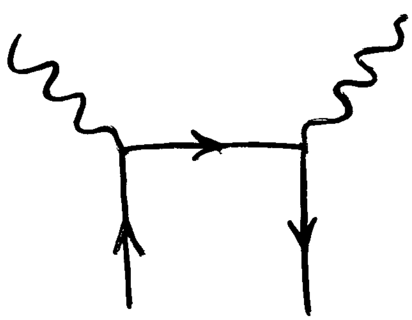
Higher twist (duality violating) matrix elements describe transition from free to confined quarks

Famous example: GDH sum rule / EJ, Bj sum rules

$$\Gamma(Q^2) = \int_0^1 dx \; g_1(x, Q^2)$$
$$= \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

twist − 2 : $\quad \mu_2 = \frac{1}{12} \, g_3 + \frac{1}{36} \, g_8 + \frac{1}{9} \Delta\Sigma$

$$\sim \langle N | \bar{\psi} \gamma_+ \gamma_5 \psi | N \rangle$$



FREE QUARK SCATTERING

$$g_3 = \Delta u - \Delta d$$
$$g_8 = \Delta u + \Delta d - 2\Delta s$$
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

TWIST - 4

$$\mu_4^N(Q^2) = \frac{M^2}{9}\left(a_2^N + 4d_2^N - 4f_2^N\right)$$

TARGET-MASS CORRECTION (TWIST-2)

$$a_2^N(Q^2) = 2\int_0^1 dx\; x^2 g_{1_{TW-2}}^N(x, Q^2)$$

g_2 CORRECTION (TWIST-3)

$$d_2^N(Q^2) = \int_0^1 dx\; x^2\left(2g_1^N + 3g_2^N\right)$$

GENUINE TWIST-4

$$f_2^N(Q^2)\, s^\mu = \frac{g_s}{2M^2}\sum_q e_q^2 \langle N|\bar{\psi}_q \tilde{F}^{\mu\nu}\gamma_\nu \psi_q|N\rangle$$

± convention-dependent

twist – 4 : $\mu_4 = \frac{M^2}{9}(a_2 + 4d_2 + 4f_2)$

$$a_2 = 2\int_0^1 dx\ x^2\ g_1$$

$$d_2 = \int_0^1 dx\ x^2\,(2g_1 + 3g_2) \propto \chi_E + 2\chi_B$$

$$f_2 \propto \chi_E - \chi_B$$

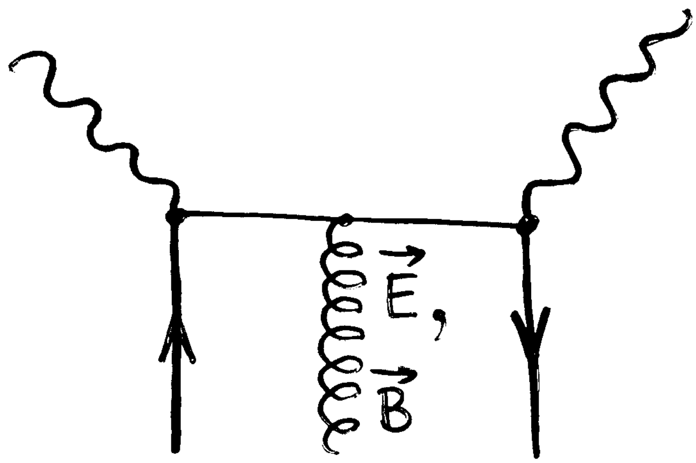
Color electric and magnetic polarizabilities:

$$\chi_E = \langle N|\psi^\dagger \left(\vec{j}^a \times \mathbf{E}^a\right)_z \psi|N\rangle$$

$$\chi_B = \langle N|\psi^\dagger\ j_0^a\ \mathbf{B}_z^a\ \psi|N\rangle$$

$$j_\mu^a = g\bar{\psi}\gamma_\mu\ \mathbf{t}^a\ \psi$$

(JI 1995 ; MANKIEWICZ, SCHÄFER ET AL 1995)



EFFECT OF COLOR FIELDS ON QUARK

HIGHER-τ PART OF Γ^N

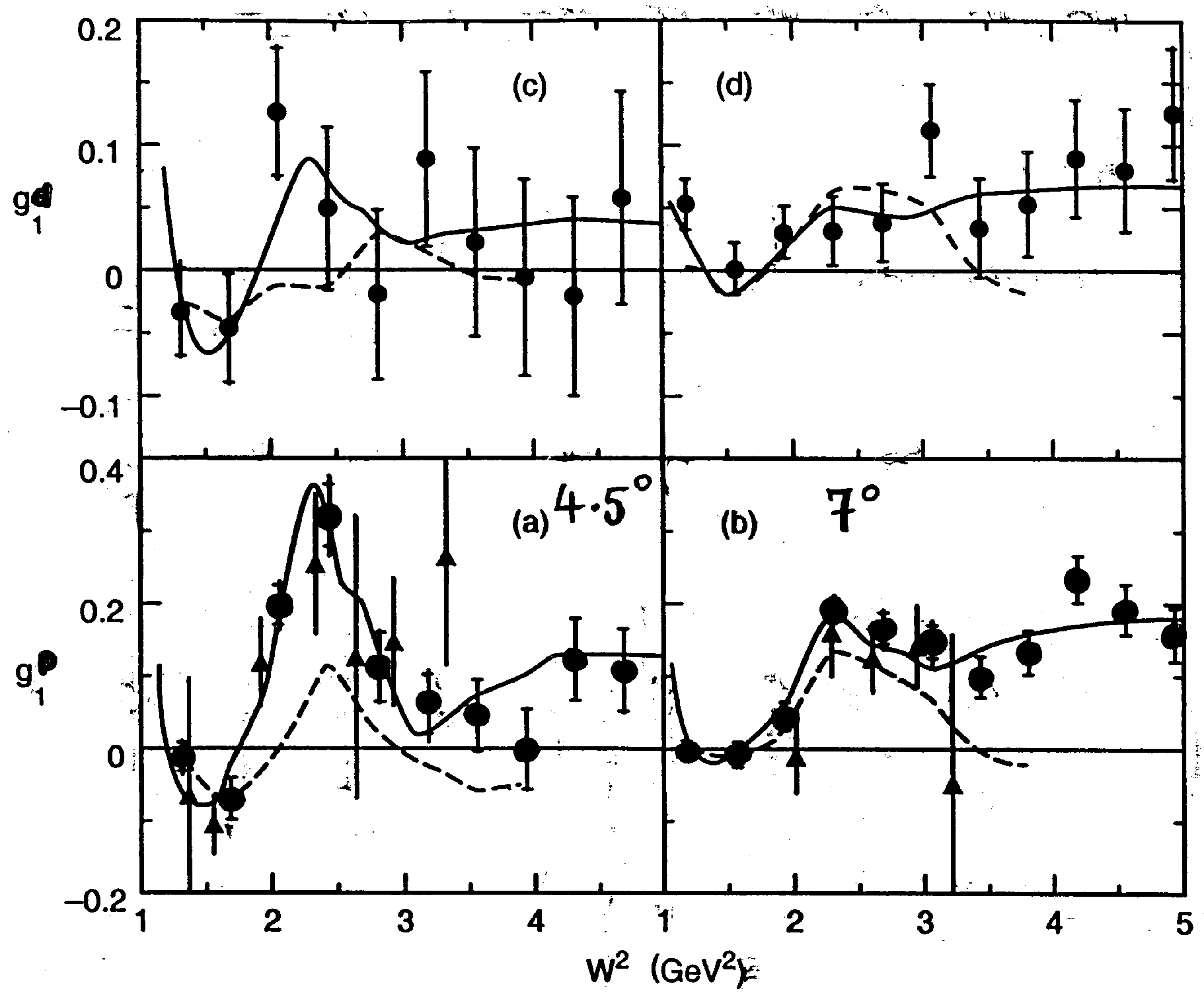
$$\Delta\Gamma^N(Q^2) \equiv \Gamma^N(Q^2) - \mu_2^N(Q^2) - \frac{M^2}{9Q^2}\left(a_2^N + 4d_2^N\right)$$

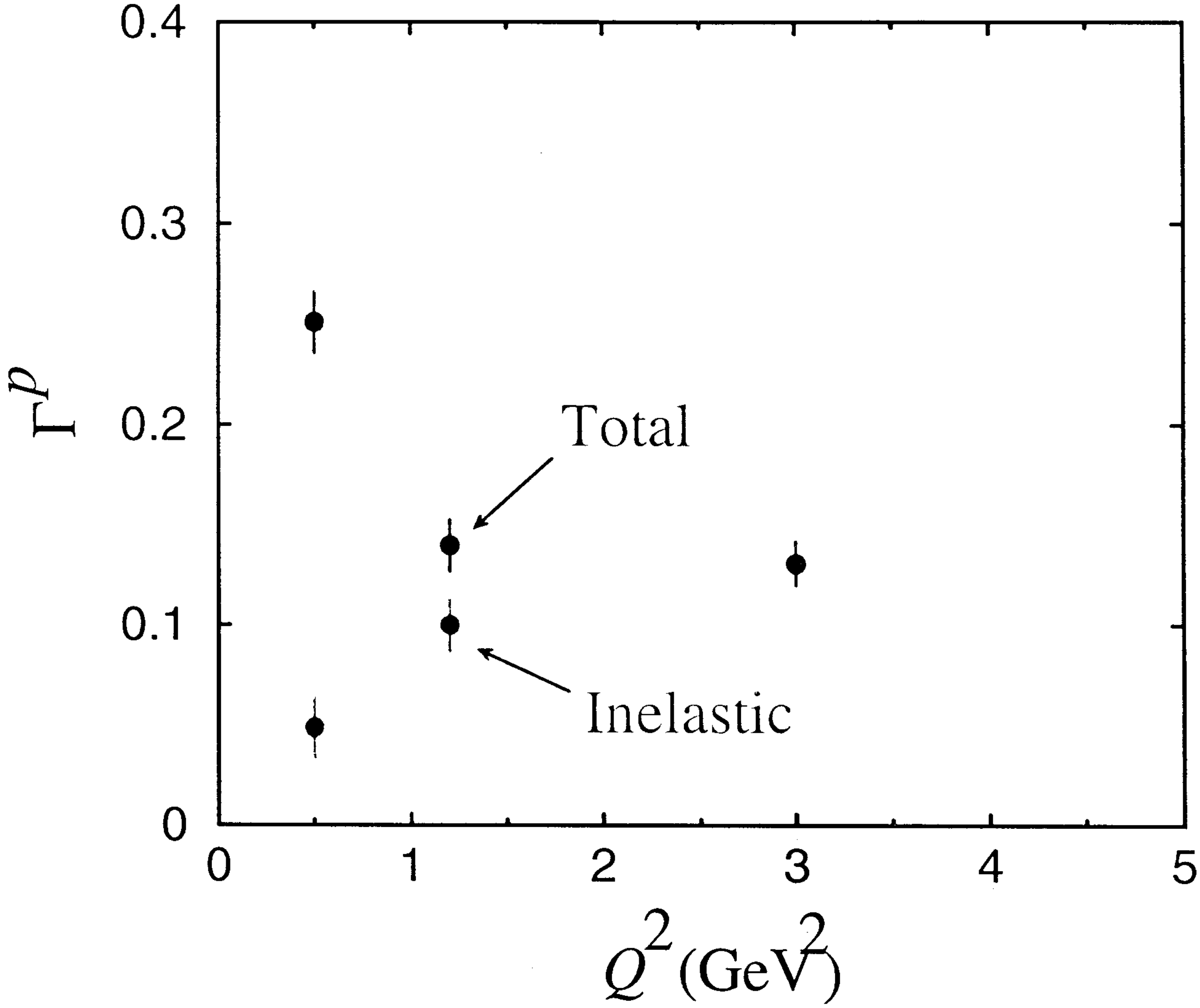
$$= -\frac{4M^2}{9Q^2} f_2^N(Q^2) + \sum_{\tau=6,8,\ldots} \frac{\mu_\tau^N(Q^2)}{Q^{\tau-2}}$$

SMALL$^?$ FOR $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

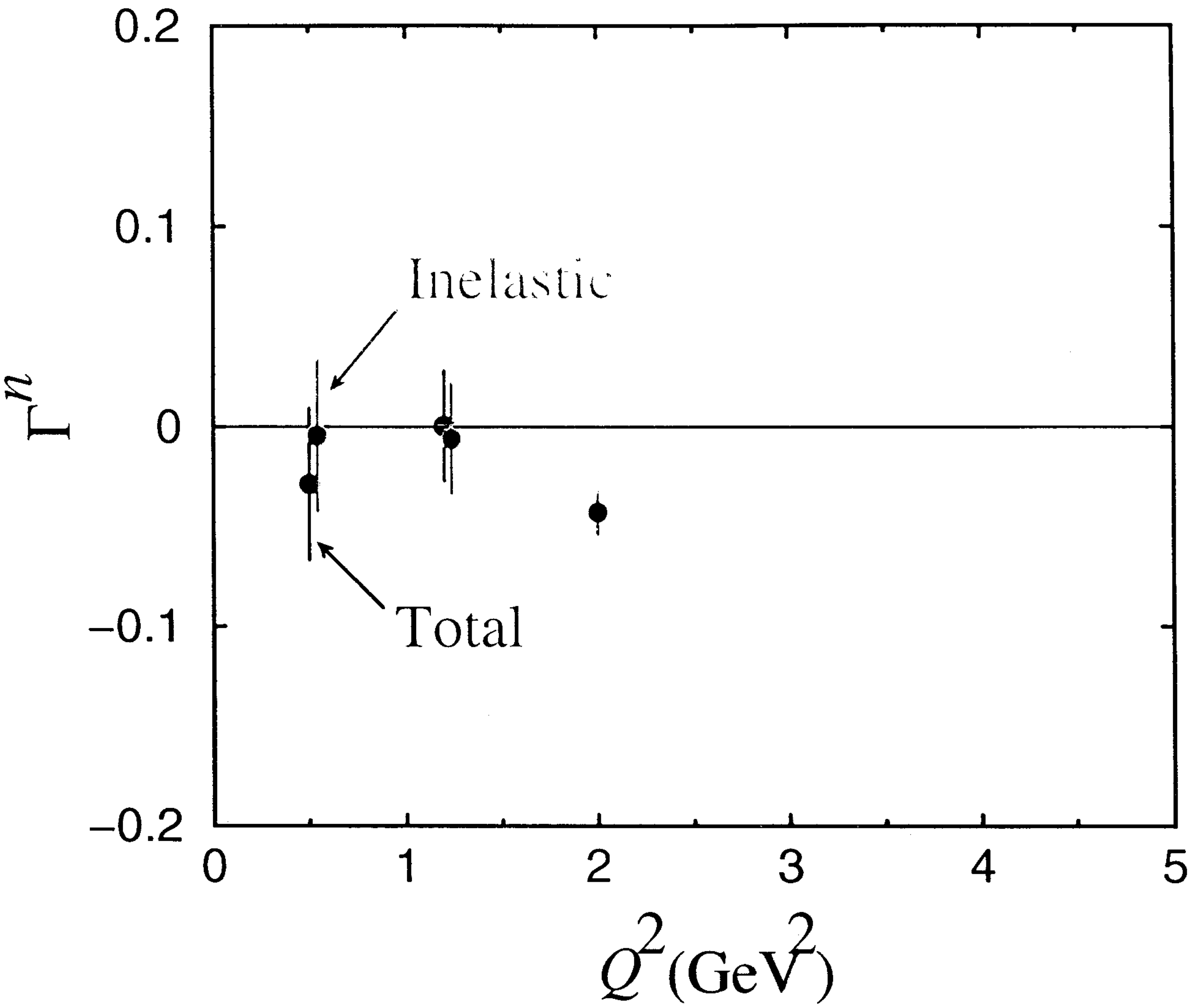
GEORGI/POLITZER PRD 14 (76) 1829

SLAC E143 (1997)



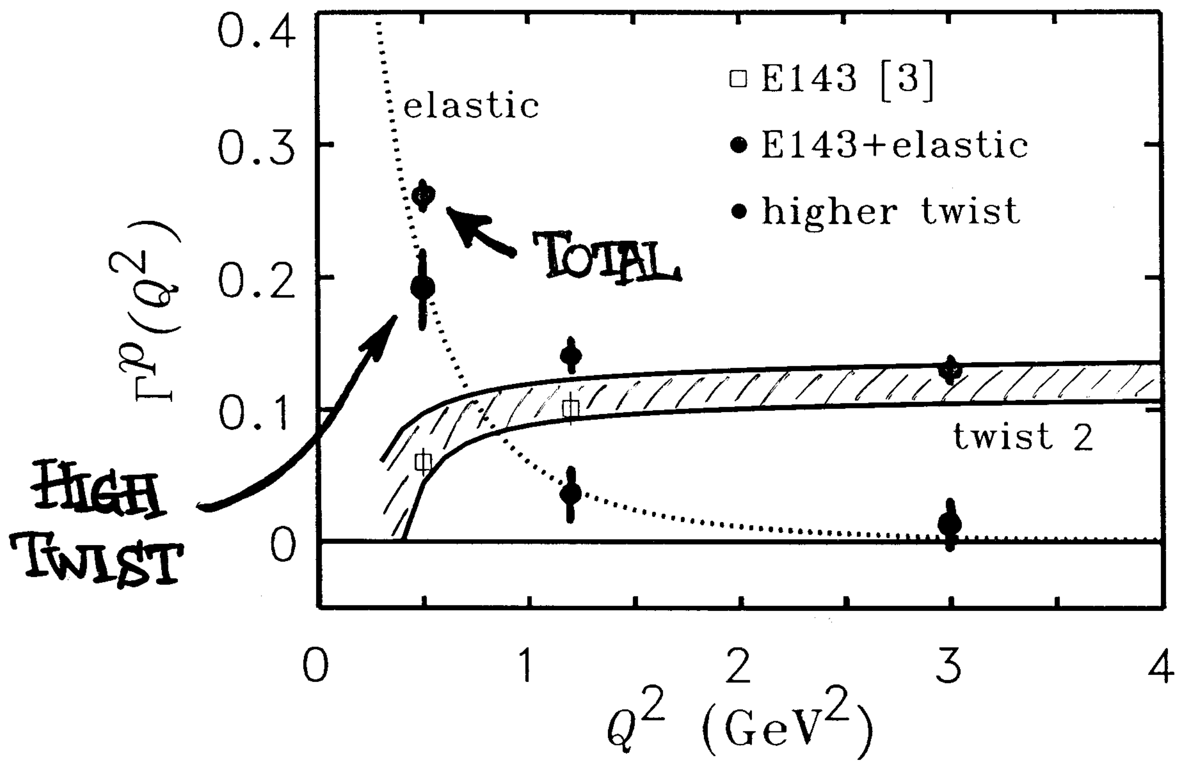


First moment of proton g_1 structure function



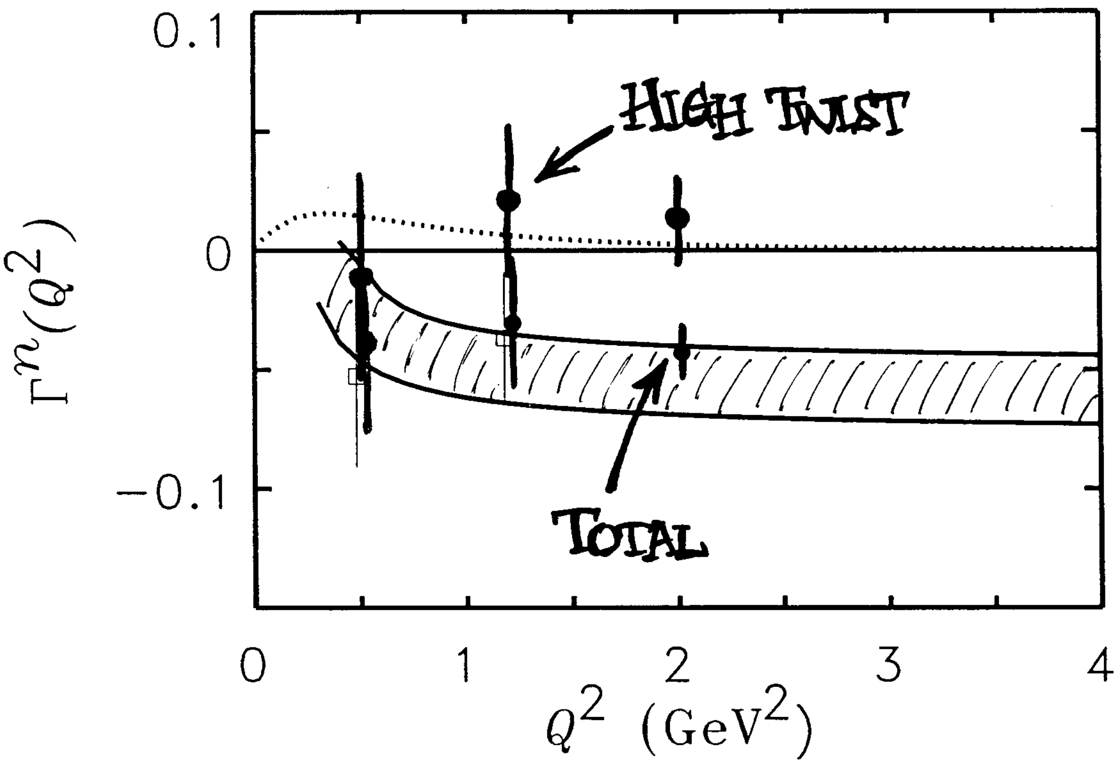
First moment of neutron g_1 structure function

JI, WM 1997



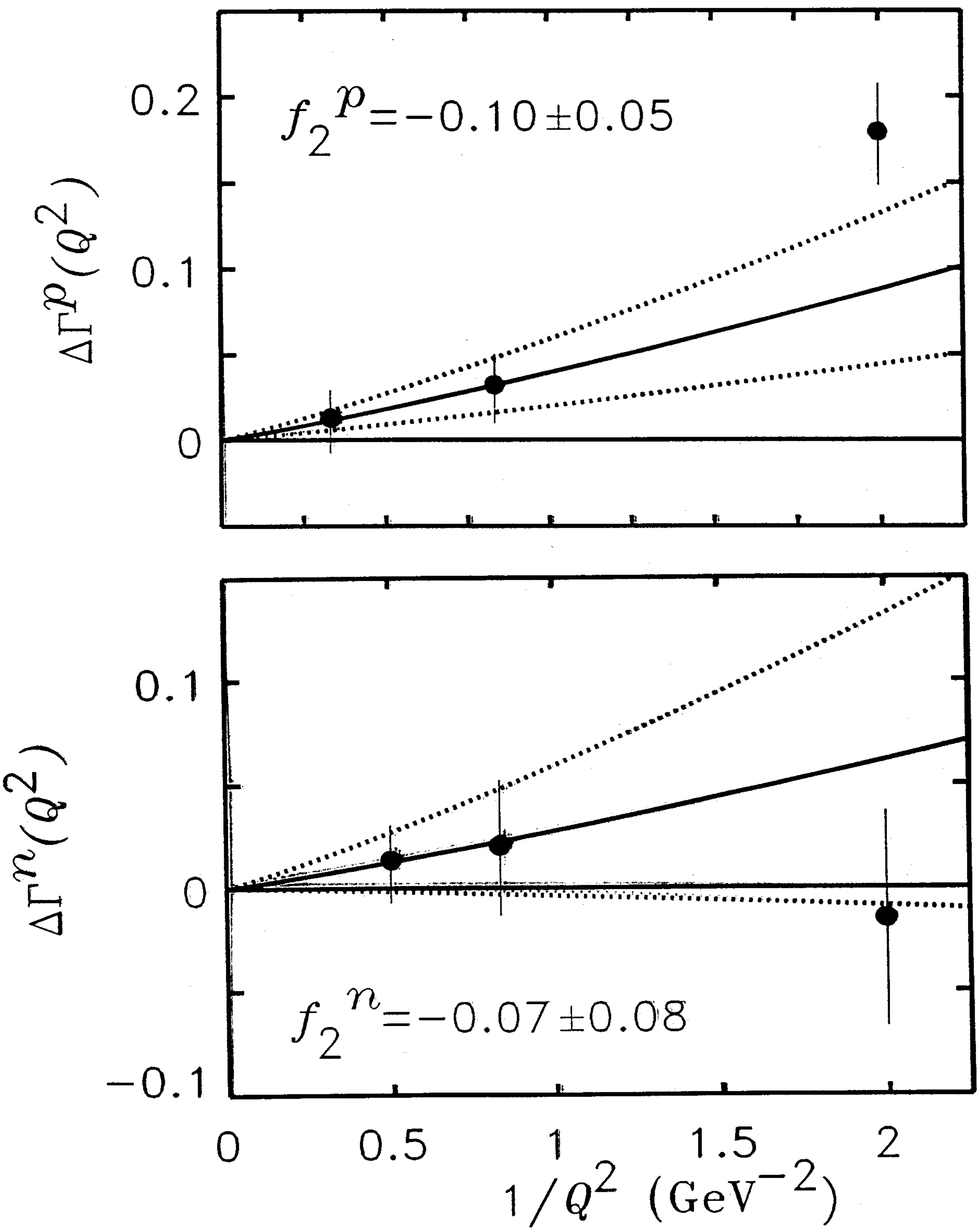
First moment of proton g_1 structure function

WM, JI 1997



First moment of neutron g_1 structure function

X. JI & W.M. PRD 56 (97) 1



MODEL CALCULATIONS

	f_2^p	f_2^n
BALITSKY, BRAUN, KOLISNICHENKO QCD SUM RULE	+0.050	−0.018
STEIN, GORNICKI, MANKIEWICZ, SCHÄFER QCD SUM RULE	+0.037	+0.013
JI, UNRAU BAG MODEL	−0.028	0

JI, WM 1991

AT SCALE $\mu = 1$ GeV

$$f_2^p = -0.10 \pm 0.05$$

$$f_2^n = -0.07 \pm 0.08$$

WITH TARGET-MASS AND TWIST-3 CORRECTIONS

$$\mu_4^p = (0.04 \pm 0.02)\ \mathrm{GeV}^2$$

$$\mu_4^n = (0.03 \pm 0.04)\ \mathrm{GeV}^2$$

UNCERTAINTIES

- HOW LOW IS TOO LOW ?
  WHEN CAN $\tau \geq 6$ BE
  NEGLECTED ?

- EFFECT OF $\mathcal{O}(\alpha_s^4)$ IN μ_2
  ESTIMATE BY "PADÉ APPROXIMANTS"

$$f_2^p : -0.10 \longrightarrow -0.17$$

$$f_2^n : -0.07 \longrightarrow -0.06$$

- Σ_{inv} : $0.15 \longrightarrow 0.3$

$$f_2^p : -0.10 \longrightarrow -0.05$$

$$f_2^n : -0.07 \longrightarrow 0.0$$

Duality for CEBAF @ 12 GeV

- OPE: validity of duality
  $\Longleftrightarrow$ size of high twists

- Why does duality work at low Q^2?
  $\longrightarrow$ cancellation of large $1/Q^{2n}$ terms?

- Spin dependence
  $\longrightarrow$ how well does duality work for F_L, g_1, g_2?

- Target dependence
  $\longrightarrow$ p, $n(D)$, A

- Onset of scaling is moment-dependent
  $\longrightarrow$ global vs. local duality

- Duality for valence vs. sea quarks
  $\longrightarrow$ resonances vs. background

- Meaning of scaling curve
  $\longrightarrow$ Q^2 dependence
  $\longrightarrow$ for F_2, g_2?