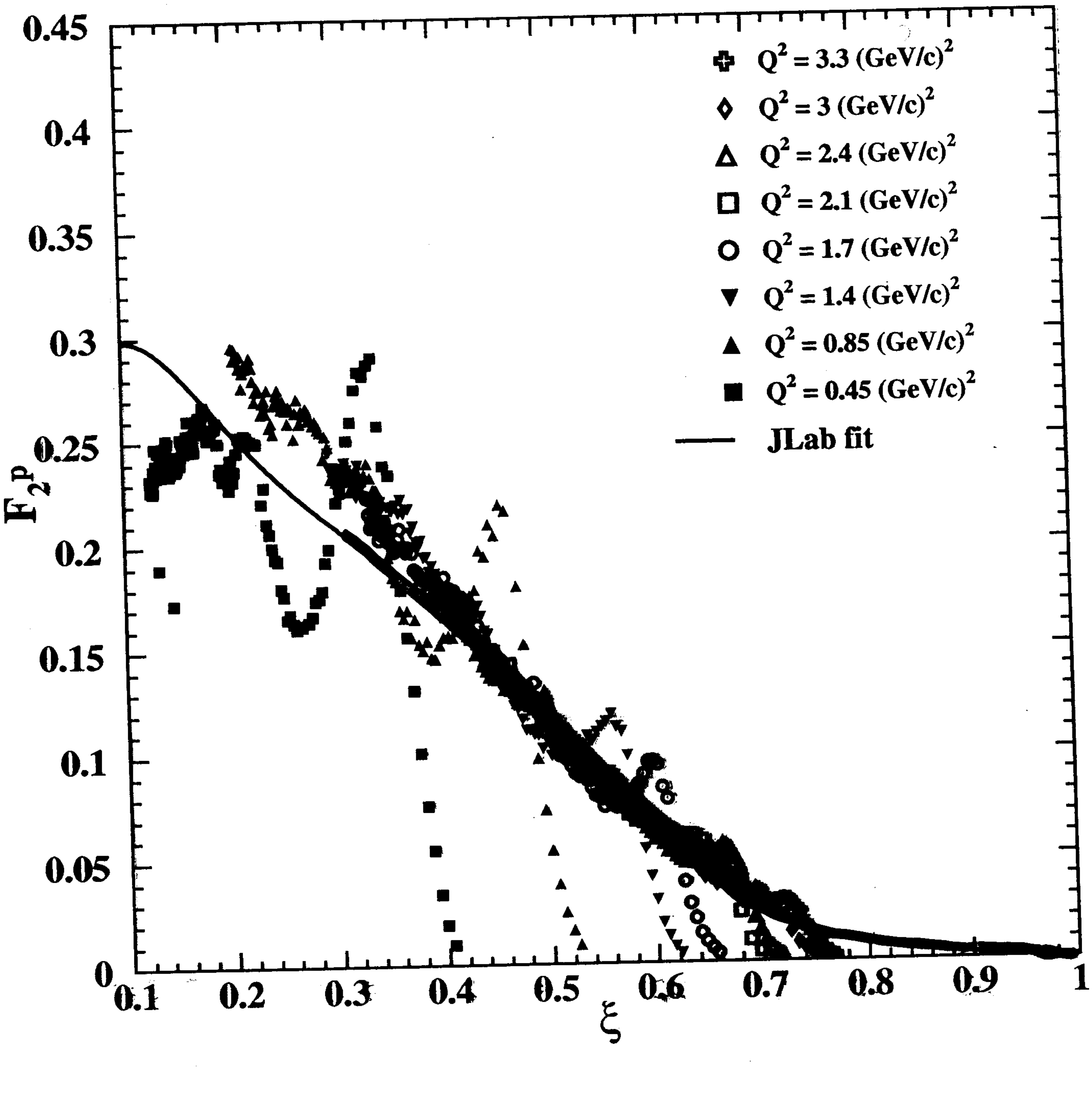
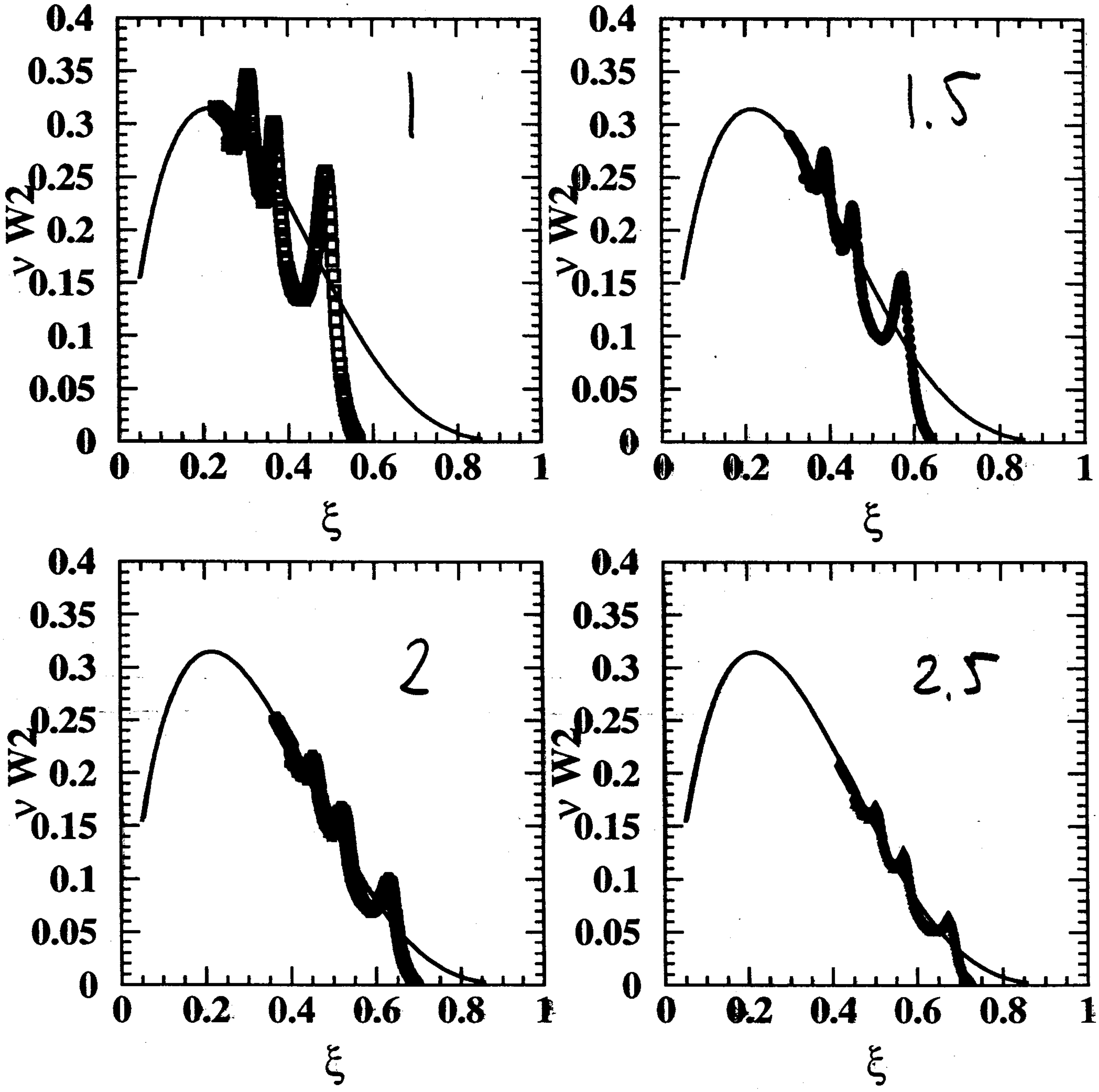
DUALITY FOR POLARIZED AND UNPOLARIZED STRUCTURE FUNCTIONS

A. RADYUSHKIN

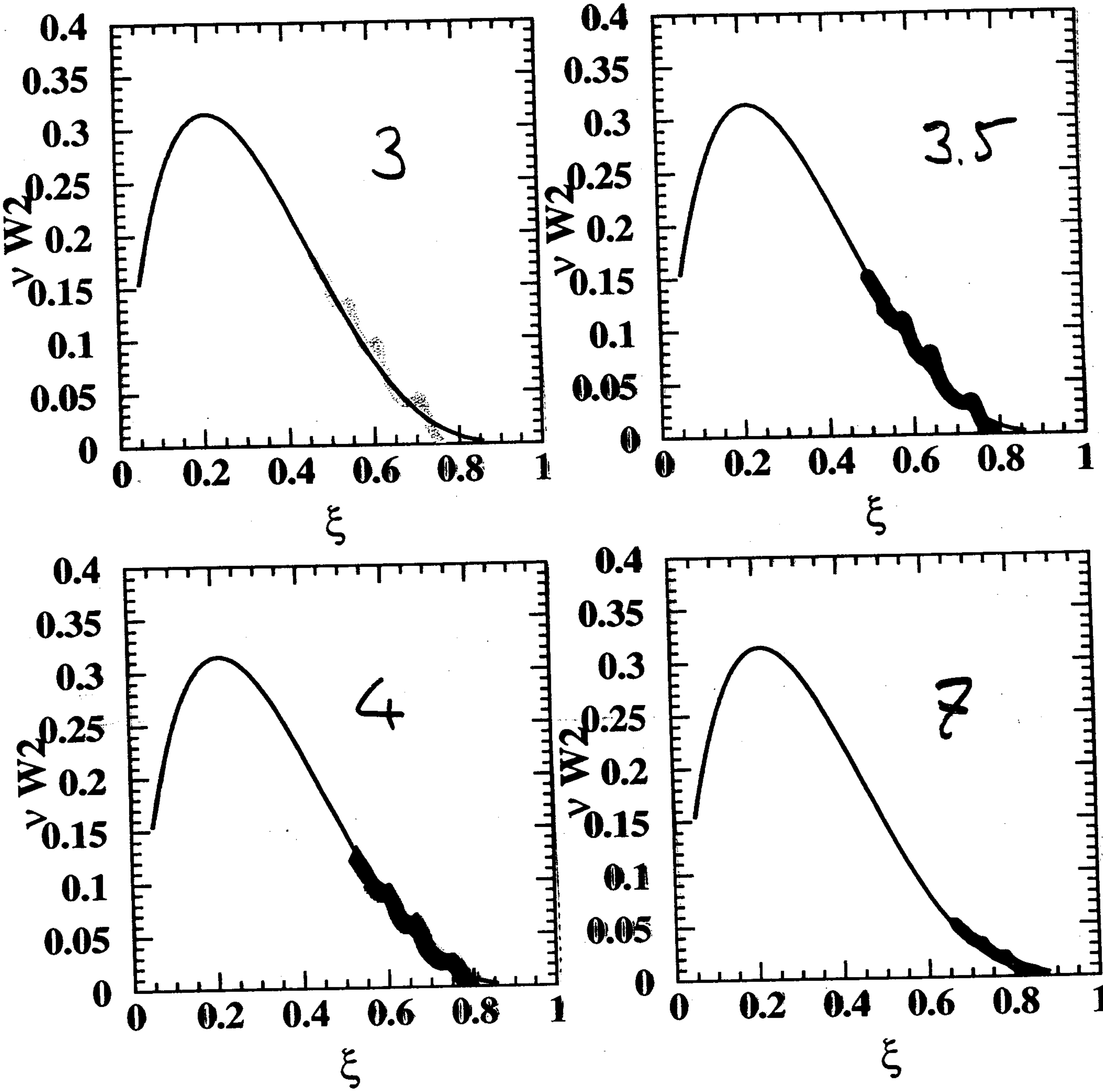


0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
F_2^p
ξ
$Q^2 = 3.3\ (GeV/c)^2$
$Q^2 = 3\ (GeV/c)^2$
$Q^2 = 2.4\ (GeV/c)^2$
$Q^2 = 2.1\ (GeV/c)^2$
$Q^2 = 1.7\ (GeV/c)^2$
$Q^2 = 1.4\ (GeV/c)^2$
$Q^2 = 0.85\ (GeV/c)^2$
$Q^2 = 0.45\ (GeV/c)^2$
JLab fit

JLab Results



JLab Results

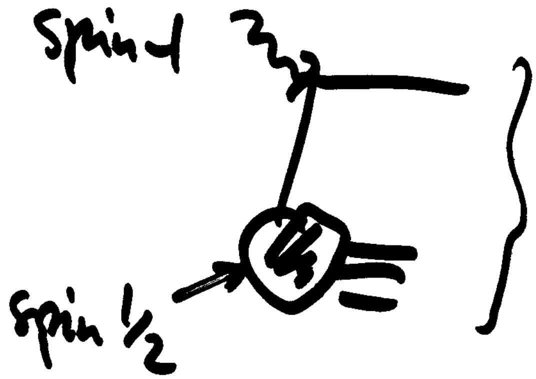


QUESTION: HOW DUALITY WILL WORK IN POLARIZED CASE?

PROBLEM: $G_1(x, Q^2)$ IS NOT ALWAYS POSITIVE

$$W_2 \sim \sigma^{1/2} + \sigma^{3/2}$$

$$G_1 \sim \sigma^{1/2} - \sigma^{3/2}$$

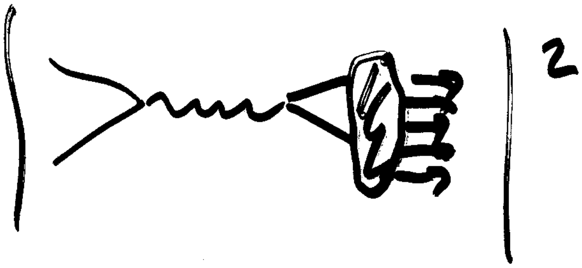


$\times$ $\begin{matrix} 1/2 \\ 3/2 \end{matrix}$

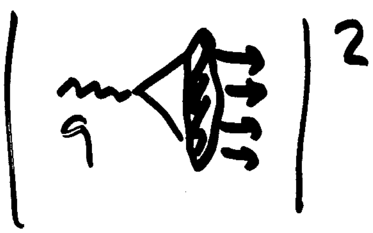
IN GENERAL: IS DUALITY POSSIBLE IF FUNCTION IS NOT POSITIVE DEFINITE?

CONSIDER SIMPLE CASES

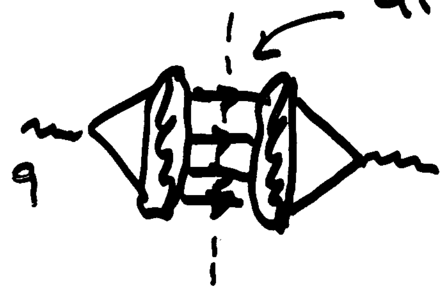
DUALITY IN $e^+e^- \to$ hadrons



$\sim \sigma^{tot}(e^+e^- \to \text{hadrons})$



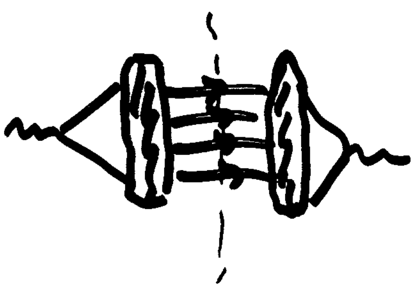
$\sim$



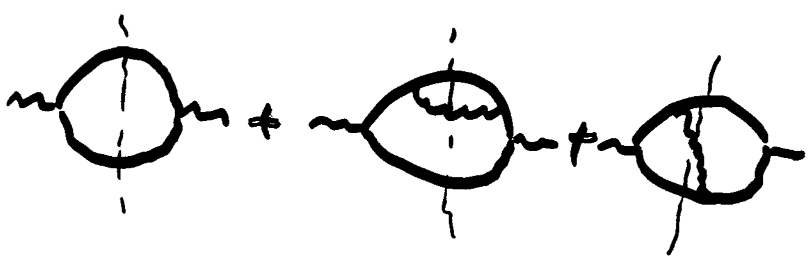
all possible hadrons

FOR LARGE $q^2 \equiv s$

$$\sum_{\text{HADRONS}} \ldots \simeq \sum_{\substack{\text{QUARKS} \\ \& \text{ GLUONS}}}$$



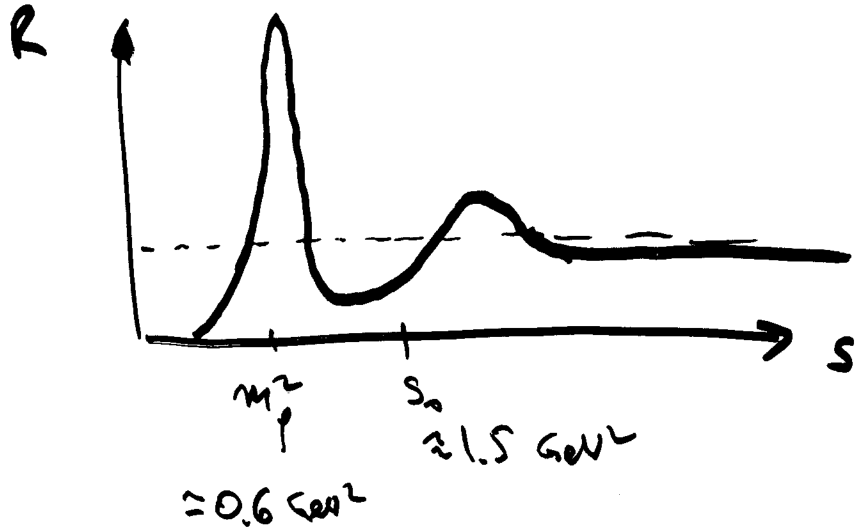
$\simeq$



$+ \ldots$

QUARKS & GLUONS WITH "100% PROBABILITY" CONVERT INTO HADRONS (?)

$$R = \frac{\sigma^{tot}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



NB: FOR $s \sim m_\rho^2$: $\sigma^{hadr} \gg \sigma^{quark}$

FOR SMALL s : $\sigma^{hadr} \ll \sigma^{quark}$

$$\int_0^{s_0} \sigma^{hadr}(s)\,ds \simeq \int_0^{s_0} \sigma^{quark}(s)\,ds$$

DOMINATED BY ρ

$s_0 \simeq 1.5\ GeV^2$

ρ IS DUAL TO PART OF QUARK "SPECTRUM"

QCD SUM RULES (SVZ)

FREE QUARKS

$$\int_0^\infty e^{-s/M^2} R^{I=1}(s)\,ds = \frac{3}{2}M^2\left[1+\frac{\alpha_s(M)}{\pi} - \frac{2\pi^2 f_\pi^2 m_\pi^2}{M^4}\right.$$

$$\left. + \frac{\pi^2}{3M^4}\left\langle \frac{\alpha_s}{\pi} GG\right\rangle - \frac{448\pi^3}{M^6}\alpha_s \langle qq\rangle^2\right]$$

GLUON CONDENSATE QUARK

$$\simeq \frac{3}{2}M^2\left[1+\frac{\alpha_s(M)}{\pi} + 0.1\left(\frac{0.6\,\mathrm{GeV}^2}{M^2}\right)^2 - 0.14\left(\frac{0.6\,\mathrm{GeV}^2}{M^2}\right)^3\right]$$

POWER CORRECTIONS (HIGHER DIMENSIONS)

TAKE $M^2 \to \infty$:

$$\int_0^\infty \left(R^{hadr}(s) - R^{quark}(s)\right) ds\, e^{-s/M^2} = O\left(\frac{1}{M^4}\right)+\ldots$$

$$\frac{3}{2} \qquad\qquad 1 \qquad\qquad \to 0$$

$$\int_0^\infty \left(R^{hadr}(s) - R^{quark}(s)\right) ds = 0.$$

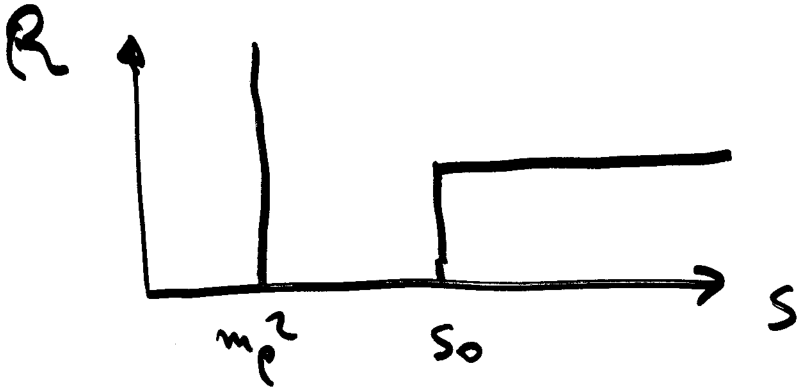
OR GLOBAL DUALITY

$$\int_0^{s_{cont}} ds\, R^{hadr}(s) = \int_0^{s_{cont}} R^{quark}(s)\,ds$$

s_{cont} > onset of continuum

APPROXIMATION:

$$R^{hadr}(s) \approx \pi f_\rho^2 \, \delta(s - m_\rho^2) + R^{quark}(s)\,\theta(s > s_0)$$



FITTING WITH KNOWN CONDENSATE VALUES: $m_\rho^2 \simeq 0.6 \text{ GeV}^2$

$s_0 \simeq 1.5 \text{ GeV}^2$

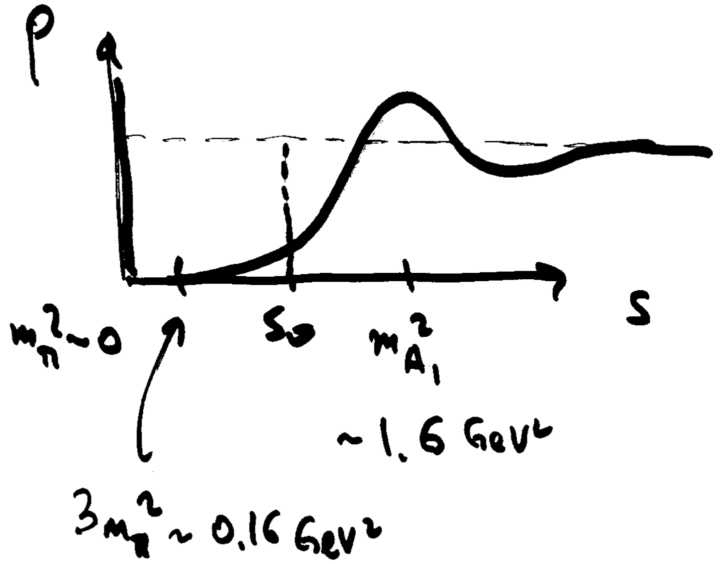
SINCE $R^{model}(s) = R^{quark}(s)$ FOR $s > s_0$,

$$\int_0^{s_0} R^{quark}(s)\, ds = f_\rho^2$$

LOCAL DUALITY

$s_0 \equiv \rho$ duality INTERVAL

SPECTRUM IN AXIAL CURRENT CHANNEL



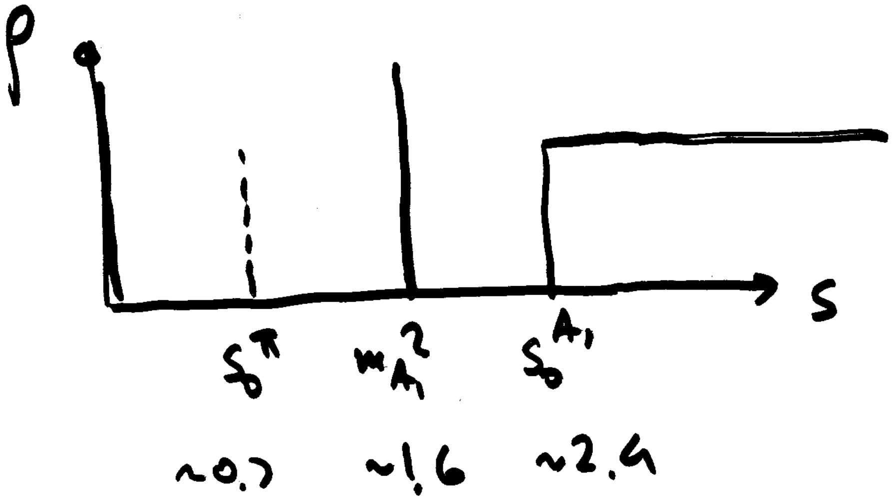
DUALITY INTERVAL:

$$s_0 = 4\pi^2 f_\pi^2 \simeq 0.67\,\mathrm{GeV}^2$$

3π THRESHOLD

$$(3m_\pi)^2 \simeq 0.16\ \mathrm{GeV}^2$$

DUALITY INTERVAL $s_0^{(\pi)}$ HAS NOTHING TO DO WITH 3π THRESHOLD

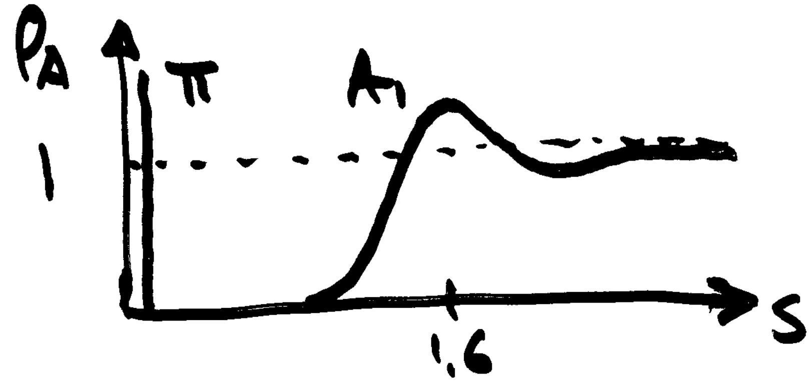


model WITH INFINITELY NARROW A_1

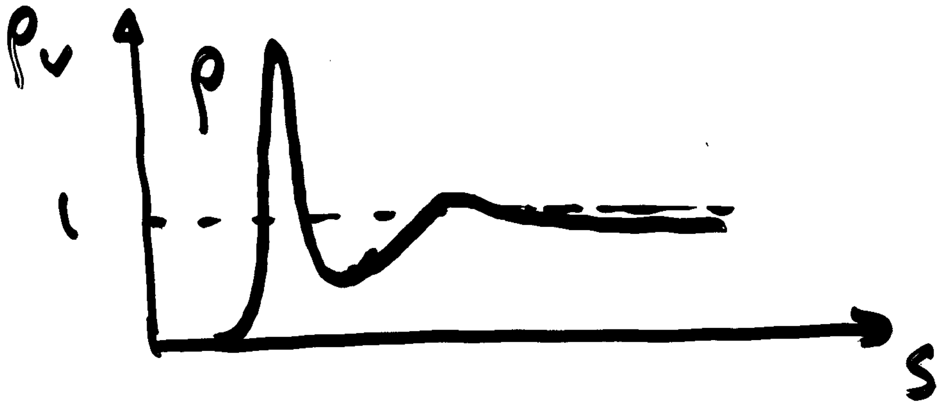
(QCD SR)

TAKE A-V (OR V-A) COMBINATION

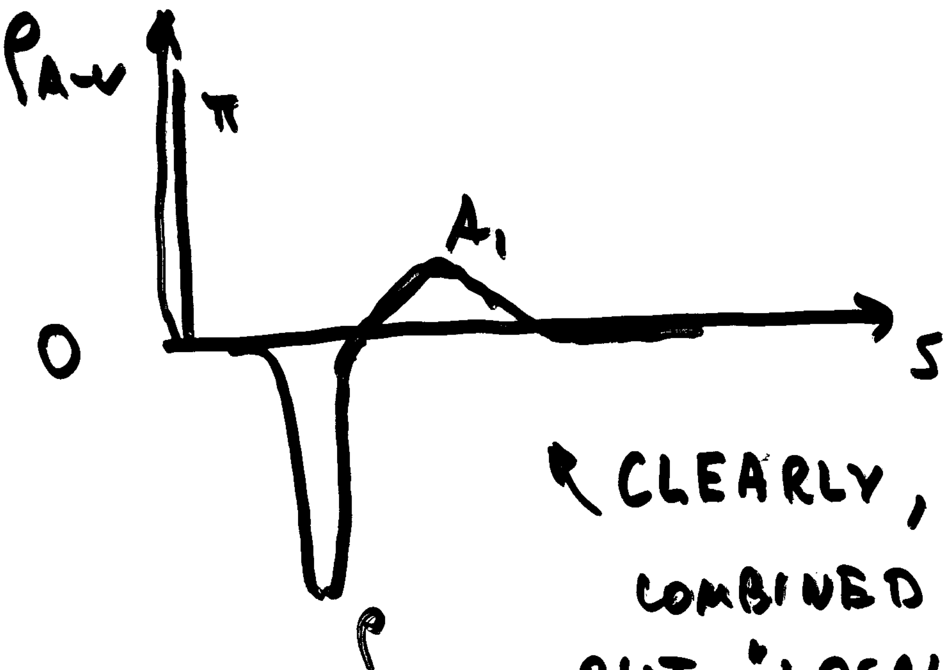
(e.g., τ-LEPTON DECAY INTO HADRONS)



$\rho_A^{quark} = \text{"1"}$



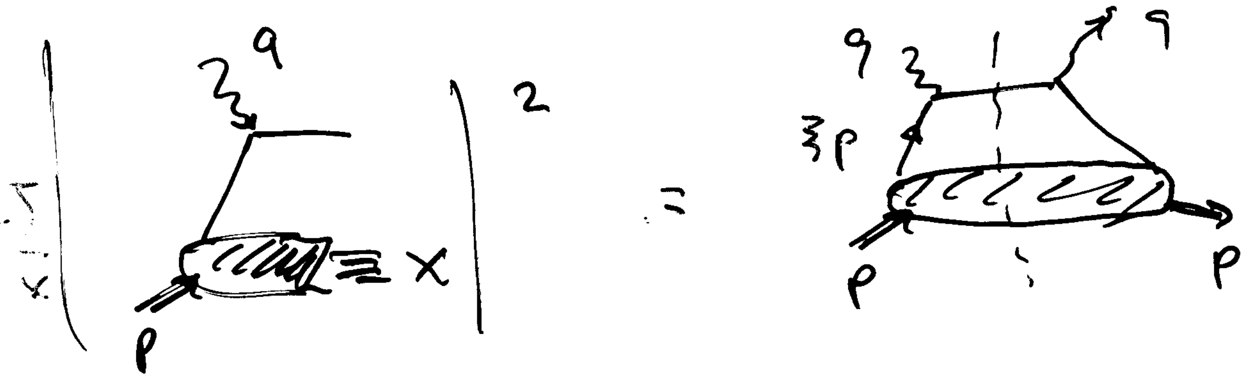
$\rho_V^{quark} = \text{"1"}$



$\rho_{A-V}^{quark} = \text{"0"}$

CLEARLY, THE π, ρ, A_1 COMBINED ARE DUAL TO 0 BUT "LOCAL" DUALITY DOES NOT WORK STATE BY STATE

STUDIES OF DUALITY AT J LAB: DIS IN RESONANCE REGION.



NACHTMANN VARIABLE

$$(\xi p + q)^2 = 0$$ — ON SHELL QUARK

$$\xi^2 m_p^2 + 2\xi (pq) + \underbrace{q^2}_{\to -Q^2} = 0$$

$$2pq = \frac{Q^2}{x_{Bj}}$$

$$\xi = \frac{2x_{Bj}}{1+\sqrt{1+4x_{Bj}^2 m_p^2/Q^2}}$$

DGP DUALITY FOR DIS

← NACHTMANN MOMENTS

$$\int_0^1 \text{"}\xi^n\text{"} F_2(x, Q^2)\, d\xi = A_n + B_n \frac{1}{Q^2} + C_n \frac{1}{Q^4} + \ldots$$

NACHTMANN/GP VARIABLE

TWIST 2 TWIST 4 TWIST 6

$$A_n = \int_0^1 \text{"}\xi^n\text{"} F_2^{\text{SCALING}}(\xi, Q^2)\, d\xi$$

LOG Q^2-DEPENDENCE DUE TO EVOLUTION ONLY

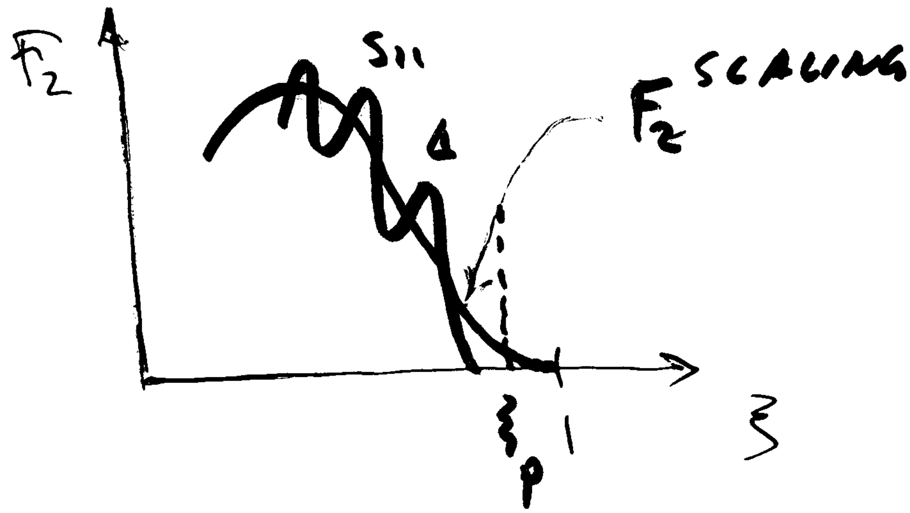
SMALL HIGHER TWISTS FOR $n=0$ →

$$\int_0^1 \left(F_2(x, Q^2) - F_2^{\text{SCALING}}(\xi)\right) d\xi \simeq 0.$$

$$\xi = \frac{2x}{1 + \sqrt{1 + 4x^2 m_N^2 / Q^2}}$$

ELASTIC POINT : $x = 1$ →

$$\xi_p = \frac{2}{1 + \sqrt{1 + 4 m_N^2 / Q^2}} < 1$$



NUCLEON ("ELASTIC") CONTRIBUTION

IS DUAL TO SOME INTERVAL

BETWEEN $\xi = 1$ AND ξ_0

CORRESPONDING TO

$$W_0^2 \sim \frac{m_N^2 + m^2_{NEXT}}{2}$$

IF $m_{NEXT} = m_\Delta$, THEN $W_0^2 \simeq 1.2\ \text{GeV}^2$

IF $m_{NEXT} \approx 1.5$ GeV THEN $W_0^2 \sim 1.6\ \text{GeV}^2$

QCD SUM RULES SUGGEST THAT DUALITY WORKS FOR CHANNELS WITH FIXED QUANTUM NUMBERS

$$W_2 \sim \sigma^{1/2} + \sigma^{3/2}$$

$$G_1 \sim \underbrace{\sigma^{1/2}}_{\text{nucleon + delta}} - \underbrace{\sigma^{3/2}}_{\text{delta}}$$

TAKE $(W_2 - G_1)$ - delta is the 1st resonance

"Δ" IS DUAL TO $F_2 - G_1$

SINCE $G_1 \sim F_2$ FOR $x \to 1$

$F_2 - G_1$ HAS AN EXTRA $\sim(1-x)$ SUPPRESSION

IF DUALITY WORKS FOR Δ IN $F_2 - G_1$,

$p \to \Delta$ FORM FACTOR SHOULD FALL FASTER THAN $p \to p$ FORM FACTOR

(CALCULATIONS IN PROGRESS)

EXTRACTING $G_M^p(Q^2)$ FROM JLAB DATA IN RESONANCE REGION & DUALITY ASSUMPTIO

"DUALITY CURVE"

$$F_2^{DUAL}(\xi) \simeq \xi^{1.12}(1-\xi)^{2.62}\left(2.7 - 21(1-\xi) + 88(1-\xi)^2 - 131(1-\xi)^3 + 67(1-\xi)^4\right)$$

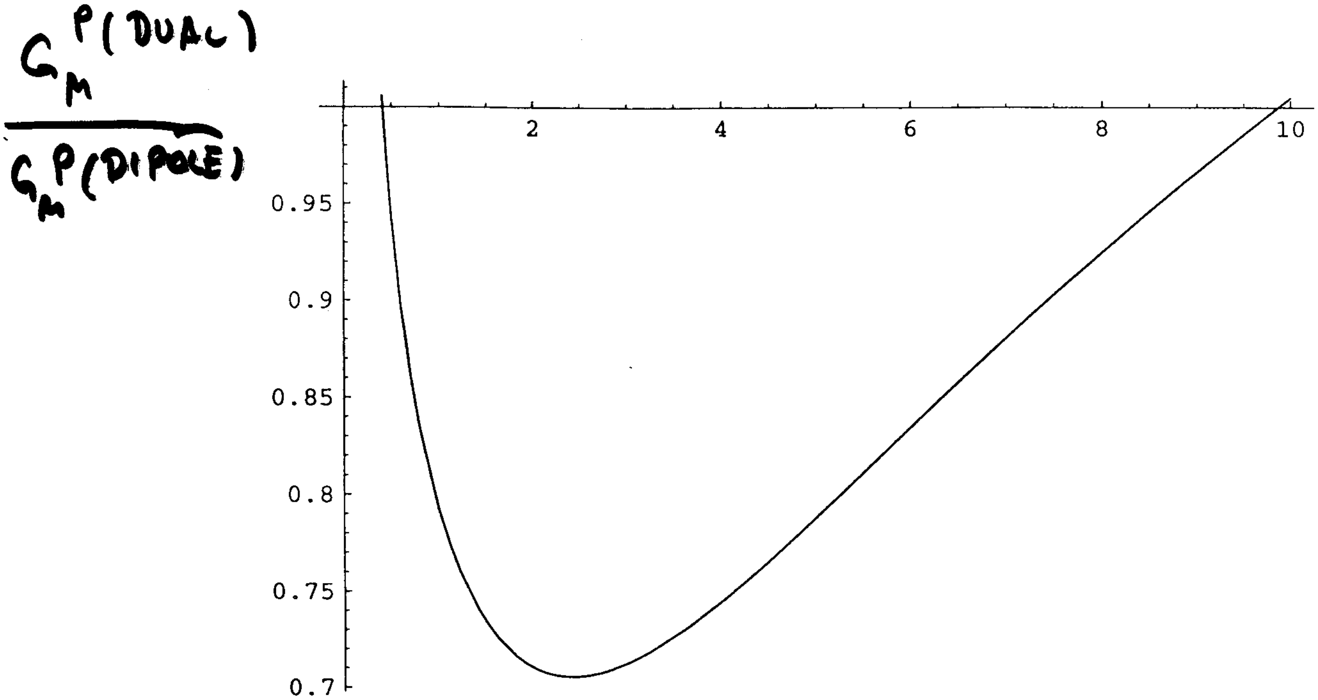
(I. NICULESCU)

$$F_2^{ELASTIC}(\xi) = \delta(\xi-\xi_p)\,\frac{1+\frac{4m^2}{\mu^2 Q^2}}{1+\frac{4m^2}{Q^2}}\,\frac{\xi_p^2}{2-\xi_p}\left(G_M^p(Q^2)\right)^2$$

$$\int_{\xi_0}^{1} F_2^{DUAL}(\xi)\,d\xi = \int_{\xi_0}^{1} F_2^{ELASTIC}(\xi)$$

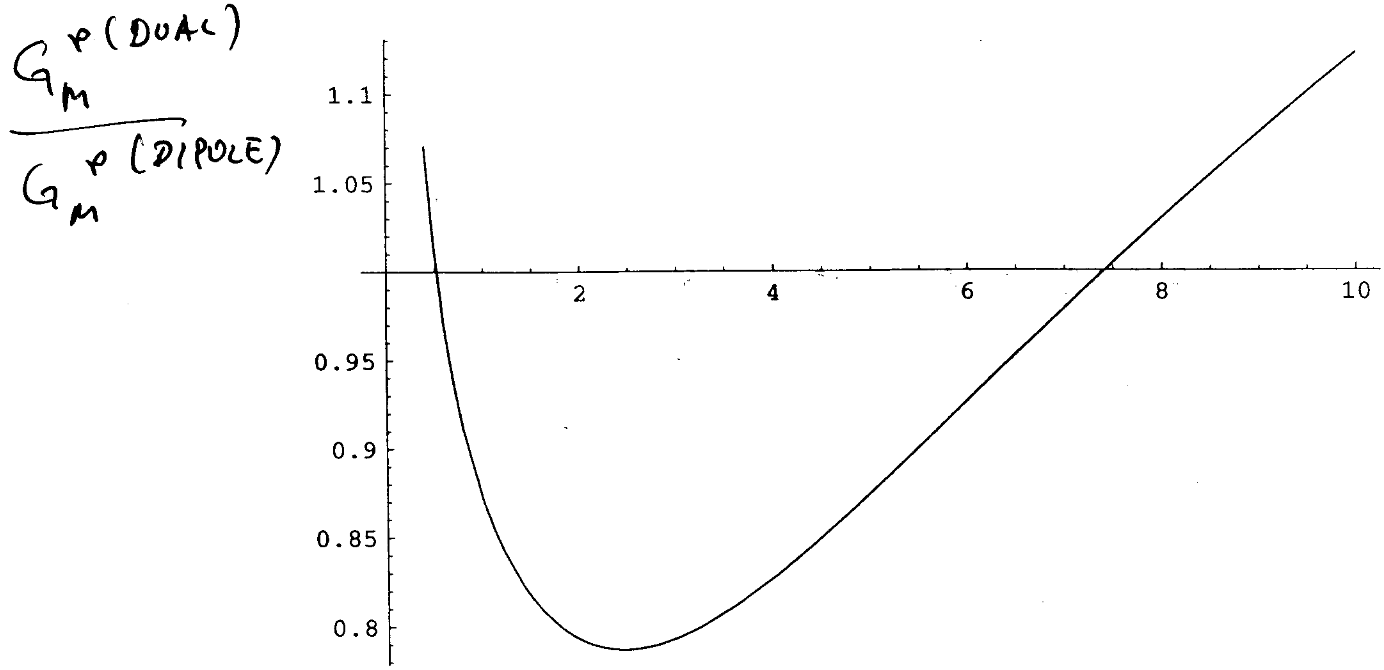
$$\xi_0 = \frac{2x_0}{1+\sqrt{1+4x_0^2 m^2/Q^2}}, \quad x_0 = \frac{1}{\frac{W^2-m^2}{Q^2}+1}$$

NUCLEON DUALITY INTERVAL

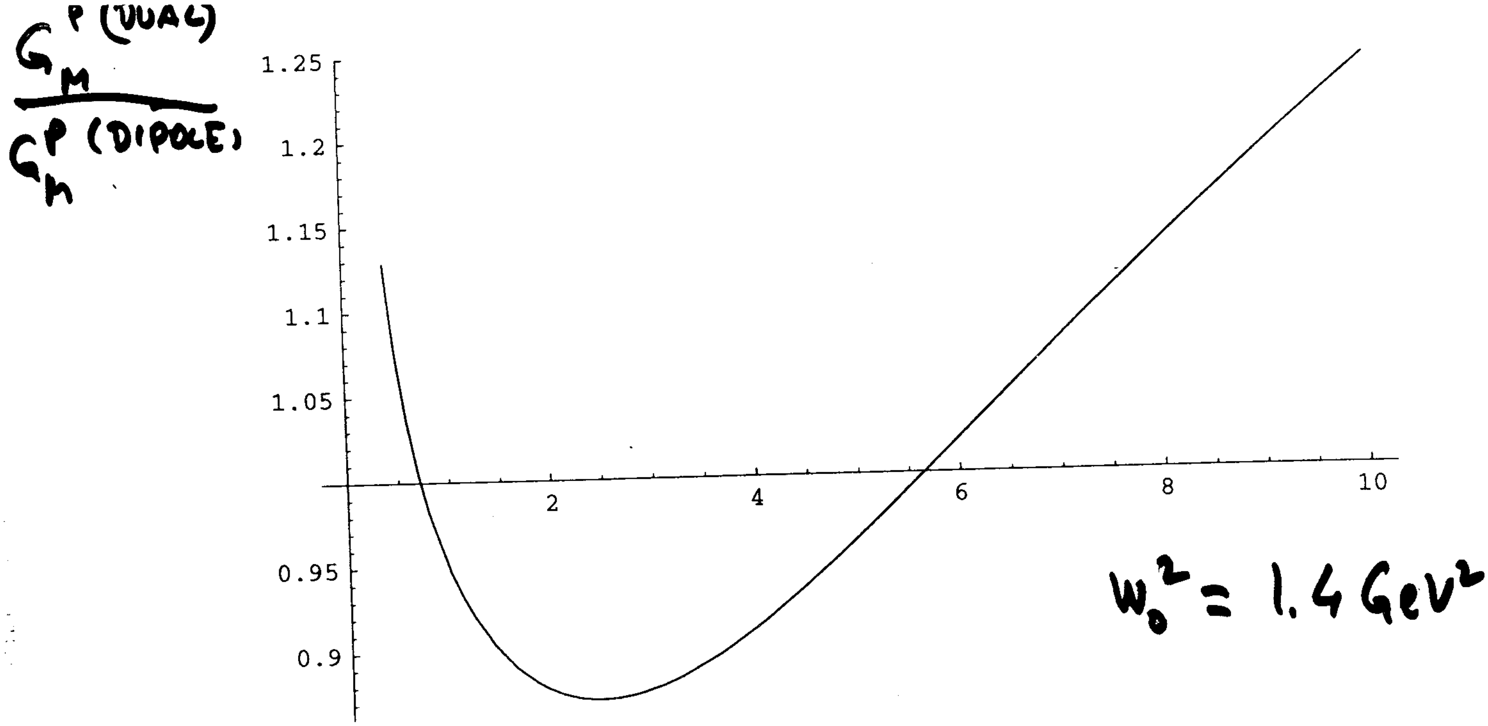


$W_D^2 = 1.2\ \text{GeV}^2$

$\simeq (m_N + m_\pi)^2$

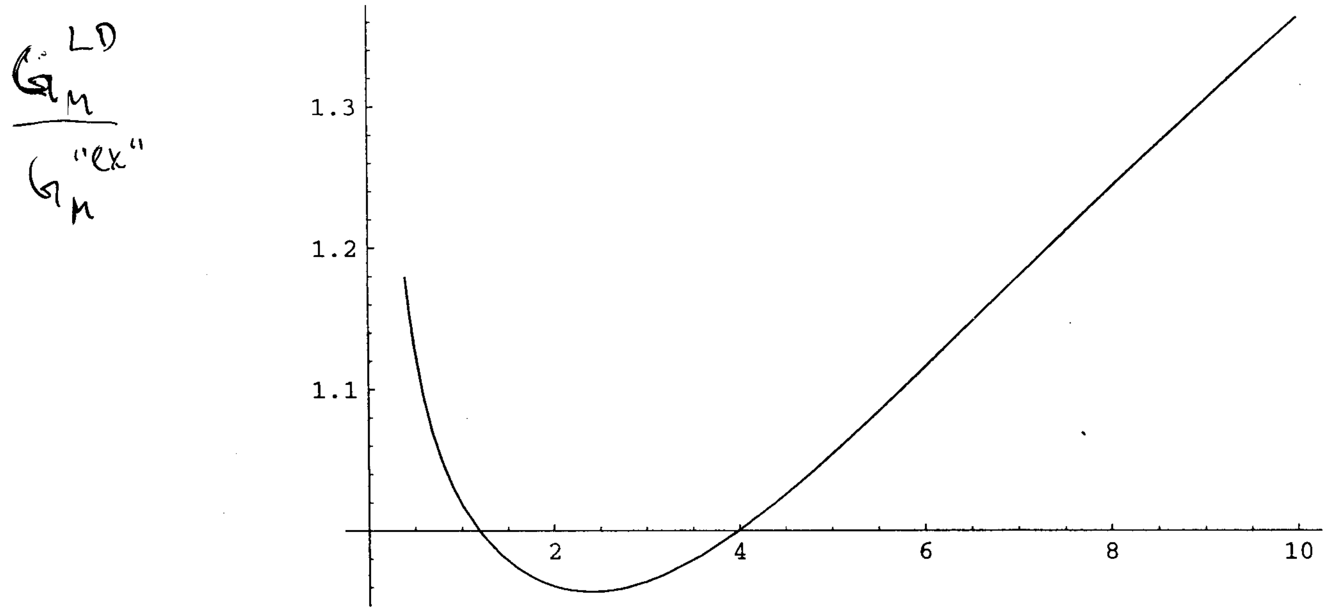


$W^2 = 6.3\ GeV^2$



Out[16]= - Graphics -

$$G_M^{p\,(\mathrm{DIPOLE})} = \frac{2.79}{\left(1 + \frac{Q^2}{0.71\,\mathrm{GeV}^2}\right)^2}$$



$W^2 = 1.5\ GeV^2$

CONCLUSIONS

- DIS DATA CAN BE / SHOULD BE USED IN GLOBAL PARAMETERIZATIONS & FOR STUDY OF EVOLUTION AT LOW Q^2 AND LARGE x
- POLARIZED DIS IN THE RESONANCE REGION CAN PROVIDE INTERESTING INFORMATION ON THE CHARACTER OF DUALITY
- SEPARATE $\sigma^{1/2}$ AND $\sigma^{3/2}$ IN FUTURE STUDIES OF LOCAL DUALITY