Pentaquark States from Lattice QCD with Overlap Fermion *

- Synopsis of overlap fermion
- Baryon excited states – Roper resonance and S_{11}
- Pentaquark States

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Lattice Formulation of QCD

Symmetries at Finite Cut-off

1. Gauge Symmetry — Wilson, 1974
   $A_\mu \longrightarrow U_\mu$

2. Chiral Symmetry ?
   — Nielsen - Ninomiya no go theorem

Difficulties of Ultralocal Actions

– Wilson Fermion

- Breaking of chiral symmetry at finite lattice spacing a
- $\bar{\Psi}\Psi$ mixes with unity
- Additive renormalization of quark masses;
  κ_c depends on the gauge configuration
- Exceptional configurations
- No unambiguous identification of fermion zero modes with topology
- Mixing of operators in different chiral sectors
- O(a) error (can be removed with improved action but with fine tuning)

– Staggered Fermion

- Breaking of flavor symmetry
- Topologically blind
- Dynamical fermion action is non-local (?)

Lattice Chiral Fermion

Synopsis of Overlap Fermion

- Domain wall Fermion, overlap fermion, and fixed-point-action fermion.
- Their Dirac operators satisfy Ginsparg-Wilson relation
  $\{\gamma_5, D\} = D\gamma_5 D \underset{a\to 0}{\Longrightarrow} 0$, $D = aD_a$
- Lattice chiral symmetry (Lüscher):
  $\delta\psi = T\gamma_5(1-\frac{1}{2}D)\psi, \delta\bar{\psi} = \bar{\psi}(1-\frac{1}{2}D)\gamma_5 T$, $\delta(\bar{\psi}D\psi) = 0$
- Continuum current algebra relations are retained upon the substitution $\psi \longrightarrow (1-\frac{1}{2}D)\psi$.
- $U(1)$ anomaly and topology is similar to continuum.
  Chiral Jacobian a la Fujikawa:
  $J = exp[-2i\epsilon Tr\gamma_5(1-1/2D)] = exp[-2i\epsilon(n_+ - n_-)]$
  $Q = n_- - n_+ = -Tr\gamma_5(1-1/2D)$
  $\Longrightarrow q(x) = -Tr\gamma_5(1-1/2D(x,x))$
- Overlap fermion (Neuberger):
  $D = 1 + \gamma_5\epsilon, \epsilon = H/\sqrt{H^2}$,
  $H = \gamma_5(D_W(m=0) - m_0), 0 < m_0 < 2.$
- Free quark: $D(p) \simeq i \not{p}$ for small p, no doublers.
- The massive fermion action is then

$$D(m) = D + ma(1-\frac{1}{2}D) \tag{1}$$

Lattice Chiral Fermion

Effective Quark Propagator on the Lattice

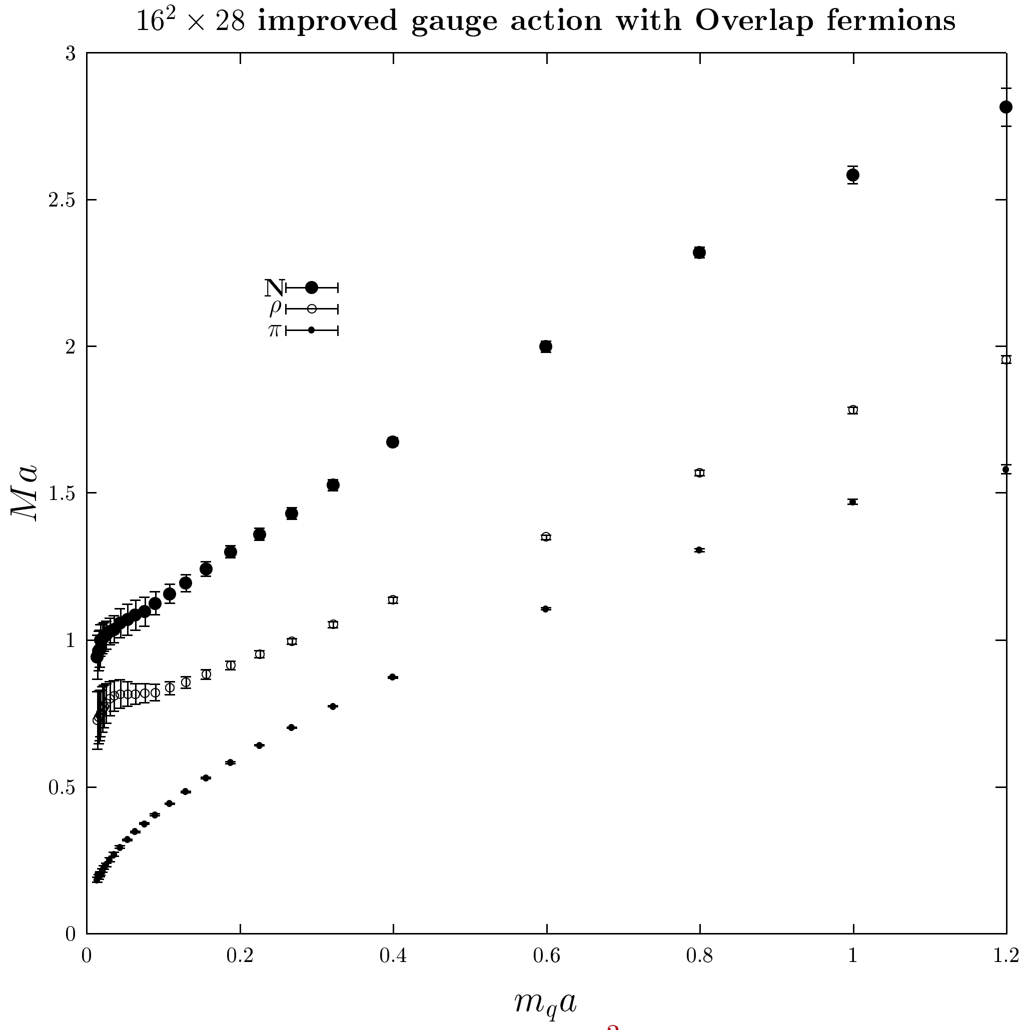
- Consider chirally regulated field $\hat{\psi} = (1 - \frac{1}{2}D)\psi$. The Grassmann integration of $\hat{\psi}\bar{\psi}$ leads to the effective propagator

$$(1 - \frac{1}{2}D)D(m)^{-1} = (D_c + ma)^{-1}. \tag{2}$$

  - $D_c = D/(1 - \frac{1}{2}D)$ is chirally symmetric in the continuum sense, i.e. $\{\gamma_5, D_c\} = 0$.
  - Unlike D, D_c is non-local.
  - $D_c^\dagger = \gamma_5 D_c \gamma_5$ and $\{\gamma_5, D_c\} = 0$
    $\Longrightarrow D_c$ is anti-hermitian.
  - Eigenvalues of $D_c = ma$ correspond to the stereographic projection of the circle onto the imaginary axis shifted by ma.
  - One-to-one correspondence between the eigenstates of $D(m)$ and $D_c + ma$, except the 'north pole' at the cut-off $\lambda = 2$ which is excluded by the chiral regulator projector.
  - $D_c + ma$ is like in the continuum formalism.
  - There is no distinction between heavy and light quarks. The quark mass is only an additive parameter like in the continuum. The only question is how large the $O(m\Lambda_{QCD}a^2)$ and $O(m^2a^2)$ errors are.

Results from Overlap Fermion

- Expected:
  - $*$ Numerically checked that there is no additive quark mass renormalization
  - $*$ No exceptional configuration.
  - $*$ No $O(a)$ error.
- Unexpected desirable features:
  - $*$ $O(a^2)$ error is small.
- 2nd unexpected feature:
  - $*$ Critical slowing down is gentle all the way to the physical pion mass region. The smallest $m_\pi \sim 180$ MeV.

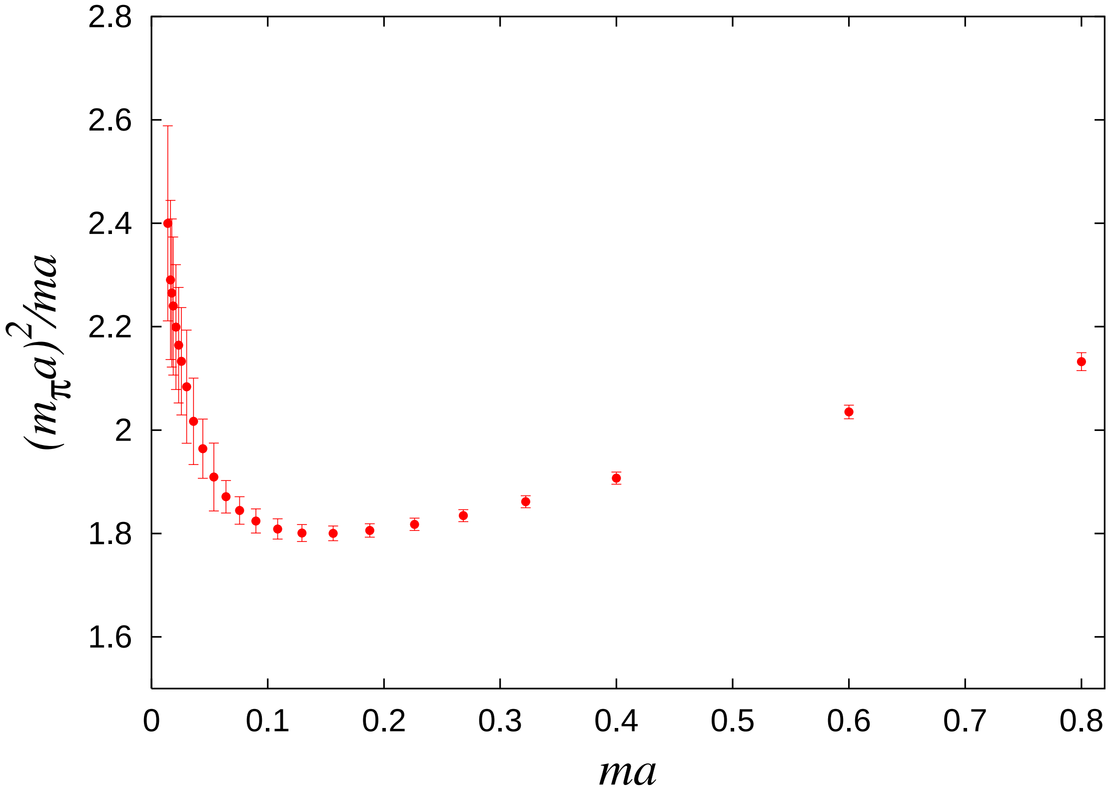


π, ρ and nucleon masses from $16^3 \times 28$ lattice with $a = 0.200$ fm. L = 3.2 fm.

Quenched Artifacts:

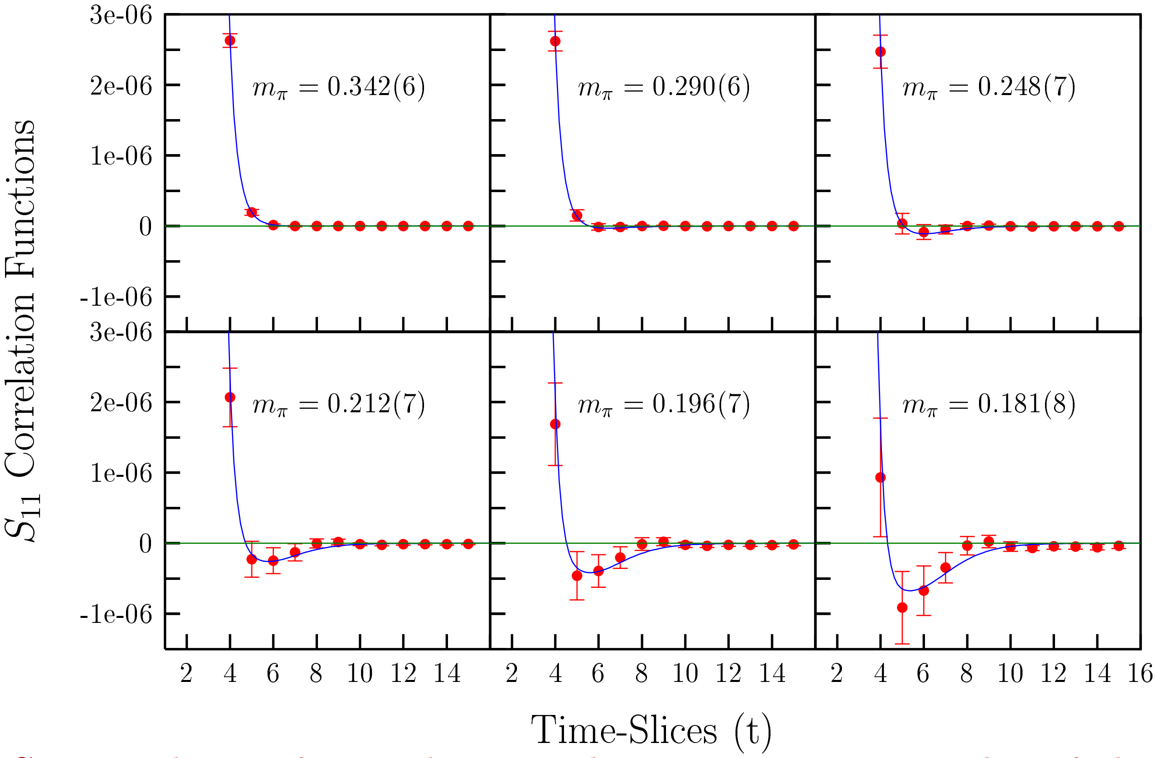
- Chiral log in m_π^2

$$m_\pi^2 = Am\{1-\delta[\ln(Am/\Lambda_\chi^2)+1]\}+A_\alpha m^2[1+2\ln(Am/\Lambda_\chi^2)]+Bm^2, \quad (3)$$



m_π^2/m_q as a function of m_q

- Ghost states in the nucleon correlator

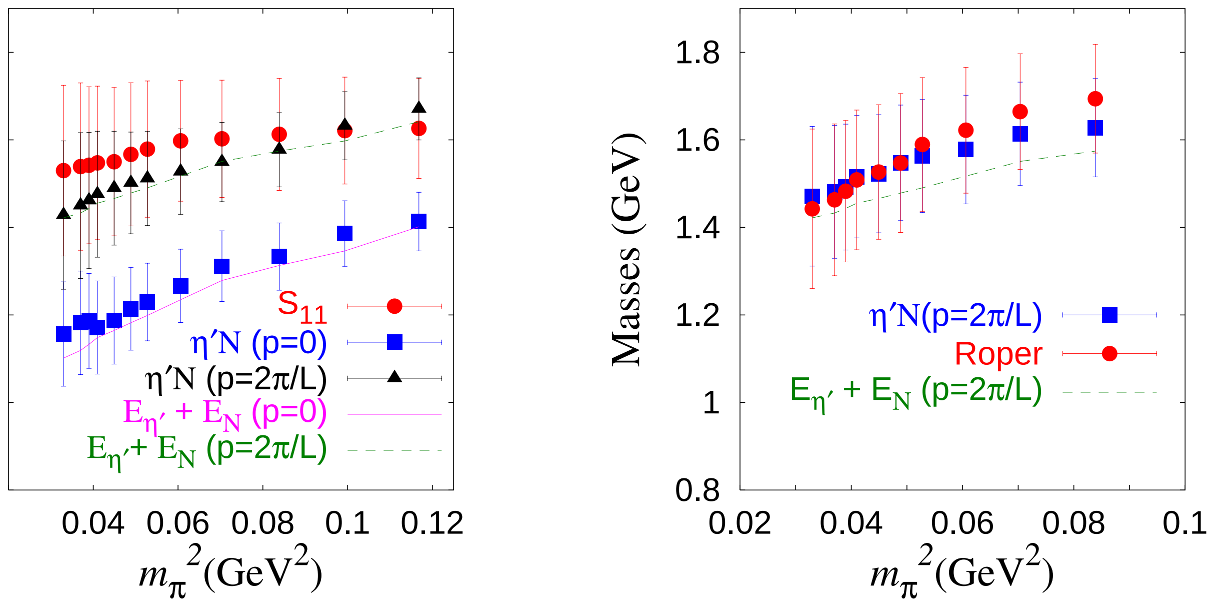


S_{11} correlators for six low quark masses. Negative dip of the correlators is an indication of the domination of the ghost $\eta' N$ state over the physical S_{11}. m_π are in GeV.

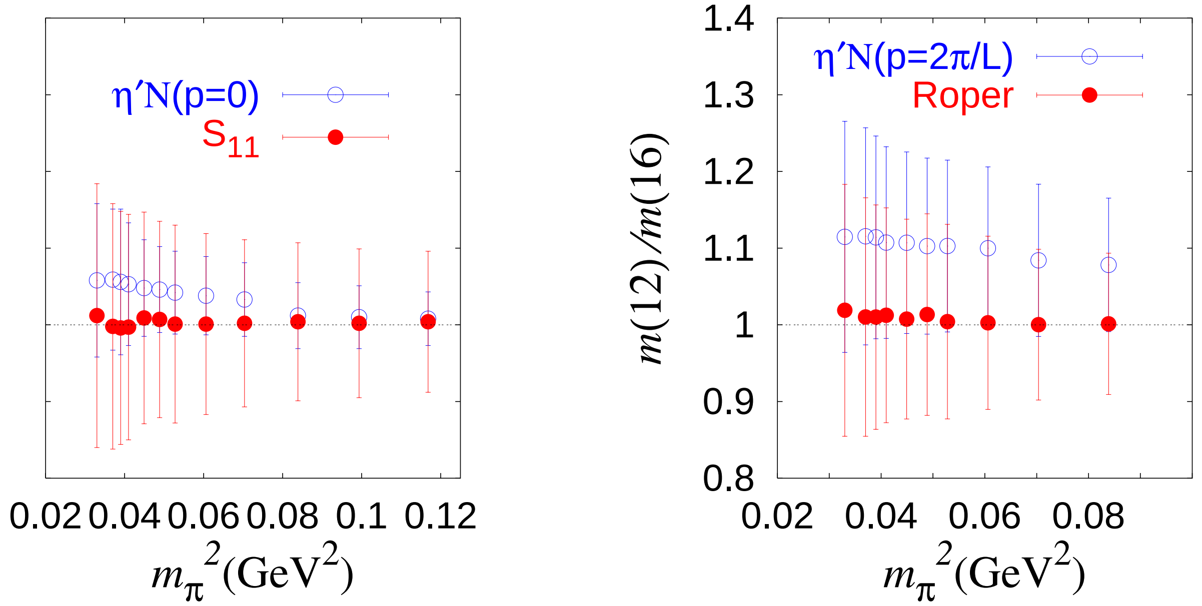
$$\Delta G_{NN}(t) = -m_0^2 \frac{|Z|^2 T}{16 L^3} \sum_{\vec{p}} \frac{E_N + E_\pi + m_N}{E_N^2 \, E_\pi^3} \times \frac{\vec{p}^{\,2}}{m_N^2} (1 + E_\pi t) e^{-(E_N + E_\pi) t}. \quad (4)$$

Roper and S_{11}

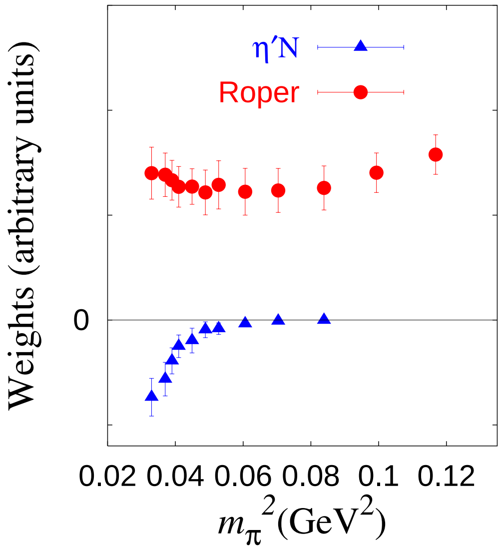
- 80 quenched configurations with Iwasaki gauge action at $a = 0.200$ fm on a $16^3 \times 28$ and $12^3 \times 28$ lattices
- Interpolation field — $\chi = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$

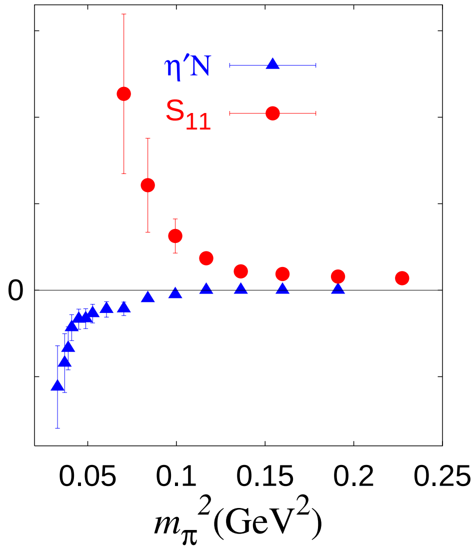


S_{11} (left) and Roper (right) masses with their associated $\eta' N$ ghost states.

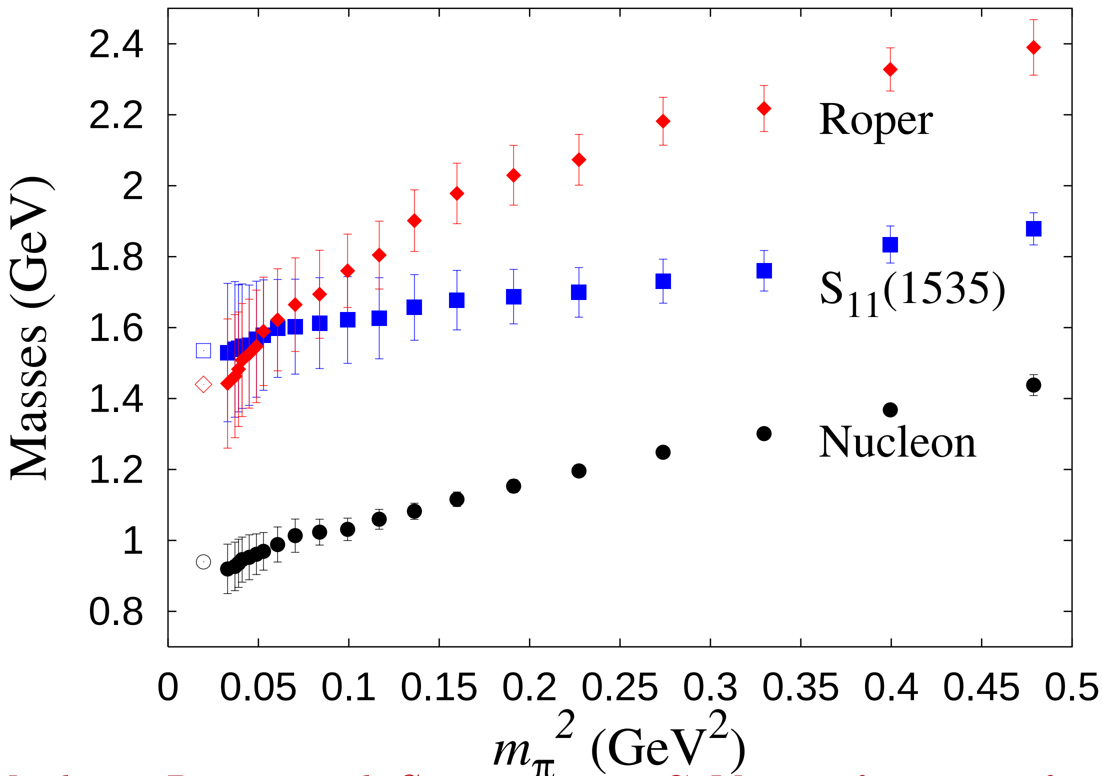


Ratio of masses from 12^3 to 16^3 lattices for S_{11} and S-wave $\eta' N$ (left) and Roper and P-wave $\eta' N$ (right).





Weights of Roper and $\eta' N$ (left), and S_{11} and $\eta' N$ (right).



Nucleon, Roper, and S_{11} masses in GeV as a function of m_π^2. The experimental values are indicated by the corresponding open symbols.

Pentaquark States

Lattice calculation:

- 80 quenched configurations with Iwasaki gauge action at $a = 0.200$ fm on a $16^3 \times 28$ and $12^3 \times 28$ lattices
- Interpolation field ($K^- P \mp K^0 n$ for $I = 0, 1$)

$$\chi = [\epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c(\bar{s}_e\gamma_5 d_e)] \mp [u \leftrightarrow d]. \tag{5}$$

- Threshold energies:

$$E(1/2^-) = m_K + m_N, \tag{6}$$

$$\tag{7}$$

$$E(1/2^+) = \sqrt{m_K^2 + (\frac{2}{a}\sin\left(\frac{p_L}{2}\right))^2} + \sqrt{m_N^2 + (\frac{2}{a}\sin\left(\frac{p_L}{2}\right))^2}, \tag{8}$$

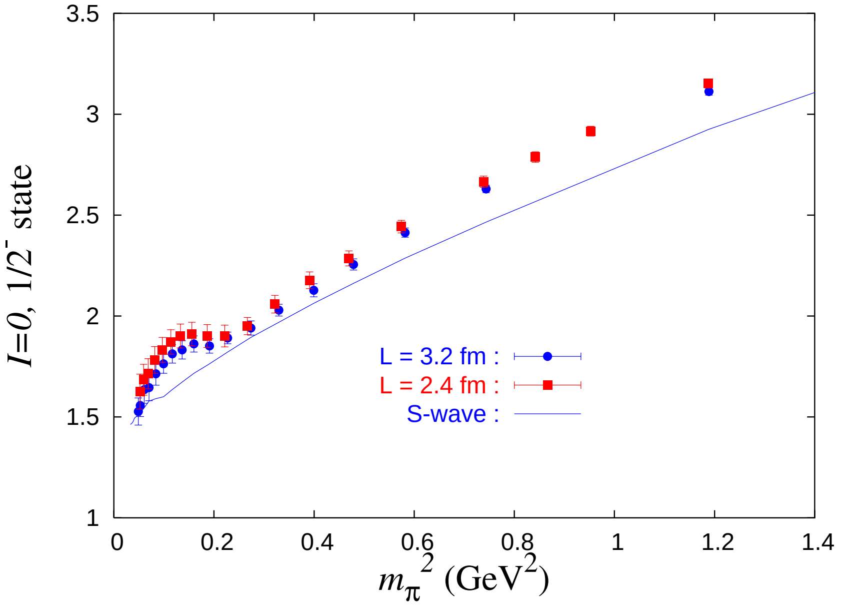
where $p_L = \frac{2\pi}{L} = 387/517$ MeV ($16^3 \times 28/12^3 \times 28$ lattice)

- Energy shift on the finite lattice:

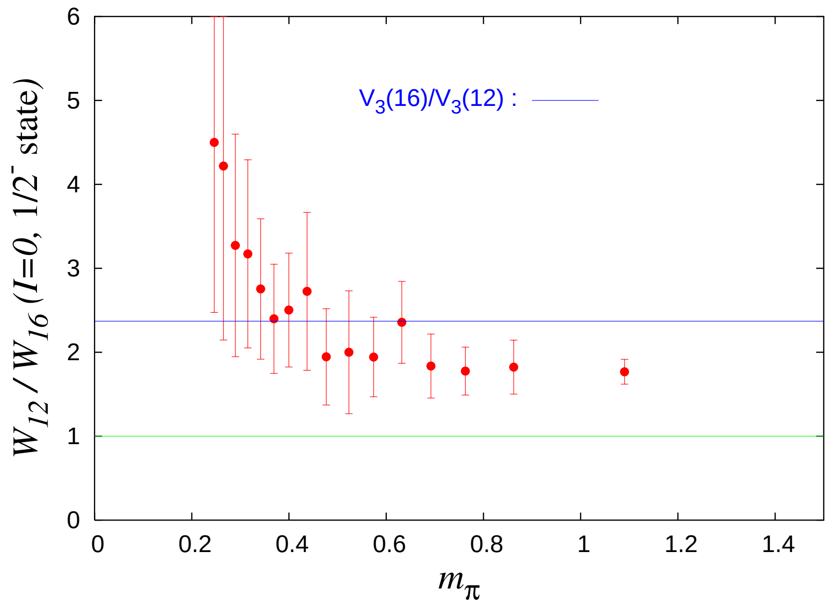
$$E_{KN} - (m_K + m_N) = -\frac{2\pi a_0}{\mu_{KN}L^3}[1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}] + O(L^{-6}). \tag{9}$$

- Expt. scattering lengths (volume):
  - S-wave (fm): 0.0 ± 0.03 (I = 0); -0.32 ± 0.02 (I = 1)
  - P-wave (j= 1/2)(fm^3): 0.08 ± 0.01 (I = 0); -0.16 ± 0.1 (I = 1)
- $E_{KN} - (m_K + m_N) \sim 8/18$ MeV for $16^3 \times 28/12^3 \times 28$ lattice for S-wave ($1/2^+$), $I = 1$ case.

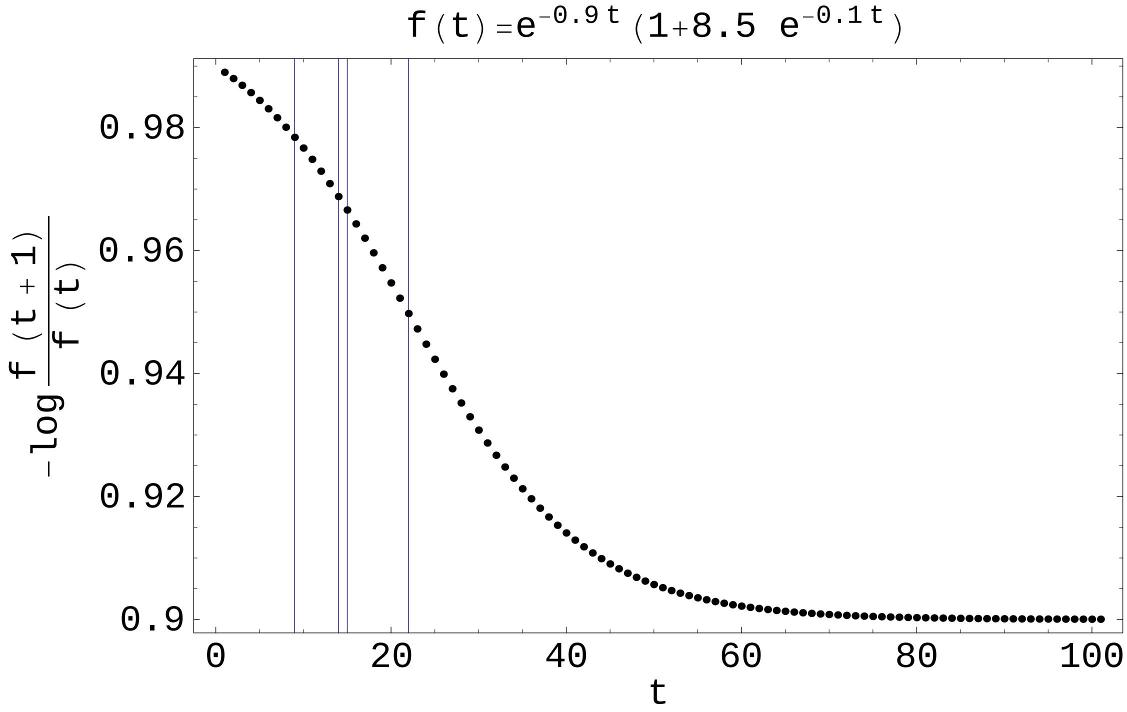
S-wave $(1/2^-)$



- No need to consider ghost states (propagators are positive)
- Lowest states in $12^3 \times 28$ are higher than those in $16^3 \times 28$ which reflect the volume dependence of the energy shift in Eq. (9).
- The first excited states are near the $E_K(p = p_L) + E_N(p = p_L)$ which are several hundred MeV higher.
- Ratio of spectral weight for two non-interacting particles $W(12)/W(16) = V_3(16)/V_3(12) = 2.37$



- Lattice calculation of S. Sasaki (hep-lat/0310014) with Wilson fermion: two plateaus in effective mass plot with $ma = 0.9$ and 1.0



- Lattice calculation of F.Csikor, Z. Fodor, S.D. Katz, and T.G. Kovacs (hep-lat/0309090) with Wilson fermion: two nearly degenerate $1/2^-$ states at the threshold.

P-wave $(1/2^+)$

- Complication due to the $KN\eta'$ ghost states which contribute near the P-wave threshold at small quark masses — pentaquark correlators turns negative.
- The volume dependence in $E_K(p=p_L)+E_N(p=p_L)$ due to the P-wave nature is seen for medium and high quark masses. Near the chiral limit, the scattering length is close to zero which is consistent with experiment.