Pentaquark in the SU(3) Skyrme Model and Chiral Quark Soliton Model

Michał Praszałowicz


M. Smoluchowski Institute of Physics, Jagellonian University, Kraków, Poland
and
Nuclear Theory Group, BNL
NOTE ON THE $S = +1$ BARYON SYSTEM

The evidence for strangeness $+1$ baryon resonances was reviewed in our 1976 edition,\textsuperscript{1} and more recently by Kelly\textsuperscript{2} and by Oades.\textsuperscript{3} Two new partial-wave analyses\textsuperscript{4} have appeared since our 1984 edition. Both claim that the $P_{13}$ and perhaps other waves resonate. However, the results permit no definite conclusion — the same story heard for 15 years. The standards of proof must simply be much more severe here than in a channel in which many resonances are already known to exist. The general prejudice against baryons not made of three quarks and the lack of any experimental activity in this area make it likely that it will be another 15 years before the issue is decided.

References

1. Particle Data Group, Rev. Mod. Phys. 48, S188 (1976).
Chiral Quark Model

\[ Z = \int D\pi^A \int D\psi^\dagger D\psi \ exp \int d^4x \ \psi^\dagger(x) [i\partial^\mu + iMU^\gamma_5(x)] \psi(x) \]

constituent quark mass \( \sim 350 \text{ MeV} \)

integrate out quarks

"Skyrme" Model

\[ \mathcal{L} = \frac{F_\pi^2}{4} \ Tr \left( \partial_\mu U^\dagger \partial^\mu U \right) + \frac{1}{32e^2} \ Tr \left( \left[ \partial_\mu U U^\dagger, \partial_\nu U U^\dagger \right]^2 \right) + \Gamma_{WZ} + \ldots \]

only pion fields, kinetic term + interaction terms
SU(3) soliton: static solution

hedgehog Ansatz:

\[ U_0 = \begin{bmatrix} e^{i n \cdot \vec{r}} P(r) & 0 \\ 0 & 1 \end{bmatrix} \]

\( P(0) = n\pi \) and \( P(\infty) = 0 \) \( n \) - winding number

variational approach: \( P(r) = P(r/r_0) \), \( r_0 \) - soliton size

Energy has minimum as a function of \( r_0 \)

In the Skyrme model winding number is interpreted as a baryon number.
Quantizing SU(3) Skyrmion

time-dependent rotation

angular velocities:

\[ U = A(t)U_0A^\dagger(t) \]

\[ A^\dagger \dot{A} = \frac{i}{2} \Omega_i \lambda_i = \frac{i}{2} \dot{\theta}_k e_{ki}(\theta) \lambda_i \]
Quantizing SU(3) Skyrmion

every quantity is a power series in $\Omega$

\[ L_0 = -M_{cl} + \frac{I_1}{2} \sum_{a=1}^{3} \Omega_a^2 + \frac{I_2}{2} \sum_{a=4}^{7} \Omega_a^2 + \frac{N_c}{2\sqrt{3}} \Omega_8 \]

no kinetic term for $\Omega_8$

\[ U_0 = \begin{bmatrix} e^{i\vec{n} \cdot \vec{\tau}} P(r) & 0 \\ 0 & 1 \end{bmatrix} \quad \text{commutes with } \lambda_8 \]

\[ U = A(t) \ h U_0 h^\dagger A^\dagger(t) \quad h = e^{i\varphi_8 \lambda_8} \]
Quantizing SU(3) Skyrmion

add small moment of inertia $\varepsilon$

$$L_0 = -M_{cl} + \frac{I_1}{2} \sum_{a=1}^{3} \Omega_a^2 + \frac{I_2}{2} \sum_{a=4}^{7} \Omega_a^2 + \frac{\varepsilon}{2} \Omega_8^2 + \frac{N_c}{2\sqrt{3}} \Omega_8$$

generalized "momenta":

$$J_a = I_1 \Omega_a \text{ for } a = 1, 2, 3$$
$$J_a = I_2 \Omega_a \text{ for } a = 4, 5, 6, 7$$

$$J_8 = \varepsilon \Omega_8 + \frac{N_c}{2\sqrt{3}}$$

Hamiltonian:

$$H_0 = M_{cl} + \frac{1}{2I_1} \sum_{a=1}^{3} J_a^2 + \frac{1}{2I_2} \sum_{a=4}^{7} J_a^2 + \frac{1}{2\varepsilon} \left( J_8 - \frac{N_c}{2\sqrt{3}} \right)^2$$

constraint for $\varepsilon \to 0$
Wave functions
analogy with a symmetric top

\[ \psi \sim N D_{mk}^{(J)}(\varphi', \theta, \varphi) \]

\( m, k - \) angular momentum projections

In SU(3) \( J \rightarrow \mathcal{R} = (p, q) \), \( m, k \rightarrow (Y, I, I_3) \)
however, because of the constraint not all \( k \)'s are allowed but only those which have \( k = (Y=1, I, I_3) \)
Wave functions and allowed states

\[ \psi_{BS}^{(\mathcal{R})} = \psi(\mathcal{R},B)(\mathcal{R}^*,S) = (-)^{Q_S} \sqrt{\text{dim}(\mathcal{R})} D_{BJ}^{(\mathcal{R})^*} \]

\[ B = (Y, I, I_3) \quad S = (Y' = -1, S, S_3) \quad J = (-Y' = 1, S, -S_3) \]
Mass formula

\[ H_0 = M_{c1} + \frac{1}{2I_1} S(S + 1) + \frac{1}{2I_2} \left( C_2(R) - S(S + 1) - \frac{N^2_c}{12} \right) \]

octet-decuplet splitting  \[ \uparrow \]

exotic-nonexotic splittings  \[ \uparrow \]

first order perturbation in the strange quark mass:

\[ \sigma = \frac{1}{3} \frac{\Sigma_{\pi N}}{m} \]

\[ H' = m_{c1} - \alpha D_{88}^{(8)}(A) = m_s\sigma - m_s\sigma D_{88}^{(8)}(A) \]
Gell-Man Okubo mass formulae

\[ \Delta M_{B}^{(8)} = F \begin{pmatrix} 8 & 8 & 8_- \\ 0 & 0 & B \\ 0 & 0 & B \end{pmatrix} + D \begin{pmatrix} 8 & 8 & 8_+ \\ 0 & 0 & B \\ 0 & 0 & B \end{pmatrix} \]

\[ \Delta M_{B}^{(10)} = C \begin{pmatrix} 8 & 10 & 10 \\ 0 & 0 & B \\ 0 & 0 & B \end{pmatrix} \]

three independent constants: \( F, C \) and \( D \)

\[ F = 379, \quad D = 79 \pm 17 \quad \text{and} \quad C = 415 \pm 15 \quad \text{MeV} \]
Chiral model relations

\[ F = -\frac{1}{2} \alpha, \quad D = -\frac{1}{2\sqrt{5}} \alpha, \quad C = -\frac{1}{2\sqrt{2}} \alpha \]

\[ \frac{F}{D} = \sqrt{5} = 2.24 \quad \text{(exp. 4.40)}, \quad \frac{F}{C} = \sqrt{2} = 1.41 \quad \text{(exp. 0.91)} \]
Yabu-Ando: higher orders in $m_s$

\[ H'_{\text{SM}} = -\alpha D^{(8)}_{88}(A) \]

second order:

\[ M_B^{(2)}(R) = -\alpha^2 \sum_{R'} \left| \frac{\langle R', B, S | D^{(8)}_{88} | R, B, S \rangle}{M^{(0)}(R') - M^{(0)}(R)} \right|^2 \]

4 free parameters: $M_{\text{sol}}, I_1, I_2$ and $\alpha$,
but now $I_2$ contributes to nonexotic splittings

\[ \Delta R'R = C_2^{R'}(SU(3)) - C_2^R(SU(3)) \]

fix $\alpha$ and then minimize $\chi^2$ with respect to the remaining parameters

JLab, Nov.7, 2003

 Michał Praszałowicz
\[ \alpha = \frac{m_s}{m_u + m_d} \frac{2}{3} \Sigma_{\pi N} \]

1987 \to \Sigma_{\pi N} \sim 60 \text{ MeV}, \quad m_s \sim 200 \text{ MeV} \to \alpha = 720 \text{ MeV}
the simplest Skyrme model predicts light $\Theta^+$

prediction for $\Xi$ matches NA49 data within model accuracy
Numerical values for all masses

<table>
<thead>
<tr>
<th></th>
<th>exp.</th>
<th>$\alpha = 720$</th>
<th>push</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>938</td>
<td>915</td>
<td>−23</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1116</td>
<td>1090</td>
<td>−26</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1193</td>
<td>1214</td>
<td>+21</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>1318</td>
<td>1323</td>
<td>+5</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>1322</td>
<td>1321</td>
<td>−1</td>
</tr>
<tr>
<td>$\Sigma^*$</td>
<td>1385</td>
<td>1389</td>
<td>+4</td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>1533</td>
<td>1535</td>
<td>+2</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>1672</td>
<td>1662</td>
<td>−10</td>
</tr>
<tr>
<td>$\Theta^+$</td>
<td>1539</td>
<td>1534</td>
<td>−5</td>
</tr>
<tr>
<td>$N^*$</td>
<td></td>
<td>1667</td>
<td></td>
</tr>
<tr>
<td>$\Sigma_{10}$</td>
<td></td>
<td>1751</td>
<td></td>
</tr>
<tr>
<td>$\Xi_{3/2}$</td>
<td>1862</td>
<td>1784</td>
<td>−78</td>
</tr>
</tbody>
</table>
χQM breaking hamiltonian

calculate next-to-leading contributions to \( H' \)

\[
H'_{\chiQM} = -\alpha' D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D_{8i}^{(8)} S_i
\]

\( O(N_c)+O(1) \quad O(1) \quad O(1) \quad \text{all } O(m_s) \)

E. Guadagnini

equivalent to Guadagnini mass formula:

\[
H_G = \alpha D_{8,8}^{(8)}(A) + \beta Y
\]
in the Skyrme model
\( \beta, \gamma = 0 \)

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

richer \( H' \):

* no handle on \( I_2 \)

* only 2 linear combinations of parameters

\( \alpha', \beta \) and \( \gamma \) enter nonexotic splittings

splittings in \( 10 \neq \overline{10} \)
one can extract constants $G_{0,1,2}$ from known decuplet decays $G_2 \sim 0$, then

$$G_{10} = G_0 + G_1/2 \sim 19 \quad \text{and} \quad G_{1\overline{0}} = G_0 - G_1 \sim 0.5$$

note that $\Gamma_R \sim G_R^2$; that explains smallness of the $\Theta$ width.

exact result is: $G_{1\overline{0}} = G_0 - G_1 - G_2/2$

hence: $G_{1\overline{0}} = 0$ !

in QM limit $G_1/G_0 = 4/5$ and $G_2/G_0 = 2/5$
Wave functions and allowed states

\[ \psi_{BS}^{(R)} = \psi(R,B)(R^*,S) = (-)^{Q_S} \sqrt{\dim(R)} D_{BJ}^{(R)^*} \]

\[ B = (Y, I, I_3) \]
\[ S = (Y' = -1, S, S_3) \]
\[ J = (-Y' = 1, S, -S_3) \]

Y

N
\[ 8 \]
Σ
\[ 10 \]
Ξ
Ω
\[ \bar{10} \]
Wave functions and allowed states

\[ \psi_{BS}^{(R)} = \psi(R, B)(R^*, S) = (-)^Q S \sqrt{\text{dim}(R)} D_{BJ}^{(R)^*} \]

- \( q = \frac{N_c - 1}{2} \)
- \( 8 = (1, q) \)
- \( 10 = (3, q - 1) \)
- \( \bar{10} = (0, q + 2) \)

G. Karl, J. Patera, S. Perantonis, Phys. Lett 172B (1986) 49,
J. Bijnens, H. Sonoda, M. Wise,
Can. J. Phys. 64 (1986) 1,
Z. Duliński, M. Praszałowicz,
Mass formula

\[ M = M_{cl} + m_{cl} - \mathcal{M} + \frac{1}{2 I_1} S(S + 1) + \frac{1}{2 I_2} \left( C_2(R) - S(S + 1) - \frac{N_c^2}{12} \right) - \alpha d_B^R \]

\[ \Delta - N \sim \mathcal{O}\left(\frac{1}{N_c}\right) \]

\[ \Theta - N \sim \mathcal{O}(1) \]
\[ \Delta \rightarrow \pi N \]

\[
\overline{M}_\Delta^2 = \frac{3}{(M_N + M_\Delta)^2} \frac{q(q + 3)}{2(q + 1)(q + 4)} \left[ G_0 + \frac{1}{2} G_1 \right]^2 p^2
\]

\[
= \frac{3}{(M_N + M_\Delta)^2} \frac{(N_c - 1)(N_c + 5)}{2(N_c + 1)(N_c + 7)} \left[ G_0 + \frac{1}{2} G_1 \right]^2 p^2
\]

\( O(1/N_c^2) \quad O(1) \quad O(N_c^3) \)

\[ \Theta^+ \rightarrow K N \]

\[
\overline{M}_\Theta^2 = \frac{3}{(M_N + M_\Theta)^2} \frac{3(q + 1)}{2(q + 2)(q + 4)} \left[ G_0 - \frac{q + 1}{2} G_1 - \frac{1}{2} G_2 \right]^2 p^2
\]

\[
= \frac{3}{(M_N + M_\Theta)^2} \frac{3(N_c + 1)}{(N_c + 3)(N_c + 7)} \left[ G_0 - \frac{N_c + 1}{4} G_1 - \frac{1}{2} G_2 \right]^2 p^2
\]

\( O(1/N_c^2) \quad O(1/N_c) \quad O(N_c^3) \)
\[ \Gamma_\Delta \sim \mathcal{O}(N_c) \times p^3 \]
\[ \Gamma_\Theta \sim \mathcal{O}(1) \times p^3 \]

where \( p \) for 1 \( \rightarrow \) 2 decay reads

\[ p = \frac{\sqrt{M_1^2 - (M_2 + m)^2} \sqrt{M_1^2 - (M_2 - m)^2}}{2M_1} \]

chiral limit:

\[ \Delta \rightarrow \pi N \quad p \sim \mathcal{O}\left(\frac{1}{N_c}\right) \]
\[ \Theta \rightarrow KN \quad p \sim \mathcal{O}(1) \]

nonzero meson masses:

\[ p \sim \mathcal{O}(1) \]

\[ \Gamma_\Delta \sim \mathcal{O}(N_c) \]
\[ \Gamma_\Theta \sim \mathcal{O}(1) \]
"The particles described in this conference are not entirely fictitious and every analogy with the particles really existing in nature is not entirely coincidental."