Pentaquarks and Radial Excitations

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Refs.: HW, Int. J. Mod. Phys. **A11** (1996) 2419,
HW, Eur. Phys. J **A2** (1998) 391,
J. Schechter, HW, Phys. Lett. **B261** (1991) 235

Introduction & Motivation

- ⋆ General interest in non–three quark baryons

- ⋆ Θ^+ $(Y=2, I=0)$ natural (lightest) candidate (Z^+)

- ⋆ $\Xi^+_{3/2}$ $(Y=-1, I=3/2)$ next to lightest non–three quark candidate

- ⋆ role of such excitations can be studied in chiral soliton models (Skyrme model and its extensions)

- ⋆ members of higher dimension SU(3) representation $(\overline{\mathbf{10}})$

- ⋆ higher dimension SU(3) representation play important role for flavor symmetry breaking

- ⋆ relation to other established baryon excitations that can be constructed from three quarks

- ⋆ exotic states as test of a model

Soliton Model & Quantization

⋆ general structure of the model

Chiral field

flavor

$$\mathcal{L} = \mathcal{L}_{\rm S}(U^\dagger \partial_\mu U) + \mathcal{L}_{\rm SB}(U) \qquad U \in SU(3)$$

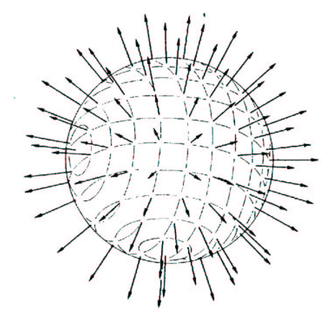
⋆ $\mathcal{L}_{\rm S}$ flavor symmetric

⋆ $\mathcal{L}_{\rm SB}$ parameterizes flavor symmetry breaking:

$$m_\pi = 138\text{MeV} \qquad m_{\rm K} = 496\text{MeV}$$
$$f_\pi = 93\text{MeV} \qquad f_{\rm K} = 114\text{MeV}$$

⋆ Baryon number one sector $\longrightarrow$ hedgehog soliton:

$$\boxed{U_H(\vec{x}\,) = \exp\left(i\hat{r}\cdot\vec{\tau}\,F(r)\right)}$$



⋆ ridig rotator quantization of rotational "zero modes":

$$U(\vec{x}\,,t) \;=\; A(t)\,U_{\rm H}(\vec{x}\,)\,A^\dagger(t)\,, \qquad A \in SU(3)$$

(in iso–subspace)

⋆ Lagrangian function for collective coordinates

$$L = \int d^3r\mathcal{L} = L(A,\Omega_a)\,, \qquad \Omega_a = -i\,{\rm tr}\left(\lambda_a\,A^\dagger(t)\,\frac{A(t)}{dt}\right)$$

angular velocities

⋆ Hamiltonian for collective coordinates

$$H = -L - \sum_{a=1}^{8} \Omega_a R_a \,, \qquad R_a = -\frac{\partial L}{\partial \Omega_a} \quad \leftarrow \ SU(3)\ \text{generators}$$

$$\begin{aligned} H \ = \ & E_{\rm cl} + \left(\frac{1}{2\alpha^2} - \frac{1}{2\beta^2}\right)\vec{J}^{\,2} + \frac{1}{2\beta^2} \underbrace{C_2\left[SU(3)\right]}_{\sum_{a=1}^{8} R_a R_a} \\ & - \frac{3}{8\beta^2} + \frac{\gamma}{2}\left[1 - \underbrace{D_{88}(A)}_{D_{ab} = \frac{1}{2}{\rm tr}\left(\lambda_a A \lambda_b A^\dagger\right)}\right] \end{aligned}$$

⋆ Casimir operator: $C_2|\mathbf{8}\rangle = 3|\mathbf{8}\rangle, \quad C_2|\mathbf{10}\rangle = 6|\mathbf{10}\rangle, \ldots$

– $E_{\rm cl}, \alpha^2, \beta^2, \gamma$ are functionals of the soliton, *i.e.* numbers

– $\gamma \propto f_K^2 m_K^2 - f_\pi^2 m_\pi^2, \ldots$ measures explicit sym. breaking

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⋆ no other symmetry breaking terms unless model is extended (*e.g.* vector mesons, chiral quarks)

⋆ γ dominant symmetry breaker in such extensions

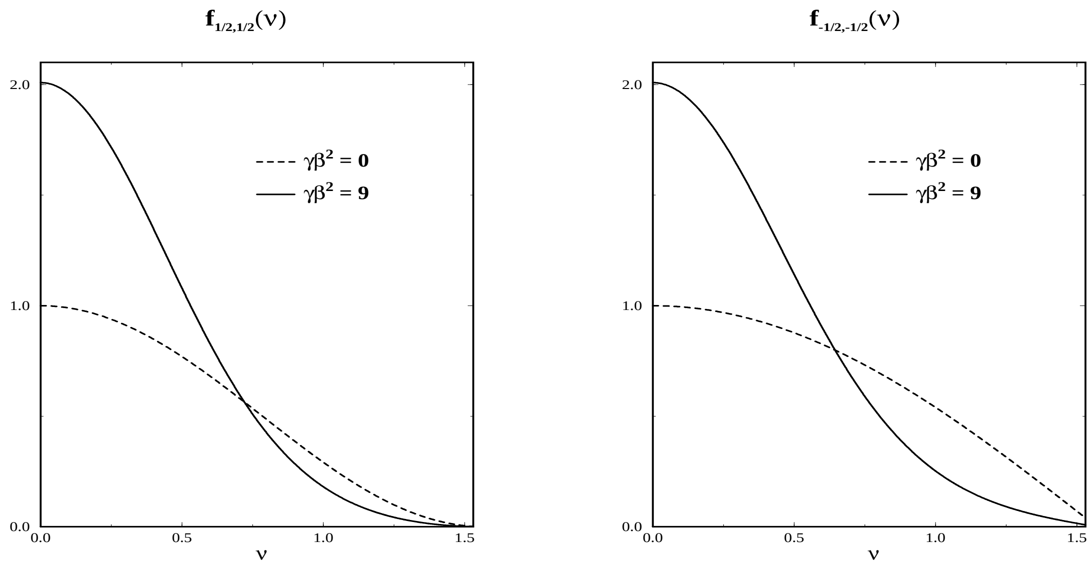
$\star$ "Euler angles" and exact diagonalization (Yabu & Ando '88):

strangeness changing angle

$$\Psi(A) = \sum_{M_L,M_R} D^{I\,*}_{I_3,M_L}(\alpha,\beta,\gamma) f_{M_L,M_R}(\nu) D^{J\,*}_{M_R,-J_3}(\alpha',\beta',\gamma') \mathrm{e}^{i\rho}$$

$$H\Psi = E\Psi \quad \longrightarrow \quad \text{diff. eqn. for } f_{M_L,M_R}(\nu)$$

nucleon:



f^2: probability for rotations into strangeness direction

$\star$ $\gamma\beta^2$: – effective symmetry breaking: (explicit) $\times$ (intertia parameter).

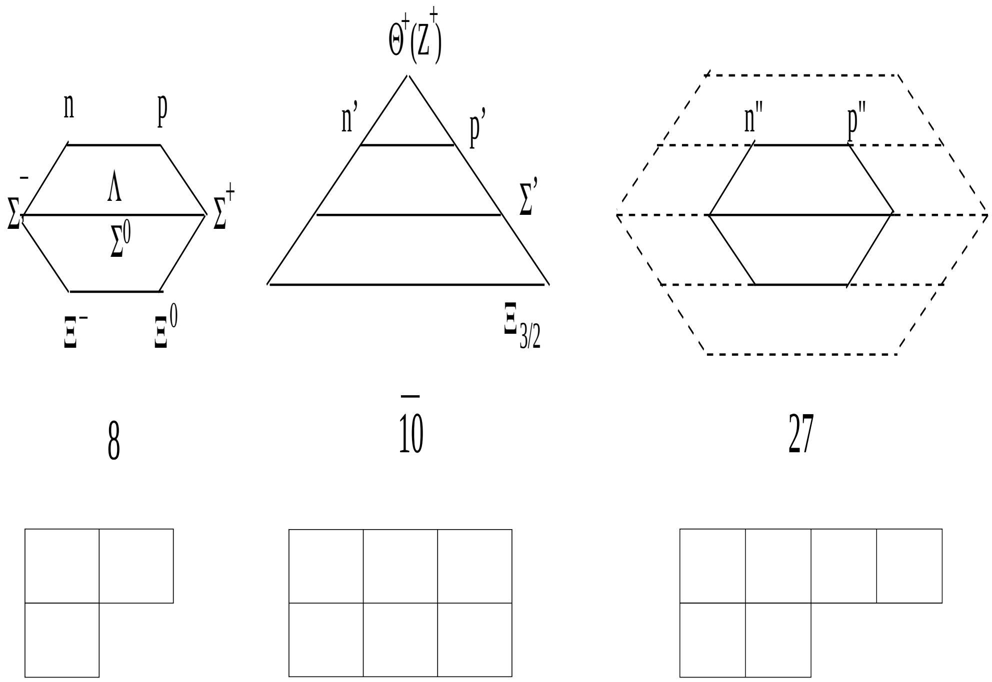
– pattern of mass differences $\omega^2 = \frac{3}{2}\gamma\beta^2$:

	$(M_\Lambda - M_N)$	:	$(M_\Sigma - M_\Lambda)$	:	$(M_\Xi - M_\Sigma)$
$1^{\rm st}$ order	1	:	1	:	1/2
ω^2=6.0	1	:	0.69	:	0.70
ω^2=8.0	1	:	0.61	:	0.77
empir.	1	:	0.44	:	0.69

– non–linear effects are important

⋆ perturbation expansion: (Park, Schechter, HW, '89)

$$|N\rangle = |N, \mathbf{8}\rangle + 0.0745\gamma\beta^2\, |N, \mathbf{\overline{10}}\rangle + 0.0490\gamma\beta^2\, |N, \mathbf{27}\rangle + \ldots$$



$3q$ $\quad$ $4q\&\bar{q}$ $\quad$ $4q\&\bar{q}$

⋆ flavor symmetry breaking unavoidably includes pentaquark baryons

⋆ rigid rotator approach has pentaquark pieces in octet:

$$\frac{\langle N, \mathbf{8}|A_3^{(s)}|N, \mathbf{8}\rangle}{\langle N, \mathbf{8}|A_3^{(3)}|N, \mathbf{8}\rangle} = 0.23 \longleftarrow \sim g_{\mathrm{A}}$$

non–vanishing strange quark polarization as indication for $q\bar{q}$ pairs.

⋆ detour: slow rotator approach

(B Schwesinger & HW, Nucl. Phys. **A540** (92) 461)

– coupling of symmetry breaking to soliton profile

– slow collective rotation $\Longrightarrow$ chiral angle reacts on symmetry breaking forces: $F = F(r, \nu)$

– $F(r, 0) \sim \exp(-m_\pi r)$

$F(r, \frac{\pi}{2}) \sim \exp(-m_K r)$ as $r \to \infty$

– $$H = E_{\rm cl}(\nu) + \tfrac{3}{4}\gamma(\nu)\sin^2\nu + \frac{1}{2\alpha^2(\nu)}\vec{J}^{\,2} + \ldots$$

– <u>exact</u> diagonalization via DEQs for $f_{M_L,M_R}(\nu)$

– proper description of U–spin violation in magnetic moments, *e.g.* $\mu_p > \mu_{\Sigma^+}$

– reasonable results for g_A/g_V of hyperon β–decay

⋆ important interplay between flavor symmetry breaking and baryon size

Radial excitations: breathing modes

⋆ some rough estimates

- $M_{|N,\bar{10}\rangle} - M_{|N,8\rangle} = \frac{3}{2\beta^2} \approx 400\text{MeV}$ Roper $N(1440)$?
- $M_{|\Delta,27\rangle} \approx 1.75\text{GeV}$ expt.: P33(1600) or P33(1920)?
- significant mixing between radial excitations and states in higher dimensional SU(3) representations
- no simple $\hbar\omega$ excitations

⋆ alternative technique to incorporate symmetry breaking effects on soliton profile: collective coordinate $\xi(t)$ for soliton extension

$$\boxed{U(\vec{x}\,,t) \;=\; A(t)\, U_{\rm H}(\xi(t)\vec{x}\,)\, A^{\dagger}(t)}$$

⋆ quantize both $A(t)$ and $\xi(t)$

momentum conjugate to ξ

$$\begin{aligned} H \;=\; & \frac{1}{2m(\xi)} {p_\xi}^2 + V(\xi) \\ & + \left(\frac{1}{2\alpha^2(\xi)} - \frac{1}{2\beta^2(\xi)}\right) \vec{J}^{\,2} + \frac{1}{2\beta^2(\xi)} C_2\left[SU(3)\right] \\ & + \underbrace{\tfrac{1}{2}{\rm tr}\left(\lambda_a A \lambda_b A^{\dagger}\right)\, s(\xi)} \end{aligned}$$

symmetry breaking term

$\star$ two step–diagonalization of H

1) construct basis states:

$$H_{\rm sym}|\mu, n_\mu\rangle = \epsilon_{\mu,n_\mu}|\mu, n_\mu\rangle \qquad (i.e.\ s(\xi) \equiv 0)$$

μ: SU(3) representation

n_μ: breathing excitation in SU(3) representation μ

differential equation in ξ yields $f_{\mu,n_\mu}(\xi)$

2) diagonalize matrix $\langle \nu, n_\nu|H|\mu, n_\mu\rangle$

$\longrightarrow$ $\epsilon_{\rm B,m}$ (spectrum)

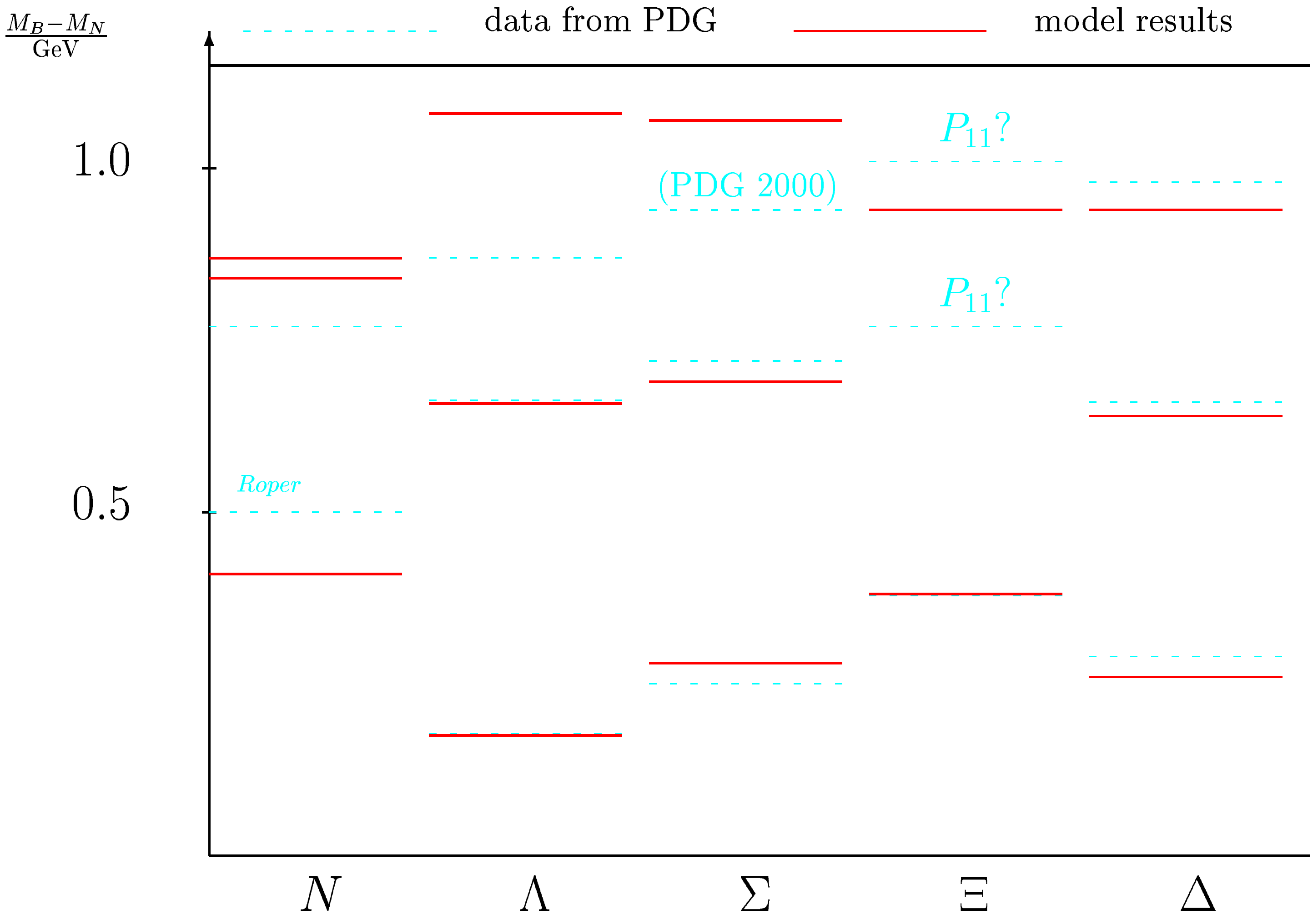
$\longrightarrow$ $|B; m\rangle = \sum_{\mu,n_\mu} C^{(B,m)}_{\mu,n_\mu}|B; \mu, n_\mu\rangle$

m: $m^{\rm th}$ excitation with quantum numbers of baryon B

e.g. Nucleon $\sim |N; 1\rangle$ Roper $\sim |N; 2\rangle$

$C^{(B,m)}_{\mu,n_\mu}$: answer to the mixing problem

Computed Spectrum



- ⋆ low–lying states agree well
- ⋆ first *radial* excitation somewhat too low
- ⋆ second *radial* excitation somewhat too high
- ⋆ model does not predict any additional states in three quark channels
- ⋆ missing states in Λ, Σ channels may be $\hbar\omega$ excitations

⋆ more detailed numbers for mass differences

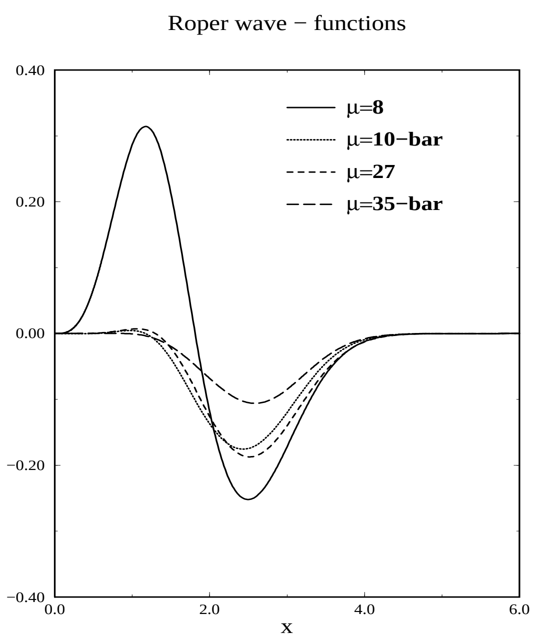
(e: Skyrme parameter)

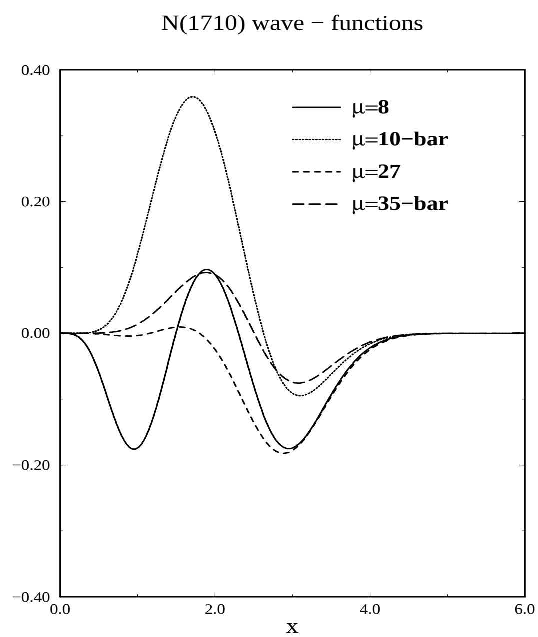
B	$m=0$			$m=1$			$m=2$		
	e=5.0	e=5.5	expt.	e=5.0	e=5.5	expt.	e=5.0	e=5.5	expt.
N	Input			413	445	501	836	869	771
Λ	175	173	177	657	688	661	1081	1129	871
Σ	284	284	254	694	722	721	1068	1096	838
Ξ	382	380	379	941	971	?	1515	1324	?
Δ	258	276	293	640	680	661	974	1010	981
Σ*	445	460	446	841	878	901	1112	1148	1141
Ξ*	604	617	591	1036	1068	?	1232	1269	?
Ω	730	745	733	1343	1386	?	1663	1719	?

agreement on the 10% level

⋆ wave–functions

– low–lying states are dominantly octet





Static Baryon Properties

⋆ magnetic moments

	this model		expt.	
Baryon	μ_B	μ_B/μ_p	μ_B	μ_B/μ_p
p	2.21	1.00	2.79	1.00
n	-1.84	-0.83	-1.91	-0.68
Λ	-0.52	-0.24	-0.61	-0.22
Σ^+	1.82	0.82	2.42	0.87
Σ^0	0.44	0.20	—	—
Σ^-	-0.94	-0.43	-1.16	-0.42
Ξ^0	-1.06	-0.48	-1.25	-0.45
Ξ^-	-0.41	-0.19	-0.69	-0.25
$\Sigma^0 \to \Lambda$	-1.37	-0.62	-1.61	-0.58

– agreement on 20% level (typical for soliton models)

– correct deviation from U–spin symmetry

(generalization of rigid rotator)

⋆ hyperon decays

	this model			emp.	SU(3)
decay	$g_{\rm V}$	$g_{\rm A}$	$(g_{\rm V}/g_{\rm A})_{\rm N}$	$g_{\rm V}/g_{\rm A}$	$(g_{\rm V}/g_{\rm A})_{\rm N}$
$\Lambda \to p$	1.215	0.615	0.797	0.79 ± 0.03	0.72
$\Sigma- \to n$	1.091	0.272	0.393	0.36 ± 0.05	0.36
$\Xi- \to \Lambda$	1.246	0.169	0.213	0.25 ± 0.05	0.18
$\Xi- \to \Sigma^0$	0.690	0.610	1.392	1.29 ± 0.16	1.26

– reasonable agreement with data

– minor effects of flavor symmetry breaking

(similar to rigid rotator)

Model predictions for exotic states

- $\star$ $\Theta^+(Z^+) : 1.57 \ldots 1.59\text{GeV}$ ($e = 5.0 \ldots 5.5$) (top of $\bar{10}$)

  earlier results: $M(\Theta^+) - M(N) \approx 600\text{MeV}$

  Biedenharn, Chemtob, Prasałowicz, Walliser, Diakonov *et. al.*, ...

  recent expts.: $M(\Theta^+) = (1540 \pm 10)\text{MeV}$

  Nakano *et. al.*, DIANA, CLAS, SAPHIR, Asratyan *et. al.*

- $\star$ $\Xi_{3/2} : 1.89 \ldots 1.91\text{GeV}$ ($e = 5.0 \ldots 5.5$) (bottom of $\bar{10}$)

  other pred.: $M(\Xi_{3/2}) \approx 1.78\text{GeV}$ (Walliser & Kopeliovich)
  $M(\Xi_{3/2}) \approx 2.07\text{GeV}$ (Diakonov *et. al.*)

  recent expt.: $(1.862 \pm 0.002)\text{GeV}$ (NA49 @ CERN)

- $\star$ radial excitations: $\Theta^+ : \; 2.02 \ldots 2.07\text{GeV}$ ($e = 5.0 \ldots 5.5$)
  $\Xi_{3/2} : \; 2.29 \ldots 2.33\text{GeV}$ ($e = 5.0 \ldots 5.5$)

$\star$ estimate of widths $\quad (\Theta^+ \longrightarrow nK^+)$

– P–wave decay: $\quad \Gamma \propto |\vec{p}_M|^3$
(indicates experimentally observed narrowness)

– open problem beyond that (in soliton models)

interaction Lagrangian must be linear in meson field

however: $\quad \left.\frac{\partial \mathcal{L}}{\partial U}\right|_{\rm sol} = 0 \quad$ by definition of soliton

– this model:

PCAC, GT & plane wave appr.: $\Gamma \approx 80\text{MeV}$

PCAC, GT & distorted wave appr.: $\Gamma = \mathcal{O}(m_\pi^2/f_\pi)$

N. B.: complete treatment of PCAC requires induced kaon fields:

$$K \sim W(r) \begin{pmatrix} \Omega_4 - i\Omega_5 \\ \Omega_6 - i\Omega_7 \end{pmatrix}$$

Summary

- ⋆ description of pentaquark states requires understanding of mixing between radially excited baryons and states in higher dimensional SU(3) representations

- ⋆ suggested approach: breathing mode quantization in SU(3) chiral soliton model

  - allows for exact diagonalization of Hamiltonian with flavor symmetry breaking
  - accounts for observed U–spin violation in baryon magnetic moments
  - agrees with SU(3) description of semi–leptonic hyperon decays
  - radially excited baryons and states in higher dimensional SU(3) representations mix via flavor symmetry breaking

- ⋆ model reproduces essential characteristics of spectra in P–wave channels of $3q$ baryons.

- ⋆ no need to call for new states in $3q$ channels

- ⋆ good agreement with observed masses for Θ^+ and $\Xi_{3/2}$ pentaquarks

- ⋆ fully dynamical calculation of width is still missing