Quantum Phase-Space Quark Distributions in the Proton

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GPDs and their Interpretation

- Common complains about GPD physics
  - Too many variables !

    e. g. , $H(x, \xi, t, \mu)$ – 4 variables

    For most of people the upper limit is 2.

    I will argue 4 is nice, the more the better from a theory point of view!

  - Too many different GPDs!

    In fact, there are eight leading-twist ones

    All GPDs are equal, but some are more equal than the others.

GPD is a Quantum Distribution!

- **What is a classical distribution?**

  A distribution that has strict classical interpretation.

  Charge denstiy, ρ(r)

  current density, j(r)

  momentum distribution, f(p), f(x)...

- **A quantum distribution?**

  A distribution that has No strict classical interpretation. But it may have a classical analogue

  Wigner Distribution W(r,p)

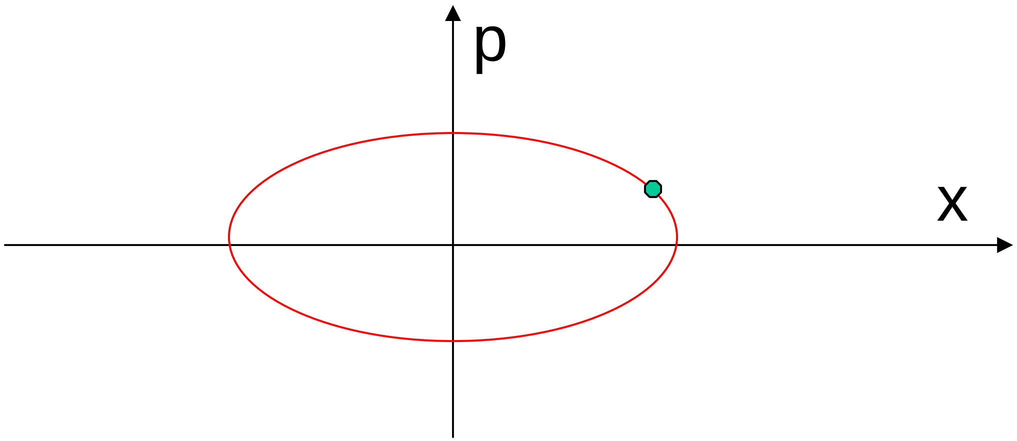
Problems with Classical Distributions

- Elastic form-factors provide static *coordinate-space* charge and current distributions (in the sense of Sachs, for example), but NO information on the dynamical motion.
- Feynman parton densities give *momentum-space* distributions of constituents, but NO information of the spatial location of the partons.
- But sometimes, we need to know BOTH the *position and momentum* of the constituents.
  - For example, one need to know **r** and **p** to calculate $\mathbf{L}=\mathbf{r}\times\mathbf{p}$!

Classical phase-space distribution

- The state of a classical particle is specified *completely* by its coordinate and momentum:
  - A point in the phase-space (x,p):

  Example: Harmonic oscillator



- A state of a classical identical particle system can be described by a phase-space distribution $f(x,p)$.

Quantum Analogue?

- In quantum mechanics, because of the uncertainty principle, the phase-space distribution seems ill-defined in principle.
- **Wigner introduced the first phase-space distribution in quantum mechanics (1932)**

  it is extremely useful for understanding the
  quantum dynamics
  using the classical language of phase-space.
  - *Heavy-ion collisions,*
  - *quantum molecular dynamics,*
  - *signal analysis,*
  - *quantum info,*
  - *optics,*
  - *image processing*

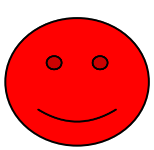
Wigner function

- Define as

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

  - When integrated over x (p), one gets the momentum (probability) density.
  - Not positive definite in general, but is in classical limit!
  - Quantum average of any dynamical variable can be calculated as

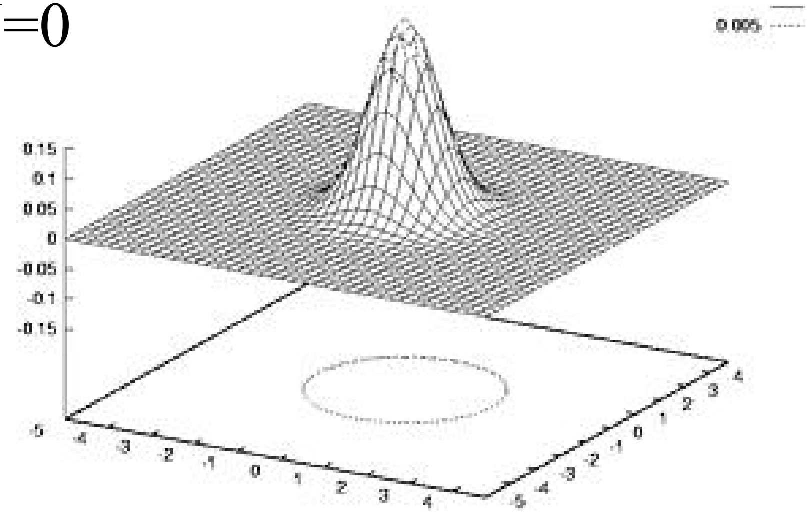
$$\langle O(x,p)\rangle = \int dxdpO(x,p)W(x,p)$$



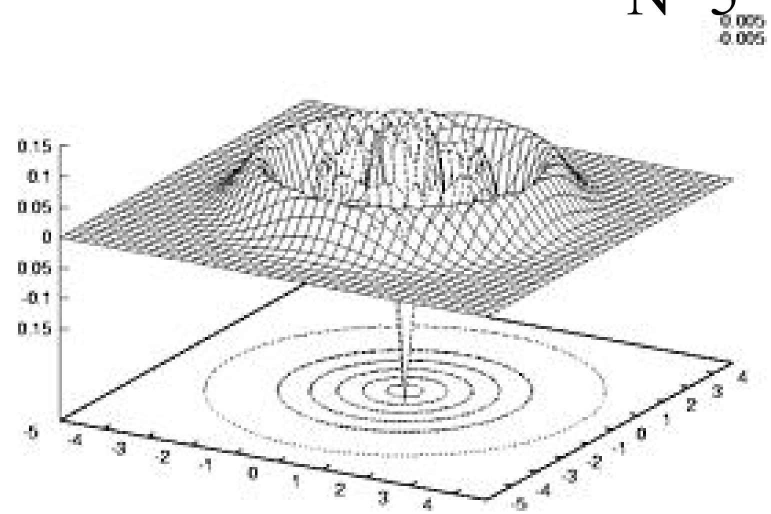
Short of measuring the wave function, the Wigner function contains the *most complete (one-body) info* about a quantum system.

Simple Harmonic Oscillator

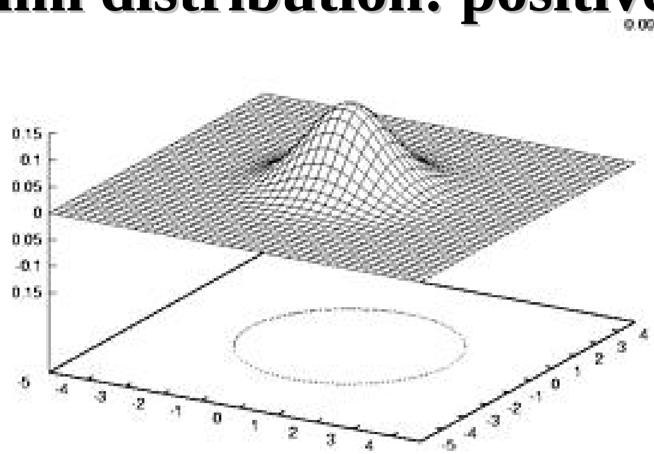
N=0

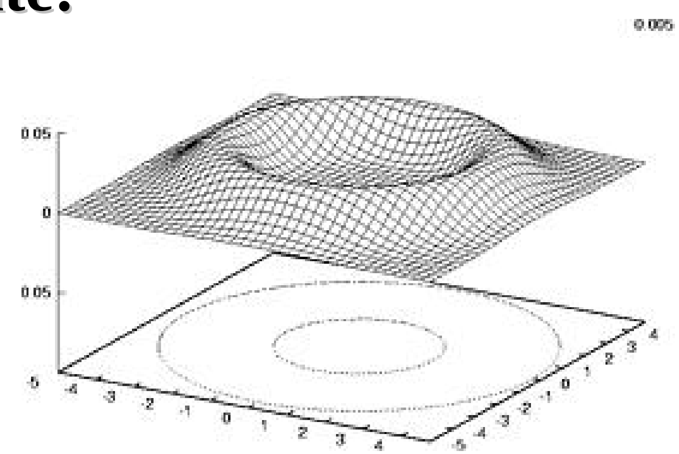


N=5



Husimi distribution: positive definite!





Measuring Wigner function of a quantum Light!

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Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

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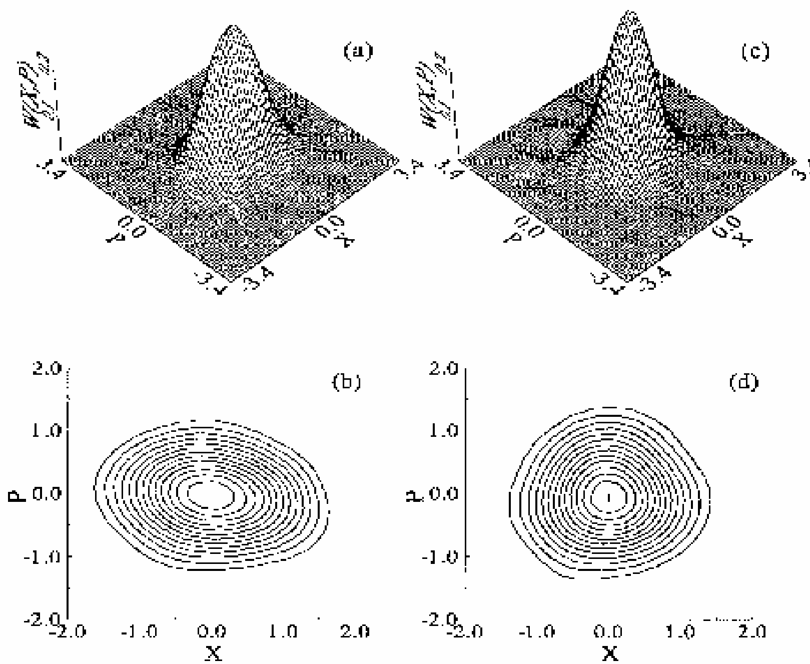


FIG. 1. Measured Wigner distributions for (a),(b) a squeezed state and (c),(d) a vacuum state, viewed in 3D and as contour plots, with equal numbers of constant-height contours. Squeezing of the noise distribution is clearly seen in (b).

Quarks in the Proton

- Wigner operator

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta \ ,$$

- Wigner distribution: "**density**" for quarks having *position r and 4-momentum k^μ* (off-shell)

$$W_\Gamma(\vec{r},k) = \frac{1}{2M}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}(\vec{r},k)\right|-\vec{q}/2\right\rangle$$

$$= \frac{1}{2M}\int\frac{d^3\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}(0,k)\right|-\vec{q}/2\right\rangle$$

a la Saches

Ji, PRL91, 062001 (2003)

7-dimensional distribtuion

No known experiment can measure this!

Custom-made for high-energy processes

- In high-energy processes, one cannot measure k^- = $(k^0 - k^z)$ and therefore, one must integrate this out.
- The reduced Wigner distribution is a function of 6 variables [$\mathbf{r}$, $\mathbf{k}=(k^+\ \mathbf{k}_\perp)$].

1. After integrating over $\mathbf{r}$, one gets transverse-momentum dependent (TDM) parton distributions.
2. Alternatively, after integrating over $\mathbf{k}_\perp$, one gets a **spatial distribution of quarks with fixed Feynman momentum** $k^+=(k^0+k^z)=xM$.

$f(\mathbf{r},x)$

Proton images at a fixed x

- For every choice of x, one can use the Wigner distribution to picture the quarks; *This is analogous to viewing the proton through the x (momentum) filters!*
- The distribution is related to *Generalized parton distributions* (GPD) through

$$f_\Gamma(\vec{r}, x) = \frac{1}{2M}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} F_\Gamma(x, \xi, t) \ .$$

$$\frac{1}{2M} F_{\gamma^+}(x, \xi, t) = [H(x, \xi, t) - \tau E(x, \xi, t)] + i(\vec{s}\times\vec{q})^z \frac{1}{2M}[H(x, \xi, t) + E(x, \xi, t)]$$

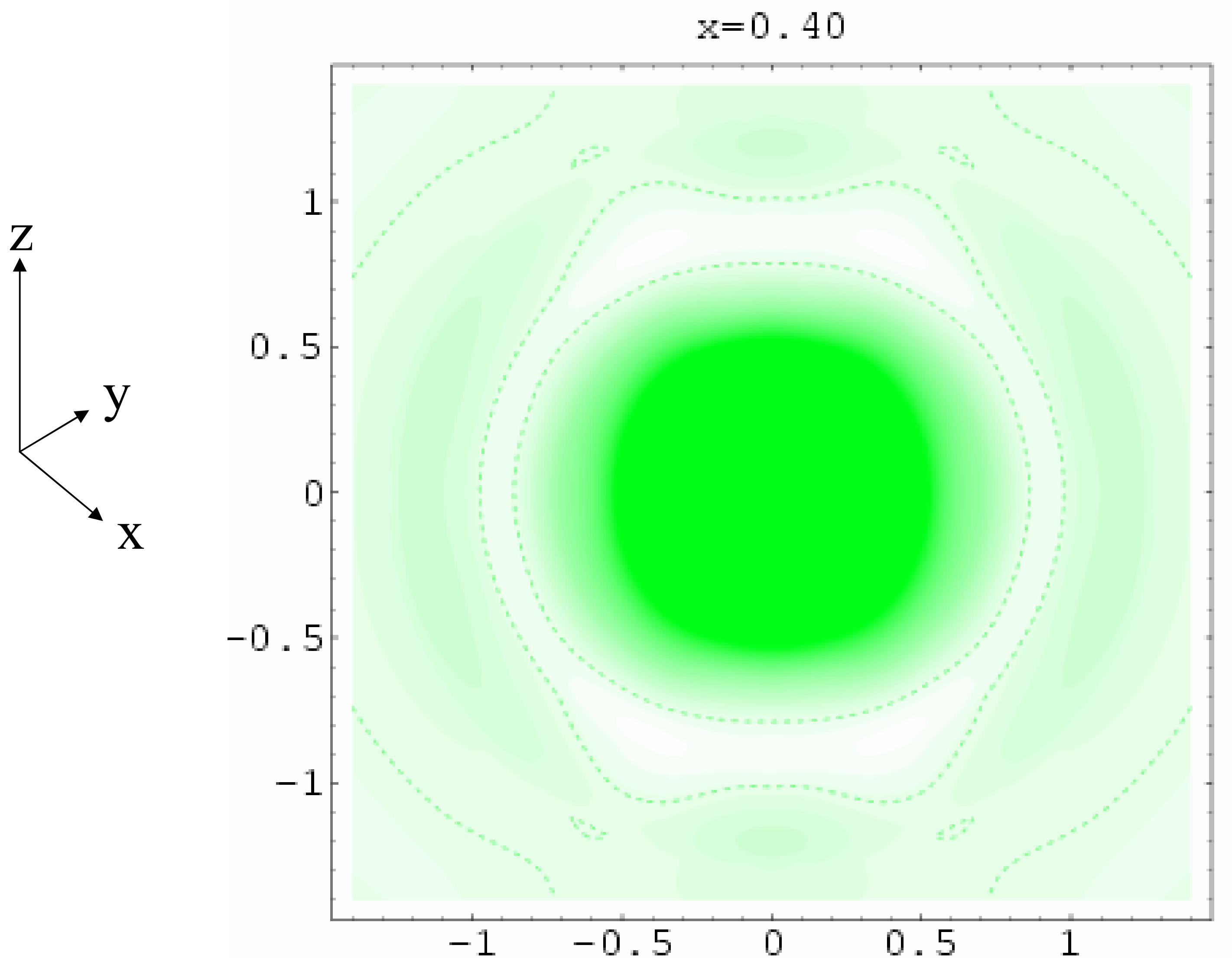
$t = -q^2$

$\xi \sim q_z$

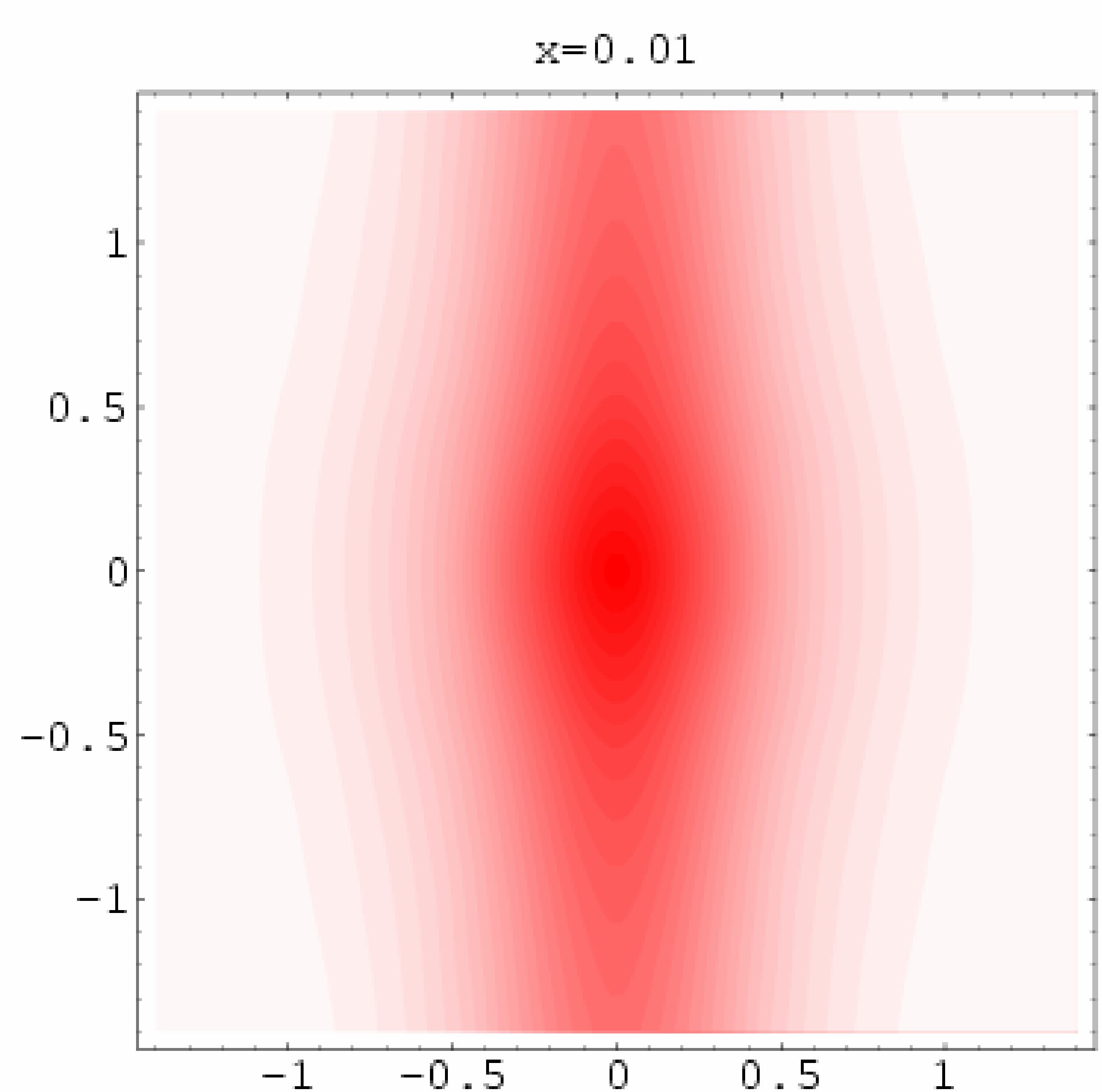
A GPD or Wigner Function Model

- A parametrization which satisfies the following *Boundary Conditions*: (A. Belitsky, X. Ji, and F. Yuan, hep-ph/0307383, to appear in PRD)
  - Reproduce measured Feynman distribution
  - Reproduce measured form factors
  - Polynomiality condition
  - Positivity
- Refinement
  - Lattice QCD
  - Experimental data

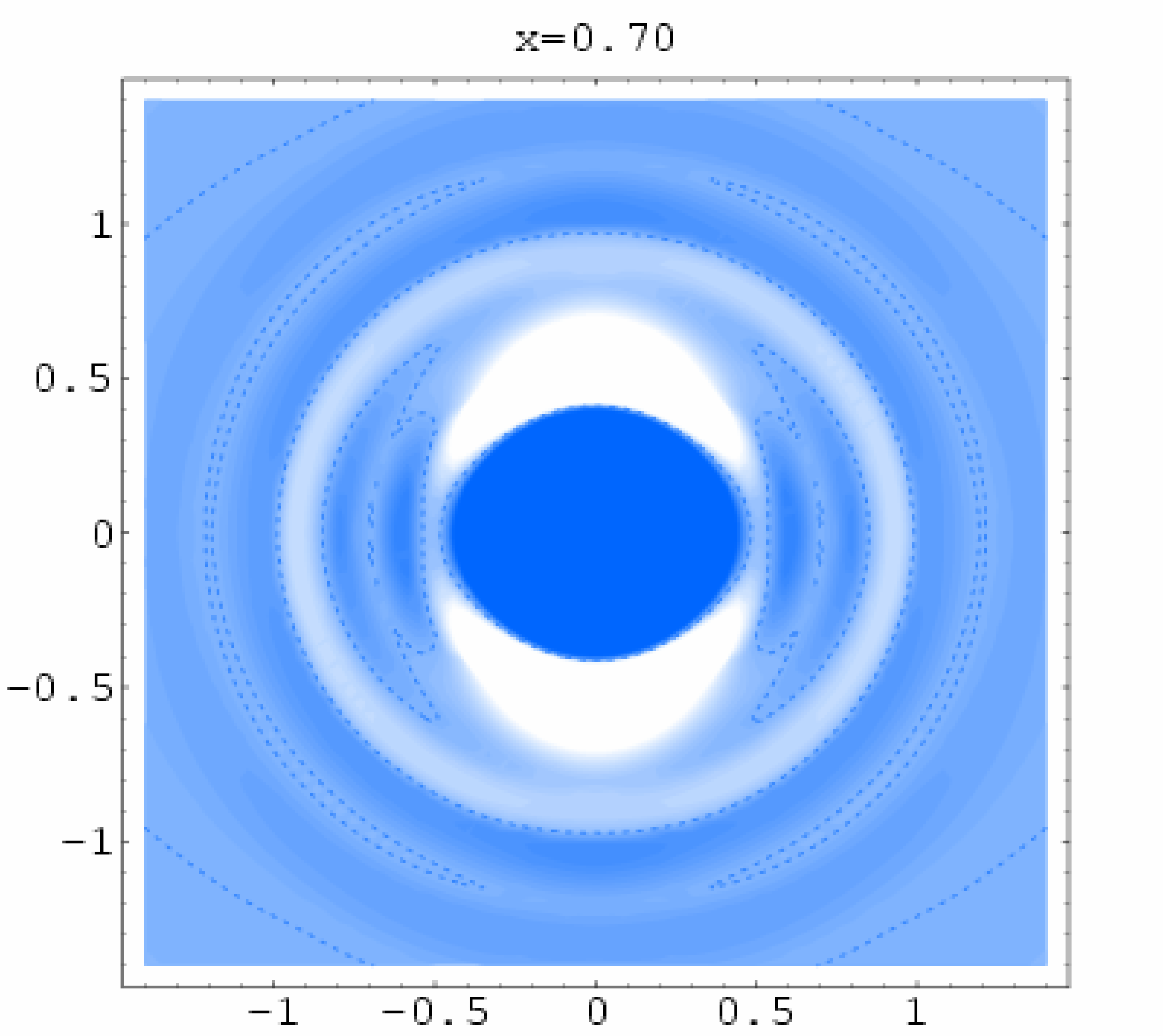
Up-Quark Charge Density at x=0.4



Up-Quark Charge Denstiy at x=0.01

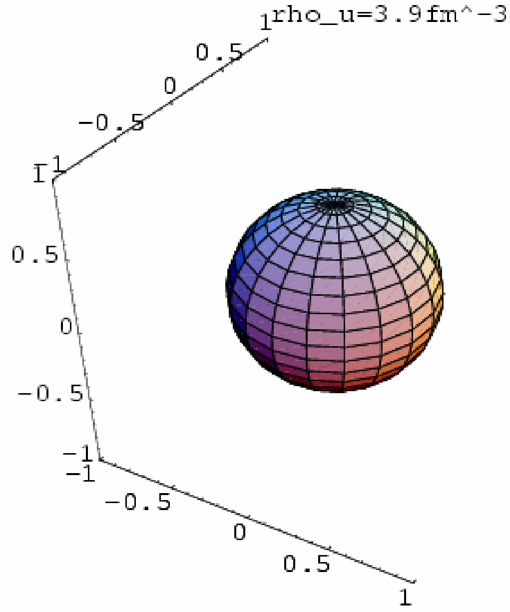


Up-Quark Density At x=0.7



Comments

- If one puts the pictures at all x together, one gets a spherically round nucleon! (Wigner-Eckart theorem)



- If one integrates over the distribution along the z direction, one gets the 2D-impact parameter space pictures of M. Burkardt (2000) and Soper.

TMD Parton Distribution

- Appear in the process in which hadron transverse-momentum is measured, often together with TMD fragmentation functions.
- The leading-twist ones are classified by Boer, Mulders, and Tangerman (1996,1998)
  - There are 8 of them

    $q(x, k_\perp)$, $q_T(x, k_\perp)$,

    $\Delta q_L(x, k_\perp)$, $\Delta q_T(x, k_\perp)$,

    $\delta q(x, k_\perp)$, $\delta_L q(x, k_\perp)$, $\delta_T q(x, k_\perp)$, $\delta_{T'} q(x, k_\perp)$

Factorization for SIDIS with $P_\perp$

- For traditional high-energy process with one hard scale, inclusive DIS, Drell-Yan, jet production,…soft divergences typically cancel, except at the edges of phase-space.
- At present, we have two scales, Q and $P_\perp$ (could be soft). Therefore, besides the collinear divergences which can be factorized into TMD parton distributions (not entirely as shown by the energy-dependence), there are also soft divergences which can be taken into account by the soft factor.

X. Ji, F. Yuan, and J. P. Ma (to be published)

Conclusion

- Wigner distribution is the unifying framework for all the distributions!