Azymuthal Spin Asymmetries for Large P_T Hadron Production at eRHIC

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Cf. Y.K. and J. Nagashima, NPB660(2003) 269

Azymuthal asymmetry for large-p_T hadron production in SIDIS

- Double spin asymmetries: twist-2

$$e + \vec{p} \rightarrow e + \vec{\Lambda}(p_T) + X$$

$$e + p^{\uparrow} \rightarrow e + \Lambda^{\uparrow}(p_T) + X$$

$$\vec{e} + \vec{p} \rightarrow e + \pi(p_T) + X, \quad or \quad e + 2\,jet + X$$

$$\vec{e} + p \rightarrow e + \vec{\Lambda}(p_T) + X$$

Cf. p_T -integrated cross sections:
LO; Ji('94).
NLO; de Florian et al ('96), Stratmann('96), Vogelsang('96),…

p_T -differential cross section with intrinsic k_T:
LO, Mulders&Tangerman('96), Boer&Mulders('98), …
This is relevant only for small p_T production.

Interest:

(1) Can p-QCD(+resummation) describe azymuthal- and p_T- dependence of these reactions? At which p_T? — Well studied for unpolarized processes.(Georgi&Politzer('78), Cahn('78), Mendez('78).... With resummation; Meng et al ('96), Nadolsky et al('00))

(2) More information on the polarized parton density and fragmentation functins.

Spin	Average	$S_{\parallel}$	$S_{\perp}$
quark distribution	$q(x)$	$\Delta q(x)$	$\underline{\delta q(x)}$
gluon distribution	$G(x)$	$\Delta G(x)$	
quark fragmentation	$\widehat{q}(z)$	$\Delta\widehat{q}(z)$	$\underline{\delta\widehat{q}(z)}$
gluon fragmentation	$\widehat{G}(z)$	$\Delta\widehat{G}(z)$	

——; chiral-odd.

Kinematics for $e(k) + p(p_A, S_A) \rightarrow e(k') + h(p_B, S_B) + X$

- Lorentz invariants:

$$S_{ep} = (p_A + k)^2 \cong 2 p_A \cdot k$$

$$x_{bj} = \frac{Q^2}{2 p_A \cdot q}$$

$$Q^2 = -q^2 = -(k - k')^2$$

$$z_f = \frac{p_A \cdot p_B}{p_A \cdot q}$$

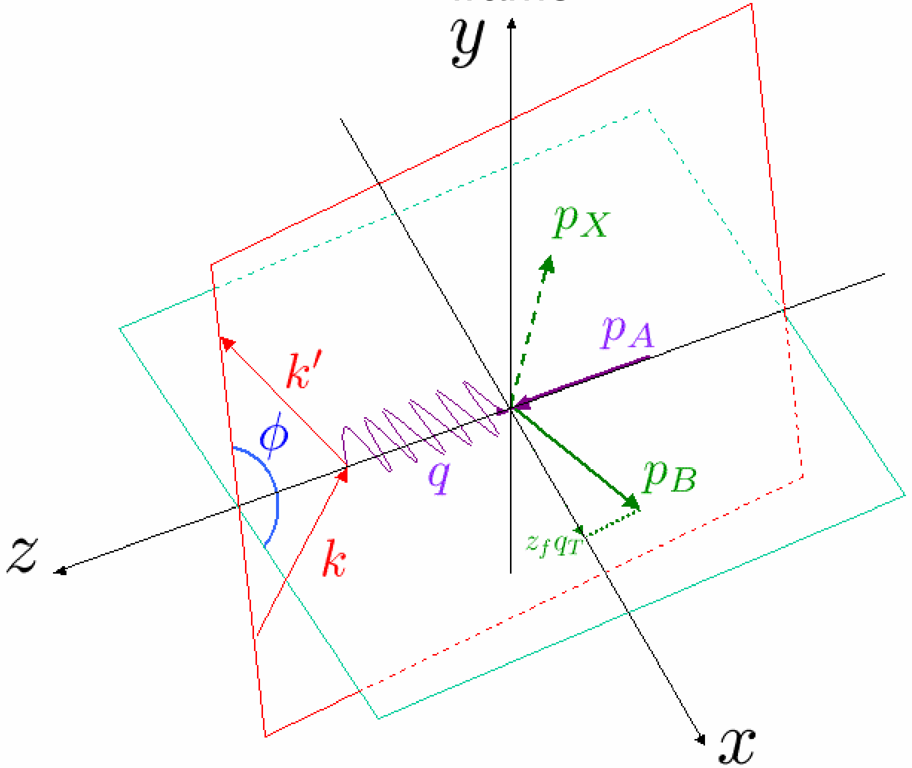
$$q_T = \sqrt{-q_t{}^2} \quad \text{with}$$

$$q_t{}^{\mu} = q^{\mu} - \frac{p_B \cdot q}{p_A \cdot p_B} p_A{}^{\mu} - \frac{p_A \cdot q}{p_A \cdot p_B} p_B{}^{\mu}$$

$$p_A \cdot q = p_B \cdot q = 0$$

- Hadron frame for $e(k)+p(p_A,S_A)\to e(k')+h(p_B,S_B)+X$

(Meng, Olness, Soper('91)) photon-proton Breit frame



$$q=k-k'=(0,0,0,-Q)$$

$$p_A=(\frac{Q}{2x_{bj}},0,0,\frac{Q}{2x_{bj}})$$

$$p_B=\frac{z_f}{2}(1+\frac{q_T^{\;2}}{Q^2},\frac{2q_T}{Q},0,\frac{q_T^{\;2}}{Q^2}-1)$$

$p_T=z_f q_T$: $\perp$ mom. of hadron B

$$k=\frac{Q}{2}(\cosh\psi,\sinh\psi\cos\phi,\sinh\psi\sin\phi,-1)$$

$$\cosh\psi=\frac{2x_{bj}S_{ep}}{Q^2}-1=\frac{2}{y}-1$$

ϕ: Azymuthal angle between lepton and hadron plane

$\perp$ Spin vector

$$S_A^{\;\perp}=(0,\cos\Phi_A,\sin\Phi_A,0)$$

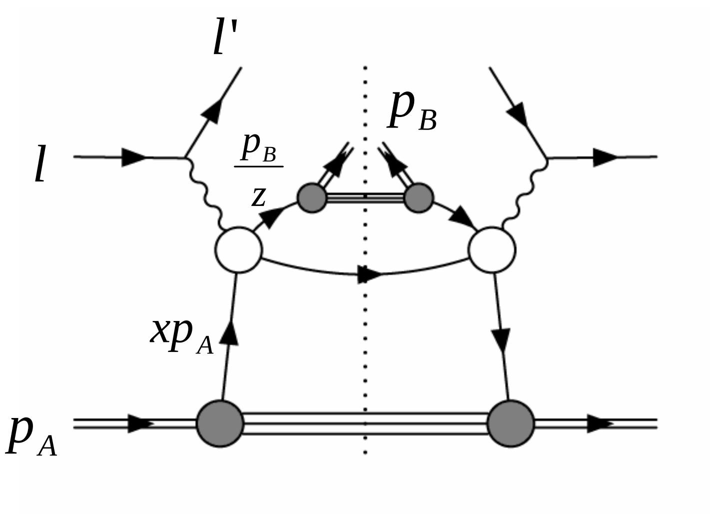
$$S_B^{\;\perp}=(0,\cos\Theta_B\cos\Phi_B,\sin\Phi_B,-\sin\Theta_B\cos\Phi_B)$$

Φ_A, Φ_B: Azymth. Measured from hadron plane.

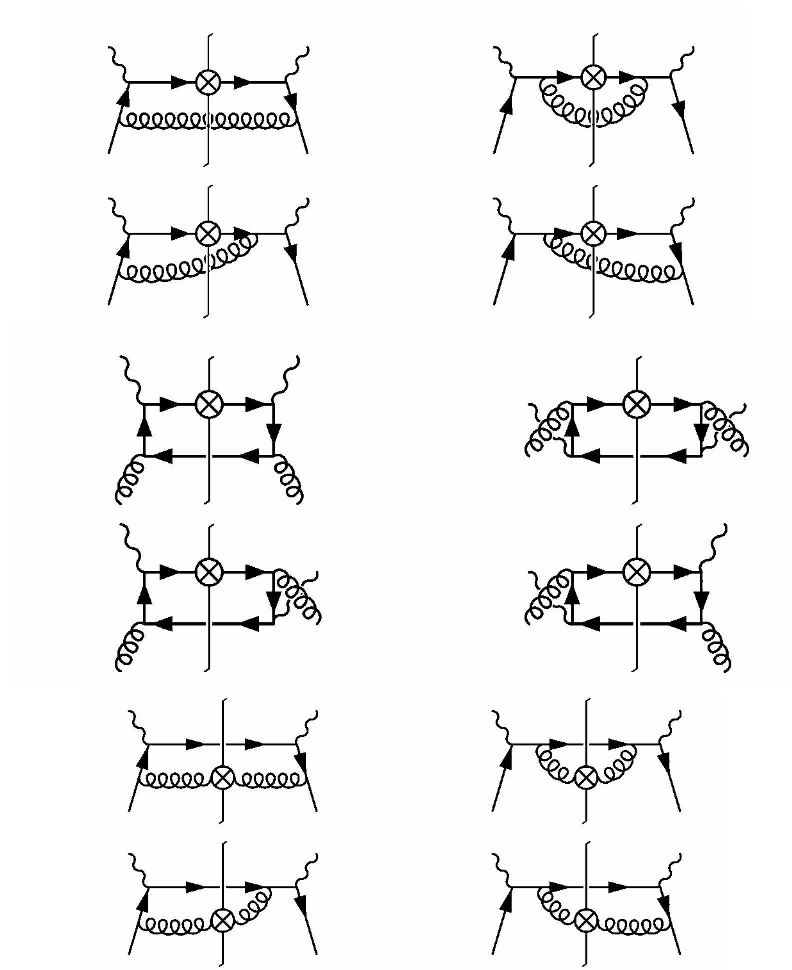
Θ_B : Polar angle of $\vec{p}_B$

$$\sin\Theta_B=\frac{2q_T Q}{q_T^{\;2}+Q^2}$$

- Large p_T hadron production in perturbative QCD (twist-2)



$O(\alpha_S)$ diagrams



⊗: Fragmentation insertion

Cross Section:

$$\frac{d^5\sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} = \sigma_0 + \cos(\phi)\sigma_1 + \cos(2\phi)\sigma_2. \qquad p_T = z_f q_T$$

for

$$e + p \to e + \pi, \Lambda(p_T) + X$$

$$e + \vec{p} \to e + \vec{\Lambda}(p_T) + X$$

$\sigma_2 \equiv 0$ for

$$\vec{e} + \vec{p} \to e + \pi(p_T) + X$$

$$\vec{e} + p \to e + \vec{\Lambda}(p_T) + X$$

Characteristics:

- Quark and Gluon contributions are the same $O(\alpha_s)$ effect. $\to$ sensitive to gluon dist./frag. functions.
- Partonic hard cross section in σ_0 has a nonintegrable $1/q_T^2$-dependence at $q_T \to 0$, but not in $\sigma_{1,2}$. $\to$ Canceled and factorized in the combination with the virtual correction diagrams.
- For the differential cross section at small q_T-region, resummation is necessary. (W. Vogelsang, J. Nagashima, YK in preparation.) $\to$ Use present formula only for large-p_T.

(iii) $e + p^{\uparrow} \to e + \Lambda^{\uparrow}(p_T) + X$:

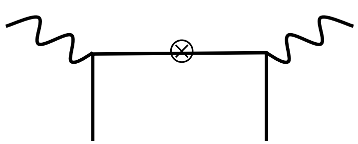
$$\frac{d^5\sigma_{TT}}{dx_{bj}\,dQ^2\,dz_f\,dq_T^2\,d\phi} \sim \sum_q e_q^2 \delta q(x) \otimes \delta\hat{q}(z) \otimes \hat{\sigma}_T,$$

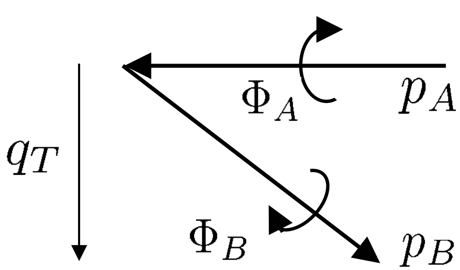
No gluon contribution!

where

$$\hat{\sigma}_T \sim (1+\cosh^2\psi)\cos(\Phi_A-\Phi_B) - \sinh(2\psi)\cos(\Phi_A-\Phi_B-\phi)\frac{Q}{q_T} + \sinh^2\psi\cos(\Phi_A-\Phi_B-2\phi)\frac{Q^2}{q_T^2}.$$

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1 = \frac{2}{y} - 1.$$

Note:  $\propto \cos(\Phi_A - \Phi_B - 2\phi)$



Recall: ϕ, Φ_A, Φ_B, measured from the hadron plane.

$\Phi_A - \Phi_B - 2\phi$ does not depend on hadron plane

at $q_T \to 0$.

Azymuthal asymmetries:

- $$\langle \cos(n\phi) \rangle_{S_A S_B} \equiv \frac{\int_0^{2\pi} d\phi \, \cos(n\phi) \, \frac{d^5\sigma^{av,pol}}{dQ^2 dx_{bj} dz_f dq_T{}^2 d\phi}}{\int_0^{2\pi} d\phi \frac{d^5\sigma^{av}}{dQ^2 dx_{bj} dz_f dq_T{}^2 d\phi}} = \frac{\sigma_n{}^{av,pol}}{(2)\sigma_0},$$

$(n = 0,1,2)$

$S_A S_B = OO, \; LL, \; LO, \; OL$ O: Unpolarized, L: Long. pol.

- $$\langle \cos(\Phi_A - \Phi_B - n\phi) \rangle_{TT} \equiv \frac{\int_0^{2\pi} d\phi \, \cos(\Phi_A - \Phi_B - n\phi) \, \frac{d^5\sigma_{TT}}{dQ^2 dx_{bj} dz_f dq_T{}^2 d\phi}}{\int_0^{2\pi} d\phi \frac{d^5\sigma^{av}}{dQ^2 dx_{bj} dz_f dq_T{}^2 d\phi}},$$

$(n = 0,1,2)$

Sample Calculation of Azymuthal Asymmetries:

In general:

For smaller x_{bj} with $\vec{p}$ and $p^{\uparrow}$, smaller asymmetries.

For polarized $\vec{e}$, smaller $y = \frac{Q^2}{x_{bj} S_{ep}}$ gives smaller asym.

Chosen Kinematic variables:

- COMPASS energy: $S_{ep} = 300\ GeV^2,\ Q^2 = 100\ GeV^2,\ x_{bj} = 0.4$ "Valence" region
- EIC energy: $S_{ep} = 10^4\ GeV^2,\ Q^2 = 100\ GeV^2,\ x_{bj} = 0.012$ "Sea" region

  $(S_{ep} = 2000\ GeV^2,\ Q^2 = 100\ GeV^2,\ x_{bj} = 0.06)$

$\longrightarrow$

These give the same $y = 0.83$ $(\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1 = 1.4)$. The difference in the asymmetries is ascribed to the difference in the probed x-region $(x > x_{bj})$ and the hard cross section.

Sample Calculation of Azymuthal Asymmetries:

- $q(x),\ G(x)$ GRV98
- $\Delta q(x),\ \Delta G(x)$ GRSV2000 (std. and val.), AAC ('00), GS('96)
- Ansatz $\delta q(x,Q^2) = \Delta q(x,Q^2)$

  N.B. Very crude estimate. Their evolution is different.
- π fragmentation function: KKP2000, Kretzer('00)
- $\Lambda(\Lambda + \overline{\Lambda})$ fragmetation function: DSV98 (de Florian et al)

  Three scenarios for long. pol. fragmentation function:

  At low energy input scale they are constructed such that

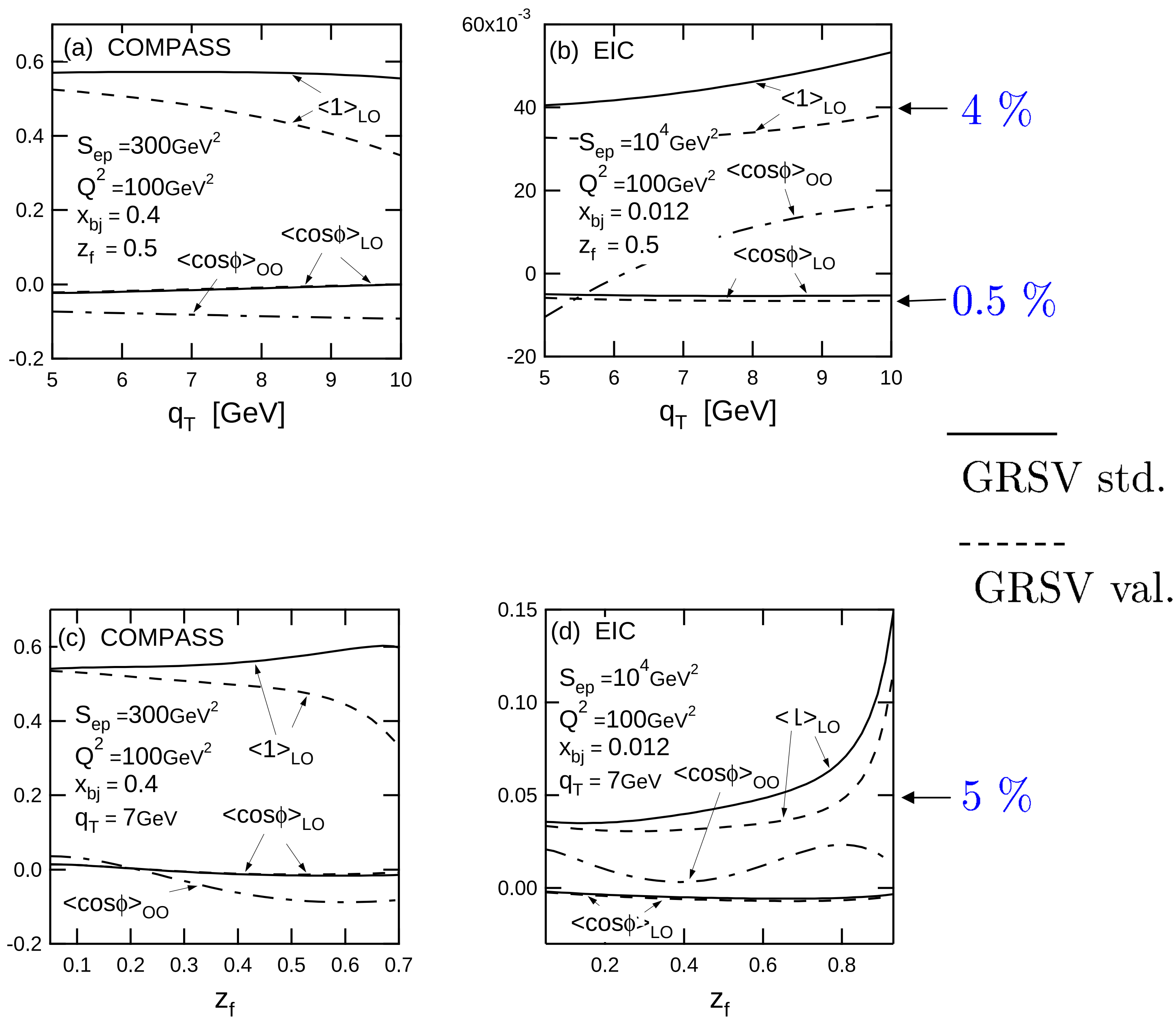
  scen.1: $\Delta\hat{u} = \Delta\hat{d} = 0$ (NR quark model)

  scen.2: $\Delta\hat{u} = \Delta\hat{d} = -0.2\,\Delta\hat{s}$ (inspired by Burkardt-Jaffe)

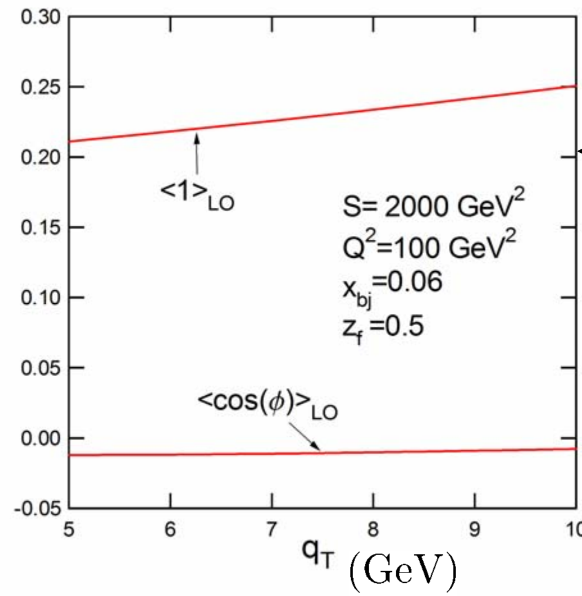
  scen.3: $\Delta\hat{u} = \Delta\hat{d} = \Delta\hat{s}$

  Transversity fragmentation function: $\delta\hat{q}(z,Q^2) = \Delta\hat{q}(z,Q^2)$

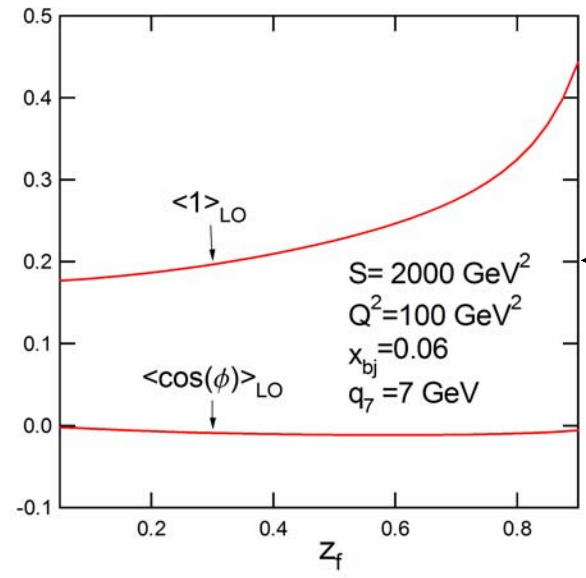
$$\vec{e} + \vec{p} \to e + \pi(p_T) + X \text{ at } x_{bj} = 0.06,\ 0.012.$$



$$\vec{e} + \vec{p} \to e + \pi(p_T) + X \text{ at } x_{bj} = 0.06.$$



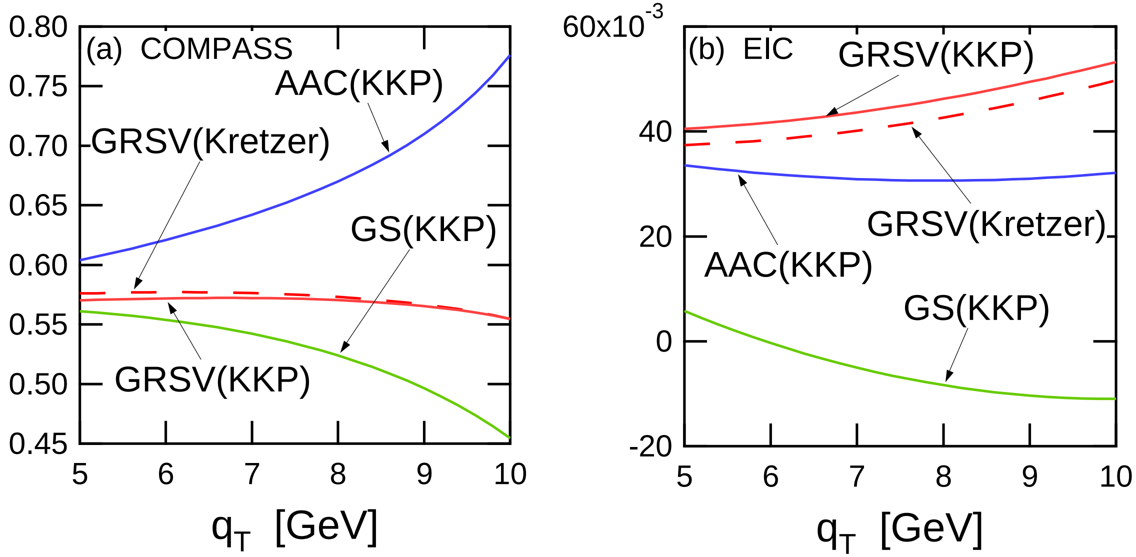
20 %



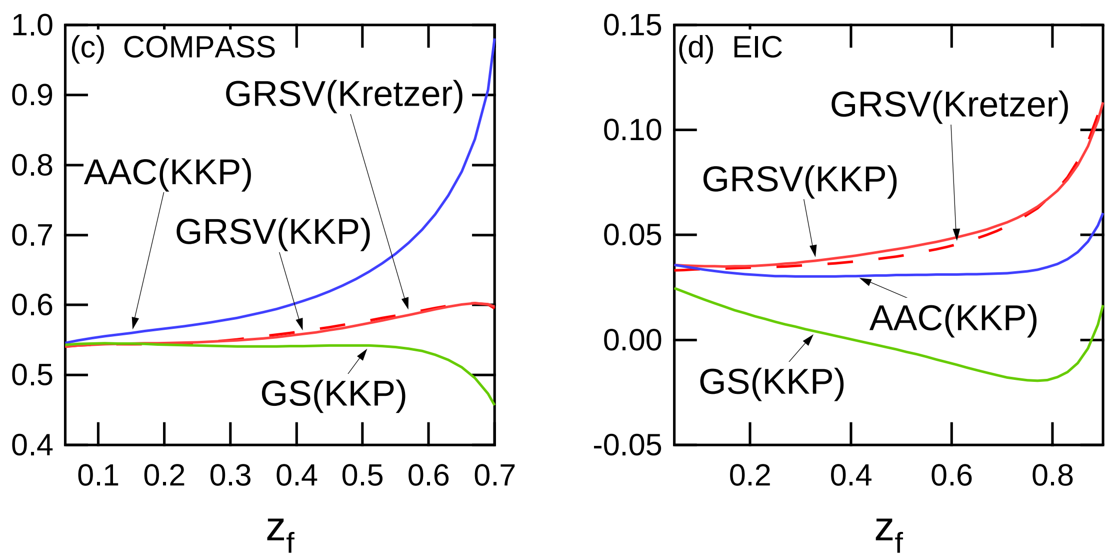
20 %

Dependence on distribution/fragmentation function:

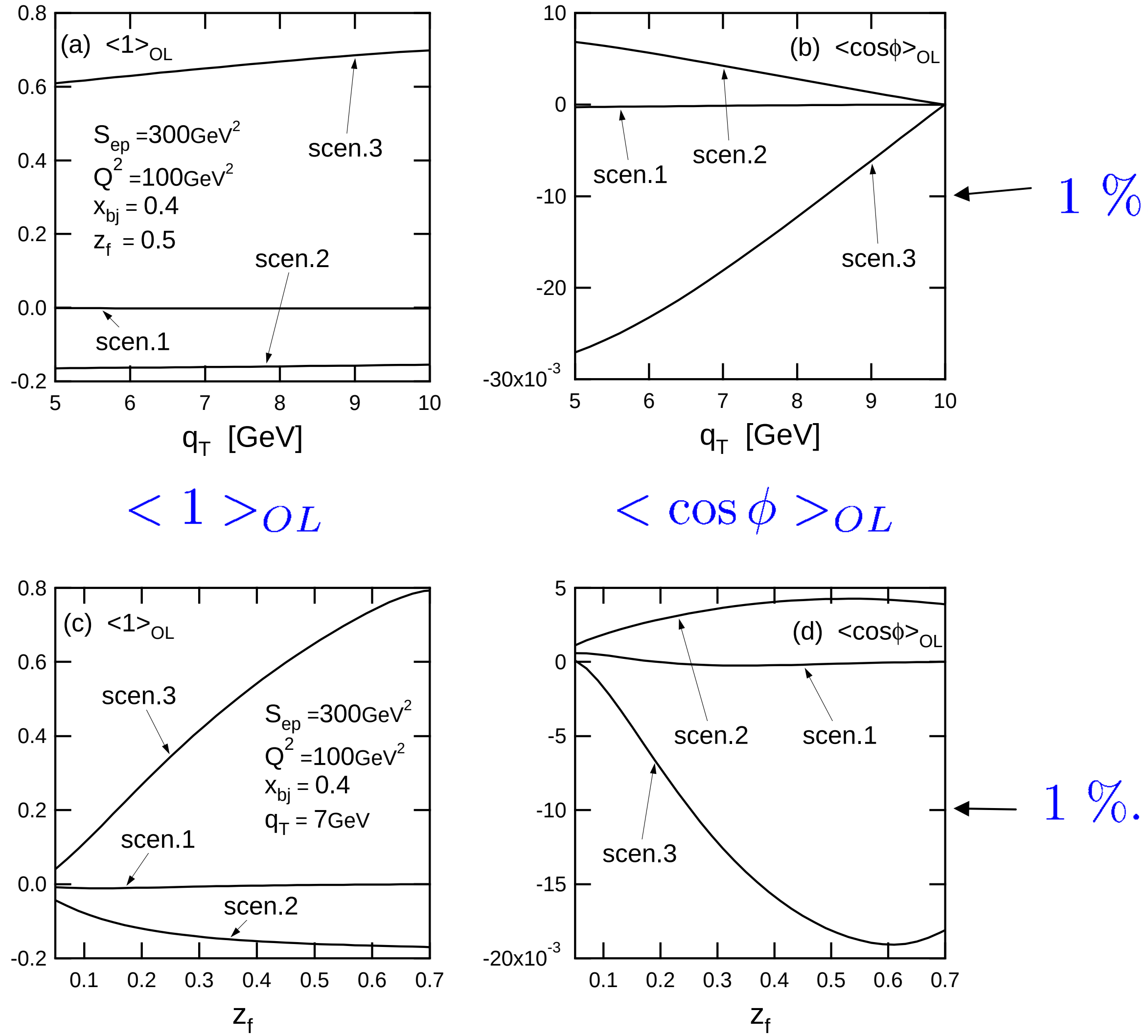
$\langle 1 \rangle_{LO}$ in $\vec{e} + \vec{p} \rightarrow e + \pi + X$.



Huge dependence on the choise of parton densities!



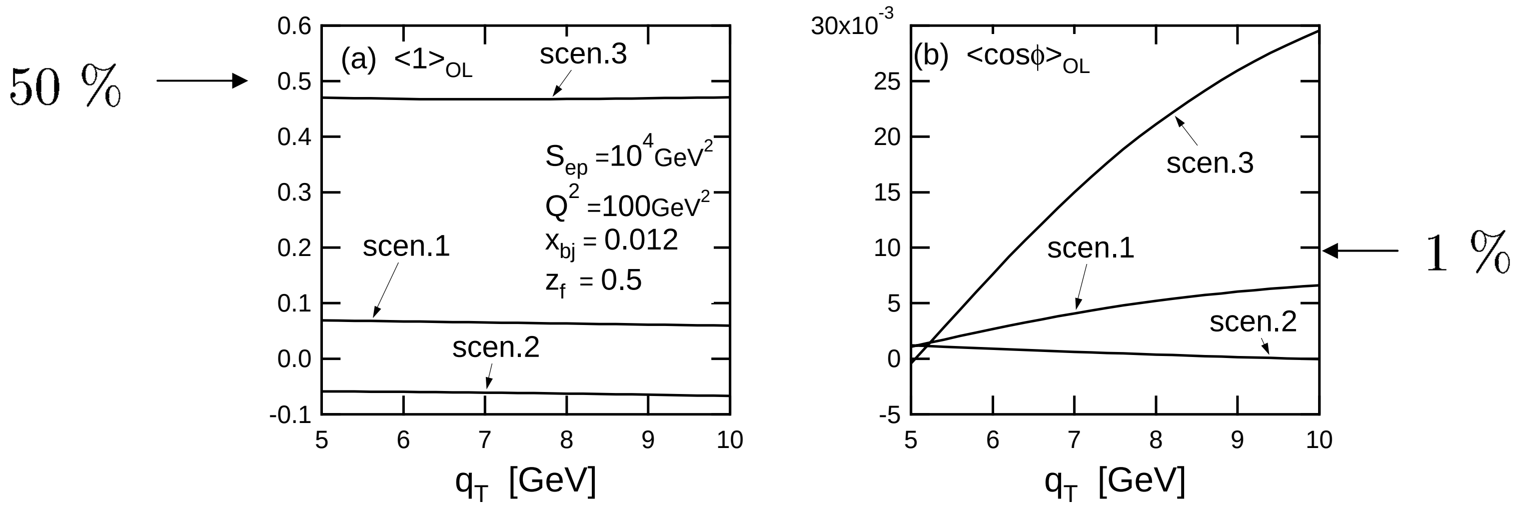
$$\vec{e} + p \to e + \vec{\Lambda}(p_T) + X \text{ at } x_{bj} = 0.4.$$



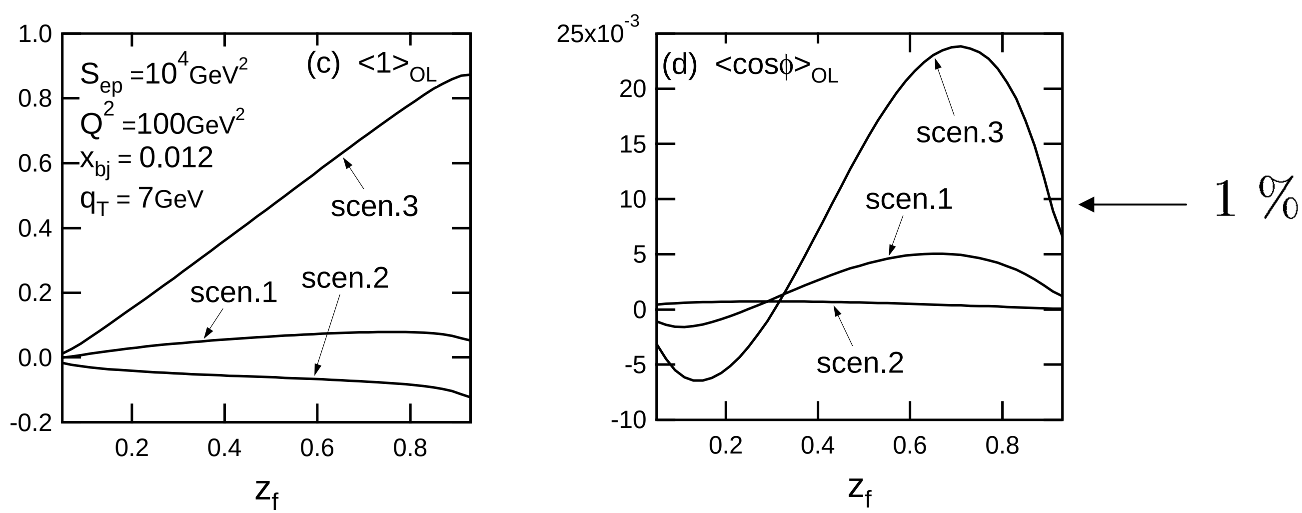
$\sim$ a few 10 %.

$\sim$1 %.

$$\vec{e} + p \to e + \vec{\Lambda}(p_T) + X \text{ at } x_{bj} = 0.012.$$



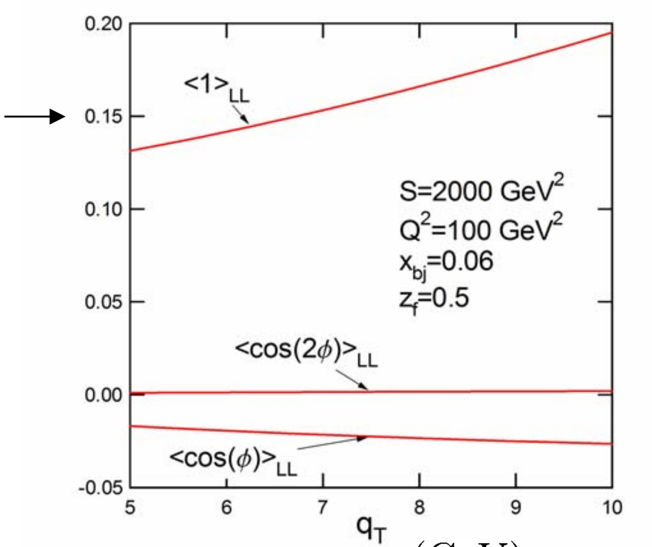
$< 1 >_{OL}$ $< \cos\phi >_{OL}$



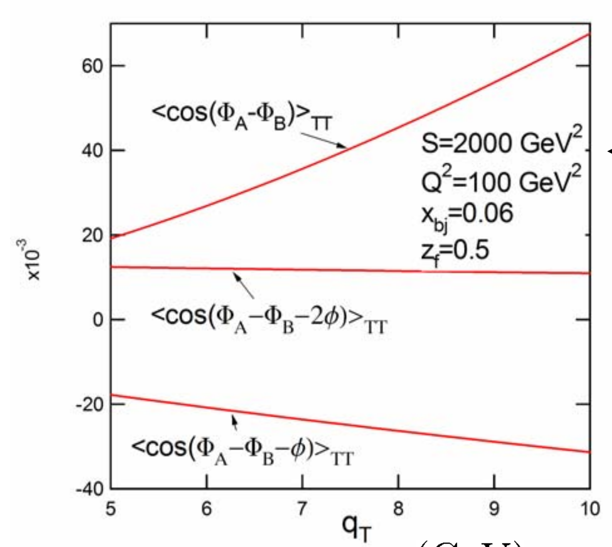
$x_{bj} = 0.06$

$e\vec{p} \to e\vec{\Lambda}X$ $\quad$ $ep^{\uparrow} \to e\Lambda^{\uparrow}X$

15 % →



(GeV)

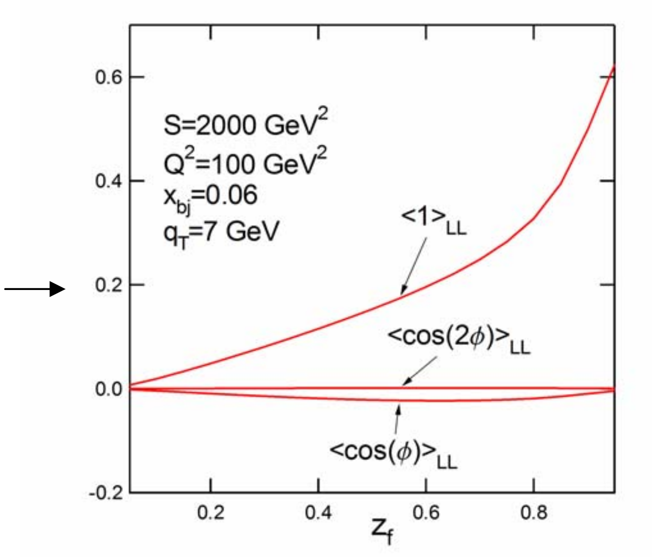


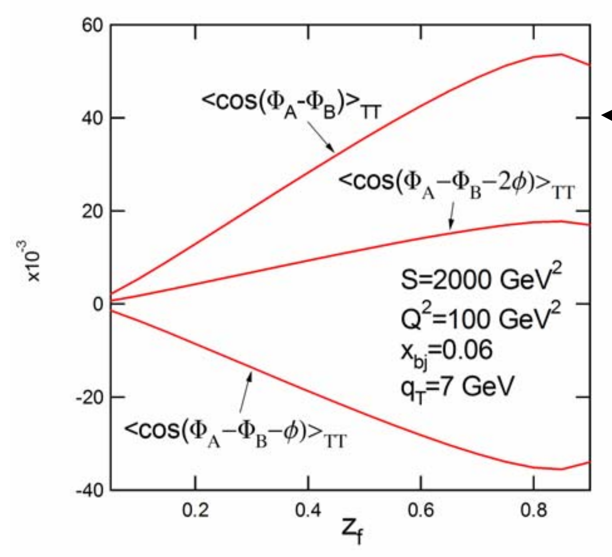
← 4 %

(GeV)

Scen. 3 for $\Delta\widehat{q}$ and $\delta\widehat{q}$.

20 % →





← 4 %

Summary

p_T-differential cross section for large-p_T polarized SIDIS was derived to $O(\alpha_s)$ in pertubative QCD:

- $$\frac{d^5\sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} = \sigma_0 + \cos(\phi)\sigma_1 + \cos(2\phi)\sigma_2. \qquad p_T = z_f q_T$$

  for $e + \vec{p} \to e + \vec{\Lambda}(p_T) + X$ $\qquad \sigma_2 \equiv 0$ for $\vec{e} + \vec{p} \to e + \pi(p_T) + X$, $\vec{e} + p \to e + \vec{\Lambda}(p_T) + X$

  $\longrightarrow \sigma_0 >> \sigma_1 >> \sigma_2$

- $$\frac{d^5\sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi}$$
  $$= \cos(\Phi_A - \Phi_B)\sigma_0 + \cos(\Phi_A - \Phi_B - \phi)\sigma_1 + \cos(\Phi_A - \Phi_B - 2\phi)\sigma_2.$$

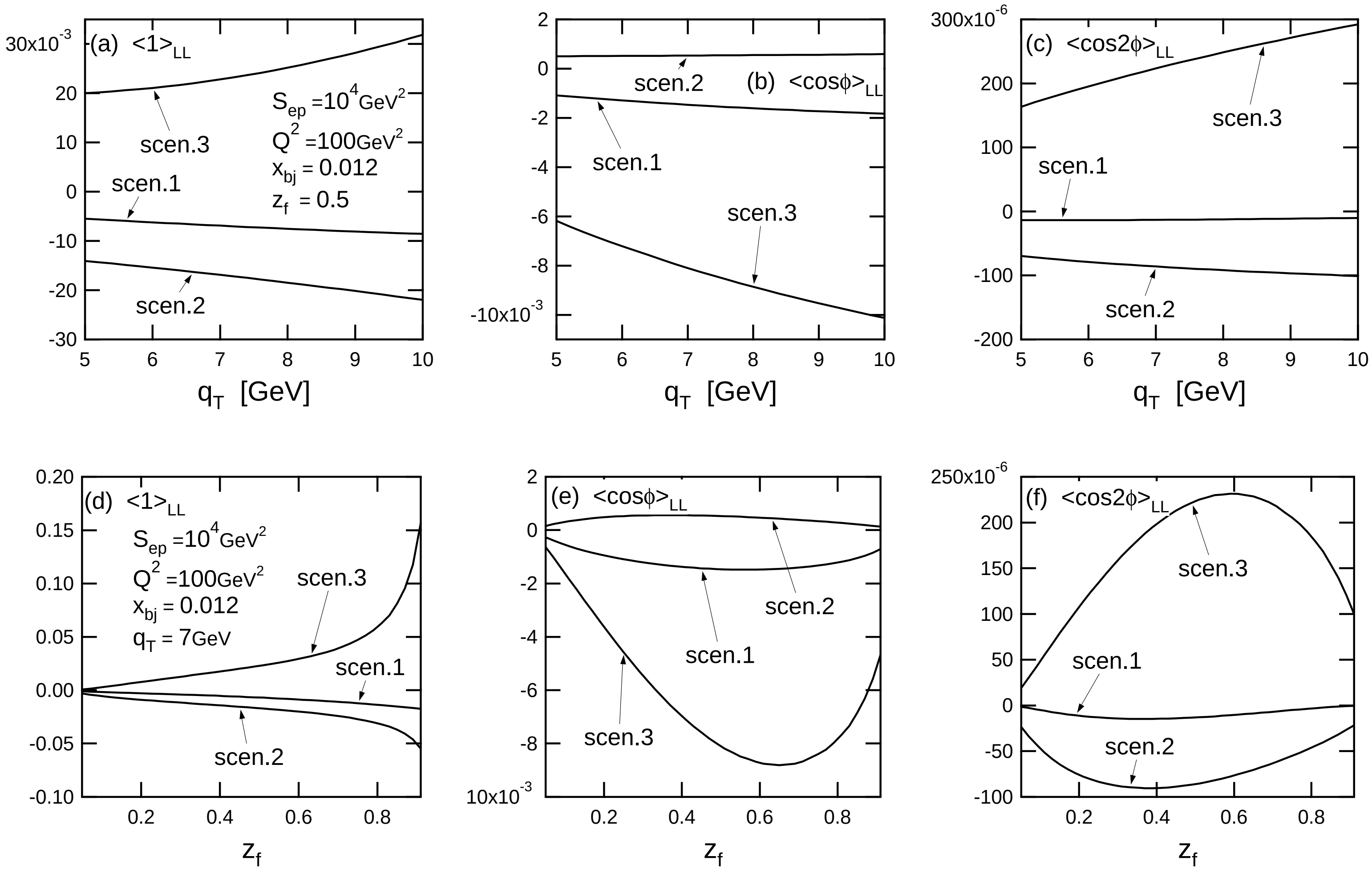
  for $e + p^{\uparrow} \to e + \Lambda^{\uparrow}(p_T) + X$ $\qquad \longrightarrow \sigma_0 \simeq \sigma_1 \simeq \sigma_2$

Huge difference in the asymmetry with different choice of polarized parton distributions!

Resummed cross section for small q_T region, underway.

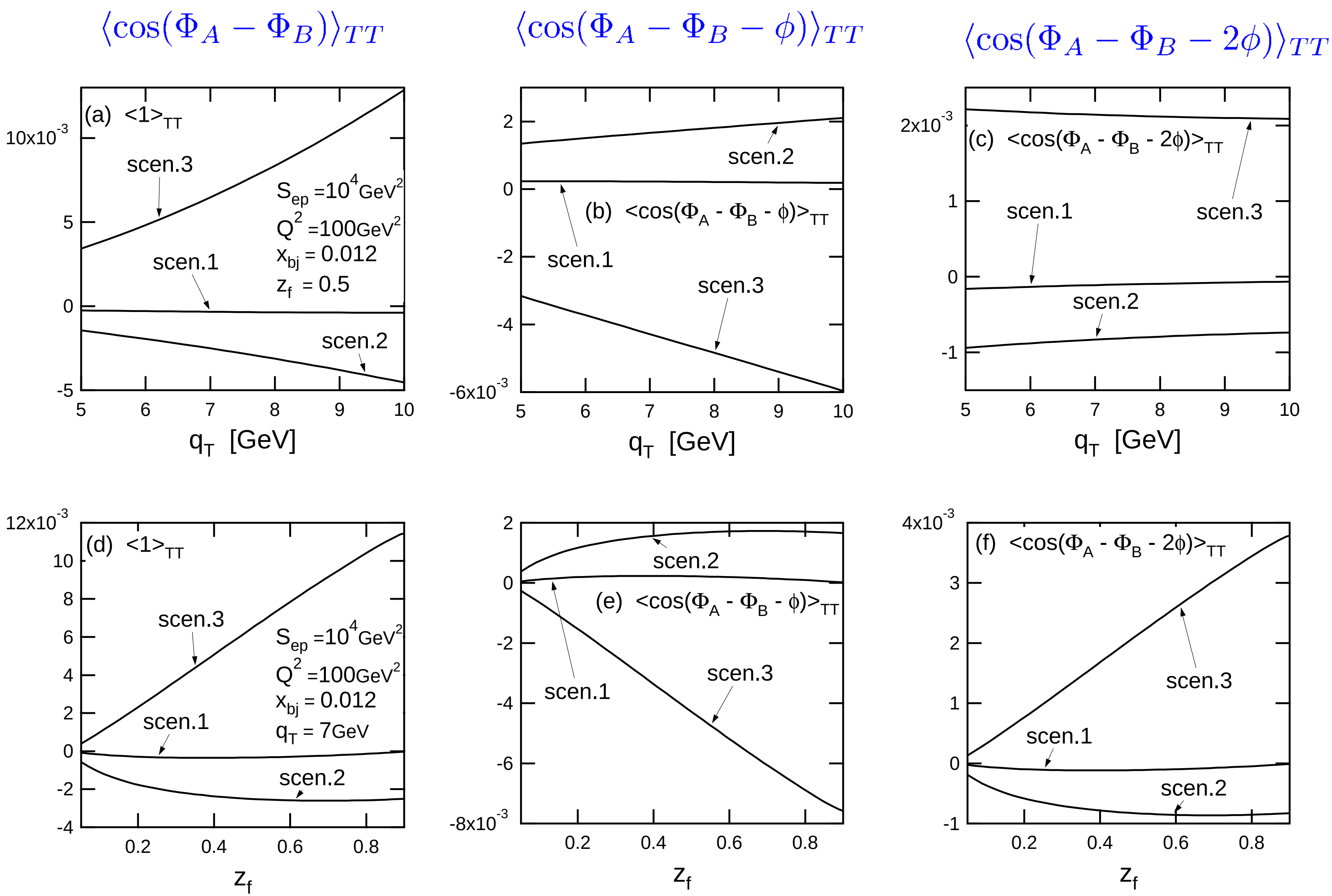
$$e + \vec{p} \to e + \vec{\Lambda}(p_T) + X \text{ at } x_{bj} = 0.012.$$

$< 1 >_{LL}$ | $< \cos\phi >_{LL}$ | $< \cos 2\phi >_{LL}$



~a few % | ≤ 1 % | ~ 0.01 %

$$e + p^\uparrow \to e + \Lambda^\uparrow(p_T) + X \text{ at } x_{bj} = 0.012.$$



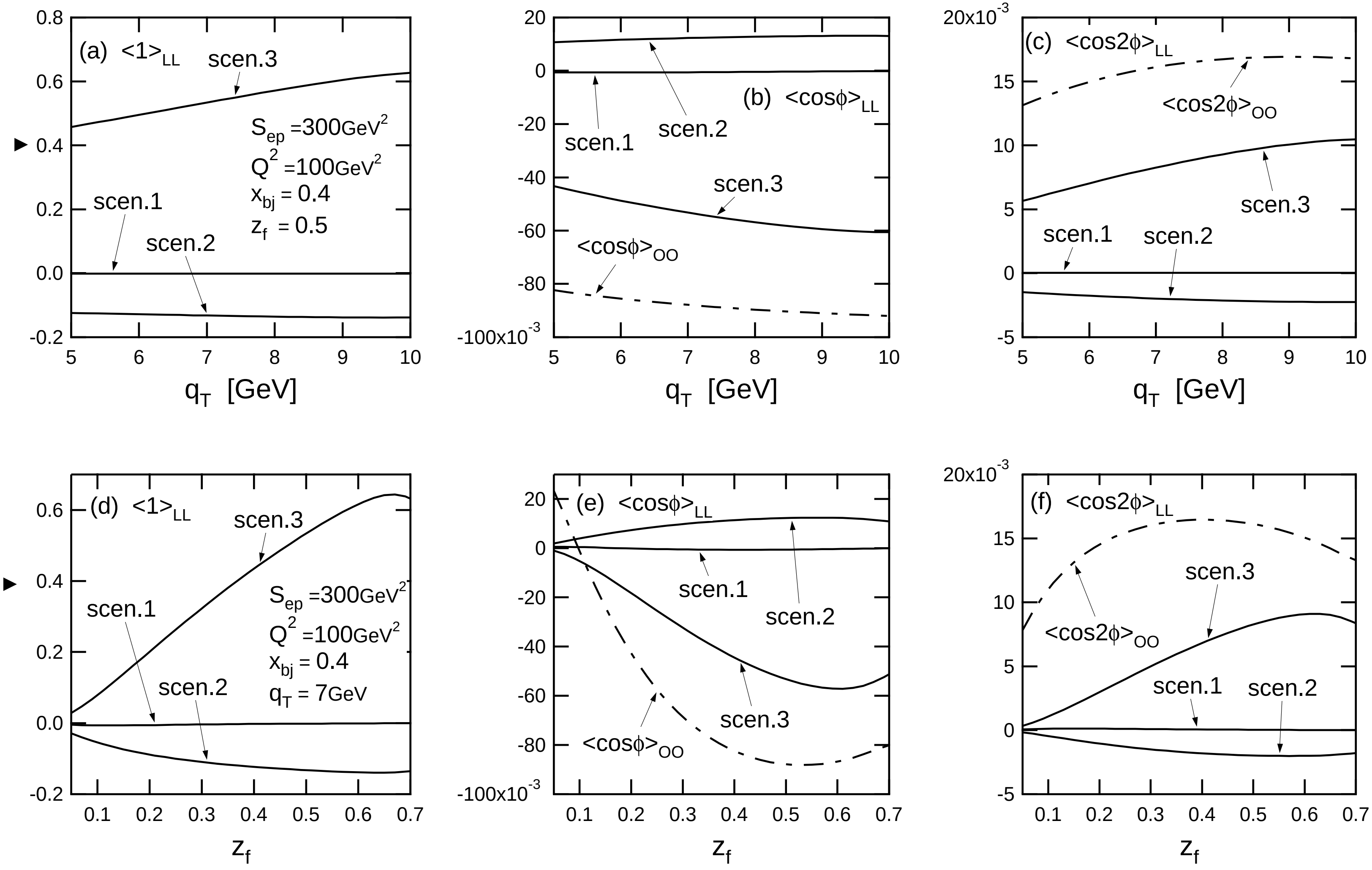
$\sim 1\ \%$ $\sim 0.2 - 1\ \%$ $\sim 0.1 - 0.3\ \%$

$$e + \vec{p} \to e + \vec{\Lambda}(p_T) + X \text{ at } x_{bj} = 0.4.$$

$< 1 >_{LL}$ $< \cos\phi >_{LL}$ $< \cos 2\phi >_{LL}$

40 % ▶

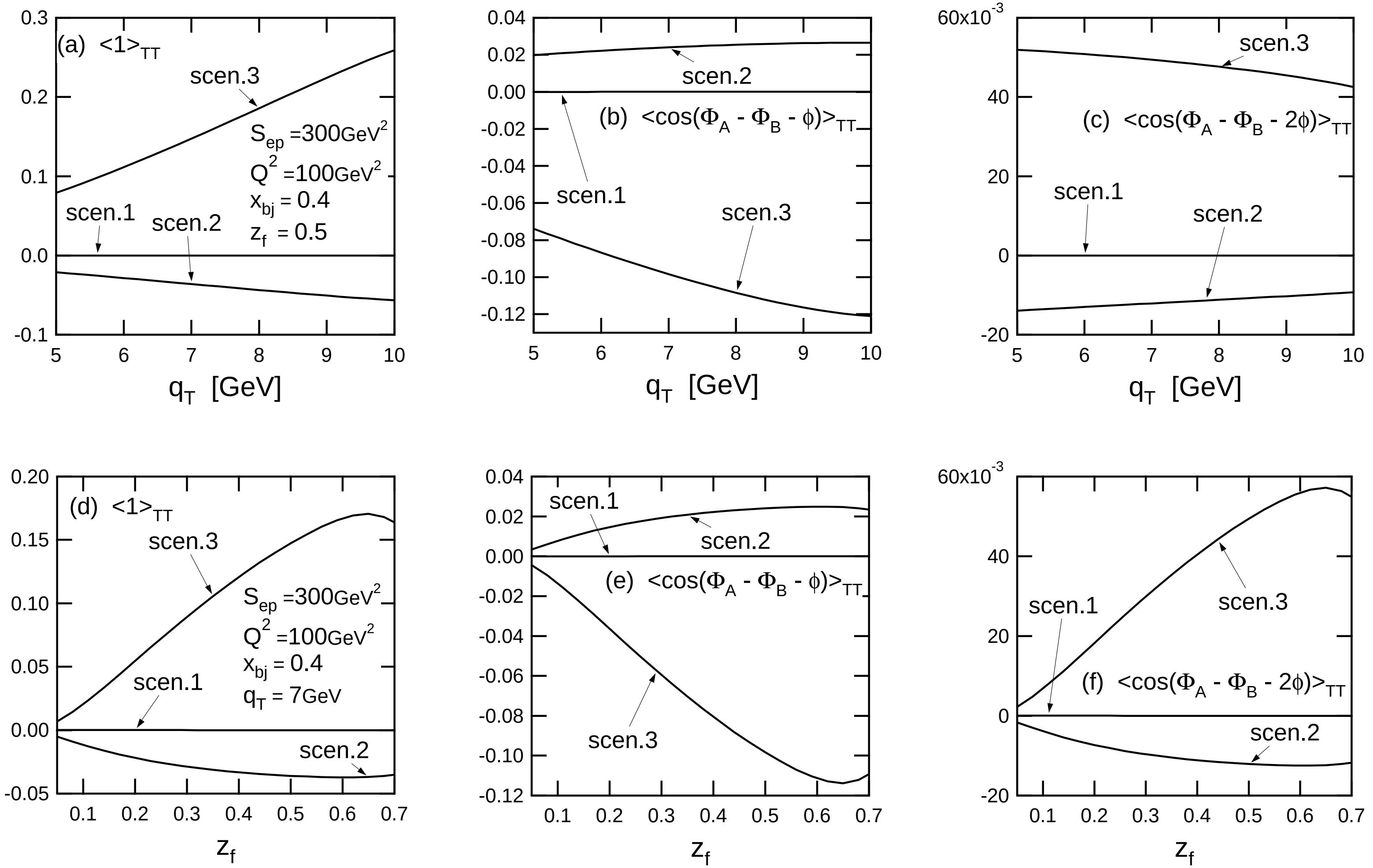
40 % ▶



$\sim 5\ \%$ $\sim 0.1\ \%$

$$e + p^{\uparrow} \to e + \Lambda^{\uparrow}(p_T) + X \text{ at } x_{bj} = 0.4.$$

$\langle \cos(\Phi_A - \Phi_B) \rangle_{TT}$ | $\langle \cos(\Phi_A - \Phi_B - \phi) \rangle_{TT}$ | $\langle \cos(\Phi_A - \Phi_B - 2\phi) \rangle_{TT}$



$\sim 20\ \%$ | $\sim 2 - 10\ \%$ | $\sim 0.2 - 0.6\ \%$

★Cross section formula

$$\frac{d^5\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi} = \frac{\alpha_e^2\alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k \int_{x_{bj}}^1 \frac{dx}{x}\int_{z_f}^1 \frac{dz}{z}\,[f\otimes D\otimes \widehat{\sigma}_k]$$
$$\times\delta\left(\frac{q_T^2}{Q^2}-\left(\frac{1}{\widehat{x}}-1\right)\left(\frac{1}{\widehat{z}}-1\right)\right),$$

where $\widehat{x}=\frac{x_{bj}}{x}$, and $\widehat{z}=\frac{z_f}{z}$. For (i) and (ii), $k=1$ to 4 with;

$$\mathcal{A}_1 = 1+\cosh^2\psi, \qquad \mathcal{A}_2=-2,$$
$$\mathcal{A}_3 = -\cos\phi\sinh 2\psi, \qquad \mathcal{A}_4=\cos 2\phi\sinh^2\psi.$$

For (iv) and (v), $k=6$ and 7 with;

$$\mathcal{A}_6=-2\cosh\psi, \qquad \mathcal{A}_7=2\cos\phi\sinh\psi$$

with $\cosh\psi=\frac{2x_{bj}S_{ep}}{Q^2}-1$.

(i) $e+p\to e'+\{\pi,\Lambda\}+X$:

$$
\begin{aligned}
[f\otimes D\otimes\widehat{\sigma}_k] &= \sum_q e_q^2 q(x)\widehat{q}(z)\widehat{\sigma}_k^{qq}+\sum_q e_q^2 G(x)\widehat{q}(z)\widehat{\sigma}_k^{gq} \\
&\quad +\sum_q e_q^2 q(x)\widehat{G}(z)\widehat{\sigma}_k^{qg},
\end{aligned}
$$

where

$$
\begin{aligned}
\widehat{\sigma}_1^{qq} &= 2C_F\widehat{x}\widehat{z}\left\{\frac{1}{Q^2q_T^2}\left(\frac{Q^4}{\widehat{x}^2\widehat{z}^2}+\left(Q^2-q_T^2\right)^2\right)+6\right\}, \\
\widehat{\sigma}_2^{qq} &= 2\widehat{\sigma}_4^{qq}=8C_F\widehat{x}\widehat{z}, \\
\widehat{\sigma}_3^{qq} &= 4C_F\widehat{x}\widehat{z}\frac{1}{Qq_T}(Q^2+q_T^2),
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\sigma}_1^{gq} &= \widehat{x}(1-\widehat{x})\left\{\frac{Q^2}{q_T^2}\left(\frac{1}{\widehat{x}^2\widehat{z}^2}-\frac{2}{\widehat{x}\widehat{z}}+2\right)+10-\frac{2}{\widehat{x}}-\frac{2}{\widehat{z}}\right\}, \\
\widehat{\sigma}_2^{gq} &= 2\widehat{\sigma}_4^{gq}=8\widehat{x}(1-\widehat{x}), \\
\widehat{\sigma}_3^{gq} &= \widehat{x}(1-\widehat{x})\frac{2}{Qq_T}\left\{2(Q^2+q_T^2)-\frac{Q^2}{\widehat{x}\widehat{z}}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\sigma}_1^{qg} &= 2C_F\widehat{x}(1-\widehat{z})\left\{\frac{1}{Q^2q_T^2}\left(\frac{Q^4}{\widehat{x}^2\widehat{z}^2}+\frac{(1-\widehat{z})^2}{\widehat{z}^2}\left(Q^2-\frac{\widehat{z}^2q_T^2}{(1-\widehat{z})^2}\right)^2\right)+6\right\}, \\
\widehat{\sigma}_2^{qg} &= 2\widehat{\sigma}_4^{qg}=8C_F\widehat{x}(1-\widehat{z}), \\
\widehat{\sigma}_3^{qg} &= -4C_F\widehat{x}(1-\widehat{z})^2\frac{1}{\widehat{z}Qq_T}\left\{Q^2+\frac{\widehat{z}^2q_T^2}{(1-\widehat{z})^2}\right\}.
\end{aligned}
$$

(iv) $\vec{e}+\vec{p}\to e'+\{\pi,\Lambda\}+X$:

$$
\begin{aligned}
[f\otimes D\otimes\widehat{\sigma}_k] &= \sum_q e_q^2\Delta q(x)\widehat{q}(z)\Delta_{LO}\widehat{\sigma}_k^{qq}+\sum_q e_q^2\Delta G(x)\widehat{q}(z)\Delta_{LO}\widehat{\sigma}_k^{gq} \\
&\quad +\sum_q e_q^2\Delta q(x)\widehat{G}(z)\Delta_{LO}\widehat{\sigma}_k^{qg},
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_{LO}\widehat{\sigma}_6^{qq} &= -2C_F\left\{\left(\frac{1}{\widehat{x}\widehat{z}}+\widehat{x}\widehat{z}\right)\frac{Q^2}{q_T^2}-\frac{\widehat{x}\widehat{z}q_T^2}{Q^2}\right\}, \\
\Delta_{LO}\widehat{\sigma}_7^{qq} &= -4C_F\widehat{x}\widehat{z}\frac{Q^2-q_T^2}{Qq_T},
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{LO}\widehat{\sigma}_6^{gq} &= \frac{2\widehat{x}-1}{\widehat{x}}\left(2\widehat{x}+\frac{\widehat{x}-1}{\widehat{z}^2}\frac{Q^2}{q_T^2}\right), \\
\Delta_{LO}\widehat{\sigma}_7^{gq} &= \frac{2Q}{q_T}\frac{(\widehat{x}-1)(2\widehat{z}-1)}{\widehat{z}},
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{LO}\widehat{\sigma}_6^{qg} &= \frac{2C_F\widehat{z}}{\widehat{x}-1}\left\{\frac{1}{\widehat{z}^2}-(\widehat{x}-1)^2+\frac{\widehat{x}^4}{(\widehat{x}-1)^2}\frac{q_T^4}{Q^4}\right\}, \\
\Delta_{LO}\widehat{\sigma}_7^{qg} &= \frac{4C_F\widehat{x}\widehat{z}}{\widehat{x}-1}\left(1-\frac{\widehat{x}}{\widehat{z}}\right)\frac{q_T}{Q}.
\end{aligned}
$$