Beam-Beam Interaction in Linac-Ring Colliders

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Talk Outline

- Beam-Beam Estimates
- Head-Tail Instability
- Luminosity-Deflection Theorem
- Simulation Methods
  - Coulomb Sum (Beard, Li)
  - PIC (Shi)
- Simulation Status
- Future needs for a complete solution
- Conclusions

Luminosity Factors

$$\mathcal{L} = f \frac{N_e N_i}{2\pi \sqrt{\sigma_{e,x}^2 + \sigma_{i,x}^2} \sqrt{\sigma_{e,y}^2 + \sigma_{i,y}^2}}$$

Ion Tune Shifts:

$$\xi_{i,x} = \beta^*_{i,x} \frac{N_e r_i}{2\pi\gamma_i \sigma_{e,x}(\sigma_{e,x} + \sigma_{e,y})} \qquad \xi_{i,y} = \beta^*_{i,y} \frac{N_e r_i}{2\pi\gamma_i \sigma_{e,y}(\sigma_{e,x} + \sigma_{e,y})}$$

"Equivalent" Electron Tune Shifts:

$$\xi_{e,x} = \beta^*_{e,x} \frac{N_i r_e}{2\pi\gamma_e \sigma_{i,x}(\sigma_{i,x} + \sigma_{i,y})} \qquad \xi_{e,y} = \beta^*_{e,y} \frac{N_i r_e}{2\pi\gamma_e \sigma_{i,y}(\sigma_{i,x} + \sigma_{i,y})}$$

Gedanken Experiment

For round, equal sized beams, the following scaling applies:

$$\mathcal{L} = \frac{I_e}{e}\frac{\gamma_e \xi_e}{\beta^* r_e}$$

Comparing linac-ring colliders and ring-ring colliders, what can change for the better?

1. **Maximum I_e/e is set by ION ring stability. The same in the two cases**
2. **γ_e set by the physics. The same in the two cases**
3. **Minimum β^* is set by IR region design issues. Can it be too much better for linac-ring? Should not be any worse than for ring-ring**
4. **r_e is set by (God, Yahweh, Allah, …); YOU cannot change it**
5. **If there are to be luminosity enhancements to be found for linac-ring designs compared to ring-ring designs, they must arise because one is allowed to make the equivalent tune shift ξ_e bigger for linac-ring colliders.**
6. **Finding the physical phenomena that determine the maximum ξ_e are extremely important for evaluating the linac-ring idea.**

Two guesses

1. *Emittance growth generated by a single beam-beam collision. Round Gaussian beam collision integrals can be performed to give*

$$\varepsilon_{n,after}^2 = \varepsilon_{n,before}^2 + (0.194 N_p)^2$$

What's the right scaling for circulator ring? If proportional to the number of turns squared, may have a significant problem recovering the beam with small loss. Halo for CR?

2. *Fast Head-Tail instability; Linear Stability Estimate (Lebedev, Yunn, Li). Assume short electron bunch*

$$W_i(s) = \frac{r_e N_e}{\gamma_e \sigma_i^2 \sigma_e^2} s$$

Fast Head-Tail instability

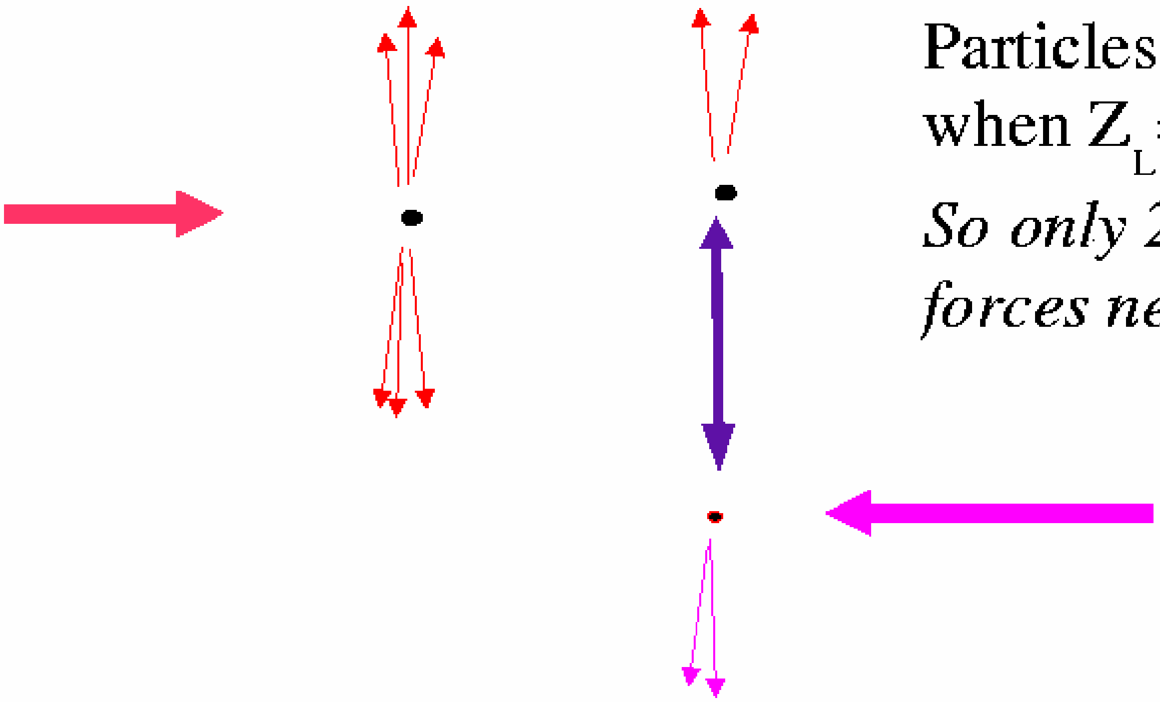
Threshold condition

$$\xi_e \xi_p \leq \frac{\beta_e \nu_s}{\pi^2 \sigma_z}$$

- Larger synchrotron frequency helps
- Different instability than in a ring because the fresh electrons come in at a fixed transverse position, without their own distortion. Makes an "impedance" model reasonable first approximation
- Full calculation needs to account for non-linear effects and the fact that the electron bunch is no longer short
- See Li, et. al, PAC2001, 2014 for a linear "long-bunch" theory, including synchro-betatron coupling

High Energy Simplification

Simplifications... v ~ c



Particles only interact when $Z_L = Z_R$

So only 2 dimensional forces need be considered

Luminosity-Deflection Theorem

Theorem from 2-D electrostatics

$$\vec{F}'_{21}\left(\vec{b}\right) = -\vec{F}'_{21} = 2Q'_1Q'_2\int n_2\left(\vec{x}_2\right)\frac{\vec{x}_2+\vec{b}-\vec{x}_1}{\left|\vec{x}_2+\vec{b}-\vec{x}_1\right|^2}n_1\left(\vec{x}_1\right)d^2\vec{x}_1 d^2\vec{x}_2$$

$$\Rightarrow \qquad \nabla_{\vec{b}}\cdot\vec{F}'_{21}\left(\vec{b}\right) = 4\pi\int\rho_2\left(\vec{x}\right)\rho_1\left(\vec{x}\right)d^2\vec{x}$$

2-D Gauss's law generalized to transversely extended macroparticles

$$\vec{D}\left(\vec{b}\right) = \Delta\gamma_1\vec{\beta}_1 = -\Delta m_2\gamma_2\vec{\beta}_2/m_1 = \frac{2q_1q_2}{m_1c^2}\int n_1\left(\vec{x}_1\right)\frac{\vec{x}_1-\vec{x}_2-\vec{b}}{\left|\vec{x}_1-\vec{x}_2-\vec{b}\right|^2}n_2\left(\vec{x}_2\right)d^2\vec{x}_1 d^2\vec{x}_2$$

$$\Rightarrow \nabla_{\vec{b}}\cdot\vec{D}\left(\vec{b}\right) = -\frac{4\pi q_1q_2}{m_1c^2}\int n_1\left(\vec{x}\right)n_2\left(\vec{x}-\vec{b}\right)d^2\vec{x} = \frac{4\pi r_e}{N_1}L\left(\vec{b}\right)$$

Luminosity-Deflection Transform Pairs

Round Beam Fast Model

$$\vec{D}(\vec{b}) = \frac{2N_2 r_e \vec{b}}{\sigma^2 + b^2} \qquad L(\vec{b}) = \frac{N_1 N_2 \sigma^2}{\pi(\sigma^2 + b^2)^2}$$

Gaussian Macroparticles

$$\vec{D}(\vec{b}) = \vec{D}_{Bassetti-Erskine}\left(\vec{b}; \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}; \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}\right)$$

$$L(\vec{b}) = \frac{N_1 N_2}{2\pi\sqrt{\sigma_{1,x}^2 + \sigma_{2,x}^2}\sqrt{\sigma_{1,y}^2 + \sigma_{2,y}^2}} \exp\left(-\frac{b_x^2}{2\sqrt{\sigma_{1,x}^2 + \sigma_{2,x}^2}}\right) \exp\left(-\frac{b_y^2}{2\sqrt{\sigma_{1,y}^2 + \sigma_{2,y}^2}}\right)$$

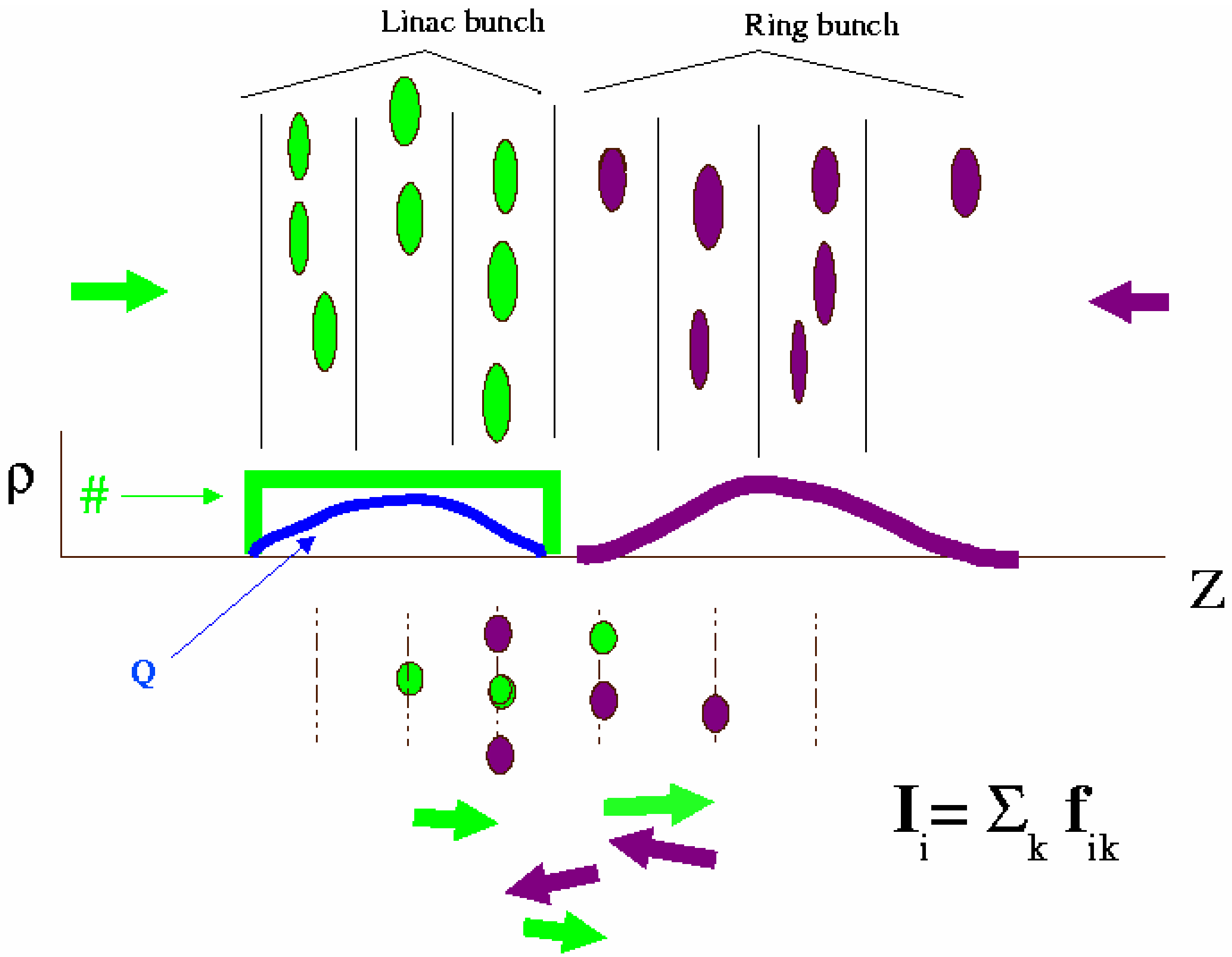
Smith-Laslett Model (no complex error function!)

$$\vec{D}(\vec{b}) = \frac{2N_2 r_e \vec{b}}{\hat{b}^2 AB}\left\{\frac{4\hat{b}^2 + 2\hat{b}^4}{\left(\hat{b}^4 + 4\hat{b}^2\right)} - \frac{4\hat{b}^2}{\left(\hat{b}^4 + 4\hat{b}^2\right)^{3/2}}\left\{\sinh^{-1}\left[\frac{\hat{b}^3}{2} + \frac{3\hat{b}}{2}\right] + \sinh^{-1}\left[\frac{\hat{b}}{2}\right]\right\}\right\}$$

$$L(\vec{b}) = \frac{N_1 N_2}{\pi AB}\left\{\frac{\left(2\hat{b}^2 - 4\right)\hat{b}^2}{\left(\hat{b}^4 + 4\hat{b}^2\right)^2} + \frac{4\hat{b}^2\left(1 + \hat{b}^2\right)}{\left(\hat{b}^4 + 4\hat{b}^2\right)^{5/2}}\left\{\sinh^{-1}\left[\frac{\hat{b}^3}{2} + \frac{3\hat{b}}{2}\right] + \sinh^{-1}\left[\frac{\hat{b}}{2}\right]\right\}\right\}$$

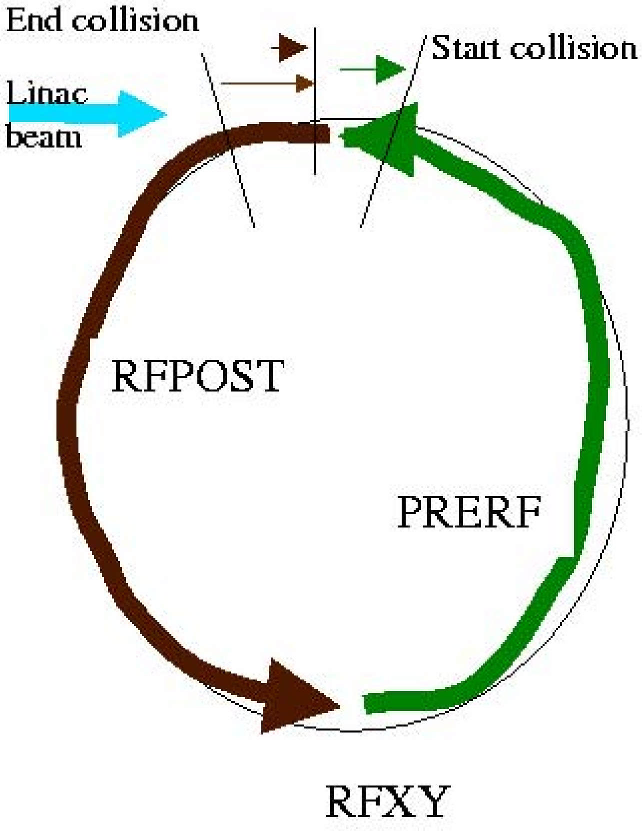
$$\hat{b}^2 = \left(\frac{b_x}{A}\right)^2 + \left(\frac{b_y}{B}\right)^2$$

Coulomb Sum Simulation Approach

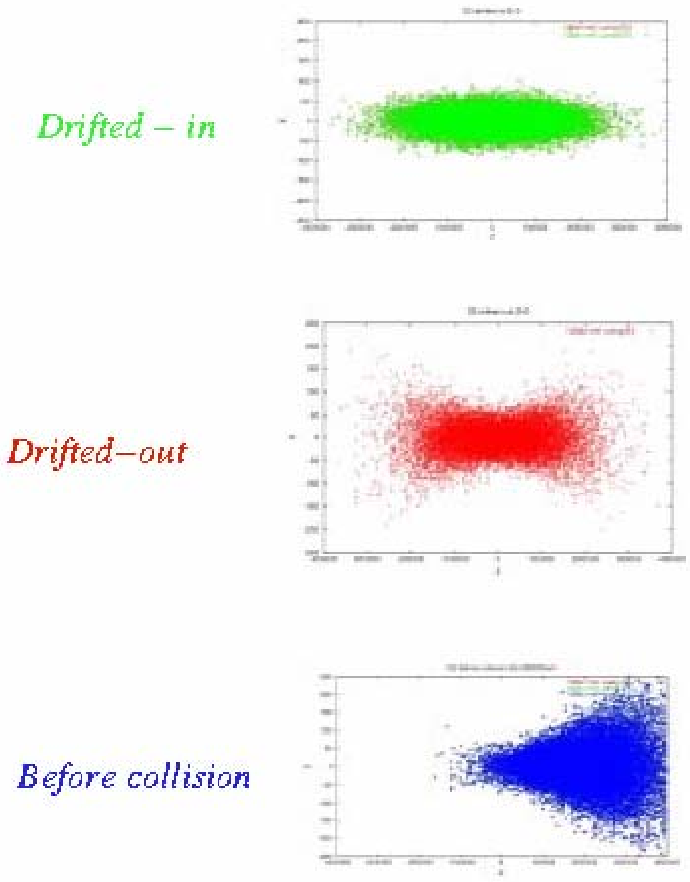


$$I_i = \Sigma_k f_{ik}$$

Matching to Reduce Mismatch Oscillations



Transverse motion



Collective Beam-Beam Effects In Hadron Colliders

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Collaboration:

University of Kansas	DESY, Cornell University
Lihui Jin	Georg Hoffstaetter
	Michiko Minty

Thanks: DOE

http://casa.jlab.org/seminars/2004/slides/shi_040213.pdf

2. Direct multi-particle tracking: the beam-beam force is calculated with particles-to-particle individually.

— Precise if N_p is large, but very slow $[O(N_p^2)]$,
typical: $N_p \leq 10^4 \Longrightarrow$ wrong physics in the nonlinear regime.

3. Particle-In-Cell (PIC): evaluate beam-beam force on a mesh.

— Precise, but very slow for separated beams.

Variations:

a. Calculate Beam-Beam Potential Without Boundary

b. Calculate The Potential With Approximated Boundary

c. Directly Calculate Beam-Beam Force on the Mesh

d. With Weighted Functions

Courtesy: Jack Shi

3. Direct Calculation of Beam-Beam Field on the Mesh

The field is calculated with

$$\vec{K}(\vec{r}) = \int d\vec{r'} \rho(\vec{r'}) \vec{G}_k(\vec{r} - \vec{r'})$$

where Green's function is

$$\vec{G}_k(\vec{r} - \vec{r'}) = \frac{(\vec{r} - \vec{r'})}{(x - x')^2 + (y - y')^2}$$

Comment:

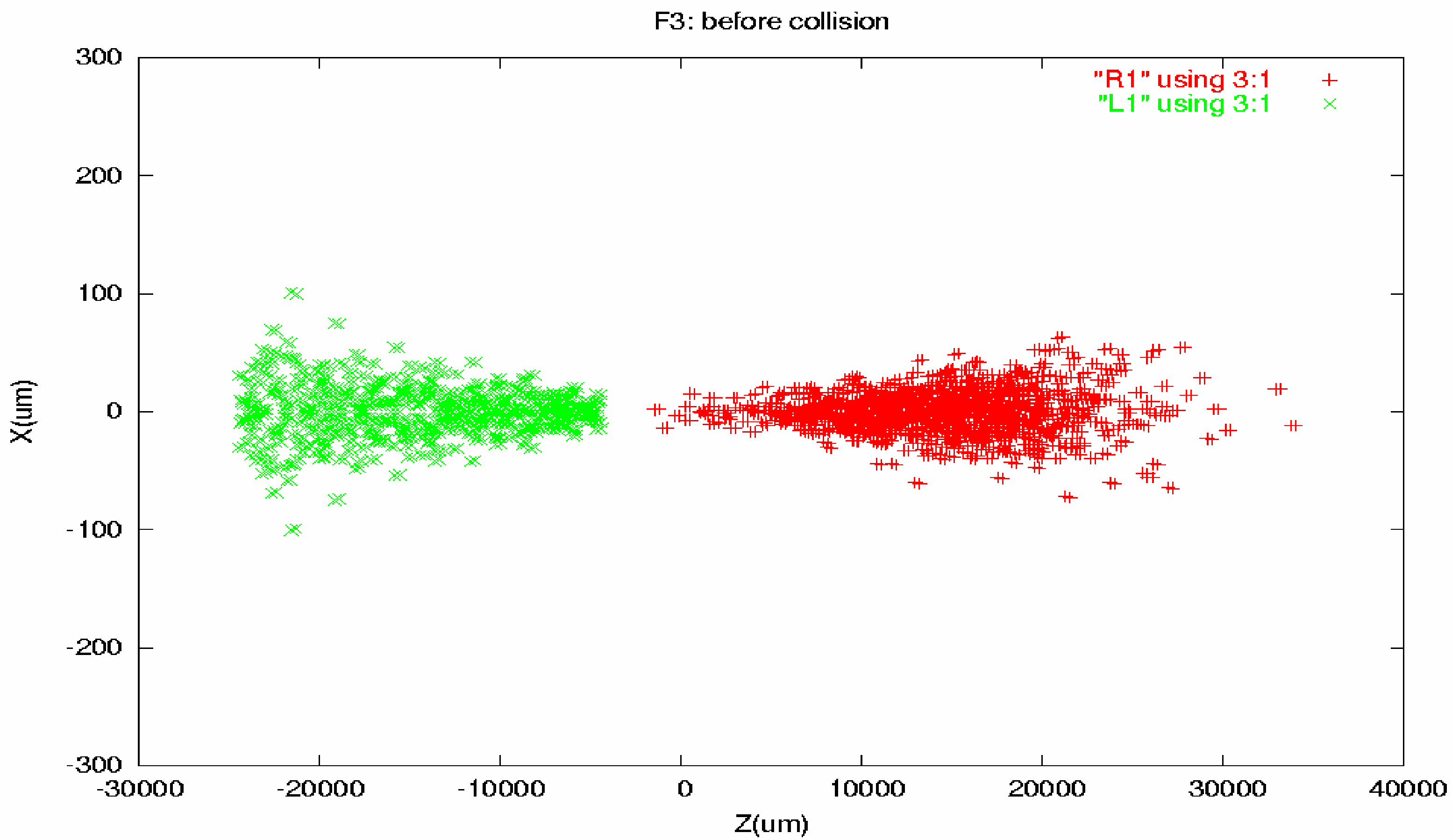
- Accurate — Exact boundary condition
  No errors due to numerical derivatives.
- Only a small number of empty cells when using adaptive mesh.
- Slow when a large mesh has to be used (mis-matched beams)
  — Computation cost $\sim N_p N_m^2$.

Courtesy: Jack Shi

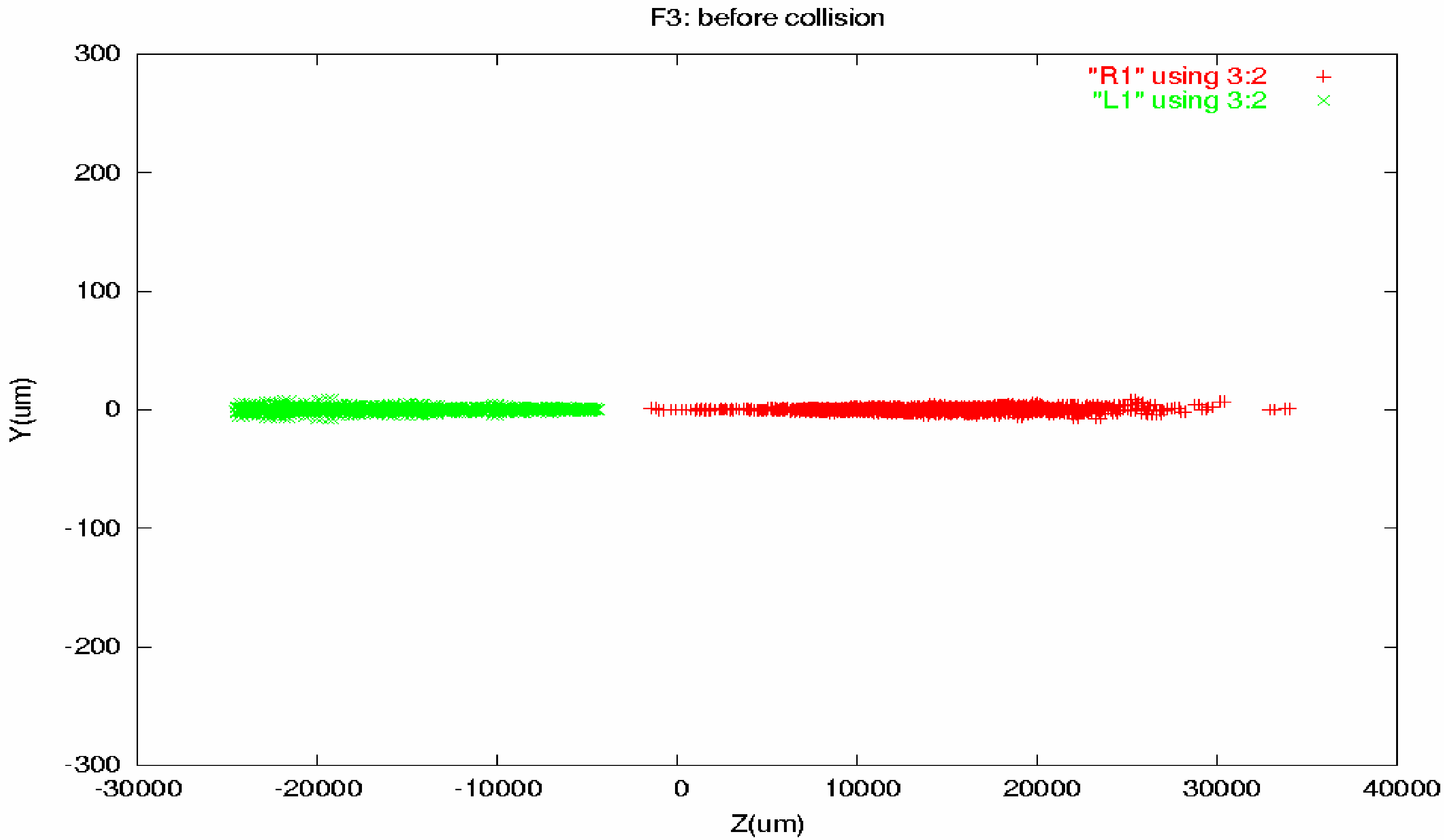
Coulomb Sum Simulation Results (K. Beard)

- Number of Beam Macroparticles
  - N_e 400
  - N_p 4000
- Number of turns 20000
- 6 hours on 2 GHz desktop
- Proper synchrotron motion and proton matched properly to ring; electron beam only geometrically matched; dipole offset suppressed
- No circulator ring (new electron bunch for each turn)
- Electrons have varying charge per macroparticle to give longitudinal charge distribution

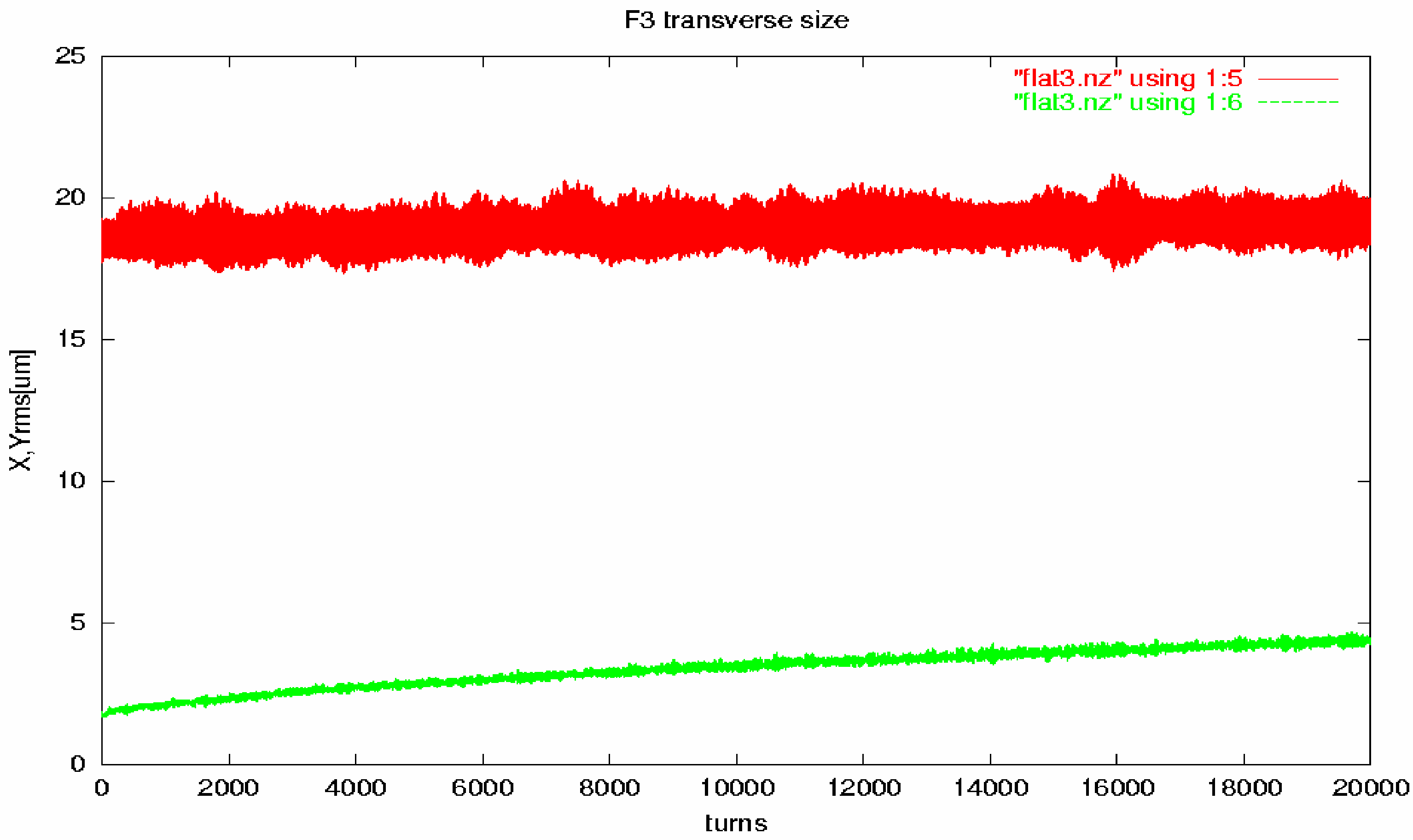
Horizontal Snapshot



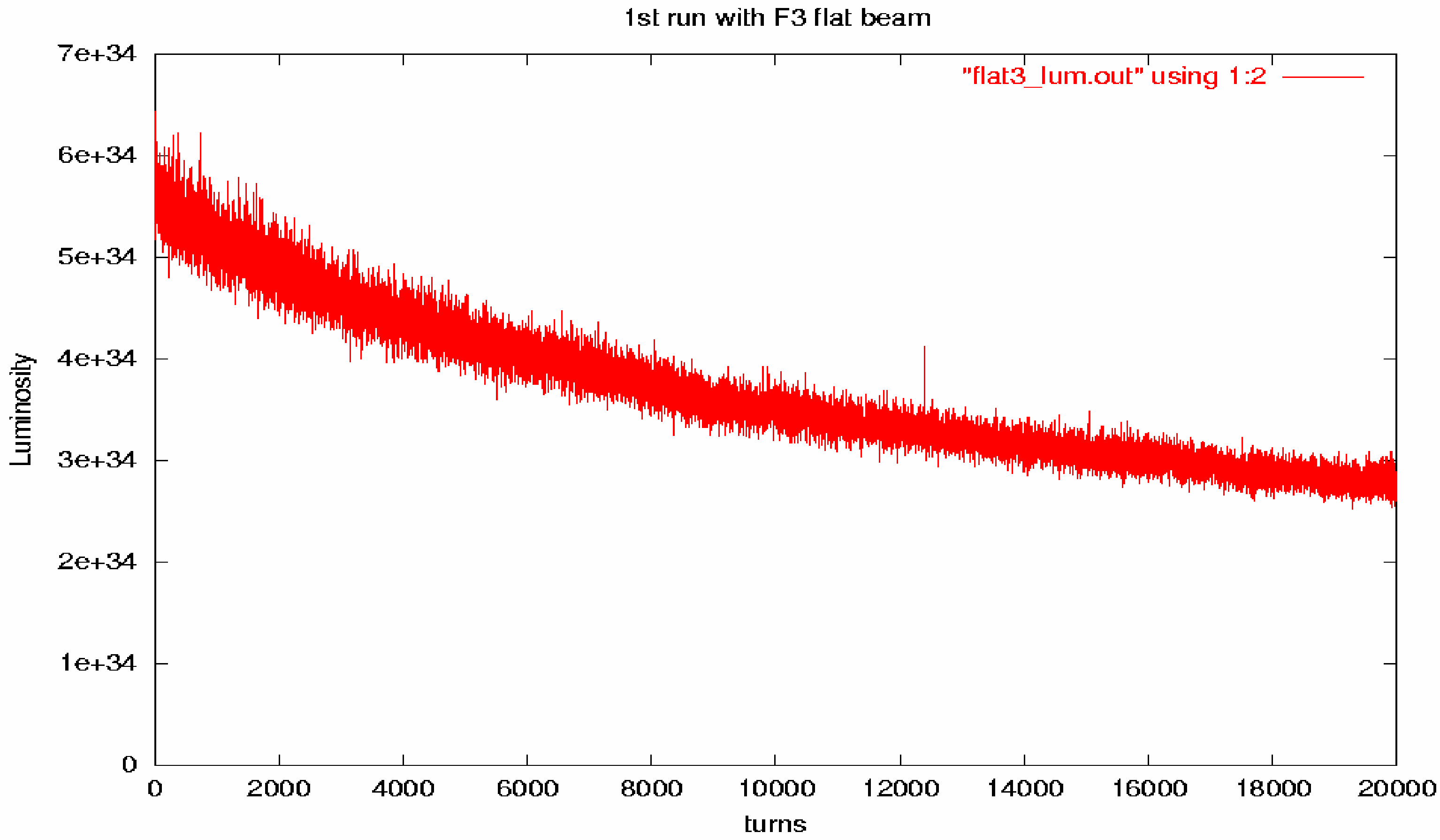
Vertical Snapshot



Proton Beam Transverse Sizes

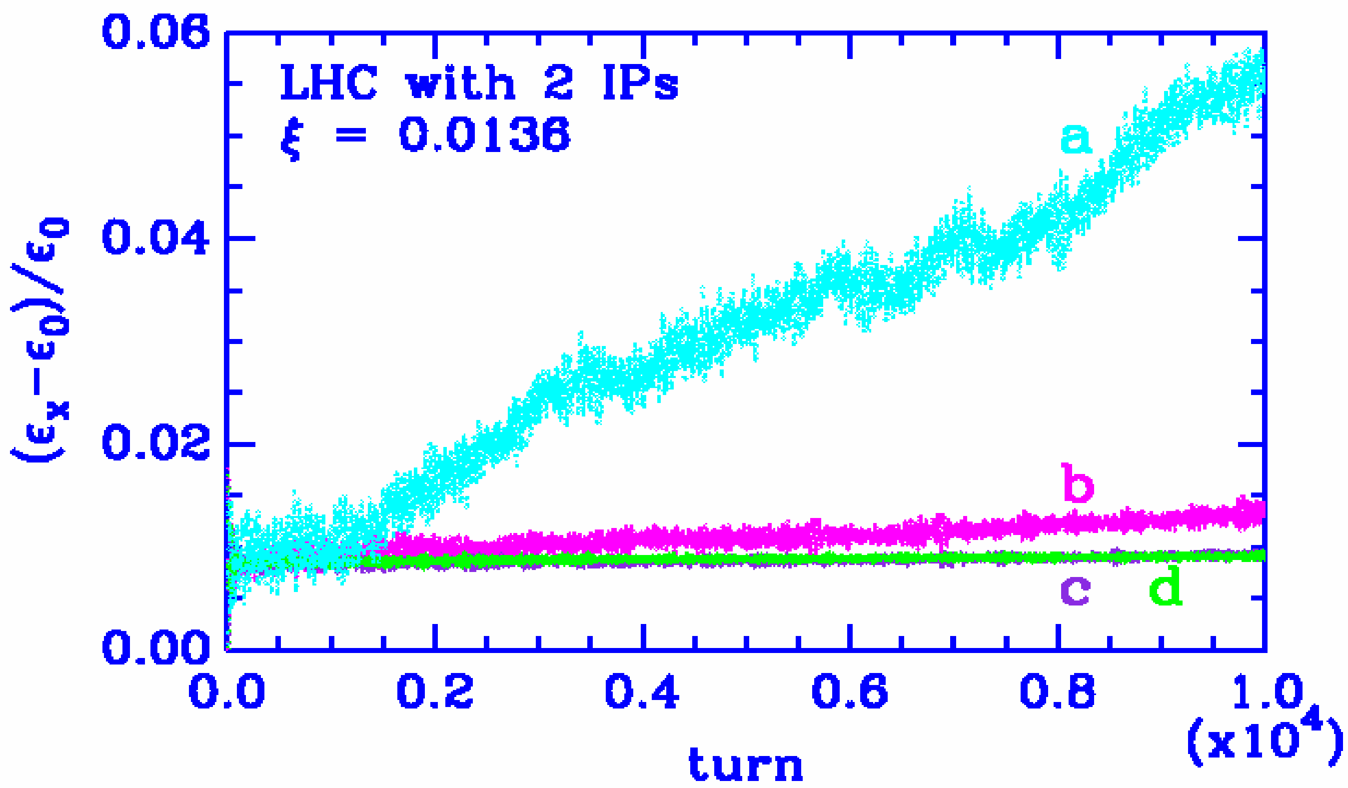


Luminosity



Thomas Jefferson National Accelerator Facility

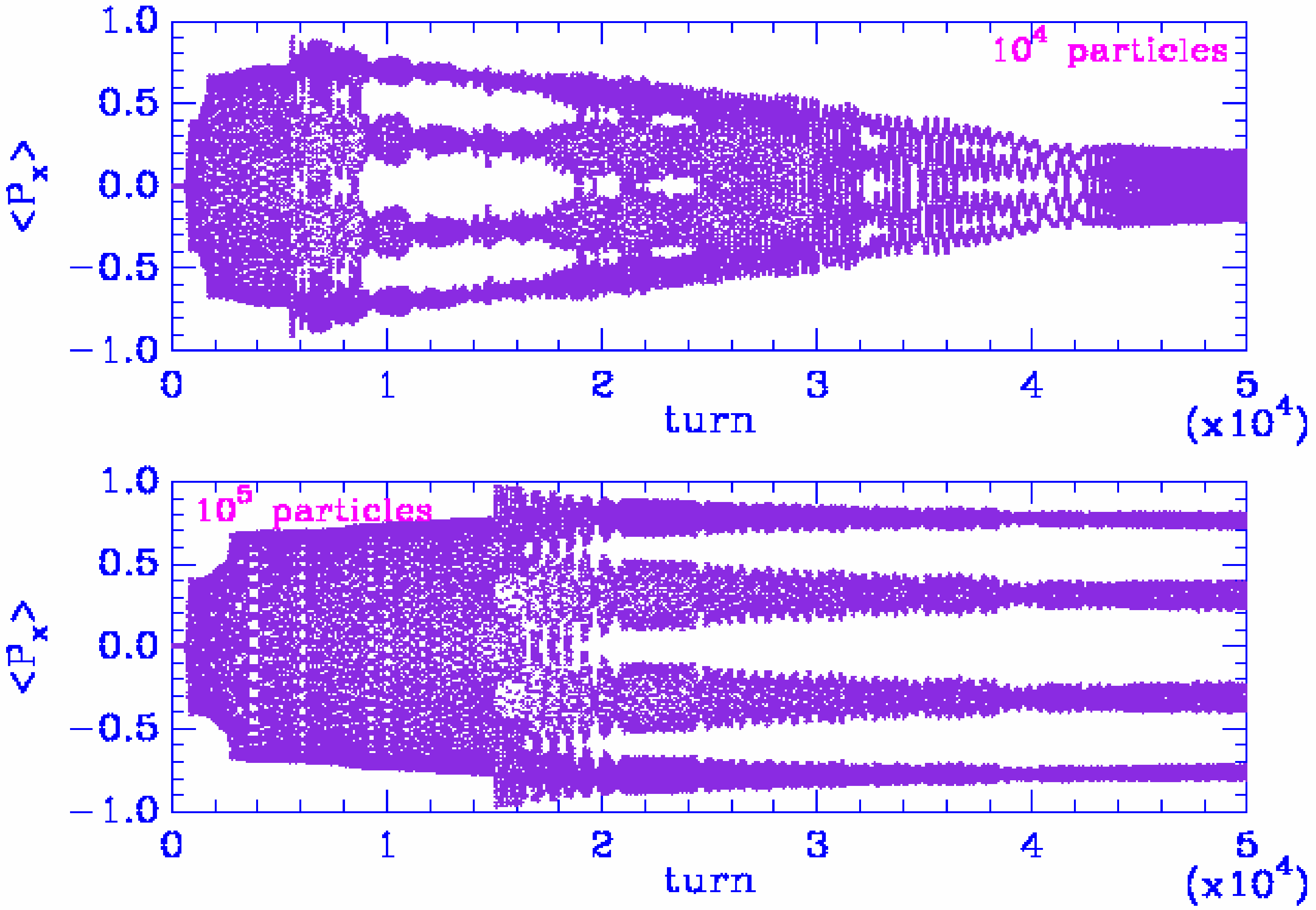
PIC Dependence on Macroparticle Number



(a) 10^4 particles; (b) 10^5 particles;
(c) 5×10^5 particles; (d) 10^6 particles.

Courtesy: Jack Shi

Comparison Between Different Numbers of Macro-Particles



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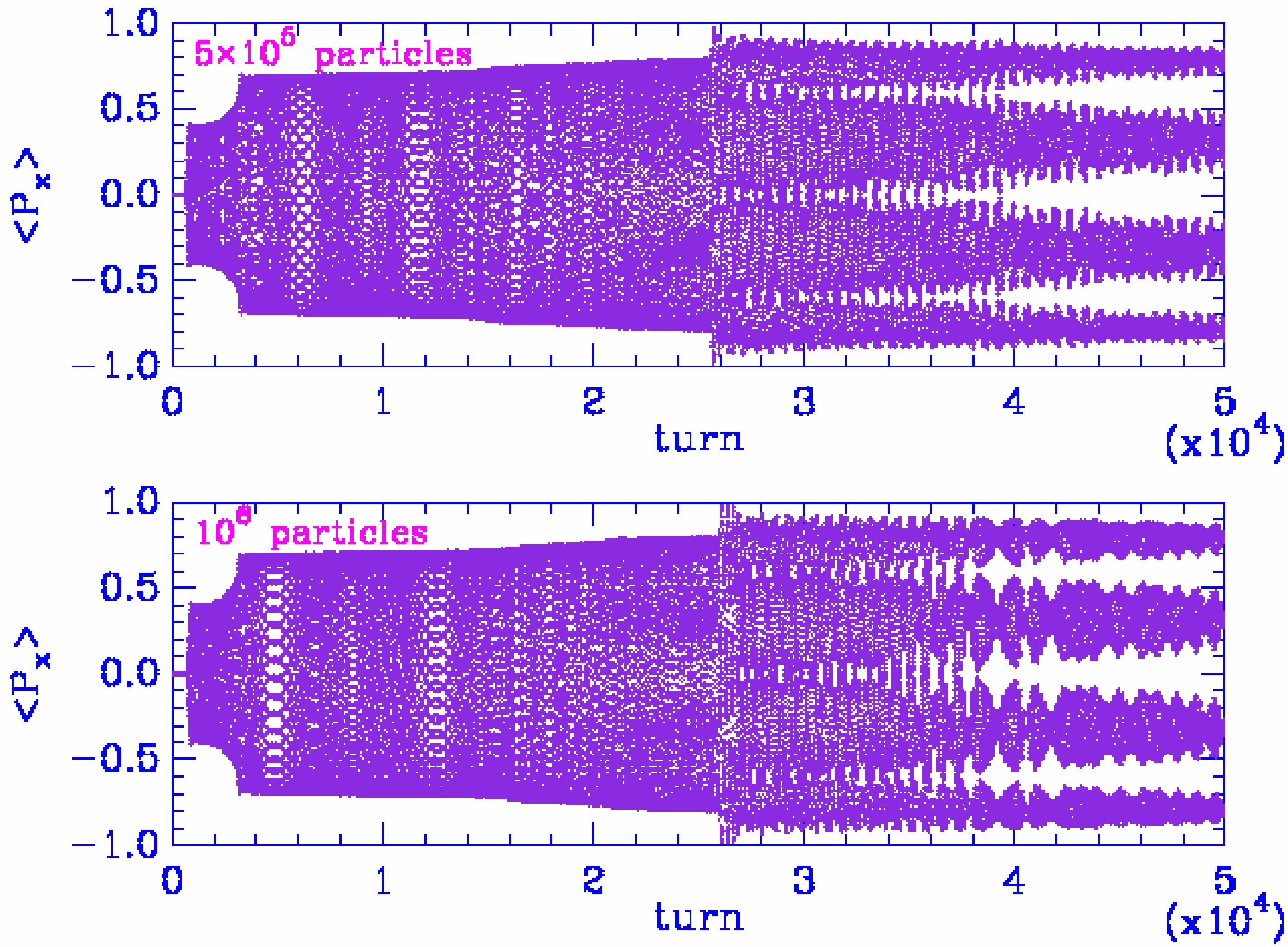

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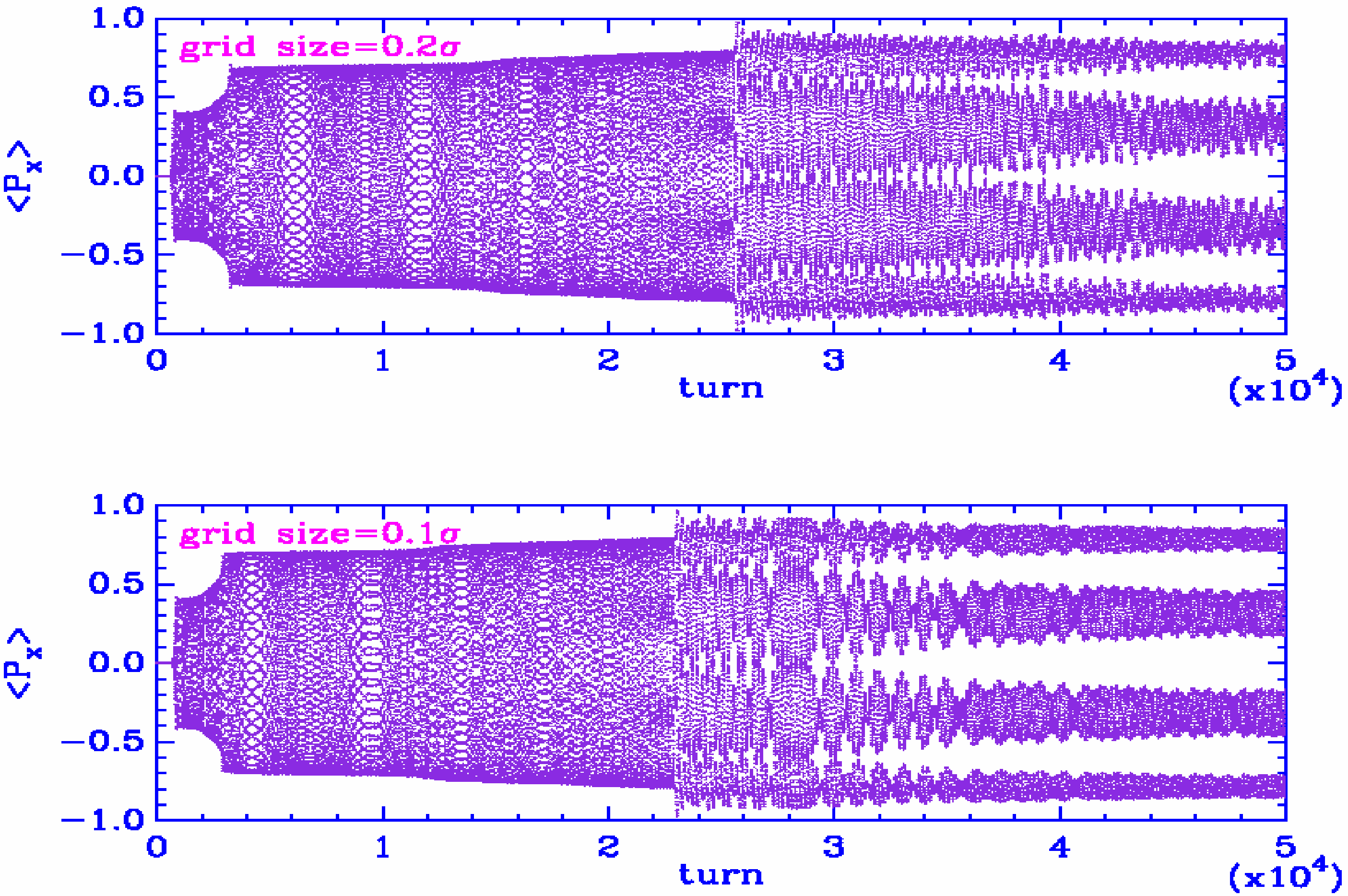
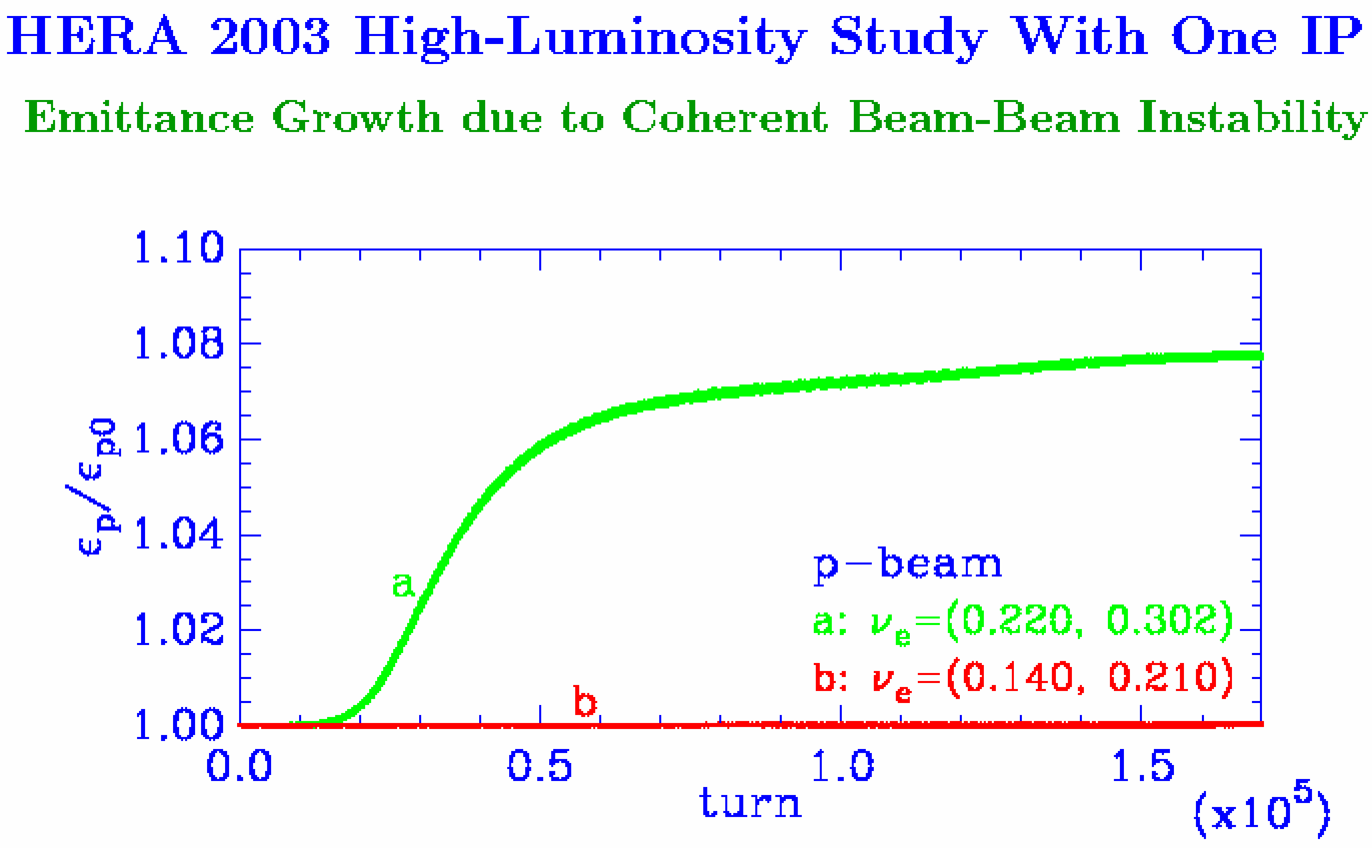
COMPARISON BETWEEN DIFFERENT GRID CONSTANTS



Courtesy: Jack Shi

Benchmarking



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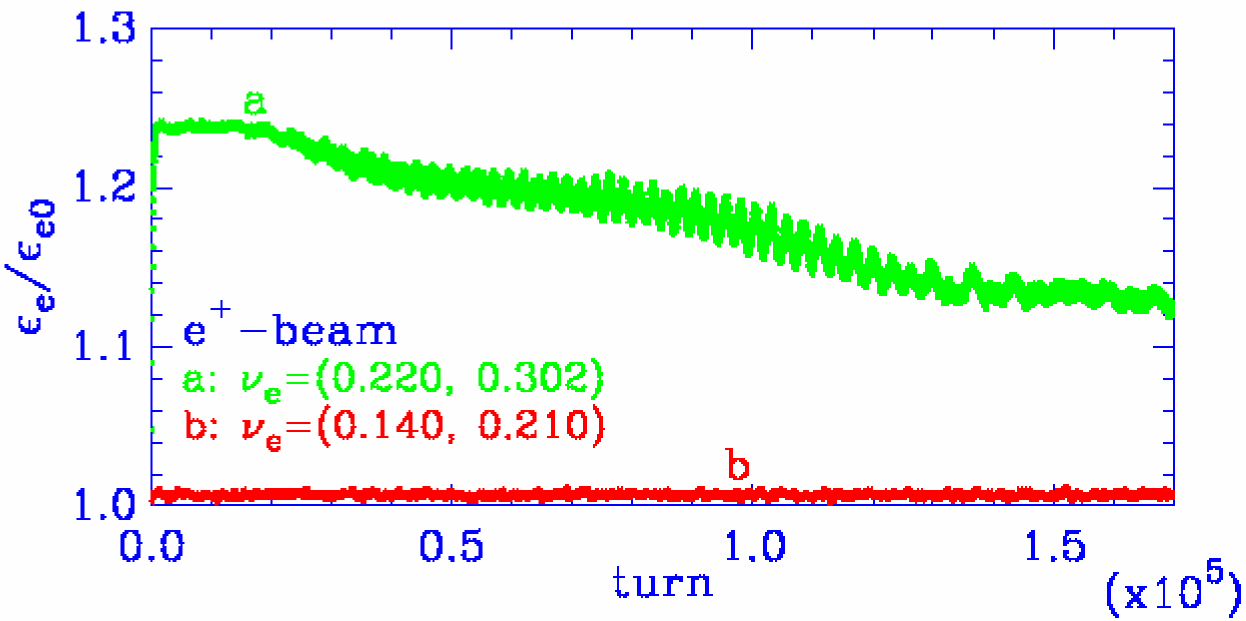

HERA 2003 Experimental Result:

In case a, the proton beam emittance increases ~30% while in case b, no emittance increase was observed.

Courtesy: Jack Shi

Simulation Stati (Stata?, Statuses?)

- Coulomb Sums
  - 1000 by 2000 macroparticle simulations, for 100000 turns are possible
  - Simple linear models for the ring transverse optics have been simulated
  - Full synchrotron motion allowed
  - Longitudinal slicing allowed
  - Single IP allowed
- PIC
  - Single slice 10^5 to 10^6 by 10^5 to 10^6 macroparticle simulations, for 100000 turns are possible
  - At least for some physics, such large macroparticle numbers seem necessary
  - At least one such simulation, of LHC, includes many of the important non-linear transverse optics effects in that storage ring
  - As of yet, no longitudinal slicing or synchrotron motion allowed; therefore must have relatively small tune shifts
  - Multiple IPs allowed

Future Needs

- Cooling Model
- Electron matching to the space charge in the ion beam
- Circulator ring simulations, including multiple electron crossings and multiple interactions in multiple IPs
- Crab Crossing

- PIC with slices and synchrotron motion

Conclusions

- We have explored the significant advantages of an energy recovered linac-ring collider
- We have pointed out the similarities and differences between the head-tail instability in such an arrangement and the more conventional ring-ring collider
- Very preliminary simulation studies driven by the CEBAF EIC parameter list, for a single IP configuration, have been undertaken.
- The next stage for obtaining a still better model may be to make a slice PIC transverse code