Current Fragmentation in SIDIS

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Work in (continuous) progress.
Based on collaborations with
E. Christova, E. Leader, F. Olness,
M. Stratmann, W.-K. Tung,
W. Vogelsang,...

http://www.pv.infn.it/~radici/FFdatabase/

maintained by M. Radici (Pavia) and R. Jakob (Wuppertal)

* FF *

*** FRAGMENTATION FUNCTIONS ***

sitemap | text on FF | references | parametrizations | links | ESOP

sitemap

text on FF

references

parametrizations

links

this database is a project of the ESOP network

maintained by Marco Radici and Rainer Jakob

Last modified: Thu, May 24, 2002 -- 11:15 --

sitemap:

sitemap (this page)

text on fragmentation functions (../FFdatabase/text.html)	references related to fragmentation functions (../FFdatabase/references.html)	parametrizations (../FFdatabase/parametrizations.html)	
parton distribution functions (PDFs)	operator definitions of PDFs and FFs	Stefan Kretzer NEW!	NEW! all three combined in one FORTRAN library (courtesy of S.Kretzer)
unintegrated PDFs	information on PDFs / parametrizations	Kniehl, Kramer, Pötter NEW!	
fragmentation functions (FFs)	information on FFs / parametrizations	Bourhis, Fontannaz, Guillet, Werlen (soon to come)	
unintegrated FFs	models for FFs		
multiple-hadron FFs	evolution of FFs / scaling violations		
models calculations	target fragmentation and fracture functions		
target fragmentation and fracture functions	more references (still not properly sorted)		

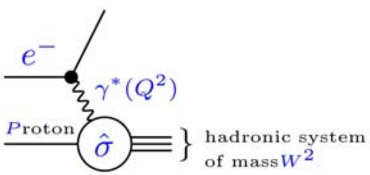
more interesting and useful links

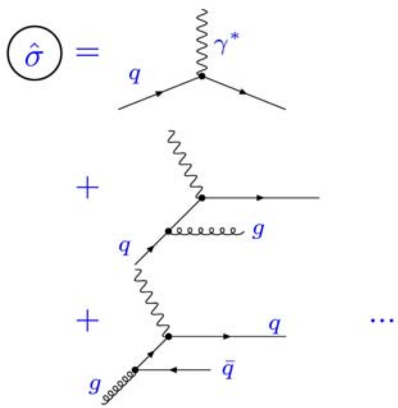
any comments, suggestions and additional information helping to complete or update this list are highly welcome and will be appreciated

send mail to: Marco Radici or Rainer Jakob

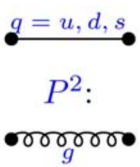
Parton-Distribution and Parton-Fragmentation Functions from Global Analysis

- Measurement of $|P\rangle$ in Deep Inelastic Scattering





- light quarks/gluons ($m = 0$):



P^2: $\mathcal{O}(Q^2) \gtrsim P^2 \gtrsim \mathcal{O}(\Lambda^2_{\rm QCD})$ $\begin{cases} P^2 > \mu^2: & \text{perturbative extrinsic} \\ P^2 < \mu^2: & \text{non} - \text{pert. intrinsic} \end{cases}$

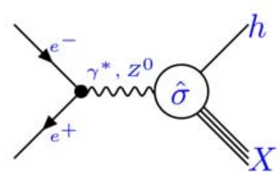
- Parton Distribution Functions:

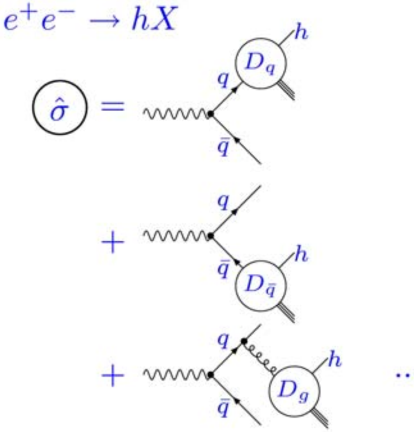
$\hat{\sigma}\Big|_{P^2<\mu^2} = q(\xi, \mu^2), g(\xi, \mu^2)$ with $\xi P_{\rm Proton} = p_{q,g}$

Parton Model: $d\sigma \sim q(\xi = x, \mu^2 = Q^2)$; $x = \frac{Q^2}{2P\cdot q}$

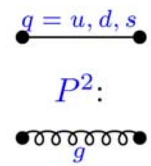
Bjorken

- Measurement of FFs in $e^+e^- \to hX$





- light quarks/gluons ($m = 0$):



P^2: $\mathcal{O}(S) \gtrsim P^2 \gtrsim \mathcal{O}(\Lambda^2_{\rm QCD})$ $\begin{cases} P^2 > \mu^2: & \text{perturbative} \\ P^2 < \mu^2: & \text{non} - \text{pert.} \end{cases}$

- Fragmentation Functions:

$\hat{\sigma}\Big|_{P^2<\mu^2} = D_q(\zeta, \mu^2), D_g(\zeta, \mu^2)$ with $P_h = \zeta p_{q,g}$

Parton Model: $d\sigma \sim D_q(\zeta = z, \mu^2 = S)$; $z = \frac{2E_h}{\sqrt{S}}$

NLO QCD:

$d\sigma = D_q^{\overline{\rm MS}}(\mu^2) \otimes d\hat{\sigma}_q^{\overline{\rm MS}}(\mu^2) + D_g^{\overline{\rm MS}}(\mu^2) \otimes d\hat{\sigma}_g^{\overline{\rm MS}}(\mu^2)$

Some Theory …

- Parton Distributions:
  - Local operator product expansion in <u>inclusive</u> DIS
  - Bilocal operator definition

$$f(x,\mu^2) \propto$$

$$\int dn\ x^{-n}\ \mathcal{O}^n(\mu^2)$$

$$\int dy^- e^{-ixP^+y^-} \langle P|\, \overline{\psi}(y^-)\gamma^+\psi(0)\, |P\rangle$$

- Fragmentation Functions:
  - No local OPE (no inclusive final state)
  - Bilocal operator definition

$$D(z,\mu^2) \propto$$

N/A

$$\int dy^- e^{iP^+/zy^-} \mathrm{Tr}\gamma^+ \langle 0|\, \psi(y^-)\, |hX\rangle \langle hX|\, \overline{\psi}(0)\, |0\rangle$$

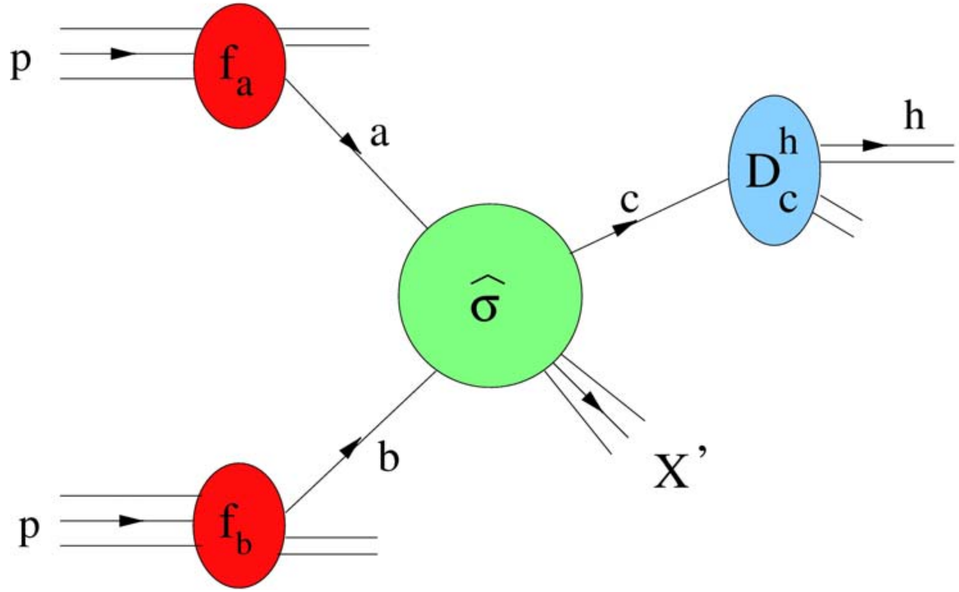
Just as PDFs, FFs are well defined in terms of $\psi \in \mathcal{L}_{\mathrm{QCD}}$

Scale dependence enters through renormalization: DGLAP

$$\frac{\partial f_i(x,\mu^2)}{\partial \ln \mu^2} = \sum_j P_{ij} \otimes f_j \qquad \frac{\partial D_i(x,\mu^2)}{\partial \ln \mu^2} = \sum_j P_{ji} \otimes D_j$$

Factorization Theorem in Practice: Inclusive Hadron Production and its Ingredients

- $(f_a \otimes f_b) \otimes \hat{\sigma} \otimes D_c^h$
- $f_a(x_1, \mu_F^{\text{initial}}), f_b(x_2, \mu_F^{\text{initial}})$
- $\alpha_s^{-2}\ \hat{\sigma} = \mathcal{O}(1) + \mathcal{O}\left[\alpha_s(\mu_R)\right] + \ldots$
- $D_c^h(z, \mu_F^{\text{final}})$



SIDIS: make f_a a $\delta(1-x)$

Factorization, Renormalization, Fragmentation:

- How many scales are there actually?
- In principle 3, but:
  - A strict handling is *very* unpractical.
  - A loose handling does not really improve on setting them all equal:

$$\mu_F^{\text{initial}} = \mu_R = \mu_F^{\text{final}} \equiv \mu \simeq p_\perp^h$$

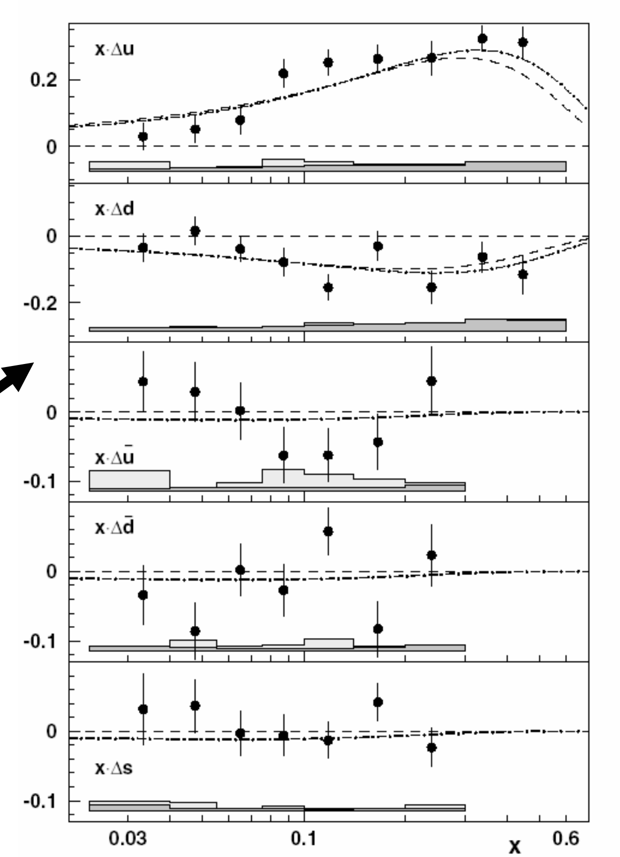
$$\alpha_s(\mu) \ln \left(p_\perp^h / \mu \right) \simeq 0$$

Operative Role of FFs

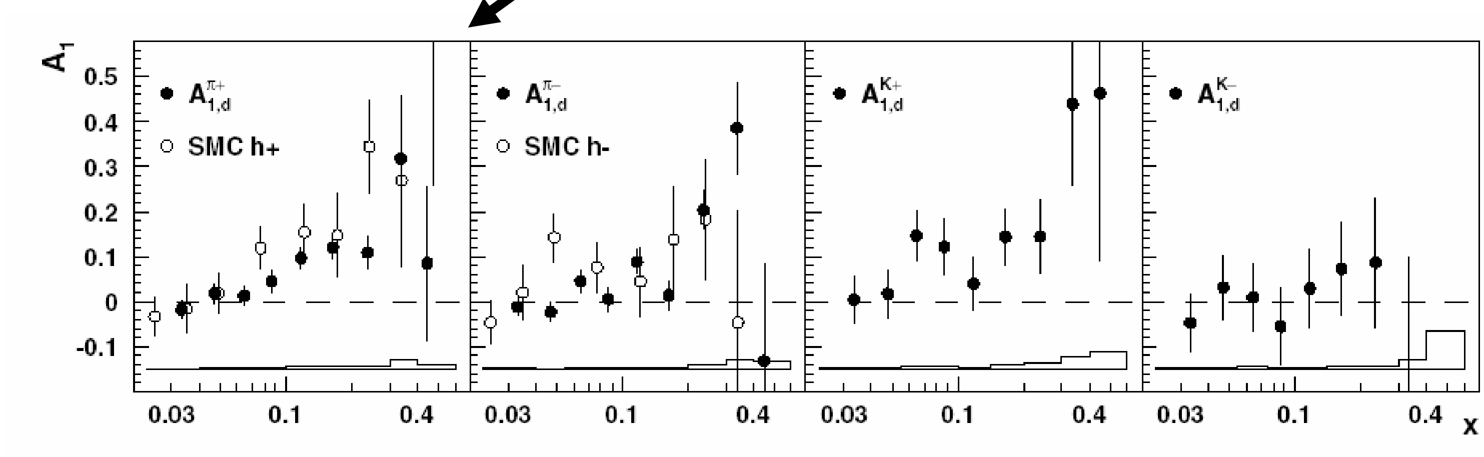
$$(\Delta)\sigma(Q) = (\Delta)\text{PDF}\left(\frac{\mu}{\Lambda}\right)\hat{\sigma}\left(\frac{Q}{\mu}\right)\text{FF}\left(\frac{\mu}{\Lambda}\right)$$

- In a semi-inclusive process, each FF q! hX weighs the contribution of PDF "q" differently
- Fix FFs in unpolarized reactions, "divide them out" from the polarized data.
- Monte Carlos are *not* (necessarily) the same:
  "Data / FF = PDF"
  "Data / MC-hadronization $\neq$ PDF"

HERMES (PRL 04) SIDIS flavour decomposition of Δq

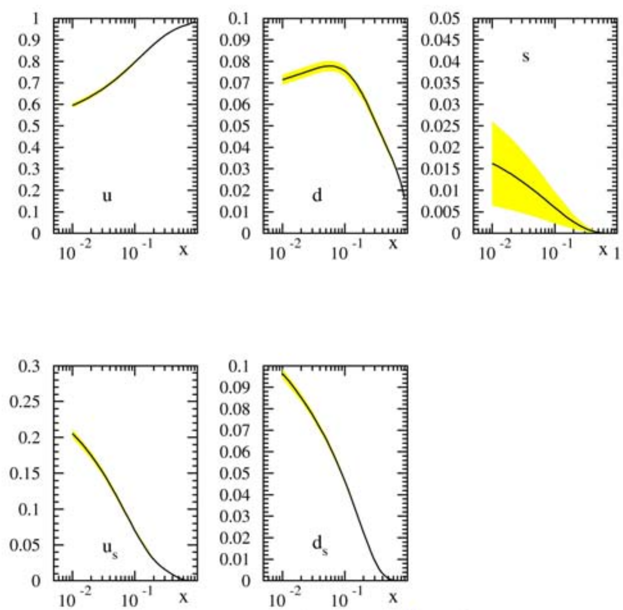


FFs



Uncertainties in FFs and their impact on polarized PDFs (here from SIDIS)

- *Purities* $P_q^{\pi^+}$ within uncertainties

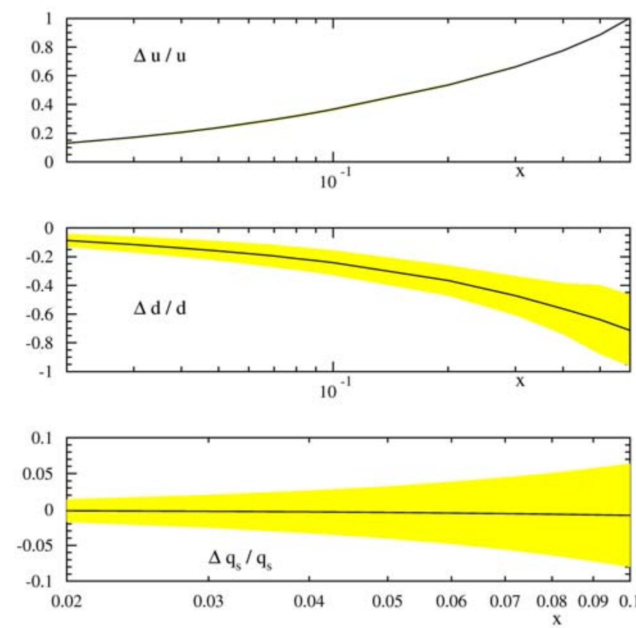


$$P_f^h(x) = \frac{e_f^2 q_f(x) \int_{0.2}^1 D_f^h(z)\, dz}{\sum_{f'} e_{f'}^2 q_{f'}(x) \int_{0.2}^1 D_{f'}^h(z')\, dz'}$$

effective coupling for SIDIS

$$A^h(x) = \sum_f P_f^h(x) \frac{\Delta q_f(x)}{q_f(x)}$$

- Systematic Uncertainties from Fragmentation on Δ PDFs



χ^2 Analysis of e^+e^- Data (LEP, SLD, TPC)

$$\chi^2 \equiv \left(\frac{\text{Theory} - \text{Measurement}}{\text{Error}} \right)^2$$

From: Nucl. Phys. **B597**, 337 (2001) (KKP)

Energy [GeV]	Flavour	Experiment	FF Set			No. of Points
			KKP	K	BFGW	
29	uds	TPC	0.178*	0.159	0.167*	7
	c		0.876*	0.911	0.923*	7
	b		2.23*	1.21	1.14*	7
91.2	all	DELPHI	1.28	1.51*	1.49	12
		SLD	1.32	0.370	0.421	21
	uds	DELPHI	3.17*	0.990*	1.95	13
		DELPHI	0.201	0.588*	1.00*	12
	c	DELPHI	0.473*	0.388*	0.401	11
	b	DELPHI	28.9*	0.887*	1.03	12
		DELPHI	0.433	9.14*	8.74	12
189	all	OPAL	0.568*	0.250*	0.414*	11

Kniehl, Kramer, Pötter
Bourhis, Fontannaz, Guillet, Werlen
Kretzer

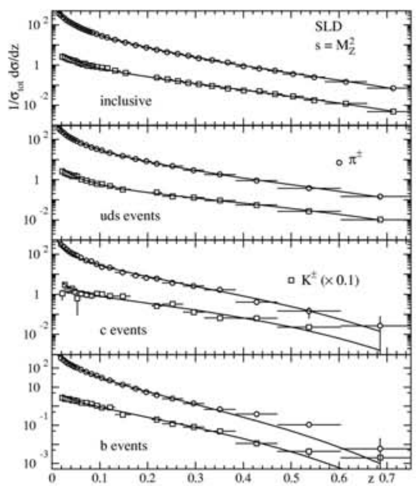
Alternative model approaches:

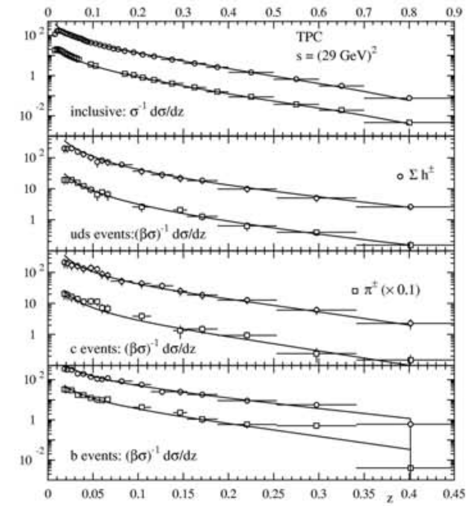
Indumathi et al.

Soffer et al.

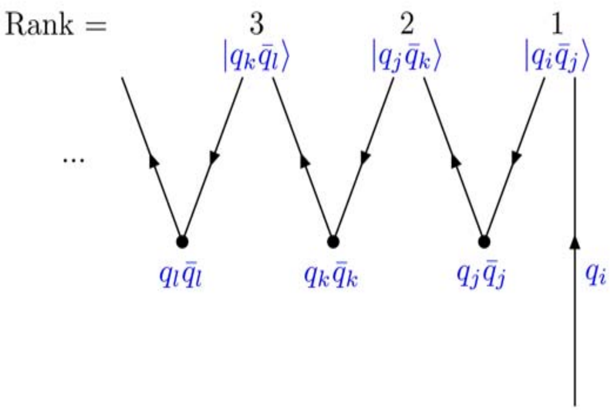
- Charged Pions, Kaons and Sum over Charged Hadrons
  @ SLD ($\sqrt{S} = M_Z$) and TPC ($\sqrt{S} = 29$ GeV^2)

  lifetime tagging unfolded to pure flavor samples





• The Feynman-Field Picture of Cascade Fragmentation



Rank = 1:	valence-type	$u \to \|u\bar{d}\rangle = \|\pi^+\rangle$
Rank $\geq$ 2:	sea-type	$u \to \|d\bar{u}\rangle = \|\pi^-\rangle$
$q_i \neq s$:	$s\bar{s}$ suppression	$u \to \|u\bar{s}\rangle = \|K^+\rangle$

Hierarchy:

$$D_q^{h^+,h^-} = D_{\bar{q}}^{h^-,h^+}; \quad h = \pi, K$$

$$D_{\bar{d}}^{\pi^+} = D_{s,\bar{s}}^{\pi^+} < D_u^{\pi^+} = D_{\bar{d}}^{\pi^+}$$

$$D_{\bar{u}}^{K^+} = D_{d,\bar{d}}^{K^+} < D_u^{K^+} < D_{\bar{s}}^{K^+}$$

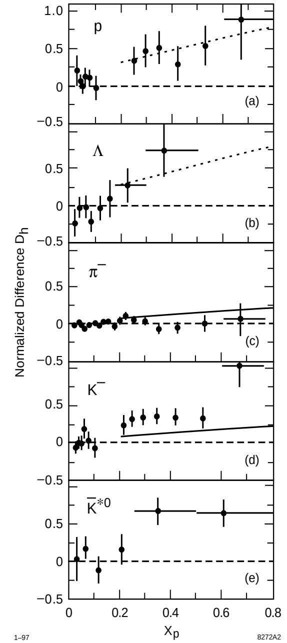
as $z \to 1$ (<u>leading particle</u> *effect*).

• Leading Particles @ SLD

$$R_h^q = \frac{1}{2N_{evts}} \frac{d}{dx_p} \left[N(q \to h) + N(\bar{q} \to \bar{h}) \right]$$

$$R_{\bar{h}}^q = \frac{1}{2N_{evts}} \frac{d}{dx_p} \left[N(q \to \bar{h}) + N(\bar{q} \to h) \right]$$

$$D_h = \frac{R_h^q - R_{\bar{h}}^q}{R_h^q + R_{\bar{h}}^q}$$



How Well Do we now FFs in practice?

- What is determined by the e^+e^- data ?

$$D_{\text{meas}}^{\pi^++\pi^-} = \sum_{q=u,d,s} \left(D_q^{\pi^++\pi^-} + D_{\bar{q}}^{\pi^++\pi^-} \right) \hat{e}_q^2(s)$$

$\hat{e}_q^2(s)$: SU(2) × U(1)

$$\hat{e}_u^2(s) = \hat{e}_d^2(s) \qquad @ \sqrt{s} = 78, 113 \text{ GeV}$$

$$\hat{e}_u^2(s)/\hat{e}_d^2(s)\Big|_{s=M_Z^2} \simeq 3/4$$

The singlet combination

$$D_\Sigma^{\pi^+} \equiv \left(D_u^{\pi^+} + D_{\bar{u}}^{\pi^+} + D_d^{\pi^+} + D_{\bar{d}}^{\pi^+} + D_s^{\pi^+} + D_{\bar{s}}^{\pi^+} \right) = 2\left(D_u^{\pi^+} + D_d^{\pi^+} + D_s^{\pi^+} \right)$$

within extreme flavour assumptions

$$0 < (D_s^{\pi^+} + D_{\bar{s}}^{\pi^+}) < (D_u^{\pi^+} + D_{\bar{u}}^{\pi^+})$$

is fixed by e^+e^- data to $\sim 5\%$:

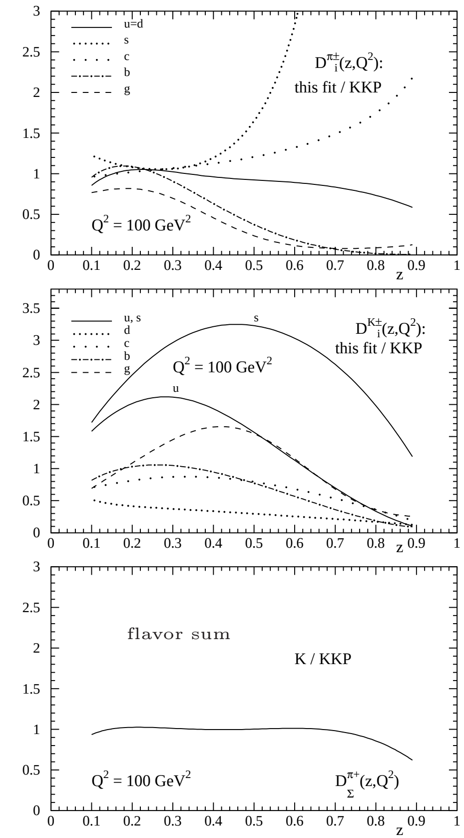
$$\tilde{D}_{\text{meas}}^{\pi^++\pi^-} = D_{\text{meas}}^{\pi^++\pi^-} \;/\; \hat{e}_d^2(s)$$

$$D_\Sigma^{\pi^+} = \frac{4}{7}\tilde{D}_{\text{meas}}^{\pi^++\pi^-} - \frac{1}{7}\left(D_s^{\pi^+} + D_{\bar{s}}^{\pi^+} \right)$$

$$\hookrightarrow \frac{4}{7}\,\tilde{D}_{\text{meas}}^{\pi^++\pi^-} < D_\Sigma^{\pi^+} < \frac{6}{11}\,\tilde{D}_{\text{meas}}^{\pi^++\pi^-}$$

And similar estimates hold for $D_\Sigma^{\Lambda,K}$.

- Comparison to Kniehl, Kramer & Pötter



Implications for Parton Phenomenology in (polarized) Semi-Inclusive-DIS: E. Leader, E. Christova; Nucl. Phys. **B607**, 369

Light flavour Separation from SIDIS: E. Christova, SK, E. Leader

Digested summary on e^+e^- annihilations:

- Determine mainly $D_\Sigma(\mu' M_Z)$
- Do not constrain:
  - Flavour decomposition
  - Gluon fragmentation
  - large z
- Precise data at low scale desirable (BELLE)
- More processes have to be included into global analysis

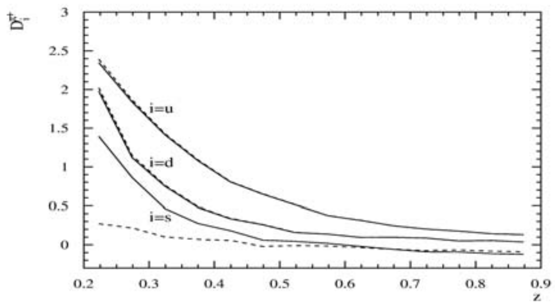
Flavour Separation from SIDIS

- Light-Flavour FFs from HERMES data:

$$D_u^{\pi^+} - D_d^{\pi^+} = \frac{9\left(R_p^{\pi^+} - R_p^{\pi^-}\right)\tilde{\sigma}_p^{DIS}}{4u_V - d_V}$$

$$D_u^{\pi^+} + D_d^{\pi^-} = \frac{9\left(R_p^{\pi^+} + R_p^{\pi^-}\right)\tilde{\sigma}_p^{DIS} - 2sD_\Sigma^{\pi^+}}{4(u+\bar{u}-s)+d+\bar{d}}$$

$$D_s^{\pi^+} = \frac{-18\left(R_p^{\pi^+} + R_p^{\pi^-}\right)\tilde{\sigma}_p^{DIS} + [4(u+\bar{u})+d+\bar{d}]D_\Sigma^{\pi^+}}{2[4(u+\bar{u}-s)+d+\bar{d}]}$$

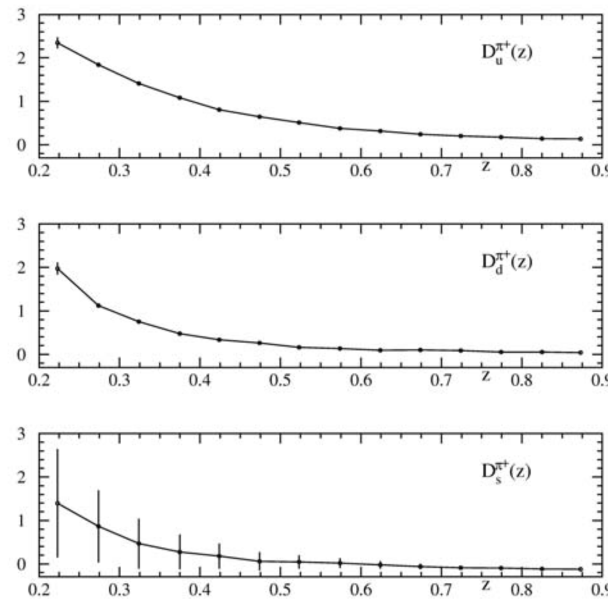


HERMES data $\oplus D_\Sigma$[K(solid), KKP(dashed)]

- Isospin:

$$D_d^{\pi^+} = D_{\bar{d}}^{\pi^-} = D_u^{\pi^-} = D_{\bar{u}}^{\pi^+}$$
$$D_{\bar{d}}^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_u^{\pi^+}$$
$$D_{\bar{s}}^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^-} = D_s^{\pi^+}$$

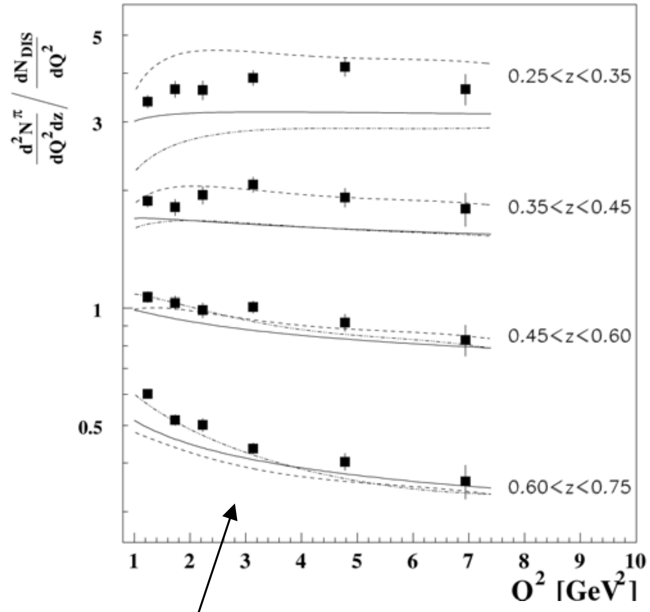
- Light-Flavour FFs within Errors (from LO extraction)



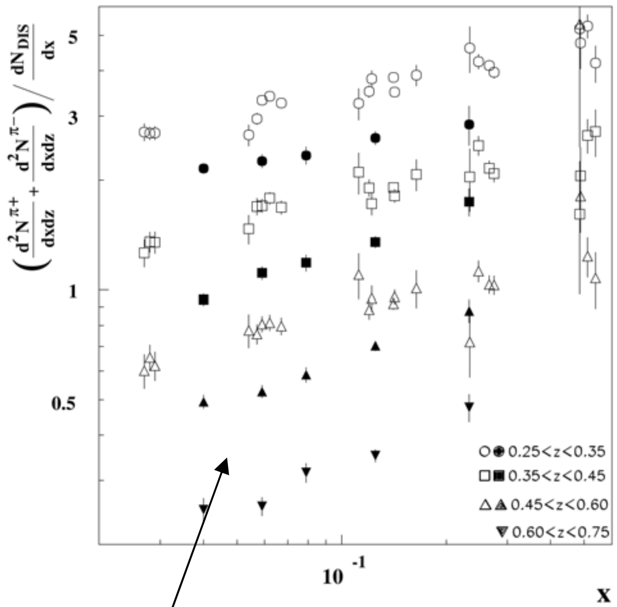
- SU(2) sector ($D_u^{\pi^+}, D_d^{\pi^+}$) well determined
- $D_s^{\pi^+}$ anything within $[0, D_d^{\pi^+}]$
- Knowledge of $D_\Sigma^{\pi^+}(\mu = \text{DIS})$ within $\sim 5\%$ would fix $D_s^{\pi^+}$ within $\sim 20-30\%$

SIDIS data and pQCD expectations

HERMES multiplicities: dσ(z) / s dσ



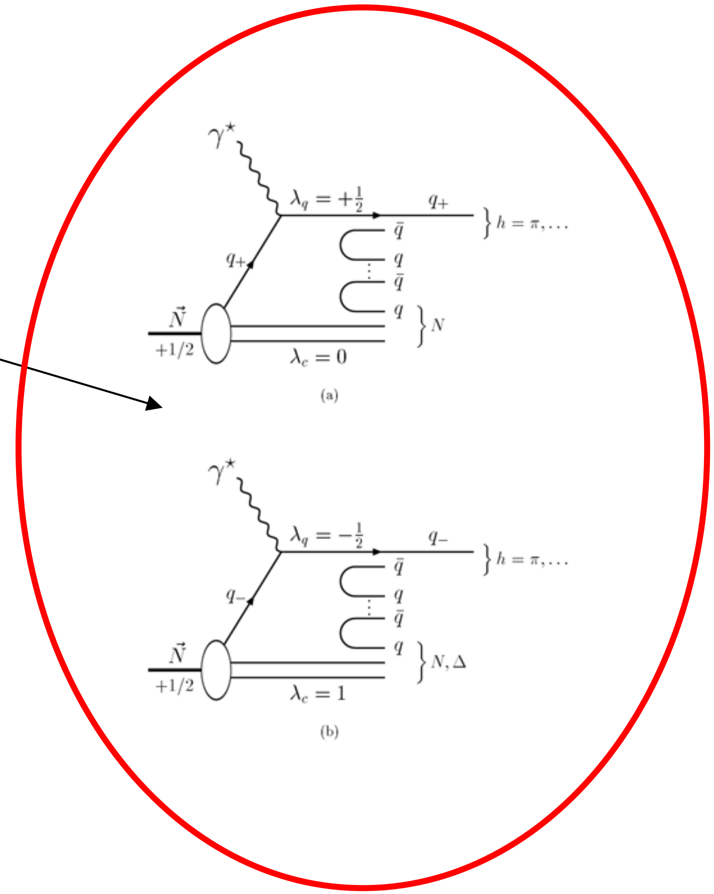
The Q-dependence seems to follow DGLAP



x-dependence at fixed z?
At LO in QCD we have
σ(x,z) / σ(x)D(z)
such that
σ(x,z)/σ(x) ' *flat in x*

Is σ_{SIDIS} ' $q(x)D(z)$ at not-so-high Q? And if not … then what?

- Process dependent higher-twist like interactions?
- E.g. Glück & Reya 02 suggest spin dependence of "fragmentation" into pions
- Strictly D_{q+}^{π} ´ D_{q-}^{π}
- Possible effects at $O(1/Q^2)$



Target Fragmentation:

$\mathbf{z} \neq \mathbf{z}$

- z ´ E^{π} / E^{π}_{max} scaled in the target frame or in the nucleon boson frame lead to different phase space integrals.
- $z = E^{h} / \nu > 0.2$ (target frame, standard definition) cuts out target fragmentation (*fracture*) contributions .
- Monte Carlo (JETSET $\neq$ pQCD) study finds such contributions (Kotzinian 03) are large.
- ***Monte Carlo approach is different from pQCD even qualitatively.***

Sanity check: Factorization!

What Factorization?

Factorization $\neq$ Factorization

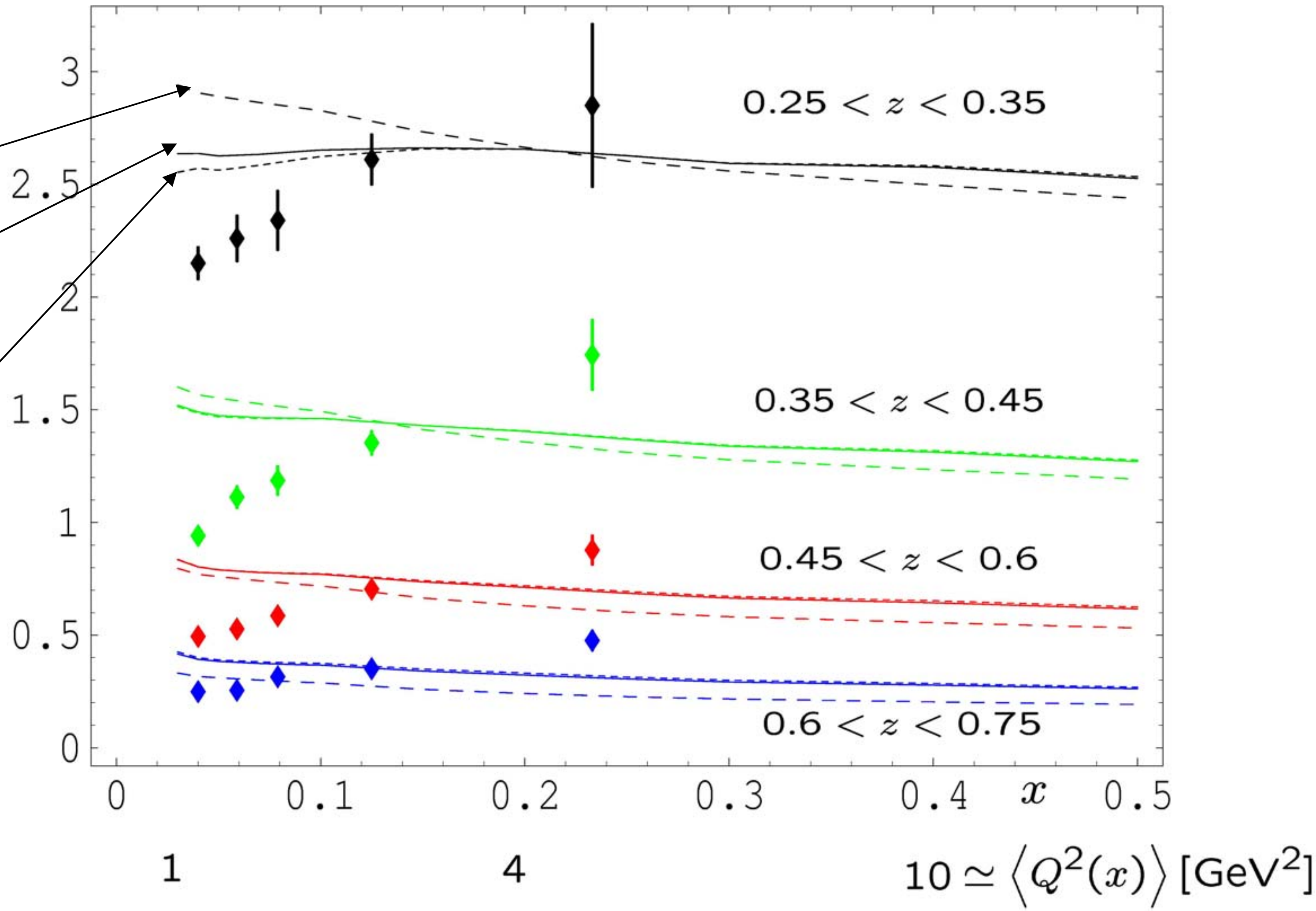
- ***The*** pQCD **Factorization** is a statement about the seperation of scales in
  $$\sigma(Q) = \mathsf{PDF}\left(\frac{\mu}{\Lambda}\right)\hat{\sigma}\left(\frac{Q}{\mu}\right)\mathsf{FF}\left(\frac{\mu}{\Lambda}\right)$$
- The LO DIS process is so simple, indeed is just a vertex / δ(1-x) δ(1-z) so that σ(x,z) / F(x)D(z) : The approximate (LO) **factorization** of **x** and **z** dependence (following from the one-particle “phase space” of LO DIS)
- Factorization ' Factorization for SIDIS

HERMES DIS π multiplicities

(unpolarized hydrogen target)

Curves:

- LO
- NLO
- ("NNLO")



HERMES DIS π multiplicities
(unpolarized hydrogen target)

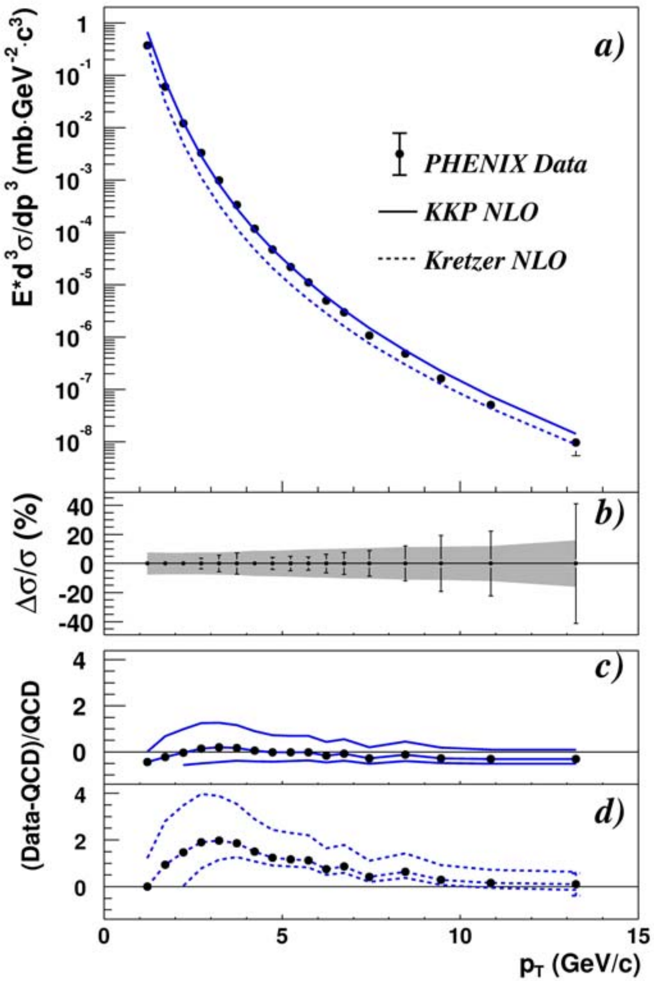
- The z dependence (averaged over x) follows roughly the expectations from $e^+ e^-$ annihilations

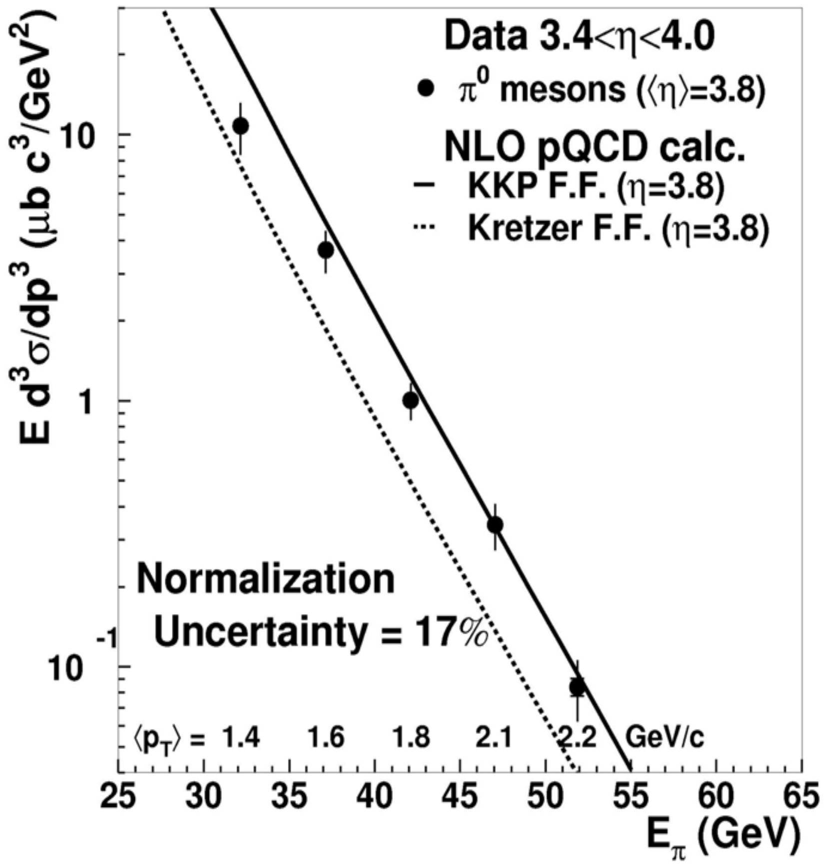
- The x dependence is too pronounced for an NLO QCD effect. The correlated $\{x, Q^2\}$ dependence shows deviations towards low x ' 0.05, i.e. low Q^2 ' 1 GeV^2 .

- Maybe we are observing stronger subleading $1/Q^2$ effects than in the inclusive $F_2(x, Q^2)$???

Hadroproduction (RHIC)

Recent p-p Data from PHENIX and STAR at central and forward rapidity





Differences between KKP and "Kretzer" FF can be traced, mostly, to $D_g^\pi(z)$

The Gluon Fragmentation Function has been measured

.....

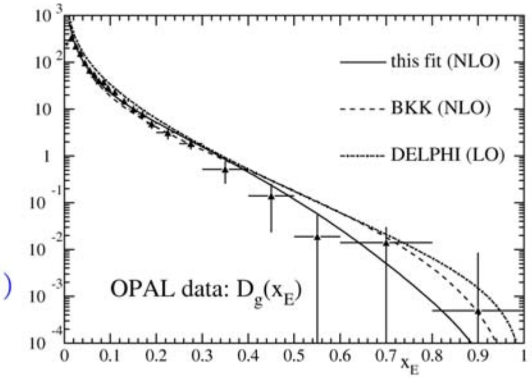
Hasn't it ?

Not quite, but we can extract it from global analysis.

- Gluon Fragmentation in $b\bar{b}g$ 3-jet topologies

Curves:

$D_g(z = x_E, \mu = E_g^{\rm jet})$



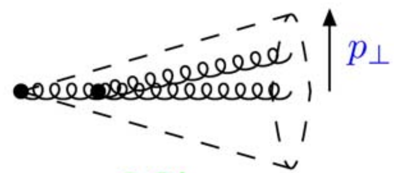
experimental Fragmentation Function

$$D_g^h(x_E, \mu^2) \equiv \frac{1}{N_{\rm tot}} \frac{\Delta N_g^h}{\Delta x_E}; \quad x_E \equiv \frac{E_h}{E_g^{\rm jet}}$$

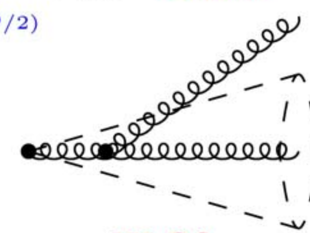
LO QCD scaling violations

$$\int_{\mu^2}^{(p_\perp^{\max})^2} \frac{dp_\perp^2}{p_\perp^2} \left[P_{ji}^{(0)}(z)\right]_{p_\perp = 0} = P_{ji}^{(0)}(z) \ln\left(\frac{p_\perp^{\max}}{\mu}\right)^2$$

$$\hookrightarrow \left.\frac{dD_g^h(x_E, \mu^2)}{d\ln\mu^2}\right|_{\mu = E^{\rm jet}\sin(\theta/2)} \quad : \ \text{LO} - \text{DGLAP}$$



LO!



NLO?

Partonometry of inclusive pion production in hadron collisions at RHIC energies

The following is a *technical* decomposition into parton processes

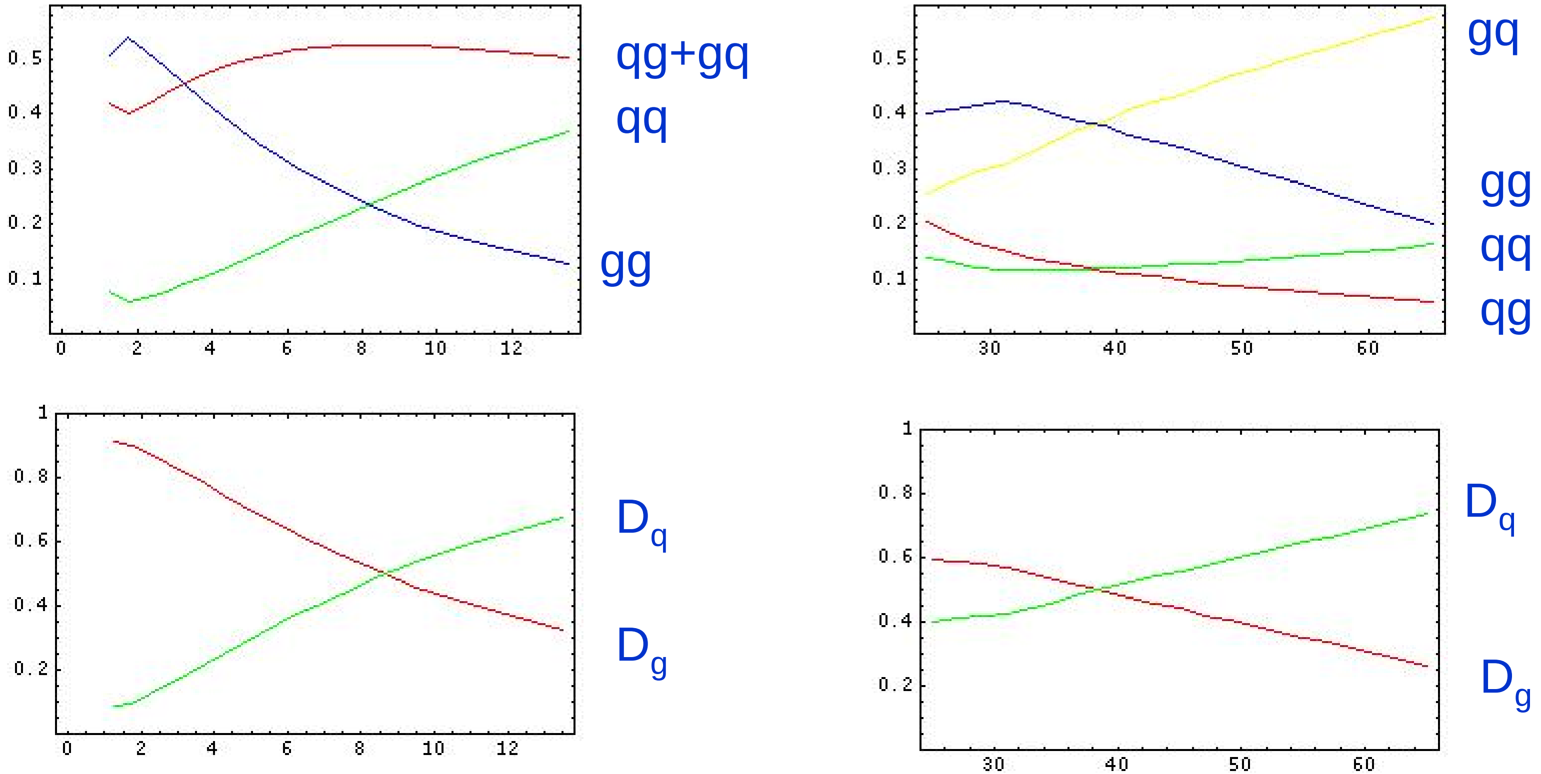
Fractional contributions from initial/final state partons

Central Rapidity

Forward Rapidity

initial

final

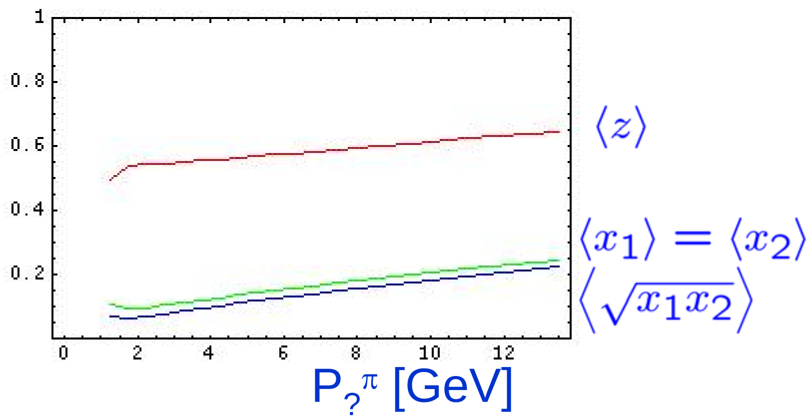


P? [GeV]

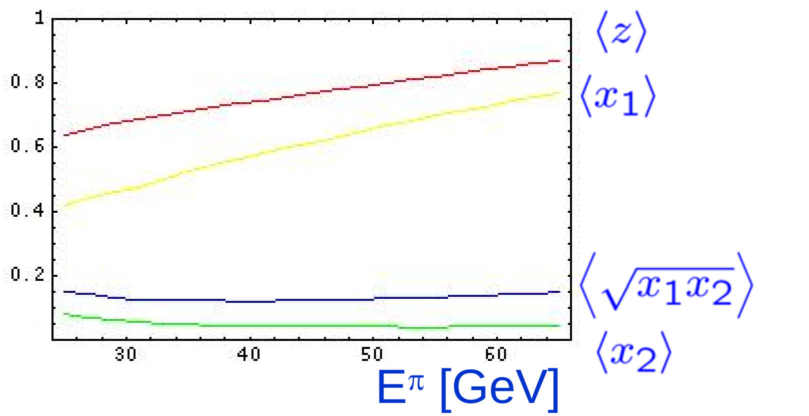
E_π [GeV]

Average Scaling Variables

Central Rapidity



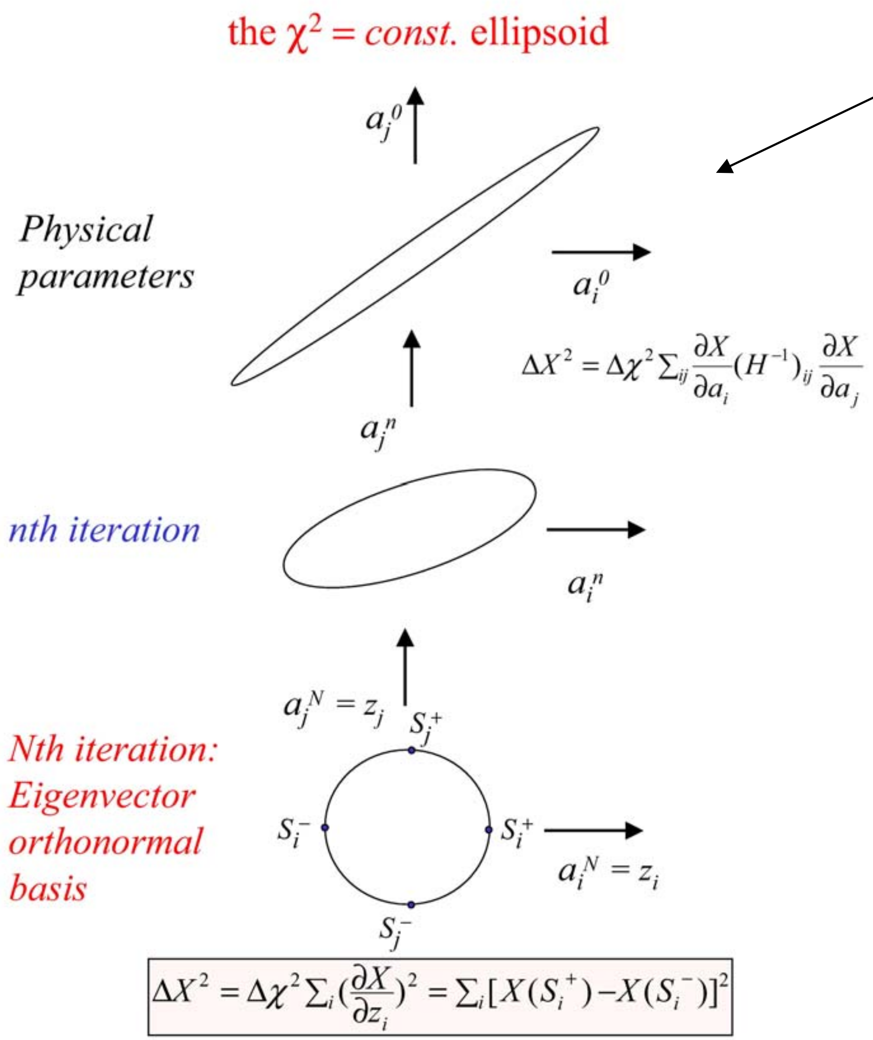
Forward Rapidity



- Symmetric / asymmetric kinematics for central / forward rapidity
- Large z fragmentation is probed. The largest z are probed by the forward rapidity data where quarks with very large x are probed as well.
- Note: There is a difference in scale as well: It varies from small to large for central rapidity, whereas it's small throughout for forward rapidity.

Iterative Method to generate Eigenvectors:
(and dramatically improve numerical reliability)

CTEQ (W.-K. Tung et al).
PDFs



Similarly exploit the neighborhood of the χ^2 minimum of FF analyses

$$\sigma(Q) = \text{PDF}\left(\frac{\mu}{\Lambda}\right) \hat{\sigma}\left(\frac{Q}{\mu}\right) \text{FF}\left(\frac{\mu}{\Lambda}\right)$$
$$\Rightarrow \delta\sigma\,[\delta\text{PDF}, \delta\text{FF}]$$

Partonic strangeness asymmetry

$$\left(s - \bar{s}\right)(x)$$

- Probes baryon-meson fluctuations and borderline between pQCD and non-pQCD
- Enters precison physics via the "NuTeV anomaly"
- Is uncertain at present, with little future perspective other than, perhaps, from ***SIDIS***

Phys. Lett. B381 (1996)

Theoretical expectations:
S. Brodsky & B.-Q. Ma (1996)
p ! Λ K^+ fluctuation

$$s(x) = \int_x^1 \frac{\mathrm{d}y}{y} f_{\Lambda/K^+\Lambda}(y) q_{s/\Lambda}\left(\frac{x}{y}\right); \quad \bar{s}(x) = \int_x^1 \frac{\mathrm{d}y}{y} f_{K^+/K^+\Lambda}(y) q_{\bar{s}/K^+}\left(\frac{x}{y}\right),$$

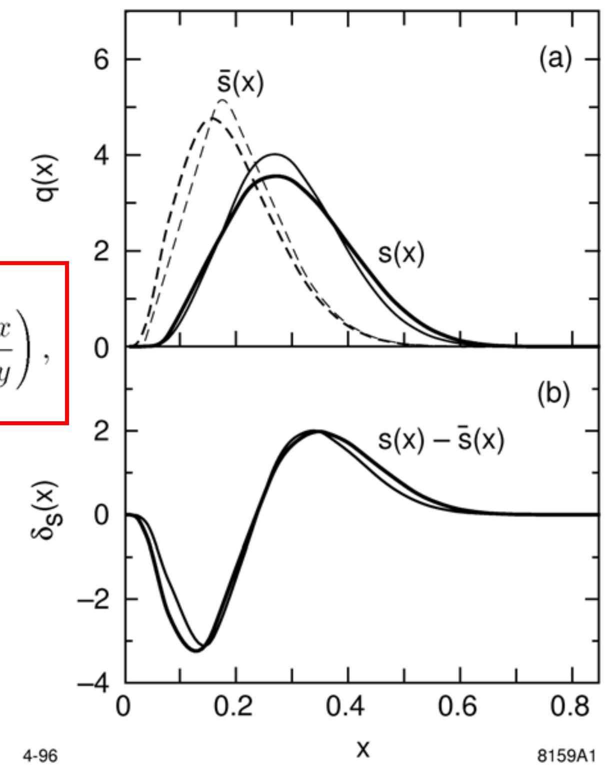


Figure 1: The momentum distributions for the strange quarks and antiquarks in the light-cone meson-baryon fluctuation model of intrinsic $q\bar{q}$ pairs, with the fluctuation wavefunction of $K^+\Lambda$ normalized to 1. The curves in (a) are the calculated results of $s(x)$ (solid curves) and $\bar{s}(x)$ (broken curves) with the Gaussian type (thick curves) and power-law type (thin curves) wavefunctions and the curves in (b) are the corresponding $\delta_s(x) = s(x) - \bar{s}(x)$. The parameters are $m_q = 330$ MeV for the light-flavor quark mass, $m_s = 480$ MeV for the strange quark mass, $m_D = 600$ MeV for the spectator mass, the universal momentum scale $\alpha = 330$ MeV, and the power constant $p = 3.5$, with realistic meson and baryon masses.

More recent results:
F.G. Cao & A.I. Signal (2003)
A.W. Thomas & W. Melnitchouk & F.M. Steffens (2000)
...

And phenomenology from:
V. Barone & C. Pascaud & F. Zomer (2000)

The Paschos-Wolfenstein relation

$$R^- = \frac{\sigma_\nu^{NC} - \sigma_{\bar\nu}^{NC}}{\sigma_\nu^{CC} - \sigma_{\bar\nu}^{CC}} \simeq \frac{1}{2} - \sin^2\Theta_W$$

has been measured (NuTeV) to deviate from the SM expectation by » 3 σ

$$R^- \simeq \frac{1}{2} - \sin^2\Theta_W - \left(\frac{1}{2} - \frac{7}{6}\sin^2\Theta_W\right)\frac{[S^-]}{[Q^-]}$$

$$[Q^-] \equiv \int_0^1 dx\ x\ \frac{u_v(x) + d_v(x)}{2}$$

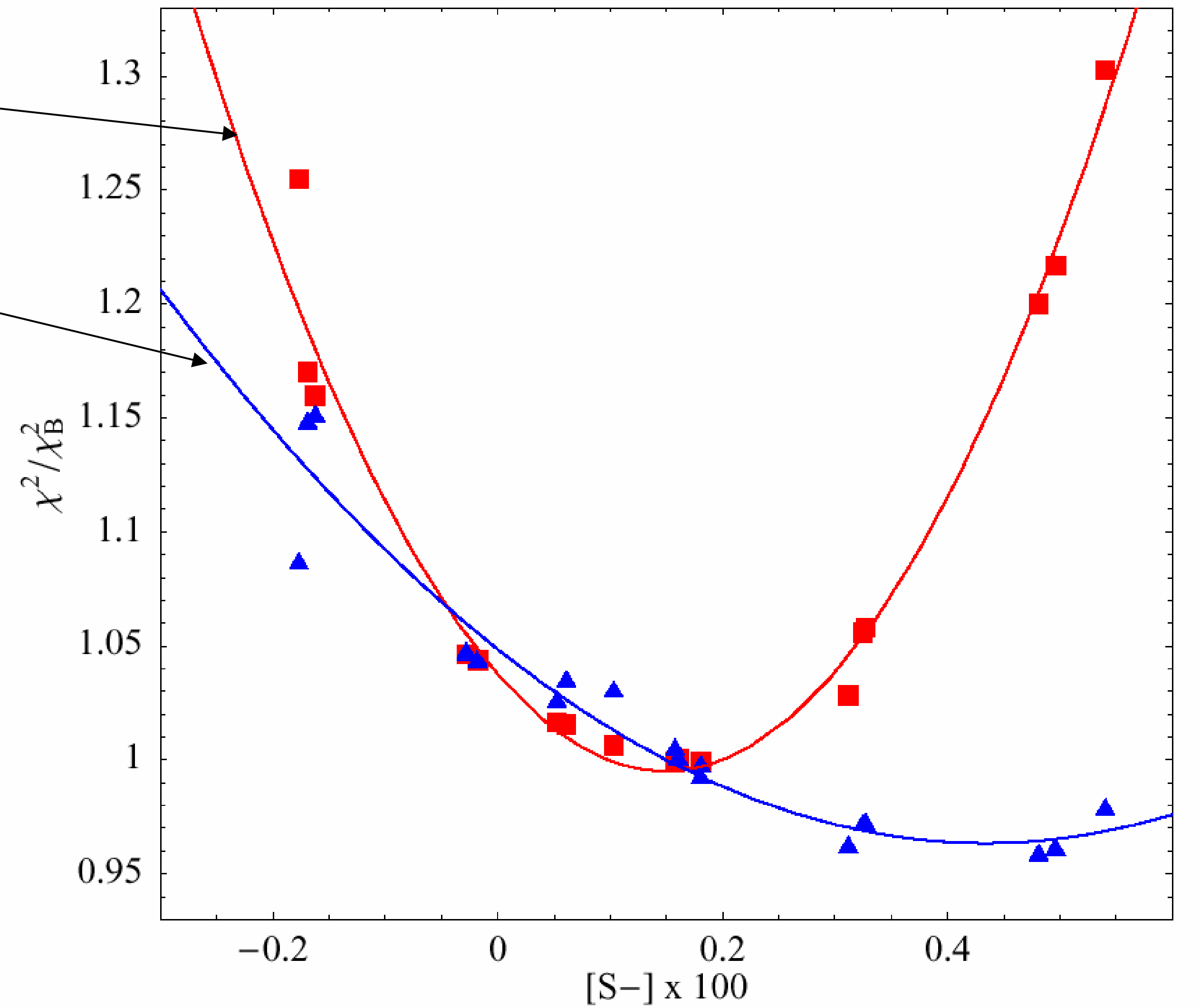
$$[S^-] \equiv \int_0^1 dx\ x\,(s - \bar{s})\,(x)$$

$(s - \bar{s})\,(x) \neq 0$ could explain or reduce the discrepancy.

Lagrangian multiplier results for [S⁻]:

μ§ data

Other (less) sensitive data



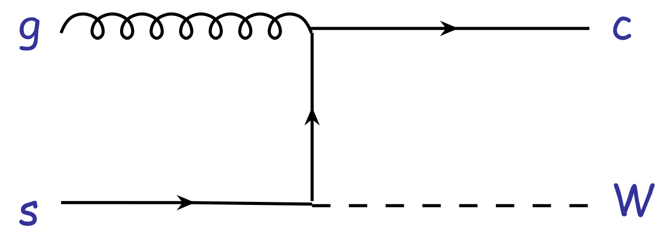
Rule of thumb:

The 3 σ anomaly corresponds to

[S⁻] £ 100 ' 0.5

Future prospects for [S⁻]?

- W and associated charm (jet) production:
conceivable @ Tevatron, RHIC, LHC
But statistics (efficiency driven) and high scale are unlikely to permit to access a small asymmetry.



Baur, Halzen, Keller, Mangano, Riesselmann

- CC charm @ HERA: ditto

- Lattice:
The moment [S⁻] itself does not correspond to a local operator.
Higher, uneven moments (n=3,5,...)

$$\int_0^1 dx\, x^{n-1}\, (s-\bar{s})(x) \sim \langle N|\, \bar{\Psi}\gamma^{\mu_1} D^{\mu_2} ... D^{\mu_n}\Psi\, |N\rangle$$

can be related to local operators and could presumably clarify the sign of the x! 1 behaviour, though not the magnitude of [S⁻].

- ***Semi-Inclusive DIS?***

What could one learn from SIDIS?

- Current fragmentation into strange hadrons:

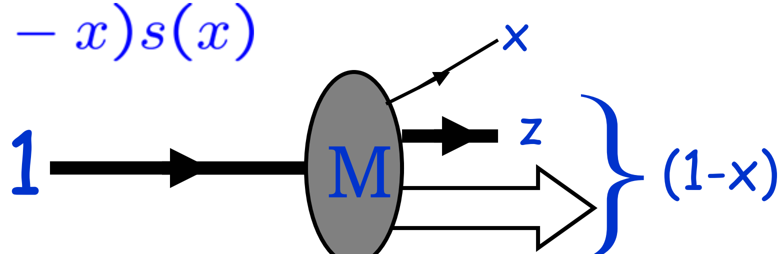
$|u\bar{s}\rangle\,, |\bar{u}s\rangle\,, |uds\rangle\,, \left|\bar{u}\bar{d}\bar{s}\right\rangle\,, \ldots$

is challenging for all the non-strange background fragmentation channels. A good knowledge of the corresponding FFs would be required.

- An asymmetry between strange and anti-strange target fragmentation / fracture products might be more promising?

Energy conservation:

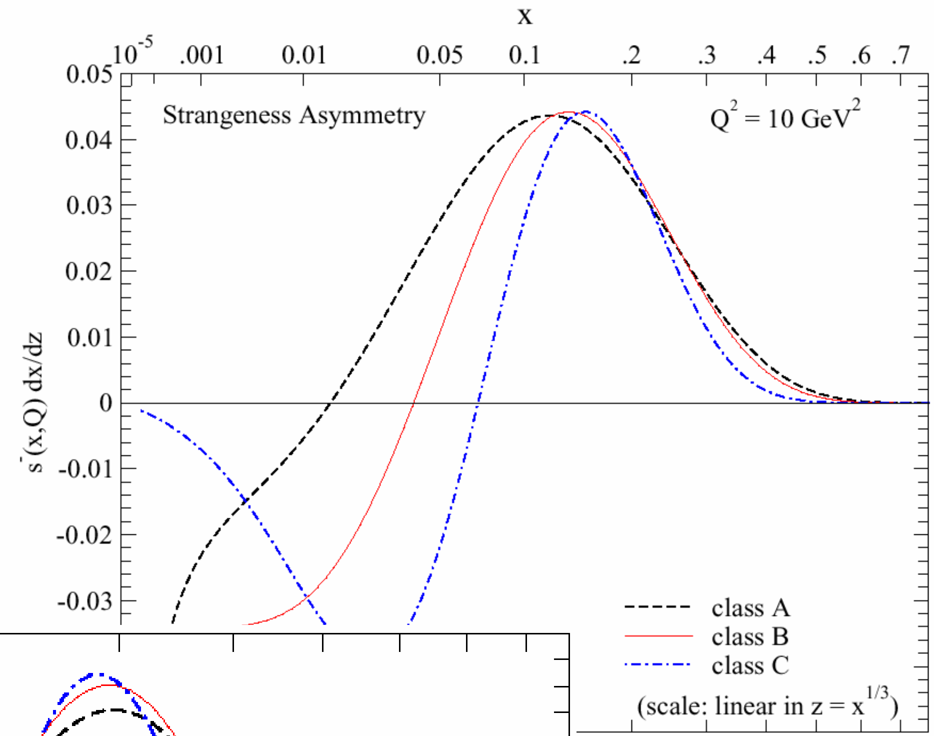
$\sum_h \int dz z M^s(x,z) = (1-x)s(x)$



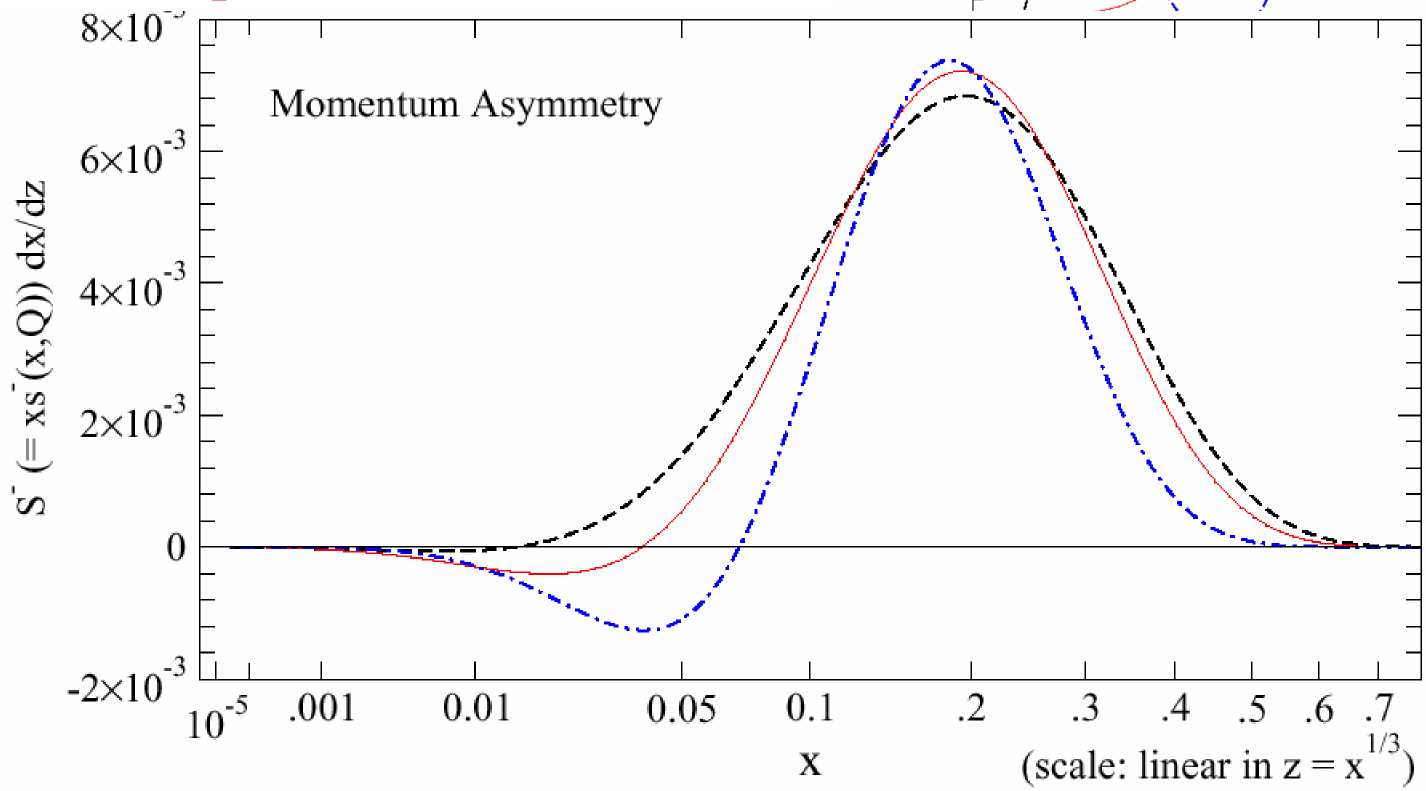
Backup Slides

Typical fit results

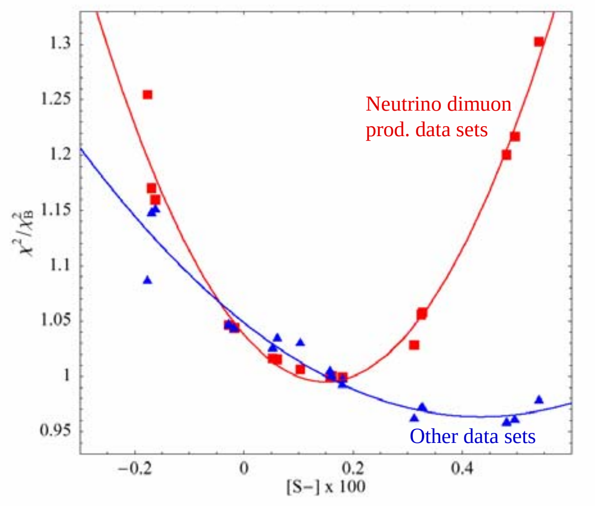
Vs. Bjorken x



positive [S-]

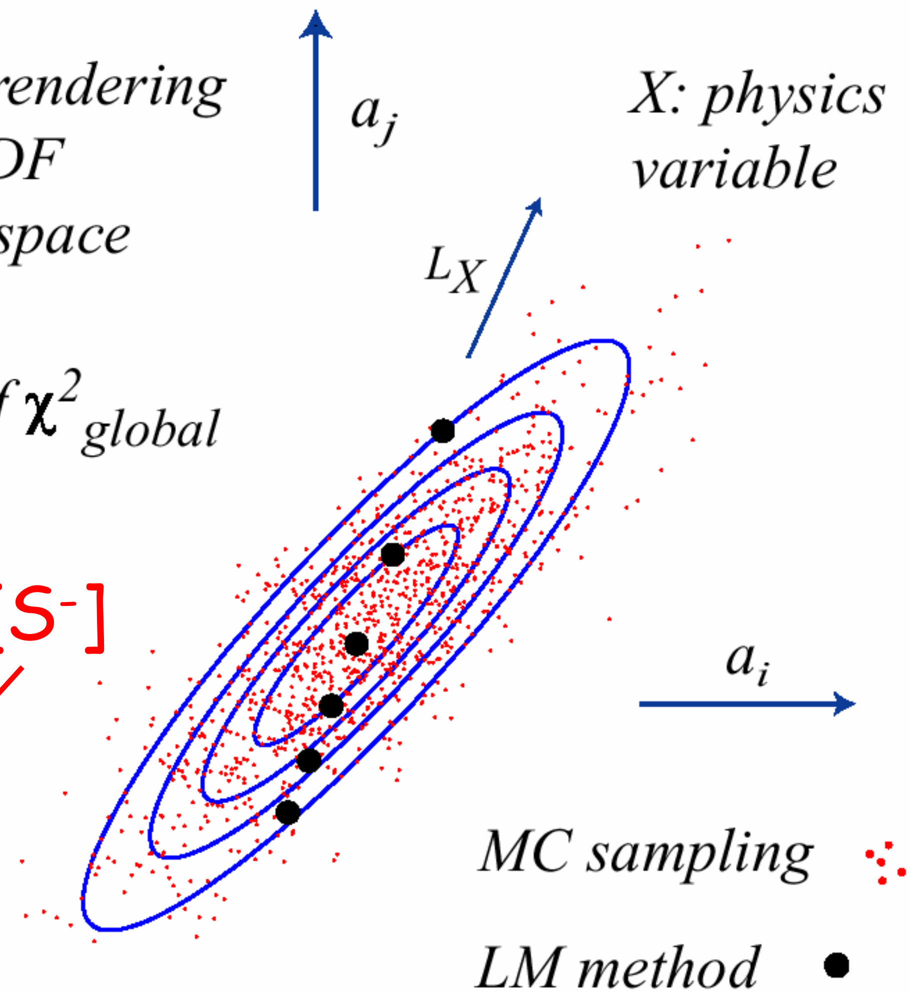


The Lagrange Multiplier Method in Global Analysis



2-dim (i,j) rendering of d-dim PDF parameter space

contours of $\chi^2{}_{global}$



[S⁻]

Constrained fits using modified χ^2 function:

$$\Psi(\lambda, a) = \chi^2_{\text{global}}(a) + \lambda X(a)$$

and vary λ over an appropriate range.