

# The Photon Content of the Unpolarized and Polarized Nucleon \*

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- Equivalent photon approximation (EPA)
- QED Compton process : (i) extract EPA  
(ii) measure  $F_{1,2}$  and  $g_{1,2}$
- QED Compton process at HERMES
- QED Compton process at eRHIC  
comparison with HERMES
- Suppression of background virtual Compton  
scattering (VCS)
- Summary and conclusion

\* In collaboration with M. Glück, C. Pisano, E. Reya  
(Dortmund)

## Equivalent Photon Approximation (EPA)

- Based on the Equivalent Photon Approximation of a pointlike charged fermion.
- The EPA can be extended to the nucleon  $N = p, n$  (non-pointlike particle) as an efficient tool to estimate high energy cross sections :

$$(\Delta)\sigma_{NX} \approx (\Delta)\sigma_{NX}^{\text{EPA}} = (\Delta)\gamma \otimes (\Delta)\hat{\sigma}_{\gamma X}$$

- In  $(\Delta)\sigma_{NX}$   
 $X = l, N$  interacts with  $N$  via virtual photon  
 $Q^2 = -t = -k^2$ : virtuality of the photon  
The EPA is a good approx. when  $Q^2$  is small
- In  $(\Delta)\sigma_{NX}^{\text{EPA}}$   
 $(\Delta)\hat{\sigma}_{\gamma X}$ : real photoproduction cross section  
 $\mu^2$ : momentum scale in  $(\Delta)\hat{\sigma}_{\gamma X}$   
 $x$ : fraction of the proton's momentum carried by the (collinear) photon
- $(\Delta)\gamma(x, \mu^2)$ : universal, scale dependent, equivalent photon distribution of the nucleon  
$$(\Delta)\gamma(x, \mu^2) = (\Delta)\gamma_{el}(x) + (\Delta)\gamma_{inel}(x, \mu^2)$$
- The kinematical region of validity of  $(\Delta)\gamma(x, \mu^2)$  has to be tested
- Has a different evolution compared to  $(\Delta)g(x, \mu^2)$ .

- In terms of the elastic form factors  $G_E(t)$ ,  $G_M(t)$  ( $m$ : nucleon mass,  $\tau = -t/4m^2$ ):

$$\Delta\gamma_{el}(x) = -\frac{\alpha}{2\pi} \int_{t_{\min}}^{t_{\max}} \frac{dt}{t} \left\{ \left[ 2 - x + \frac{2m^2x^2}{t} \right] G_M^2(t) - 2 \left[ 1 - x + \frac{m^2x^2}{t} \right] G_M(t) \frac{G_M(t) - G_E(t)}{1 + \tau} \right\}$$

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$$\gamma_{el}(x) = -\frac{\alpha}{2\pi} x \int_{t_{\min}}^{t_{\max}} \frac{dt}{t} \left\{ 2 \left[ \frac{1}{x} \left( \frac{1}{x} - 1 \right) + \frac{m^2}{t} \right] \times \frac{G_E^2(t) + \tau G_M^2(t)}{1 + \tau} + G_M^2(t) \right\}$$

Kniehl

$$t_{\min} \approx -\infty, \quad t_{\max} \approx -m^2x^2/(1-x)$$

$\rightsquigarrow$   $(\Delta)\gamma_{el}(x)$  can be integrated analytically

- \* The inelastic part  $(\Delta)\gamma_{inel}$  obeys the LO evolution equation

$$\frac{d}{d \ln \mu^2} (\Delta)\gamma_{inel}(x, \mu^2) = \frac{\alpha}{2\pi} \sum_{q=u,d,s} e_q^2 \int_x^1 \frac{dy}{y} \times (\Delta)P_{\gamma q} \left( \frac{x}{y} \right) [(\Delta)q(y, \mu^2) + (\Delta)\bar{q}(y, \mu^2)]$$

Glück, Stratmann, Vogelsang; Glück, Pisano, Reya

$$\Delta P_{\gamma q}(y) = 2 - y, \quad P_{\gamma q}(y) = [1 + (1 - y)^2]/y$$

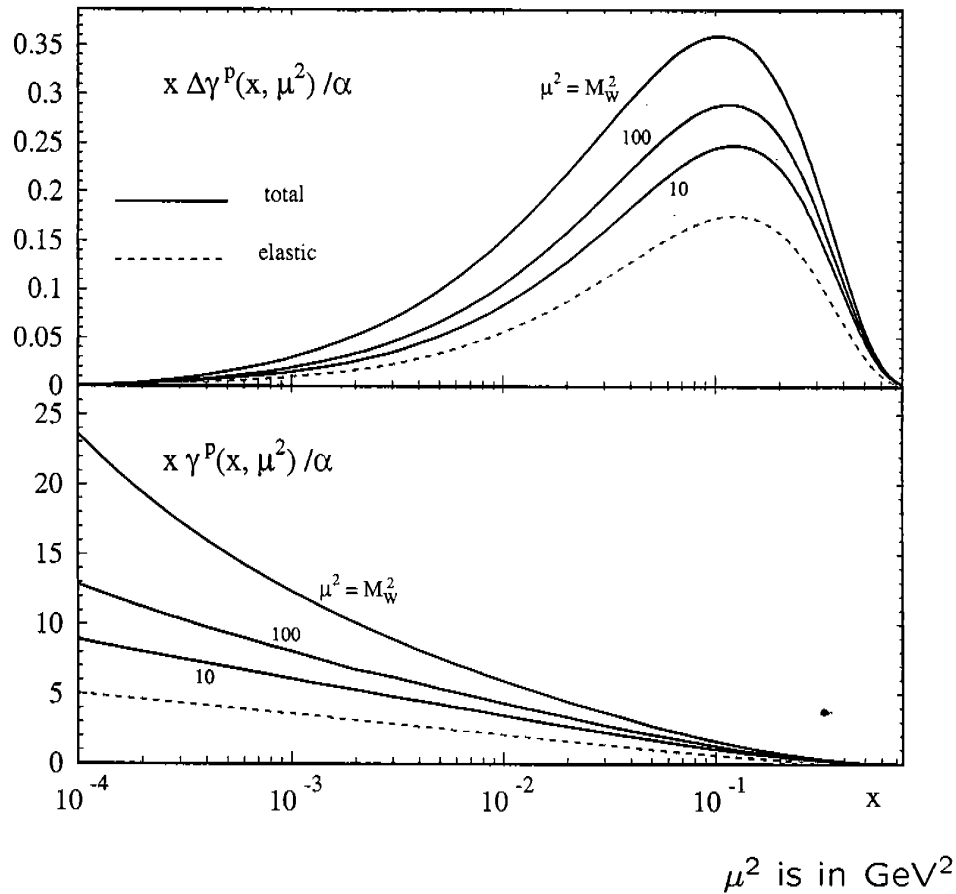
$\Delta \overset{(-)}{q}$  : LO pol. parton distr. GRSV2001 (v.s.)

$\overset{(-)}{q}$  : LO unpol. parton distr. GRV98

'minimal' not compelling boundary condition

$$(\Delta)\gamma_{inel} = 0 \text{ at } \mu_0^2 = 0.26 \text{ GeV}^2$$

## Photon distribution of the proton



$x \Delta\gamma^p(x, \mu^2)$  vanishes at small  $x$ ,  $x \gamma^p(x, \mu^2)$  increases:

$$\Delta\gamma^p(x, \mu^2) \ll \gamma^p(x, \mu^2), \text{ for } x \leq 10^{-3}$$

$$x \Delta\gamma_{el}^p(x), x \Delta\gamma_{inel}^p(x, \mu^2) \rightarrow 0 \text{ as } x \rightarrow 0$$

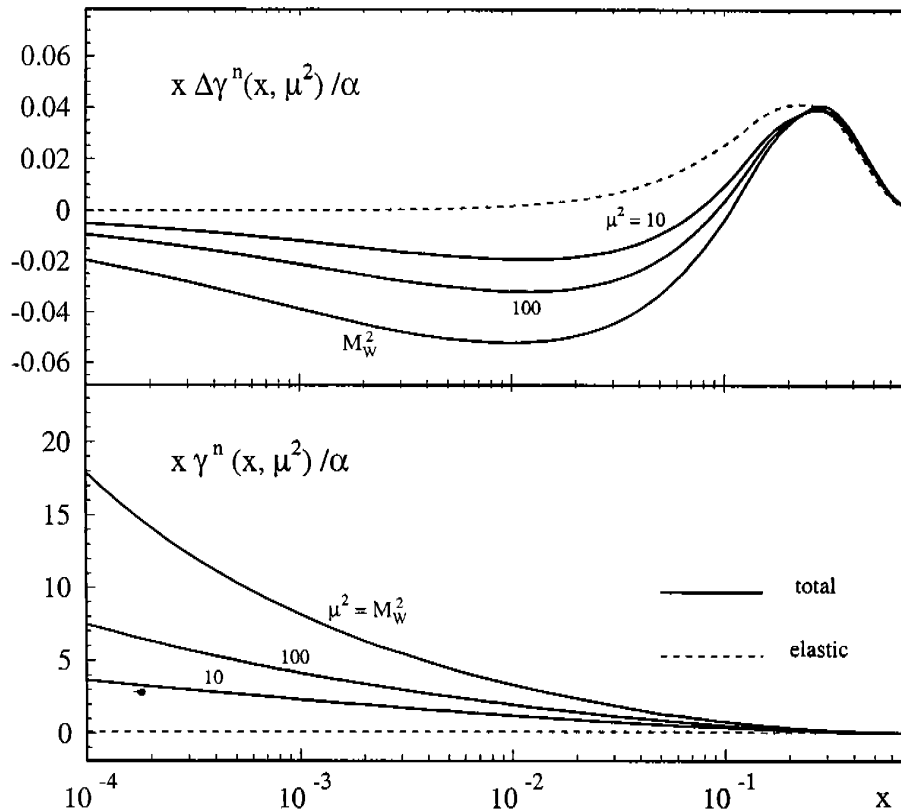
$$x \gamma_{el}^p(x), x \gamma_{inel}^p(x, \mu^2) \rightarrow \infty \text{ as } x \rightarrow 0$$

$$\Delta\gamma_{el}^p(x)/\gamma_{el}^p(x) \rightarrow 1 \text{ as } x \rightarrow 1$$

\* The elastic contribution dominates at moderate values of  $\mu^2$ :

$$(\Delta)\gamma_{el}^p(x) \geq (\Delta)\gamma_{inel}^p(x, \mu^2), \text{ for } \mu^2 \leq 100 \text{ GeV}^2$$

## Photon distribution of the neutron



$\Delta\gamma^n(x, \mu^2)$  is sizeable smaller than  $\Delta\gamma^p(x, \mu^2)$

$\Delta\gamma_{inel}^n(x, \mu^2)$  is marginal for  $x \geq 0.2$ ,

$\gamma_{el}^n(x)$  is marginal and non-singular as  $x \rightarrow 0$

At small  $x$ ,  $(\Delta)\gamma^n(x, \mu^2)$  behaves as  $(\Delta)\gamma^p(x, \mu^2)$ :

$x \Delta\gamma_{el}^n(x), x \Delta\gamma_{inel}^n(x, \mu^2) \rightarrow 0$  as  $x \rightarrow 0$

$x \gamma_{el}^n(x) \rightarrow 0.078, x \gamma_{inel}^n(x, \mu^2) \rightarrow \infty$  as  $x \rightarrow 0$

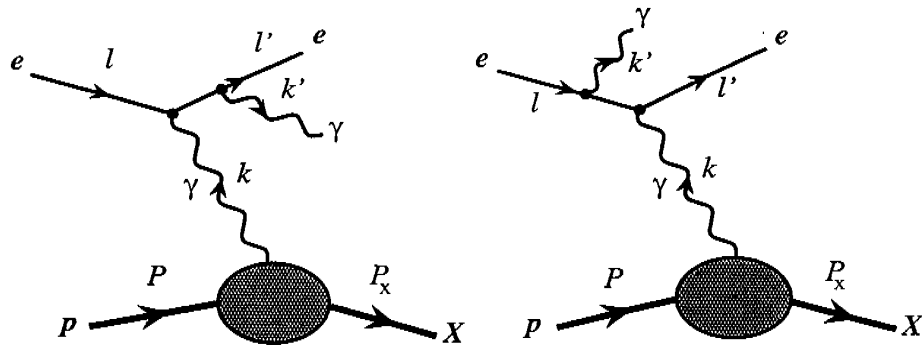
$\Delta\gamma_{el}^n(x) / \gamma_{el}^n(x) \rightarrow 6/7$  as  $x \rightarrow 1$

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## QED Compton Scattering

$$e(l) + p(P) \rightarrow e(l') + \gamma(k') + X(P_X),$$

$X$  is a proton with momentum  $P'$  for elastic scattering.



- Virtual Compton Scattering  $\rightarrow$  photon radiated from the proton : background

- Cross section has several peaks :

Outgoing photon is emitted along the incident electron line (ISR).

Outgoing photon is emitted along the final electron line (FSR).

- QED Compton Peak :  $Q^2 = -k^2 \approx 0$  but the electron propagators are not too small

Distinctive experimental signature: outgoing electron and photon are detected at large polar angles and almost back-to-back in azimuth, with little or no hadronic activity at the detectors.

- QED Compton events are singled out by imposing a cut on the total transverse momentum of the  $e - \gamma$  system or, on the acoplanarity

$$A = 180^\circ - |\Delta\phi|,$$

where  $\Delta\phi$  is the angle between the transverse momenta  $\vec{p}_{t,e}$  and  $\vec{p}_{t,\gamma}$ .

- QED Compton peak in unpol.  $e - p$  scattering :  
measurement of  $F_2(x_B, Q^2)$

Courau, Kessler; Blümlein, Levman, Spiesberger;  
Lendermann, Schultz-Coulon, Wegener : low- $Q^2$ , medium  $x_B$   
(not well known from other experiments)

- QED corrections to polarized DIS :

Bardin, Blümlein, Christova, Kalinovskaya

- Fixed target experiments :  $F_2$  and  $g_1$  at low  $Q^2$  and low  $x_B$ ;

CLAS :  $Q^2 = 0.15 - 1.6 \text{ GeV}^2$ .

- Polarized  $e - p$  collider: broader range of  $x_B$  and  $Q^2$   
→ measurement of  $g_1$  by measuring the QED Compton peak.

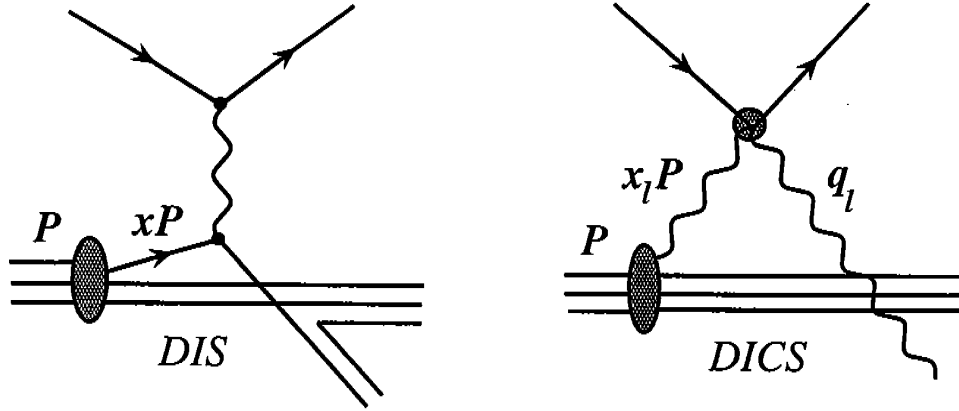
- Determination of  $g_1$  at lower  $Q^2$  region is important to test various models → nonperturbative effects → resonance contributions, higher twist; resummation effects at low  $x_B$

Also important is the transition region from from soft to hard physics.

- Another interesting aspect : Extraction of the equivalent photon distribution of the polarized proton and to find the kinematical region of validity of the equivalent photon approximation.

unpol. Courau, Kessler; Blümlein, Levman, Spiesberger; Rujula, Vogelsang; AM, Pisano; pol. Glück, Pisano, Reya, Schienbein; AM, Pisano

## QED Compton Process in the EPA



- Probing the photon in the proton by a lepton
- Event kinematics is constrained by two variables like inclusive ep scattering.

Define  $Q_l^2 = -\hat{t} = -(l - l')^2$ ;  $x_l = \frac{Q_l^2}{2P \cdot (l - l')}$

$$\frac{d^2(\Delta)\sigma}{dx_l dQ_l^2} = \int_0^1 \frac{dz}{z} (\Delta)\gamma(z, Q_l^2) \frac{d^2(\Delta)\hat{\sigma}^{e\gamma \rightarrow e\gamma}}{d(\frac{x_l}{z}) dQ_l^2}$$

$$\gamma_{\text{inel}}(x, xS) = \frac{\alpha}{2\pi} \int_x^1 dy \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} \frac{dQ^2}{Q^2} \frac{y}{x} \left[ F_2\left(\frac{x}{y}, Q^2\right) \left( \frac{1 + (1-y)^2}{y^2} - \frac{2m^2 x^2}{y^2 Q^2} \right) - F_L\left(\frac{x}{y}, Q^2\right) \right]$$

$$\Delta\gamma_{\text{inel}}(x, xS) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} \frac{dQ^2}{Q^2} \left( 2 - y - \frac{2m^2 x^2}{Q^2} \right) 2g_1\left(\frac{x}{y}, Q^2\right),$$

$$Q_{\text{min}}^2 = \frac{x^2 m^2}{1-x}, \quad Q_{\text{max}}^2 = \hat{s}$$

$$\hat{s} = (l' + k')^2, S = (l + P)^2, \quad x = \frac{\hat{s}}{S}; \quad S \gg m^2, \quad \hat{s} \gg Q^2$$



## QED Compton process at HERMES

- The 'exact'  $(\Delta)\sigma$  compared with the EPA; EPA expressed in terms of structure functions

AM, Pisano

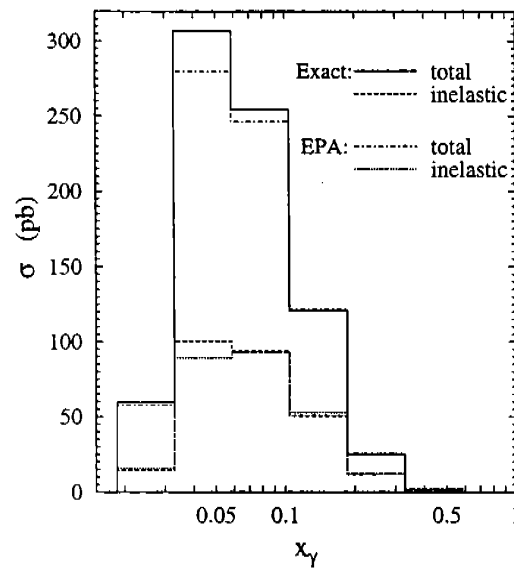
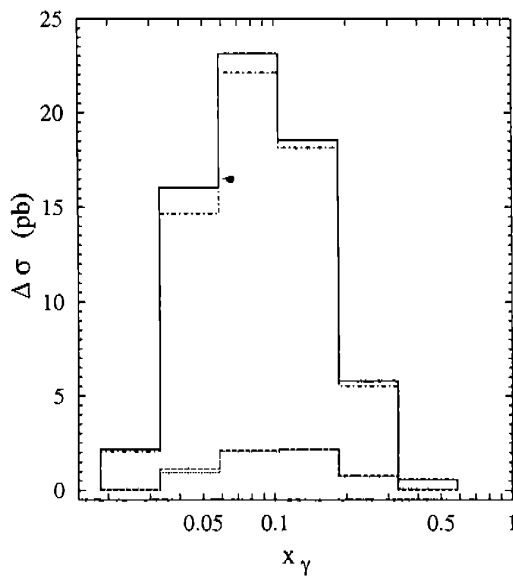
- $(\Delta)\sigma$  and  $(\Delta)\sigma^{\text{EPA}}$  are in  $x_\gamma$  bins,  $x_\gamma = \frac{l \cdot k}{P \cdot l}$

$x_\gamma$  : fraction of longitudinal momentum of the proton carried by the photon

- Bins in  $x_l = \frac{-\hat{t}}{2P \cdot (l-l')}$  can also be used

$\leadsto x_\gamma \simeq x_l \simeq x = \frac{\hat{s}}{S}$  as  $Q^2 \simeq 0$ ;  $\hat{s} = (l' + k')^2, S = (l + P)^2$

- $F_2$  param. ALLM97 (Also valid at low- $Q^2$ ) ;  
Callan-Gross reln.  $E_e = 27.5 \text{ GeV}$



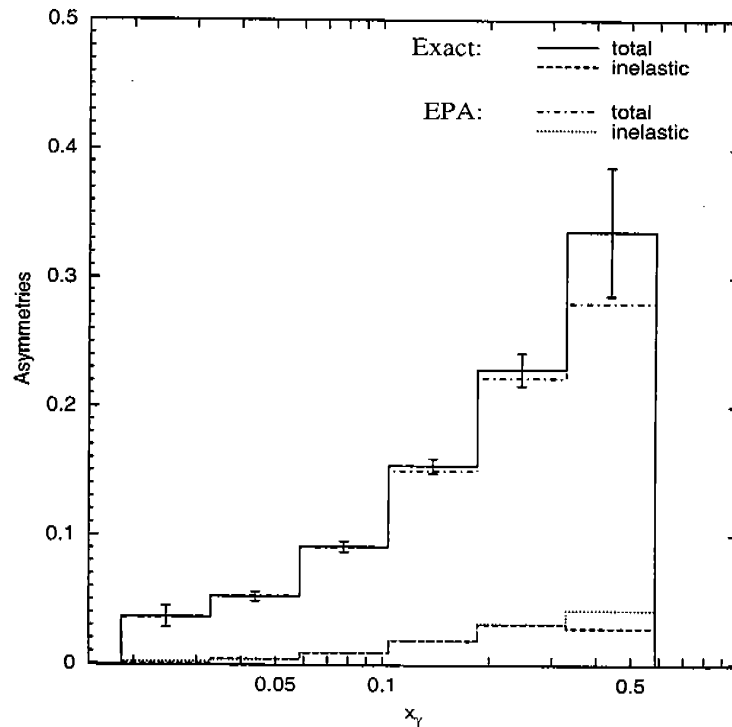
$g_2 \approx 0, g_1$ : Badelek, Kwiecinski, Ziaja.

- Kinematical cuts:  $E'_e, E'_\gamma > 4 \text{ GeV}, \hat{s} > 1 \text{ GeV}^2,$   
 $0.04 < \theta_e, \theta_\gamma < 0.2, \hat{s} > Q^2$

Suppress the radiative events unrelated to QED Compton scattering

Major background from VCS is suppressed by the exp. condition of no observable hadronic activity at the detectors.

## QED Compton process at HERMES (II)



- Asymmetry  $A_{LL} = \frac{d\Delta\sigma}{d\sigma}$

$$\Delta\sigma = \frac{1}{2}(\sigma_{++} - \sigma_{+-}); \quad \sigma = \frac{1}{2}(\sigma_{++} + \sigma_{+-})$$

- Discrepancy of 'exact' with the EPA small for small to medium  $x_\gamma$
- Expected statistical error increases in higher  $x_\gamma$  bins:

$$\text{Error: } \delta A_{LL} \approx \frac{1}{P_e P_p \sqrt{\mathcal{L} \sigma_{bin}}}; \quad P_e = P_p = 0.7, \quad \mathcal{L} = 1/fb.$$

$$\frac{\Delta\sigma^{EPA} - \Delta\sigma}{\Delta\sigma} = -4.8\%,$$

$$\frac{\sigma^{EPA} - \sigma}{\sigma} = -4.7\%$$

$$\frac{\Delta\sigma_{ef}^{EPA} - \Delta\sigma_{ef}}{\Delta\sigma_{ef}} = -5.0\%,$$

$$\frac{\sigma_{ef}^{EPA} - \sigma_{ef}}{\sigma_{ef}} = -5.7\%$$

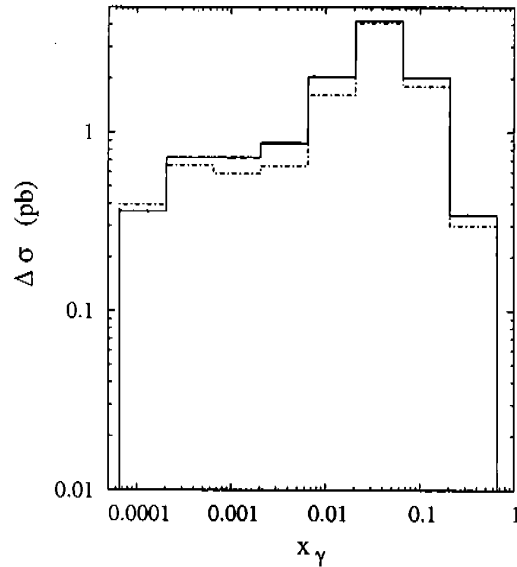
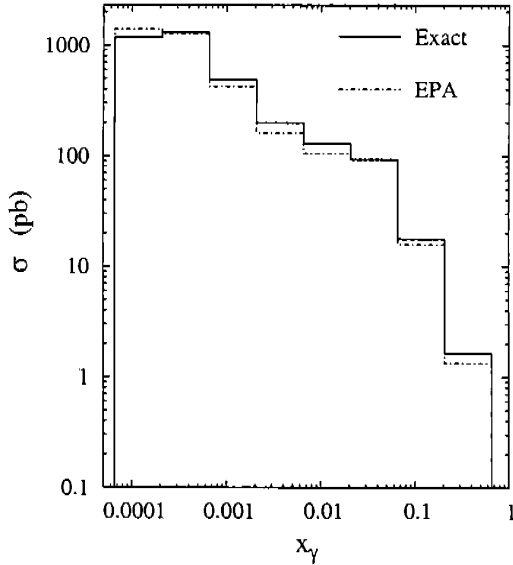
- $g_1(x_B, Q^2)$  Measurement at HERMES using QED Compton peak : events can be observed

$$x_B : 0.02 - 0.5$$

$$Q^2 : 0.007 - 7 \text{ GeV}^2$$

(subject to the constraints discussed before)

## QED Compton process at eRHIC



- $(\Delta)\sigma$  and  $(\Delta)\sigma^{\text{EPA}}$  are in  $x_\gamma$  bins,  $x_\gamma = \frac{l \cdot k}{P \cdot l}$

- $E_e = 10 \text{ GeV}$ ,  $E_p = 250 \text{ GeV}$

- Kinematical region :

$$E'_e, E'_\gamma > 4 \text{ GeV}; \quad \hat{s} > 1 \text{ GeV}^2$$

$$0.06 \leq \theta_e, \theta_\gamma \leq \pi - 0.06$$

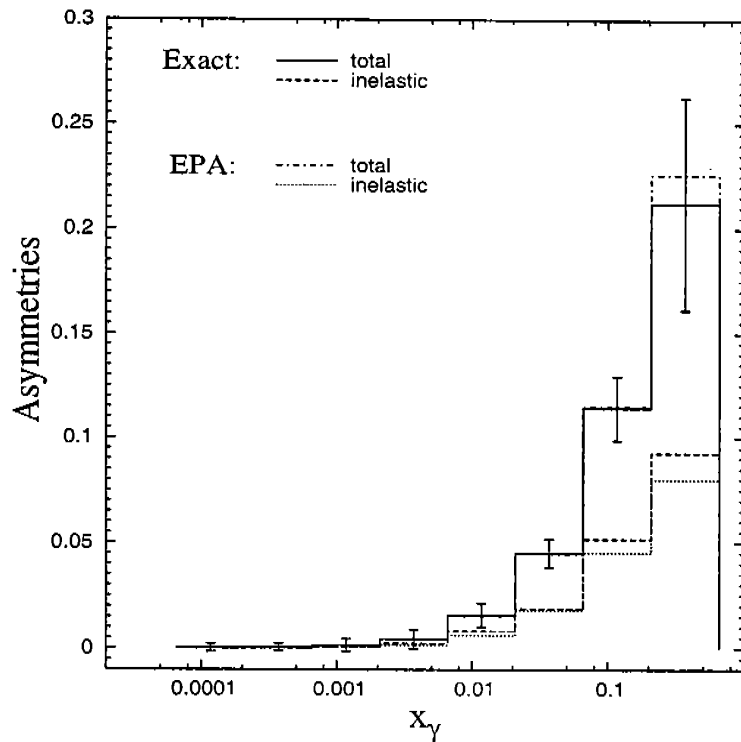
$$\hat{s} > Q^2$$

- $F_2$  param. ALLM97 (Also valid at low- $Q^2$ ) ; Callan-Gross reln.

$g_2 \approx 0$ ,  $g_1$ : Bådelek, Kwiecinski, Ziaja.

- $\frac{\Delta\sigma^{\text{EPA}} - \Delta\sigma}{\Delta\sigma} = -9.8\%$ ,  $\frac{\sigma^{\text{EPA}} - \sigma}{\sigma} = 1.2\%$

## QED Compton Process at eRHIC (II)



- Asymmetry very small for smaller  $x_\gamma$  bins; increases as  $x_\gamma$  increases
- Agreement with the EPA better except in the last bin
- Expected statistical error is large for high  $x_\gamma$  bins:

$$\text{Error: } \delta A_{LL} \approx \frac{1}{P_e P_p \sqrt{\mathcal{L} \sigma_{bin}}}; P_e = P_p = 0.7, \mathcal{L} = 1/fb.$$

- $g_1$  measurement at eRHIC using QED Compton peak :

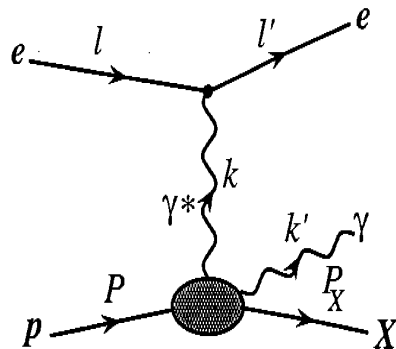
Broader  $x_B$ ,  $Q^2$  range subject to the kinematical cuts shown before

$$x_B : 1 \times 10^{-4} \text{ and above}$$

$$Q^2 : .003 - 1000 \text{ GeV}^2$$

Suitable to test models of  $g_1$  at low- $Q^2$  as well as in the transition region

## Virtual Compton Scattering Background



Elastic contribution : in terms of generalized parton distributions. Monte Carlo estimate for HERA : suppressed

To estimate the inelastic contribution, convolute cross section of subprocess

$$e(l) + q(p) \rightarrow e(l') + \gamma(k') + q(p'),$$

with an 'effective' parton distribution of the proton:

$$\tilde{q}(x_B, Q^2) = \frac{Q^2}{Q^2 + a Q_0^2} q(x_B, Q^2 + Q_0^2),$$

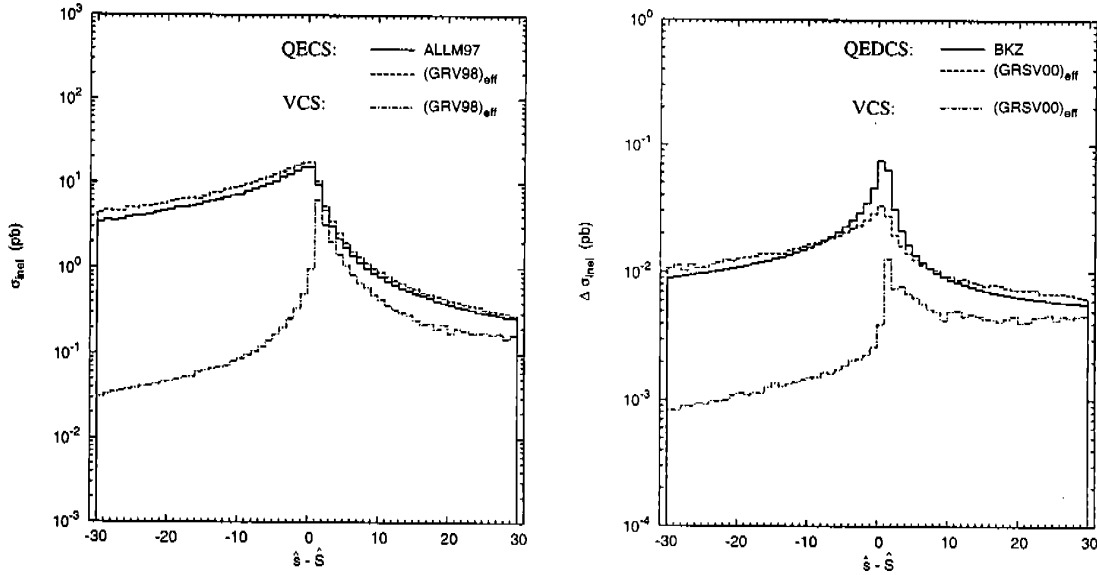
Parameters :  $a = 1/4$  and  $Q_0^2 = 0.4 \text{ GeV}^2$ ,  $q(x_B, Q^2)$  : NLO GRV98.

Polarized : convolute with the effective polarized distribution

$$\Delta \tilde{q}(x_B, Q^2) = \Delta q(\bar{x}, Q^2 + Q_0^2),$$

with  $\bar{x} = \frac{x_B(Q^2 + Q_0^2)}{Q^2 + x_B Q_0^2}$ ;  $Q_0^2 = 2.3 \text{ GeV}^2$ ; NLO GRSV00 s.s.

## Suppression of VCS Background (eRHIC)



- Integrated cross section:

$$\sigma_{\text{inel}}(S) = \sum_q \int \frac{dx_B}{x_B^2} \int d\hat{s} \int dQ^2 \int d\hat{t} \int d\varphi^* \frac{1}{(\hat{s} + Q^2)} \overline{|M_{\text{inel}}|^2} \tilde{q}(x_B, Q^2)$$

$$\overline{|M_{\text{inel}}|^2} = \overline{|M_{\text{inel}}^{\text{QEDCS}}|^2} + \overline{|M_{\text{inel}}^{\text{VCS}}|^2} - 2 \Re \overline{M_{\text{inel}}^{\text{QEDCS}} M_{\text{inel}}^{\text{VCS}*}}$$

- 'Effective' parton model estimate of VCS background

- Invariants :  $\hat{S} = (p' + k')^2 = \frac{\hat{t}(x_l - x_B)}{x_l}$ ;

with  $\hat{t} = (l - l')^2$ ,  $x_l = \frac{-\hat{t}}{2P \cdot (l - l')}$  and  $\hat{s} = (l' + k')^2$ .

- Inelastic interference between QEDCS and VCS: negligible.
- VCS is suppressed when  $\hat{s} < \hat{S}$ .

## Summary and Conclusion

- The photon content of polarized and unpolarized nucleon  $(\Delta)\gamma^N(x, \mu^2)$  evaluated in the EPA gives an estimate of the photon-induced subprocesses in elastic/deep inelastic  $eN$  and hadronic ( $pN, \dots$ ) reactions.
- Polarized : Can be extracted experimentally at HERMES/eRHIC: through QED Compton process in  $ep \rightarrow e\gamma p$  and  $ep \rightarrow e\gamma X$ : distinctive experimental signature.
- Unpolarized : at HERA
- QED Compton peak in polarized  $ep$  scattering : Will provide informations concerning the structure functions  $g_{1,2}$  in the low  $Q^2$  region at HERMES; future polarized  $ep$  collider: over a broad range of  $x_B, Q^2$  .
- Discussed relevant kinematical cuts in order to select QEDC events from experiments; both for fixed target (HERMES) and collider (eRHIC) kinematics.