Introduction to $g_1(x, Q^2)$ at small x

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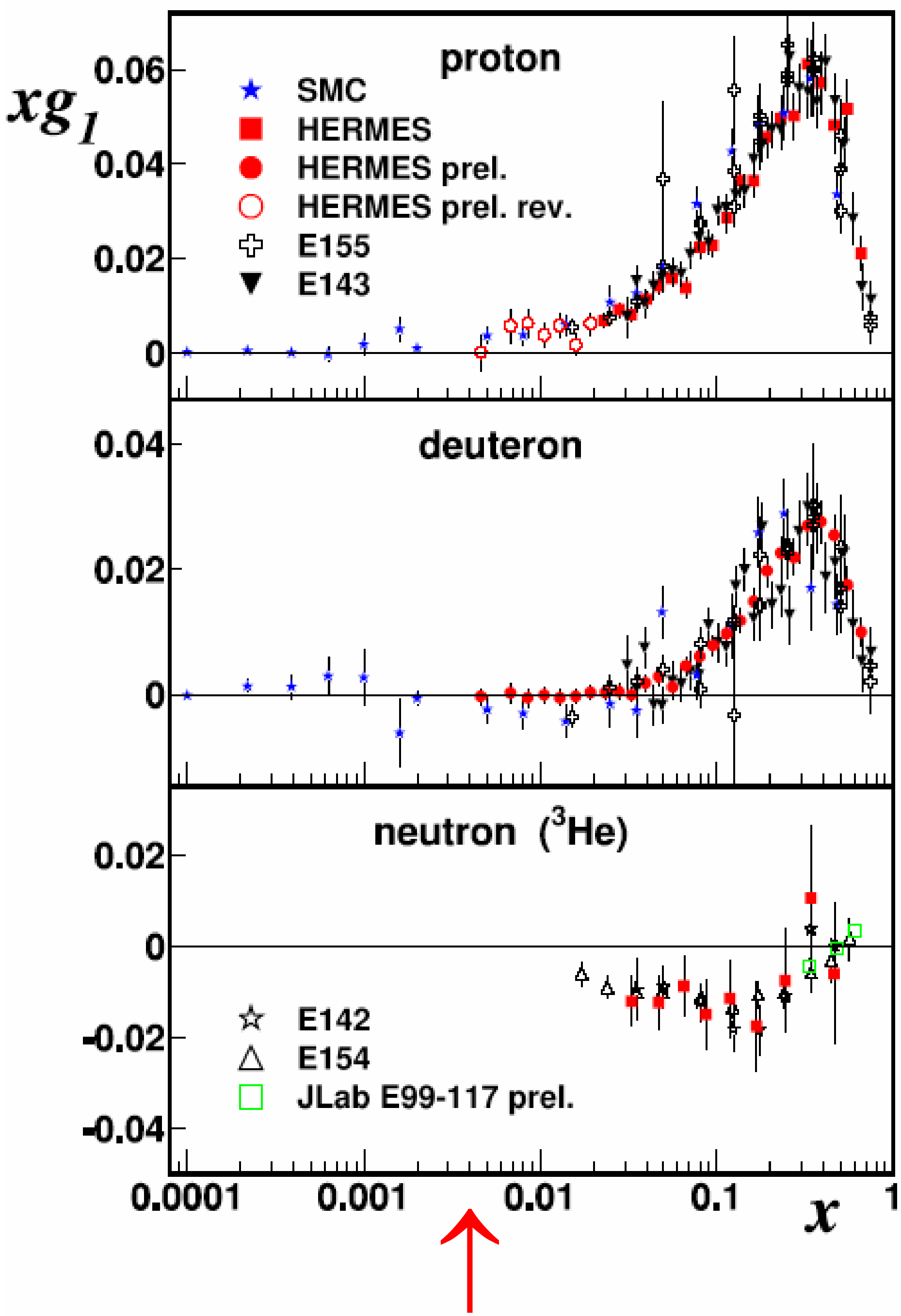
Why interest in small x behavior of $g_1(x, Q^2)$?

- by its own right: evolution, resummation

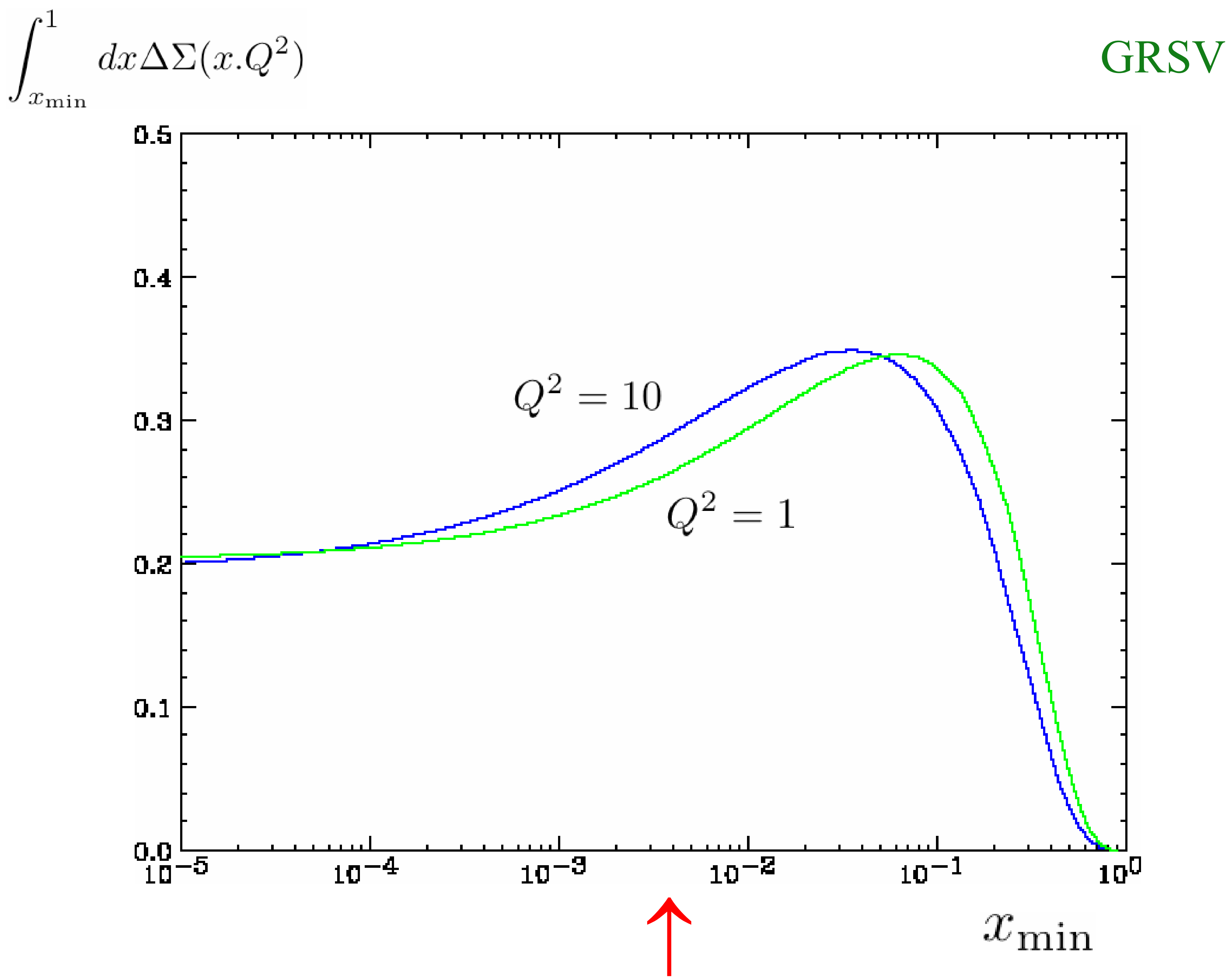
- first moments

  GRSV: >10% of $\Delta\Sigma$ from $x \leq 4 \times 10^{-3}$

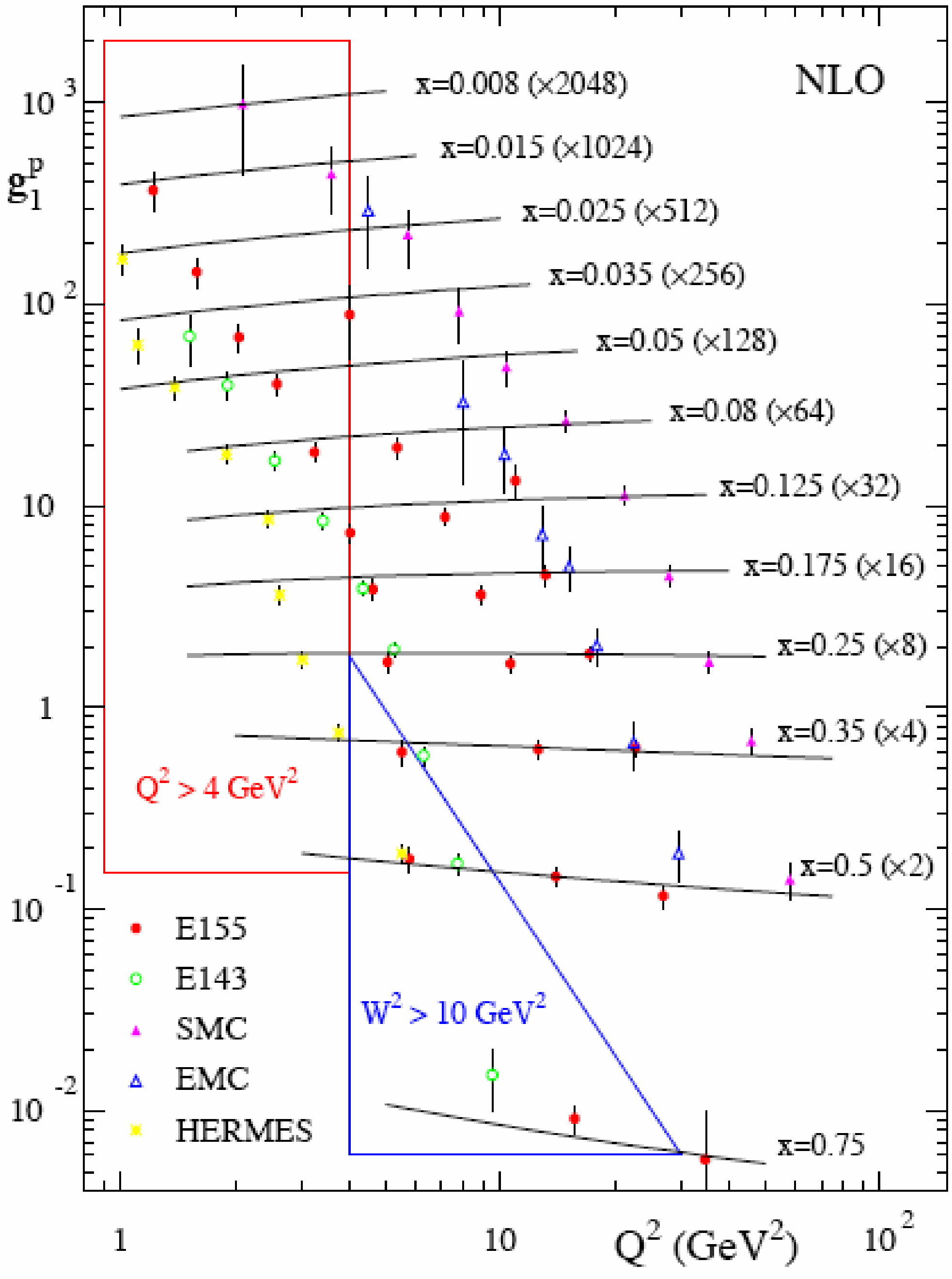


proton
SMC
HERMES
HERMES prel.
HERMES prel. rev.
E155
E143
deuteron
neutron (3He)
E142
E154
JLab E99-117 prel.
xg1
0.06
0.04
0.02
0
-0.02
-0.04
0.0001
0.001
0.01
0.1
1
x

U. Stösslein



$\int_{x_{\min}}^{1} dx \Delta\Sigma(x.Q^2)$
GRSV
0.5
0.4
0.3
0.2
0.1
0.0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$Q^2 = 10$
$Q^2 = 1$
$x_{\min}$



NLO
x=0.008 (×2048)
x=0.015 (×1024)
x=0.025 (×512)
x=0.035 (×256)
x=0.05 (×128)
x=0.08 (×64)
x=0.125 (×32)
x=0.175 (×16)
x=0.25 (×8)
x=0.35 (×4)
x=0.5 (×2)
x=0.75
g_1^p
$Q^2 > 4$ GeV^2
$W^2 > 10$ GeV^2
E155
E143
SMC
EMC
HERMES
Q^2 (GeV^2)

GRSV

- what does DGLAP evolution tell us ?

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \Delta\Sigma(x,\mu^2) \\ \Delta g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} \left(\frac{x}{z},\mu^2\right)$$

- at $x \longrightarrow 0$ (and to lowest order):

$$\begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix} (x,\alpha_s) \approx \frac{\alpha_s}{2\pi} \begin{pmatrix} C_F & -n_f \\ 2C_F & 4C_A \end{pmatrix} + \mathcal{O}(x)$$

- Mellin moments: $f^N = \int_0^1 dx\, x^{N-1} f(x)$

$$\begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix}^N \approx \frac{\alpha_s}{2\pi} \frac{1}{N} \begin{pmatrix} C_F & -n_f \\ 2C_F & 4C_A \end{pmatrix}$$

- can obtain solution:

$$\begin{pmatrix} \Delta\Sigma^N(Q^2) \\ \Delta g^N(Q^2) \end{pmatrix} \approx \frac{e^{(2/\beta_0)\lambda_+/N \ln\left(\alpha_s(Q_0^2)/\alpha_s(Q^2)\right)}}{\lambda_+ - \lambda_-} \begin{pmatrix} C_F - \lambda_- & -2n_f \\ 2C_F & 4C_A - \lambda_- \end{pmatrix} \cdot \begin{pmatrix} \Delta\Sigma^N(Q_0^2) \\ \Delta g^N(Q_0^2) \end{pmatrix}$$

$$\lambda_+ \approx 11.2\,,\ \ \lambda_- \approx 2.1$$

- interplay between input and pert. evolution:

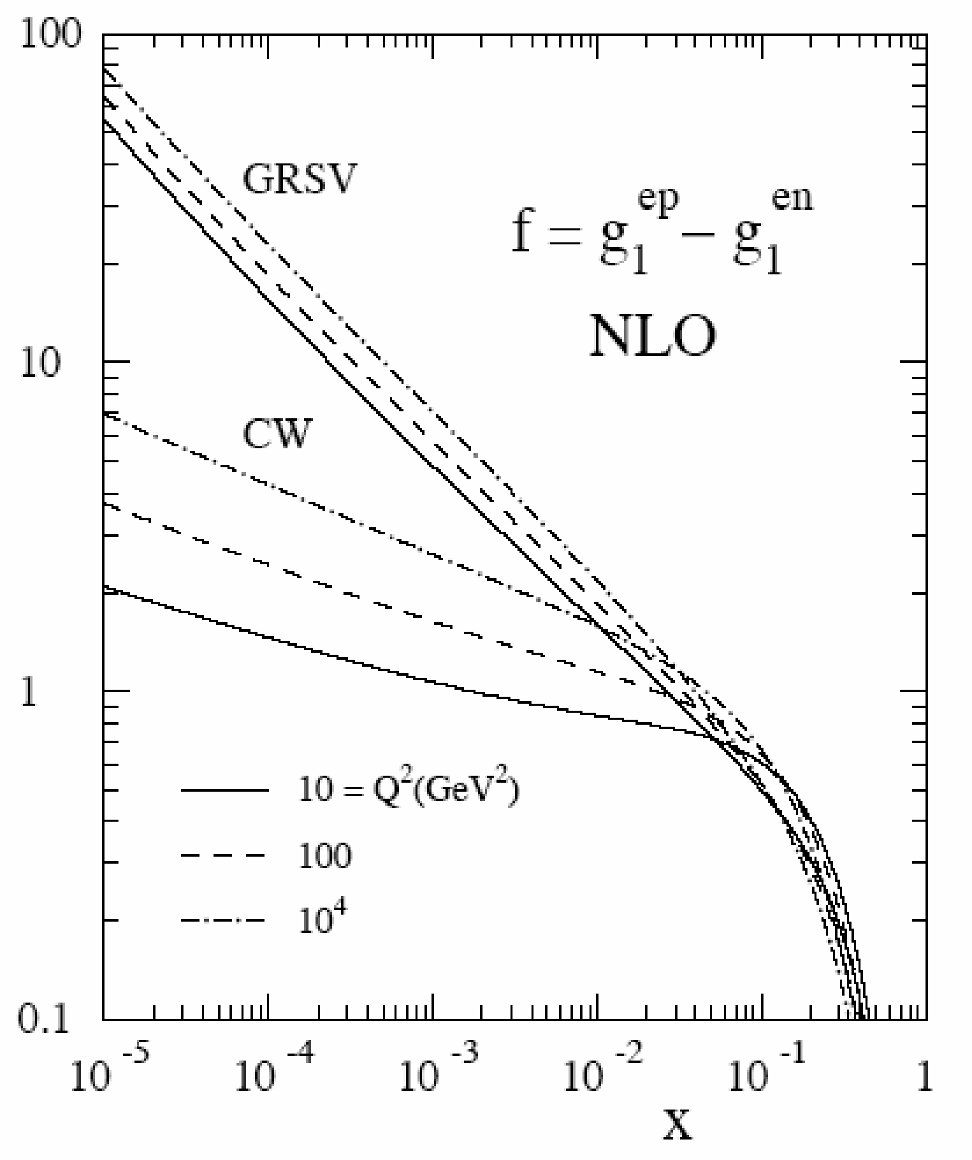
  * "flat" input : small-x behavior driven by evolution

$$\Delta\Sigma(x,Q^2) \ \propto\ \exp\left[2\sqrt{\frac{2}{\beta_0}\lambda_+ \ln(\alpha_s(Q_0^2)/\alpha_s(Q^2))\, \ln\frac{1}{x}}\,\right]$$

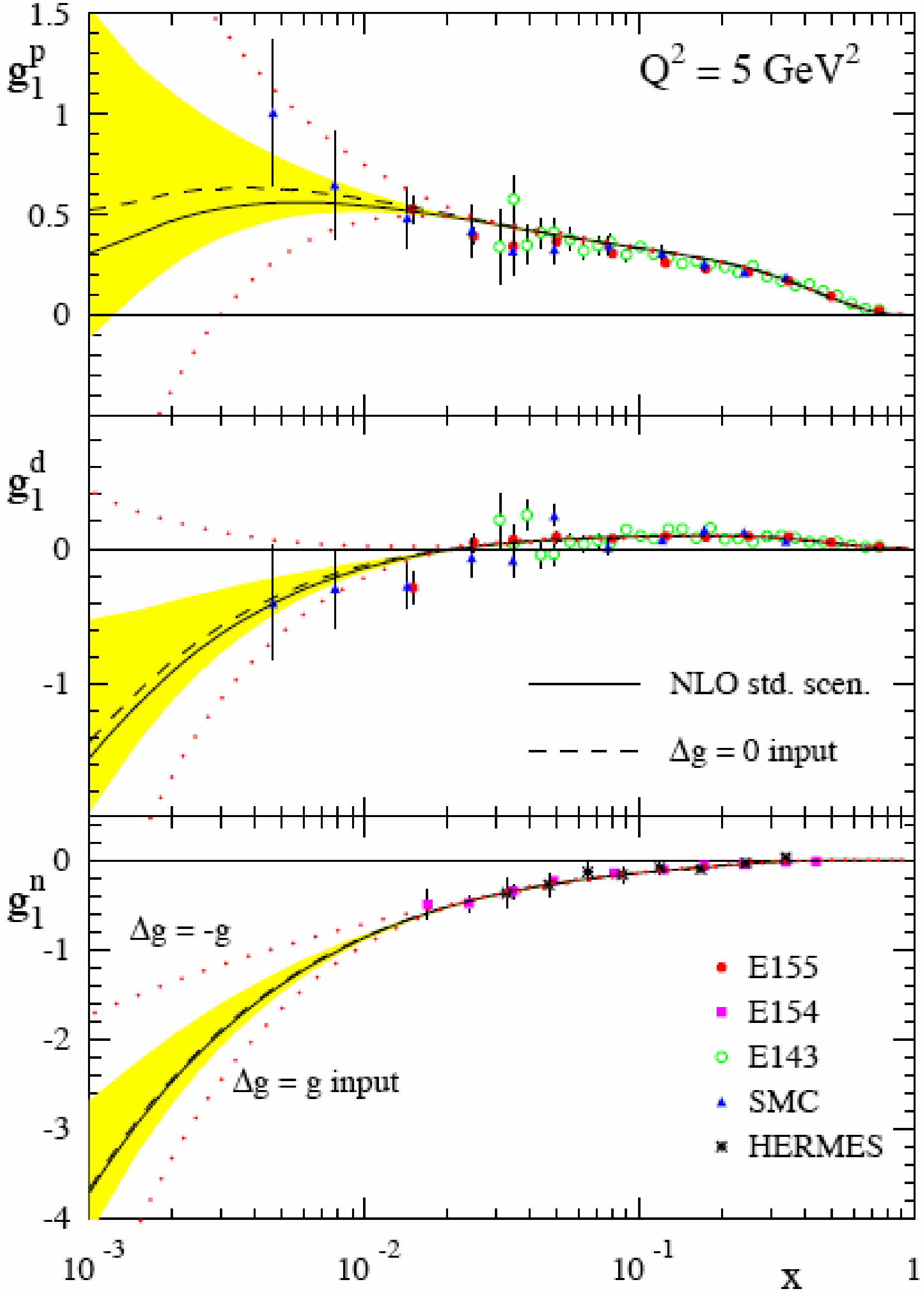
  * power-like input $\Delta\Sigma\,,\ \Delta g \ \sim\ x^{-\alpha}$
    preserved under evolution -- rise indep. of Q^2

  * in any case, size & sign of $\Delta g(x,Q_0^2)$ crucial for $\Delta q(x,Q^2)$
    and hence for $g_1(x,Q^2)$

Ball, Forte, Ridolfi

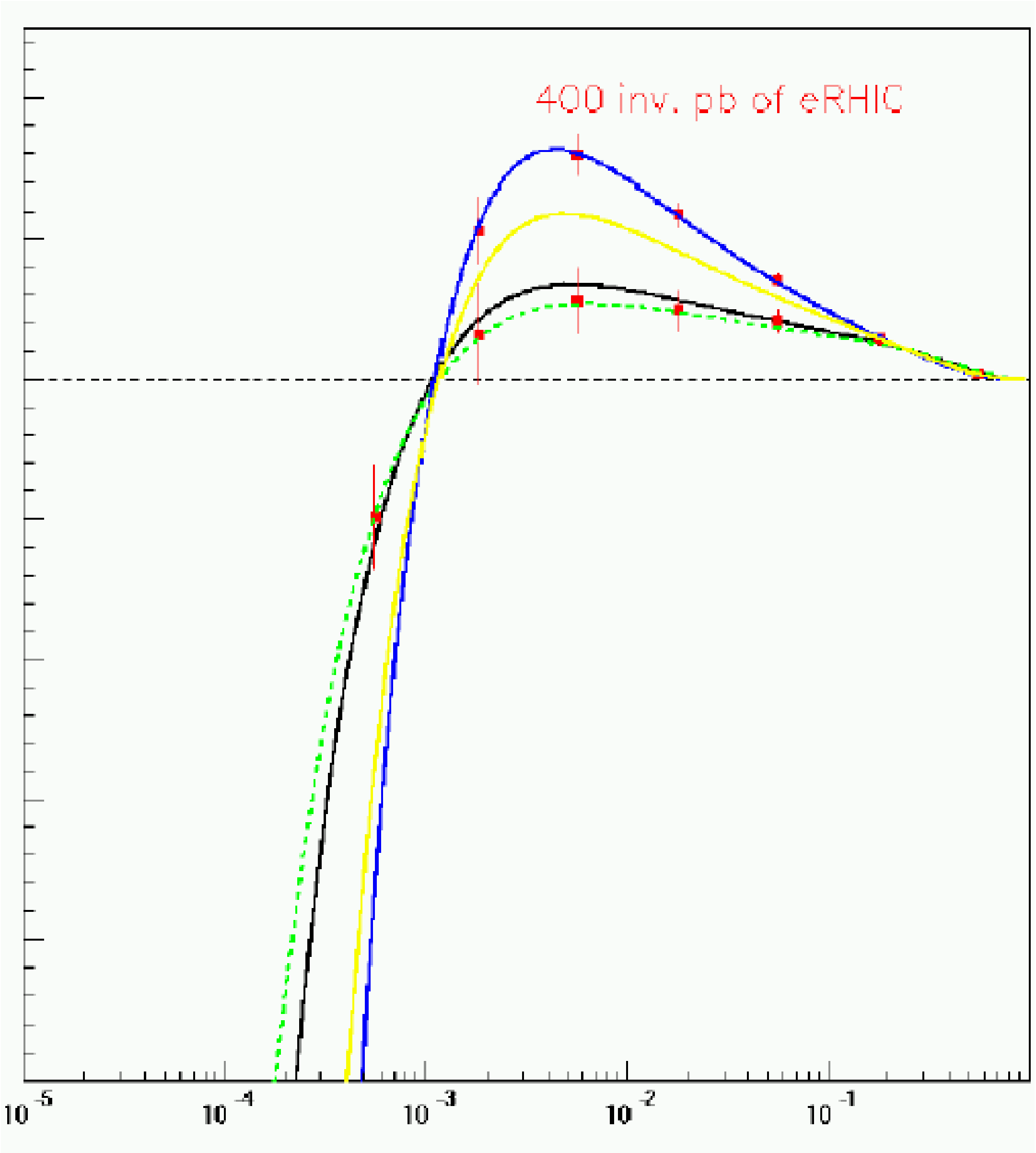


Blümlein,Riemersma,Vogt

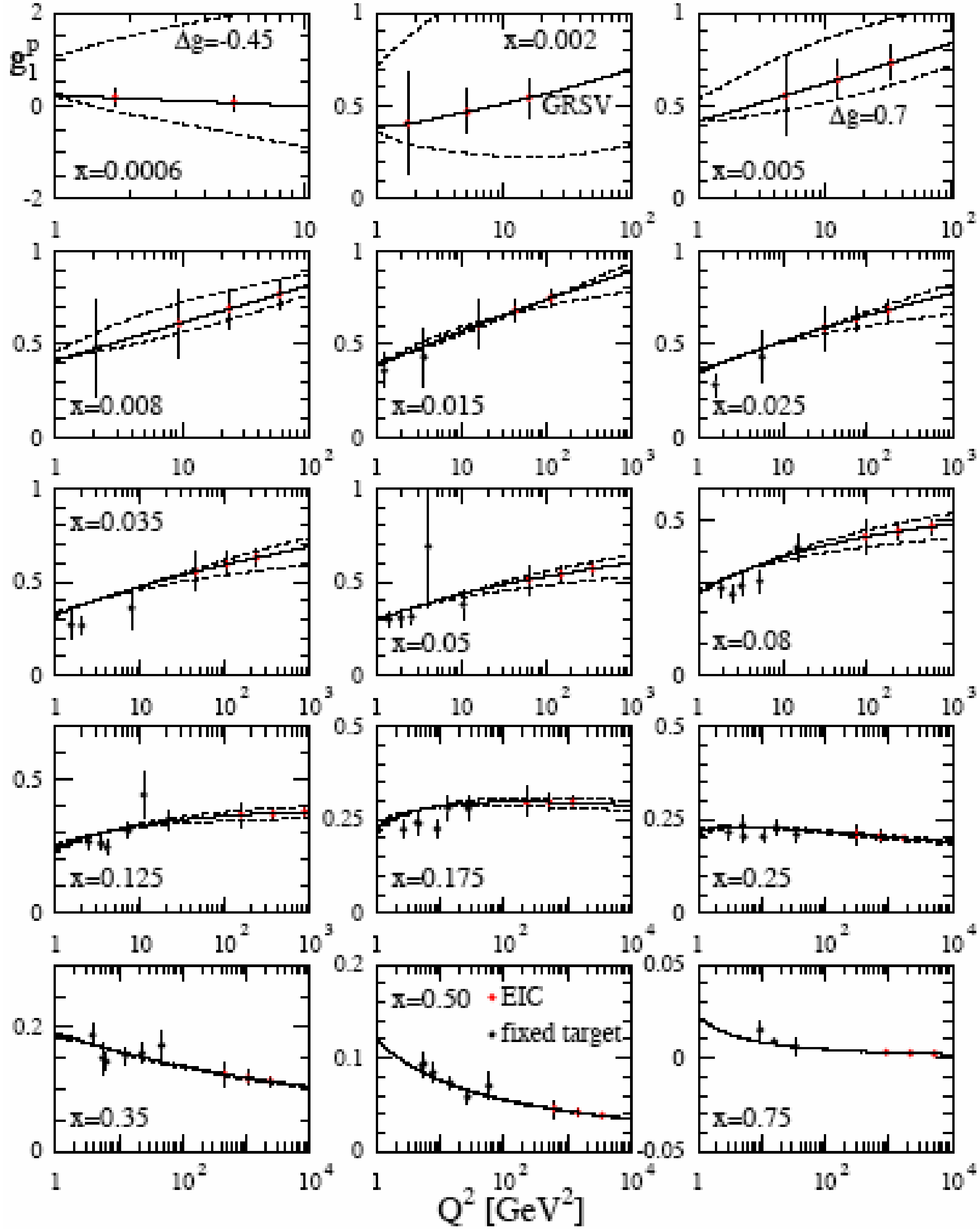


g_1^p
$Q^2 = 5\ GeV^2$
g_1^d
NLO std. scen.
$\Delta g = 0$ input
g_1^n
$\Delta g = -g$
$\Delta g = g$ input
E155
E154
E143
SMC
HERMES
x

GRSV



Deshpande et al.



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Stratmann

- higher orders in the splitting functions at small x:

$$\Delta P(x) = \frac{\alpha_s}{2\pi} c_0 + \left(\frac{\alpha_s}{2\pi}\right)^2 c_1 \ln^2 x + \ldots + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} c_k \ln^{2k} x + \ldots$$

- compare unpolarized case: (gq, gg spl. fcts.)

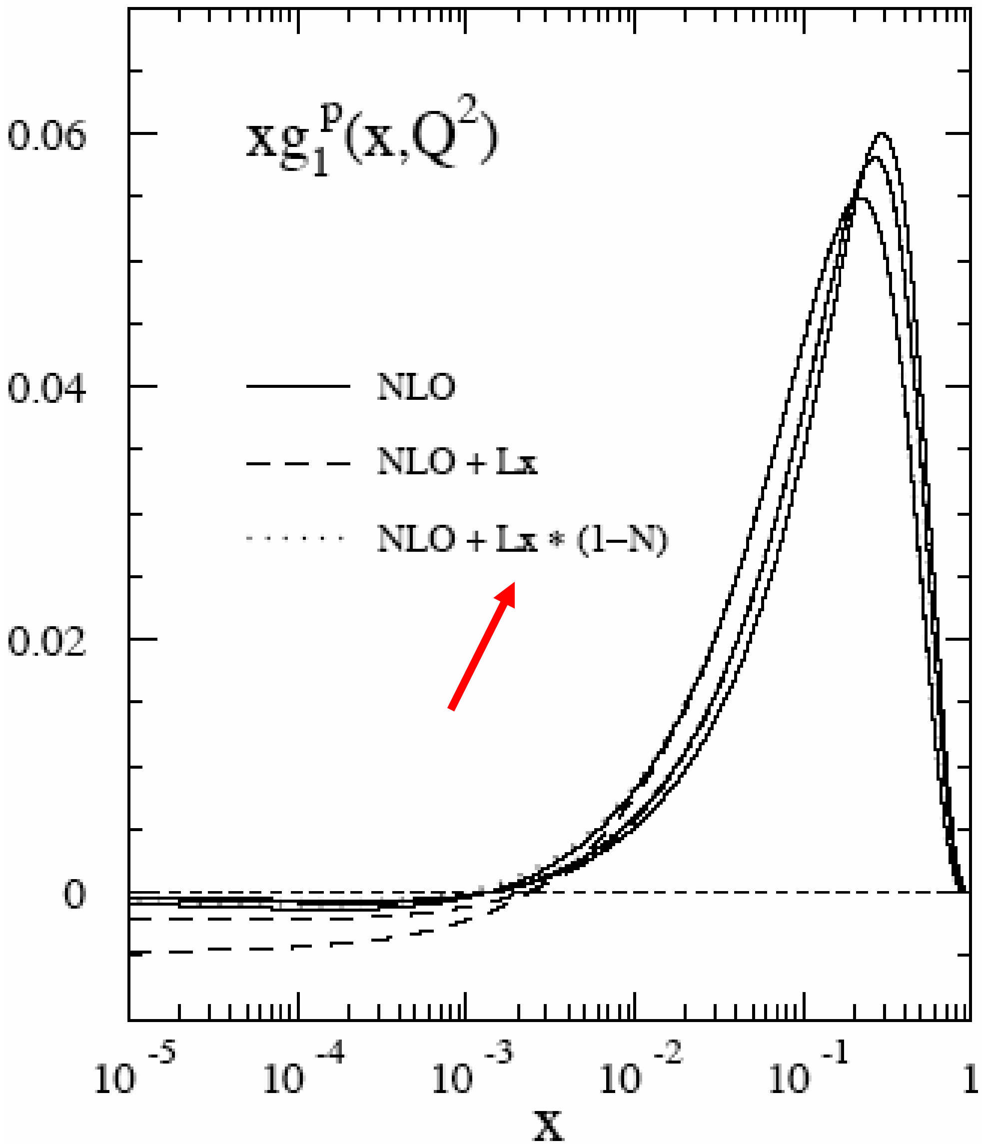
$$P(x) = \frac{\alpha_s}{2\pi} \frac{c_0'}{x} + \left(\frac{\alpha_s}{2\pi}\right)^2 c_1' \frac{\ln x}{x} + \ldots + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} c_k' \frac{\ln^k x}{x} + \ldots$$

- all-order resummation in polarized case:

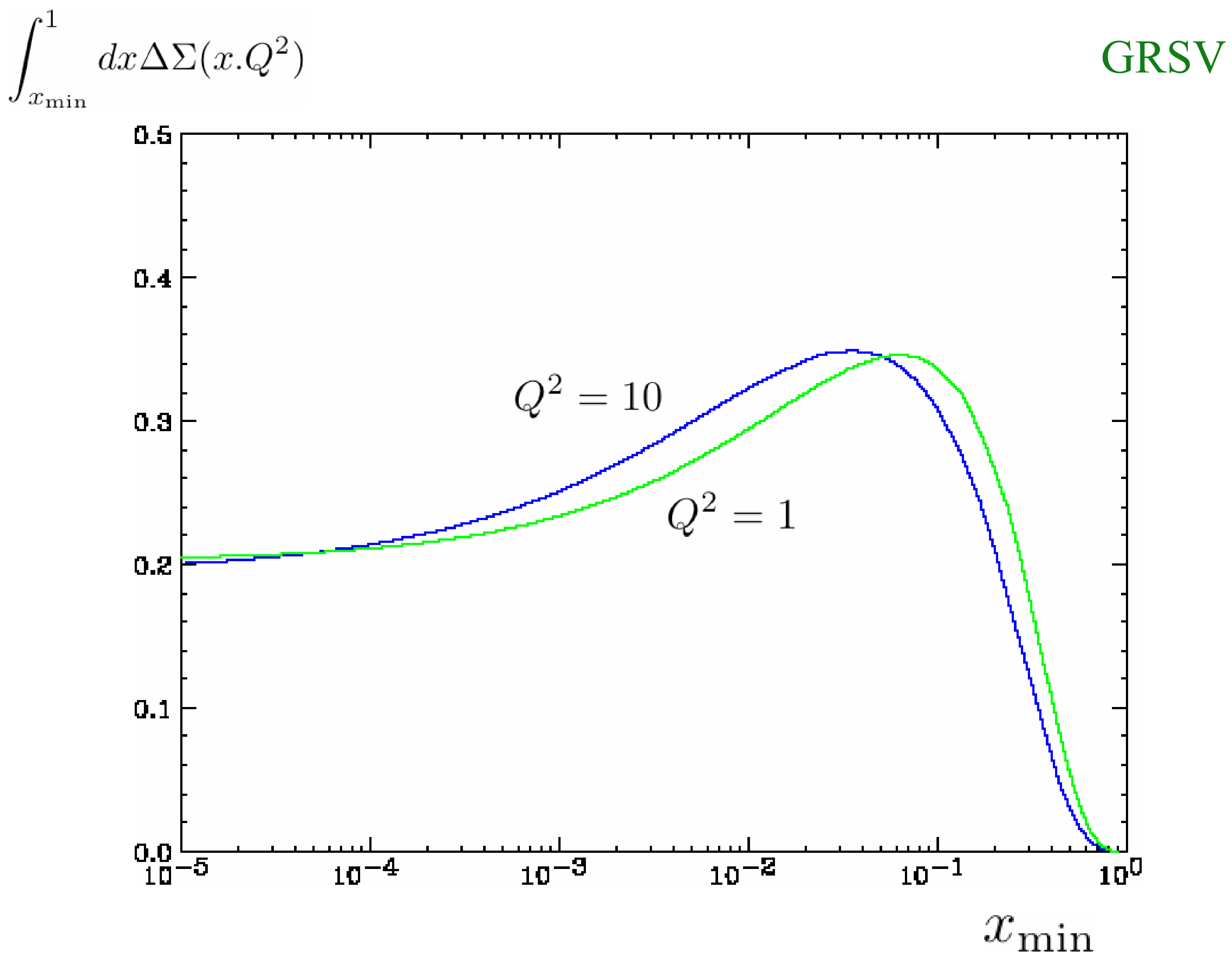
Kirschner,Lipatov; Bartels,Ermolaev,Ryskin; Kwiecinski,Ziaja; Ermolaev,Greco,Troyan; Maul

- small-x behavior of input remains relevant

- subleading terms probably remain crucial Blümlein,Riemersma,Vogt



Blümlein,Vogt



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10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$x_{\min}$

$$\int_{x_{\min}}^{1} dx \Delta\Sigma(x.Q^2)$$

GRSV max. Δg