Generalized Parton Distributions at Large x

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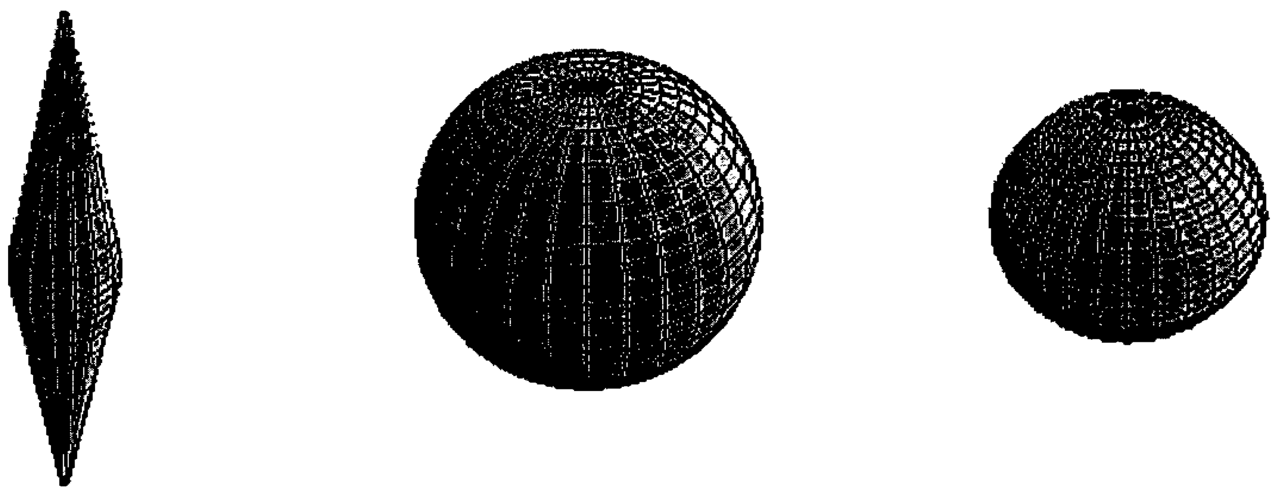
hep-ph/0311288, to appear in PRD (2004)

What GPDs tell us about the Nucleon?



X=0.01 X=0.4 X=0.7

Wigner distributions: quarks' 3-d Imaging

- Ji, PRL91,062001(2003);Belitsky,Ji,FY,hep-ph/0307383
- see also Burkardt, 2000; Diehl, 2002

How to measure GPDs?

- Deeply Virtual Compton scattering (DVCS)
- Deeply virtual meson exclusive production
- Doubly-virtual Compton scattering
- Many others, ….

Theoretical Constraints

General ones,

- Polynomiality condition

  -x^n moment is only polynomial function of ξ

- Positivity constraints

$$\left|H_q(x,\xi,t)-\frac{\xi^2}{1-\xi^2}E_q(x,\xi,t)\right|^2+\left|\frac{\sqrt{t_0-t}}{2M\sqrt{1-\xi^2}}E_q(x,\xi,t)\right|^2\leq\frac{q(x_1)q(x_2)}{1-\xi^2}$$

- Boundary condition from forward PDF and Form Factors

Perturbative QCD provides additional constraints

- Large t

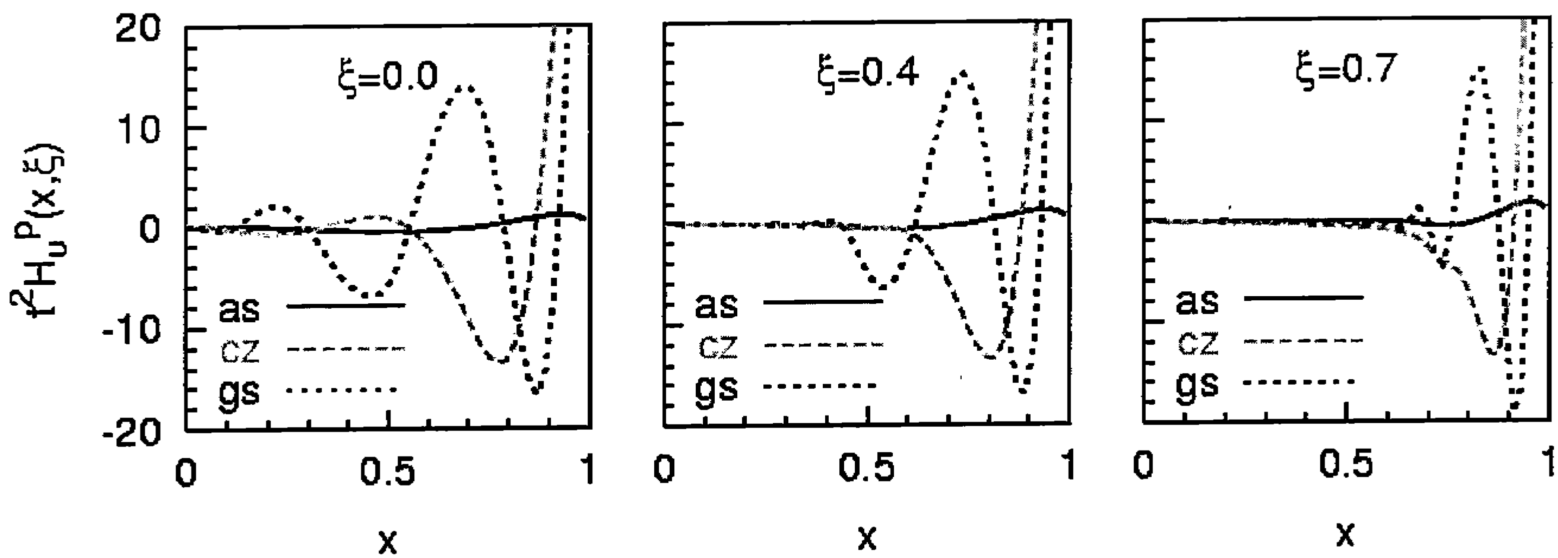
- Large x

GPDs at Large t

Factorization Formula

$$H_q(x,\xi,t,\mu) = \int dx_1 dy_1 \phi^*(y_1,\mu)\phi(x_1,\mu)T_{H_q}(x_1,y_1,x,\xi,t,\mu)$$

Distribution amplitudes



-Hoodbhoy,Ji,FY,PRL92,012003(2004)

Power counting of Large x structure

- Drell-Yan-West (1970)

$$F_1(q^2) \xrightarrow[q^2 \to -\infty]{} (-1/q^2)^n \leftrightarrow \nu W_2(x) \xrightarrow[x \to 1]{} (1-x)^{2n-1}$$

- Farrar-Jackson (1975)

$$\nu W_2^{\pi} \sim (1-x)^2 \quad \text{and} \quad \nu W_2^{p} \sim (1-x)^3$$

- Brodsky-Lepage (1979)

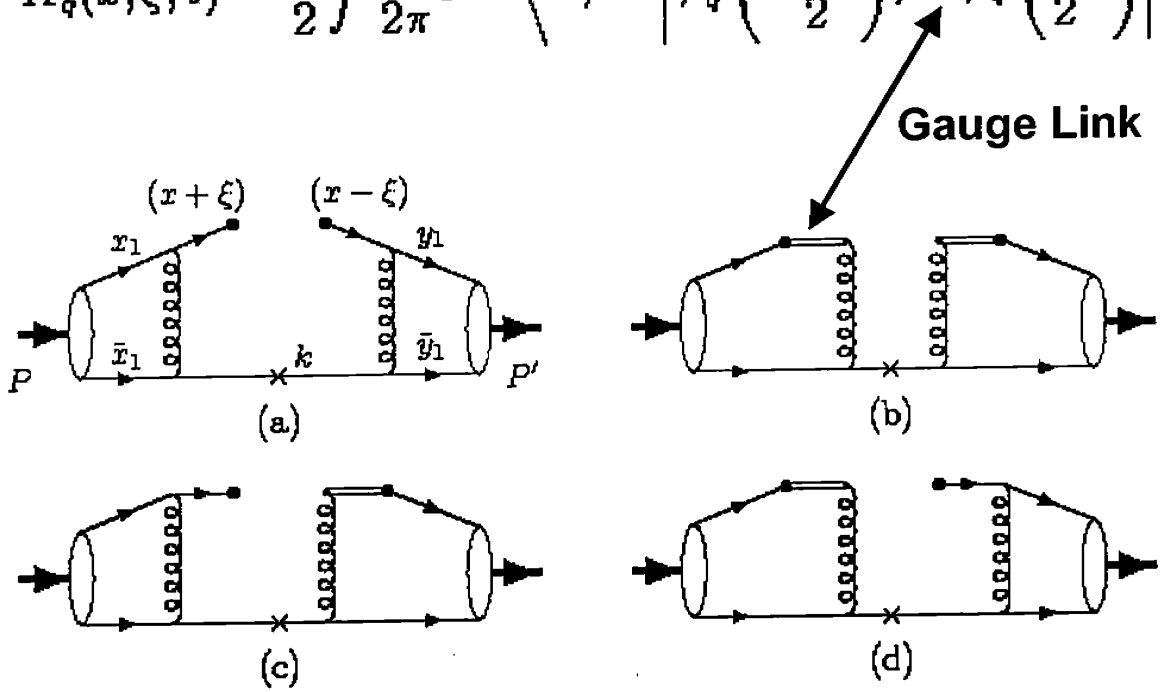
$$G_{q\uparrow/p\uparrow} \sim (1-x)^3 \quad ; \quad G_{q\downarrow/p\uparrow} \sim (1-x)^5$$

- Brodsky-Burkardt-Shmidt (1995)

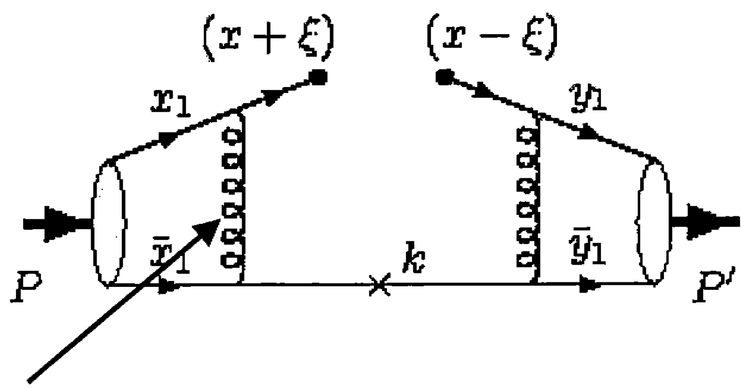
fit the polarized structure functions.

GPD for Pion

$$H_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \pi; P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{L} \psi_q\left(\frac{\lambda}{2}n\right) \right| \pi; P \right\rangle$$



Why Perturbative calculable for large x



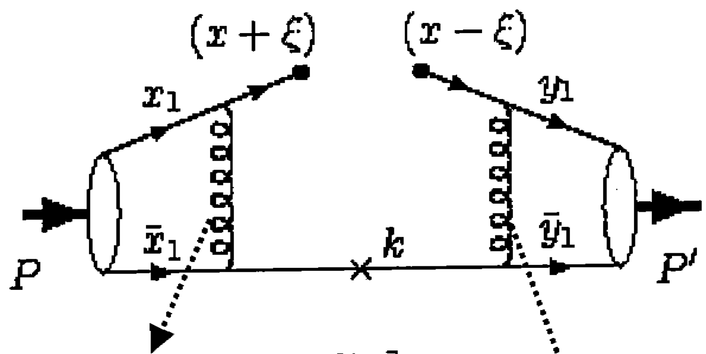
The propagators are far off-shell

$$(k - \overline{x}_1 P)^2 = 2\overline{x}_1 k \cdot P \propto \frac{k_\perp^2}{1-x} \gg \Lambda_{\rm QCD}^2$$

How do we do the power counting

- $(1-x)$ is a small parameter, ξ and t are fixed
- $\xi < x$
- $\vec{k}_\perp^2/(1-x) \gg \vec{k}_\perp^2 \gg \Lambda_{\rm QCD}^2$, and $\vec{k}_\perp^2/(1-x) \gg (-t)$

Where is the *t*-dependence



$$\frac{1}{2k\cdot P} = \frac{1-x}{\vec{k}_\perp^2(1+\xi)}\left[1+\frac{(1-x)^2(1-\xi^2)t}{4(1+\xi)^2\vec{k}_\perp^2}\right]$$

$$\frac{1}{2k\cdot P'} = \frac{1-x}{\vec{k}_\perp^2(1-\xi)}\left[1+\frac{(1-x)^2(1-\xi^2)t}{4(1-\xi)^2\vec{k}_\perp^2}\right]$$

- In the leading order, there is no *t*-dependence
- Any *t*-dependence is suppressed by a factor *(1-x)*^{2} (See also, Burkardt, hep-ph/0401159)

Power counting results for pion GPD

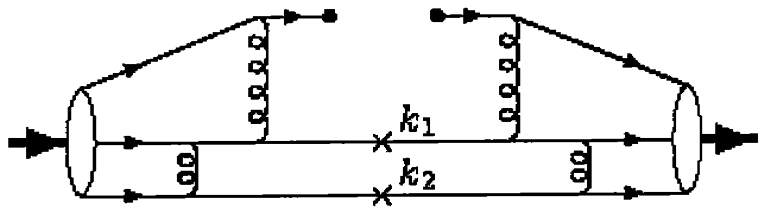
- in the limit of *x->1*,

$$H_q^{\pi}(x,\xi,t) \propto \frac{(1-x)^2}{1-\xi^2}$$

- We can approximate the GPD with forward PDF at large *x*,

$$H_q^{\pi}(x,\xi,t) = \frac{1}{1-\xi^2} q^{\pi}(x)$$

GPDs for nucleon



$$\mathcal{H}_{\lambda'\lambda} = \frac{1}{2\sqrt{1-\xi^2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P', \lambda' \left| \overline{\psi}_q \left(-\frac{\lambda}{2} n\right) \not{n} \mathcal{L} \psi_q \left(\frac{\lambda}{2} n\right) \right| P, \lambda \right\rangle$$

Helicity nonflip $\quad \mathcal{H}_{\uparrow\uparrow} = \mathcal{H}_{\downarrow\downarrow} = H_q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E_q(x,\xi,t) \; ,$

Helicity flip $\quad \mathcal{H}_{\downarrow\uparrow} = -\mathcal{H}^*_{\uparrow\downarrow} = \frac{\Delta^x + i\Delta^y}{2M_p(1-\xi^2)} E_q(x,\xi,t) \; .$

Helicity non-flip amplitude

- The propagator

$$\frac{1}{2P\cdot(k_1+k_2)} = \frac{1-x}{\langle\vec{k}_\perp^2\rangle(1+\xi)}\left[1+\mathcal{O}((1-x)^2)\frac{t}{\langle\vec{k}_\perp^2\rangle}+\cdots\right]$$

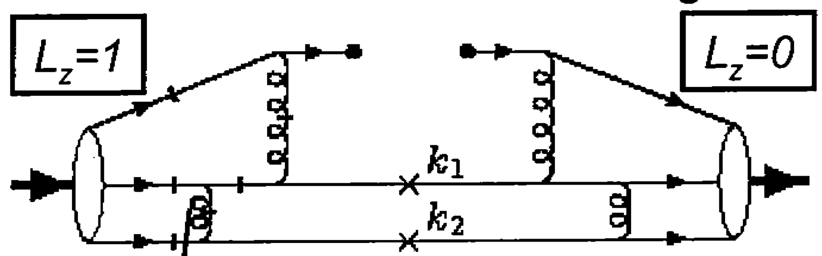
- Power behavior

$$\mathcal{H}_{11} = \frac{1}{(1-\xi^2)^2}\underline{q(x)} \sim \frac{(1-x)^3}{(1-\xi^2)^2}$$

Forward PDF

Helicity flip amplitude

- Since hard scattering conserves quark helicity, to get the helicity flip amplitude, one needs to consider the hadron wave function with one-unit of orbital angular momentum



- In the expansion of the amplitude at small transverse momentum $l_\perp$, additional suppression of $(1-x)^2$ will arise

$$\frac{1}{(k_2 - x_3 P - l_\perp)^2} = \frac{1}{(k_2 - x_3 P)^2}\left[1 - \frac{\beta(1-x)^2 \vec{\Delta}_\perp \cdot \vec{l}_\perp}{(1+\xi)^2 \vec{k}^2_{2\perp}}\right]$$

- Two kinds of expansions

  Propagator: $(1-x)^5(1+\xi^2)/(1-\xi^2)^4$

  Wave function: $(1-x)^5/(1-\xi^2)^4$

- The power behavior for the helicity flip amplitude

$$\mathcal{H}_{+1} \sim (\Delta_\perp^x + i\Delta_\perp^y)\frac{(1-x)^5}{(1-\xi^2)^4}f(\xi)$$

- GPD *E(x, ξ, t)*

$$E_q(x,\xi,t) \sim \frac{(1-x)^5}{(1-\xi^2)^3}f(\xi)$$

- GPD *H(x, ξ, t)*

$$H_q(x,\xi,t) = \frac{1}{(1-\xi^2)^2}q(x)$$

Forward PDF

Summary for the GPDs' power prediction

- No *t*-dependence at leading order
- Power behavior at large x

$$H_q^{\pi}(x,\xi,t) = \frac{1}{1-\xi^2} q^{\pi}(x) \quad \propto (1\text{-}x)^2$$

Forward PDF

$$H_q(x,\xi,t) = \frac{1}{(1-\xi^2)^2} q(x) \quad \propto (1\text{-}x)^3$$

$$E_q(x,\xi,t) = \frac{(1-x)^5}{(1-\xi^2)^3} f(\xi)$$