ERL 2005

Overview of space charge (and CSR) codes

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Outline

- **Space charge "situations"**
- **Solutions out of the box**
  - Differential methods
  - Integral methods
    - Lienard Wiechert Potentials
    - "Quasi static" codes
    - Envelope model: HOMDYN
- Comparison between codes
- Some examples of TREDI simulations
  - Start to end simulation ☺
  - Q.E. inhomogeneity & emittance degradation
- Conclusions

RF- photoinjector

- External forces – RF – Solenoids
- Collective Self Fields
- Boundary conditions (cathode – walls)

$$\alpha \equiv \gamma'/k_{RF} \cong 1..2$$

- High gradient
- Short pulse $\sigma_t << T_{RF}$
- The beam becomes relativistic in the first half cell
- Transition from laminar to emittance dominated motion in few meters

Scaled problem for emittance compensation

Other typical situations

- DC guns
  - Lower gradients & larger emittances
- Merger
  - Mix of high energy + low energy beams
  - Emittance compensation for the low energy beam
- Bending magnets
  - SPC
  - CSR

Non-axisymmetric

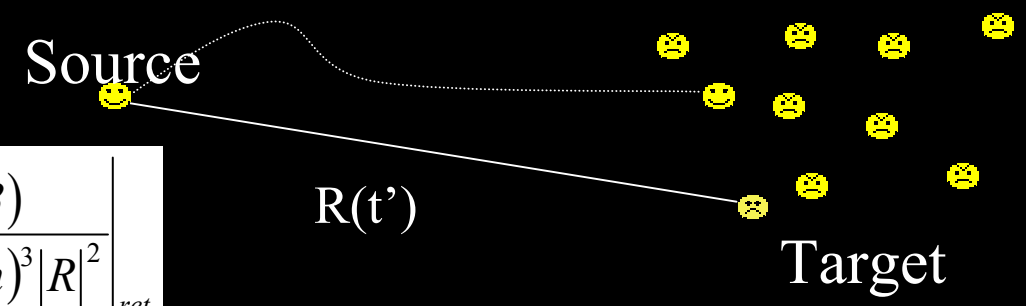
CSR & SPC

(A) radiative term

$$E = \frac{n \times \left[(n-\beta)\times\dot{\beta}\right]}{(1-\beta\cdot n)^3 |R|} + \left.\frac{(n-\beta)}{\gamma^2 (1-\beta\cdot n)^3 |R|^2}\right|_{ret}$$

$$B = n \times E\big|_{ret}$$

(B) space charge term

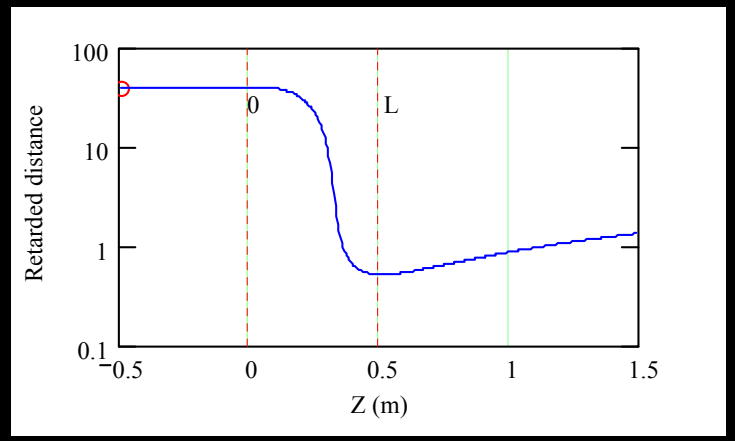


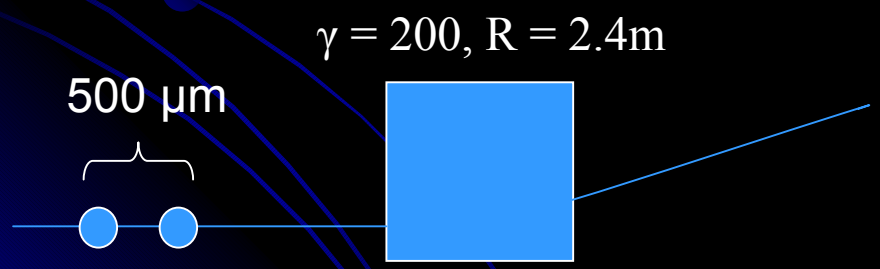
$$n = \frac{R}{|R|}$$

Retarded time $t' = t - \frac{R(t')}{c}$

Term (B) in a bend :

1) Geometric - kinematic factor +
2) Curved trajectory → small retarded delay
→ small retarded distance





Genuine space charge effects may appear at high peak current & moderate energy

Note: The "line charge method" includes only part of the space charge effects in (B), e.g. in a drift SPC are neglected

First principles

Solution Methods

Examples

- Differential Methods — *fields are independent variables of the problem*
  - Maxwells Lorentz Equations — *2 dimensions* — ITACA, Spiffe, …
- Integral Methods — *fields are derived at each time step from the particles coordinates and velocities*
  - Lienard Wiechert retarded potentials — *Relax on boundary conditions which are treated only for flat, perfectly conducting walls (e.g. cathode).* — TREDI, Trafic^4 & CSRTrack
  - Quasi Static approximation. Poisson eq. in the moving frame — *+ no accel. fields; + no-retarded times; + relax on velocity spread $\Delta\beta/\beta \ll 1$* — PARMELA, GPT, BEAMPATH, ASTRA*, …
  - Semi-Analytic — *+ paraxial approximation $\sigma' \ll 1$; + small slice energy spread $\sigma\gamma \ll 1$; + uniform bunch distribution* — HOMDYN

*ASTRA accounts for a particles velocity spread in the moving frame

PDE METHODS

Given charges and currents distributions

solve Maxwell's Equations with a PDE Solver (e.g. LeapFrog)

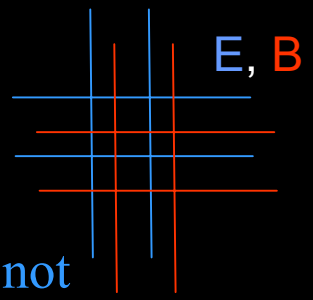
Calculate new charge & current distributions with the known fields

- The fields are evaluated on a mesh with spacing (δ_x,δ_y,δ_z)
  - The highest k vector supported by the mesh is given by $k_{x,y,z} = \frac{1}{2\delta_{x,y,z}}$
  - The highest frequency is given by $_{\max} = c\max(k_x, k_y, k_z) = \frac{c}{2\min(\delta_x, \delta_y, \delta_z)}$
  - Integration Time step $\delta t < \frac{1}{\omega_{\max}}$

TIME / SPACE decoherence

Magnetic and Electric Fields are not known in the same place at the same time.

Interpolation is required.



At large γ the Lorentz force equation is sensitive to relative errors between E and B components. Cancellation $\propto 1/\gamma^2$ is not correctly reproduced. The requirements on the mesh becomes more strict at large γ

Alternative: Maxwell eq. in 2D - Itaca (L.Serafini), Spiffe (M. Borland)

A *closed sub-set* of Maxwell equations, where the field propagation (driven by the source [ρ,J]) can be fully described in terms of a *scalar pseudo-potential* $\Phi \equiv r \cdot H_{\varphi}(r,z,t)$

Wave equation for Φ

$$\left[\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right]\Phi-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=r\frac{\partial J_z}{\partial r}-r\frac{\partial J_r}{\partial z}$$

calculate E_r, E_z, H_φ from Φ

$$\frac{dE_r}{cdt}=-\frac{1}{r}\frac{\partial\Phi}{\partial z}-J_r,\quad \frac{dE_z}{cdt}=-\frac{1}{r}\frac{\partial\Phi}{\partial r}-J_z,\quad \text{in}\,(\mathrm{r},\mathrm{z})=(\mathrm{r_m},\mathrm{z_n})$$

Scalar wave equation + ODE for electric field

From Differential methods to integral methods

Transition from laminar to emittance dominated motion

$$L_{inj} \cong \frac{\mu}{4\pi}\sqrt{\frac{2}{3}}\frac{I}{I_0}\frac{\lambda_{RF}^2}{\varepsilon_{th}}$$

typical μ=1, I=100 A, λRF$\approx$ 10 cm, εth $\approx$ 0.3 mm-mrad - the simulation length is $\approx$ 4 m (more with vel. bunching) + "High Resolution" required to recover εth after acceleration

- "Emittance compensation" extends the spatial range where space charge effects must be taken into consideration – increased mesh size
- Difficulty in separating the evolution of the accelerating field and of the particles dynamics
- Extension to 3D domain not straightforward for the "decoherence" problem and for the increased mesh size

→ Integral Methods

Lienard Wiechert Retarded Potentials

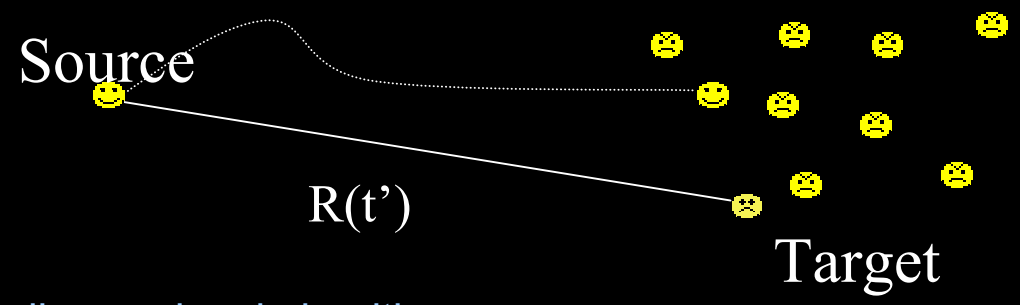
Traffic4, CSRTrack, R.Li code, TREDI

$$E = \frac{n \times \left[(n-\beta) \times \dot{\beta} \right]}{(1-\beta \cdot n)^3 |R|} + \frac{(n-\beta)\left[1-|\beta|^2\right]}{(1-\beta \cdot n)^3 |R|^2} \Bigg|_{ret}$$

$$B = n \times E \Big|$$

$$n = \frac{R}{|R|}$$

Retarded time $t' = t - \frac{R(t')}{c}$



- Three dimensoional algorithm
- Includes radiation effects and space charge effects in bends
- Trajectories are stored and the fields evaluation requires bracketing of the retarded condition.
- The implementation of “Extended particles” requires careful treatment of retarded condition
- The problem scales with n_e^2 or n_exM with and M vertices mesh

Evaluation of fields in "Quasi Static" approx. (Parmela, GPT, Astra, Beampath …)

- A space-charge mesh centered on a reference particle moves at the γ of the particle.
- Particle coordinates and momenta are transformed to the frame of this mesh and are assumed at rest*. We have indeed ($\alpha = (\lambda_{rf}/2\pi)(d\gamma/dz) \approx 1$)

$$\frac{\Delta\gamma}{\gamma} \approx \frac{\alpha\Delta\phi}{1 + \pi\alpha z/\lambda_{rf}} < 1,$$

$$\Delta\phi = \frac{2\pi\sigma_{bunch}}{\lambda_{RF}} << 1$$

- The charge is assigned to mesh cells.
- The electrostatic field at the particles coordinates are given by the sum of Green functions of a charged ring (2D scheff) for each mesh node
- Momentum kicks are applied to particles
- The coordinates and momenta of the new particle coordinates are transformed back to the lab frame.

*in ASTRA the particle velocities are used as sources of magnetic fieds in the moving frame

Quasi static approximation

- Cartesian geometry is required in bends
- In curved trajectories $$\Delta \beta_{\perp} \approx \frac{l_b}{R} \langle \beta \rangle$$
  - Magnetic field components in the moving frame appears
  - If $\Delta\beta_{\perp}$ is relativistic, the time of signal propagation within the bunch is not negligible
- Not adequate to simulate the interaction of a high energy and a low energy beams in a "merger"

HOMDYN *(M. Ferrario)*

multi-envelope model: *time dependent space charge* of a uniform charged bunch

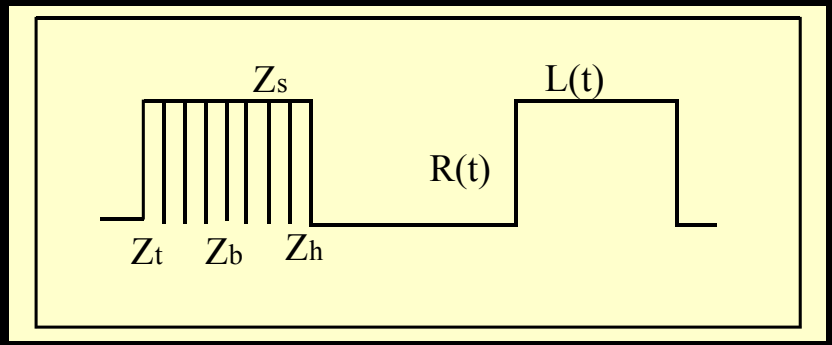
Originally developed for BBU studies in SC linacs
Could easily include the Line charge method for CSR

- Paraxial approximation $\sigma' \ll 1$
- Small energy spread $\sigma_\gamma \ll 1$
- Uniform bunch distribution, no wave breaking

$$\frac{d^2\sigma(z,\zeta_i)}{dz^2}+\frac{p'}{p}\sigma'(z,\zeta_i)+K\sigma(z,\zeta_i)=\frac{I(z,\zeta_i)g(z,\zeta_i)}{2I_0p^3\sigma(z,\zeta_i)}+\frac{\varepsilon_{n,th}^2}{p^2\sigma^3(z,\zeta_i)} \quad i=1,N$$

$$\zeta_i \equiv z_i-\beta_i ct_i \quad g(\zeta_i)=\frac{1-\zeta_i/L}{\sqrt{(1-\zeta_i/L)^2+A_{r,s}^2}}+\frac{\zeta_i/L}{\sqrt{(\zeta_i/L)^2+A_{r,s}^2}} \quad A_{r,s}\equiv\frac{R}{\gamma L}$$

$$\Delta\varepsilon_n^{cor}=\frac{1}{N}\sqrt{\sum_{i=1}^{N}\sigma(z,\zeta_i)^2\sum_{i=1}^{N}\sigma'(z,\zeta_i)^2-\left[\sum_{i=1}^{N}\sigma(z,\zeta_i)\sigma'(z,\zeta_i)\right]^2}$$



Extremely fast – allows a fast relaxation of the parameters

Code comparison

Proceedings of the 2003 Particle Accelerator Conference

CODE COMPARISON FOR SIMULATIONS OF PHOTO-INJECTORS

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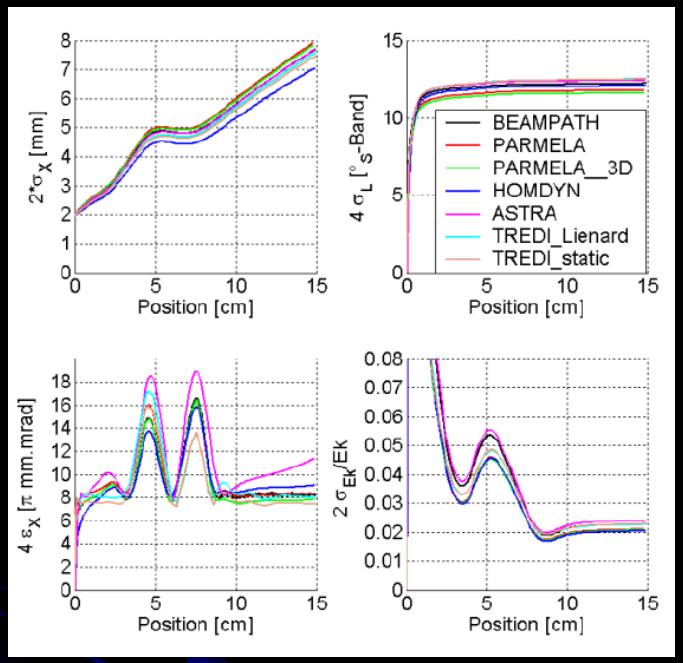
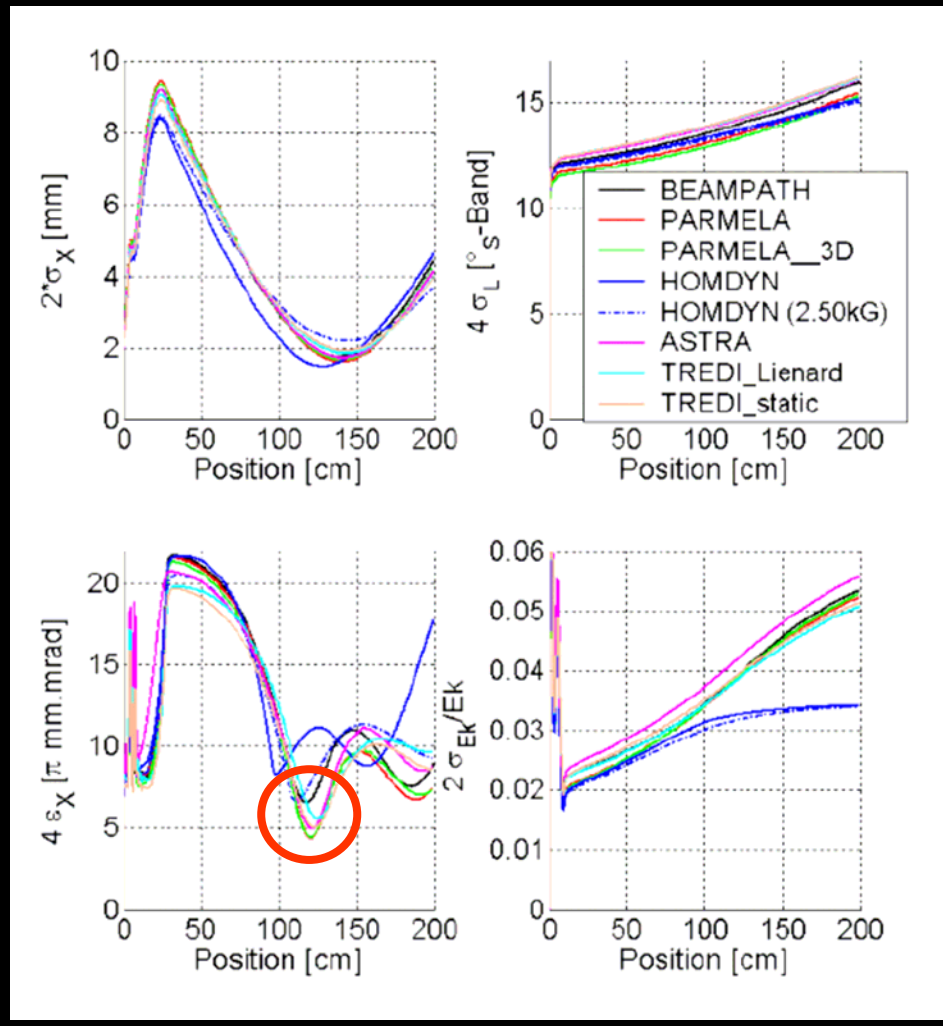
Test case

- S-Band Gun
- Solenoid 2.541 kG
- Drift
- 1 nC – 1mm, 10 ps square pulse
- Same field maps from Superfish or Poisson for all the codes

Code comparison



RF-Gun + Solenoid



RF-Gun + Solenoid + Drift

- The **image charge model** is appropriate to represent the boundary conditions at the cathode
- The **quasi-static approximation** give good results for the BNL-like gun

CPU times

Code	Platform	CPU	Num. particles	Mesh points N_r x N_z	Mesh size h_r x h_z	Integration step	CPU time (s)
HOMDYN	PC Win		75 slices			0.13 °	45
BEAMPATH	PC Win	1 GHz	10^4	256 x 2048	50x50μm^2	0.1°, 1°	8000
PARMELA	"	1 GHz	2.5 10^4	25 x 75	"	"	9846
ASTRA	"	1.8 Ghz	1.5 10^4	20 x 60	Automatic	Adaptative	420
Tredi Stat.	16 nodes	1.8 GHz	5.0 10^4	20 x 30	Automatic	Adaptative	*7.5 10^3

*The CPU time for Tredi STATIC is integrated over the 16 nodes. The "waiting" time is 1/16.

TREDI quasi static - assumptions – 2D +

- The velocity of the source particle doesn't change on a time scale comparable to the retarded time;
- The contribution of acceleration fields is negligible.

TREDI is now a factor ~ 3 faster – Optimized compiler (factor 1.6) + code (factor 2)

Cathode Q.E. inhomogeneity studies

(Parmela spch3D / TREDI)

Fel 2004

SPECTRAL ANALYSIS OF CHARGE EMISSION SPATIAL INHOMOGENEITIES AND EMITTANCE DILUTION IN RF GUNS

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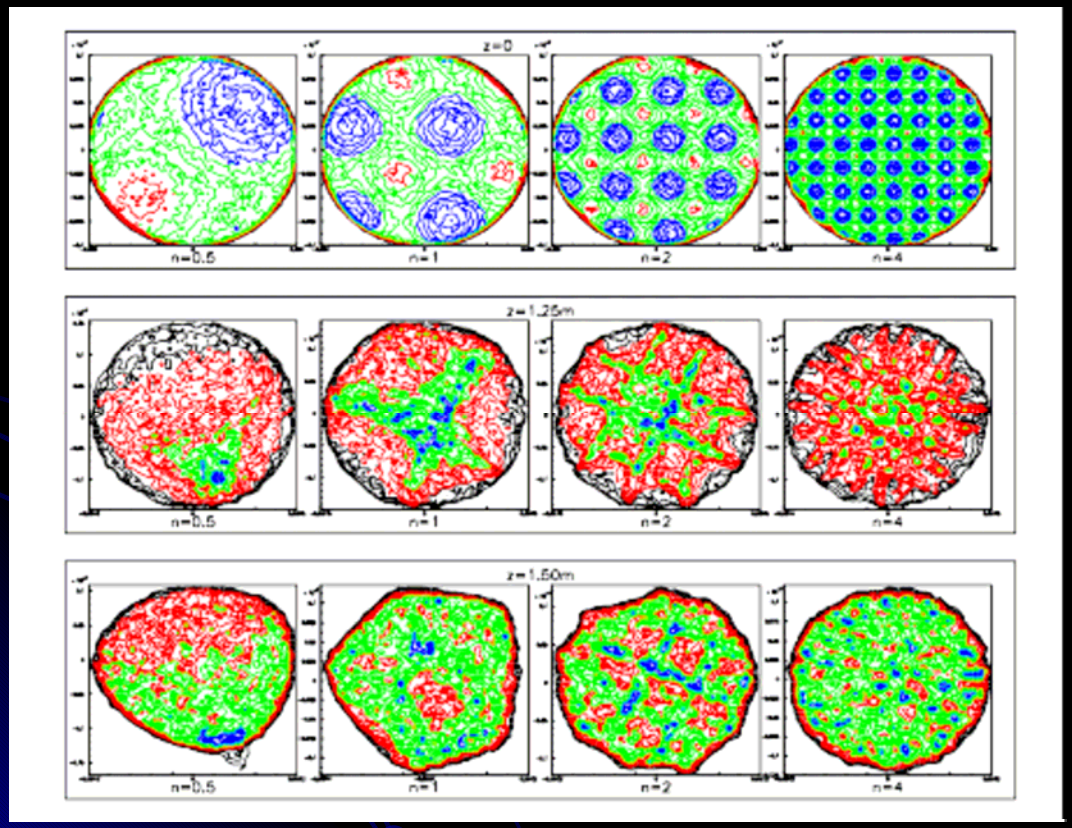
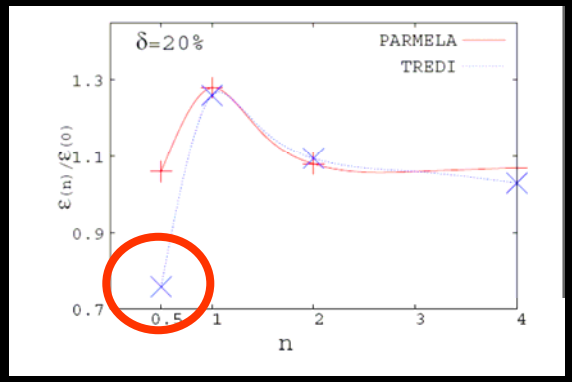
$$\epsilon\left(k_n, \delta\right) = \epsilon_0 + \sum_n a_{n,j}\delta^j$$

Spatial Frequency →

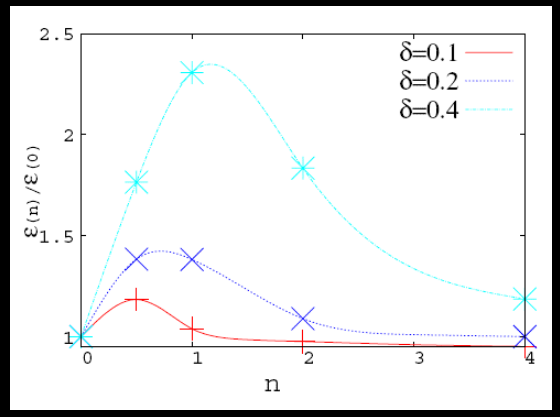
← Longitudinal Coordinate



Even harmonics



Odd harmonics (Tredi)



Compression at low energy


VOLUME 91, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 15 AUGUST 2003

Horizontal Phase-Space Distortions Arising from Magnetic Pulse Compression of an Intense, Relativistic Electron Beam

S. G. Anderson, J. B. Rosenzweig, P. Musumeci, and M. C. Thompson

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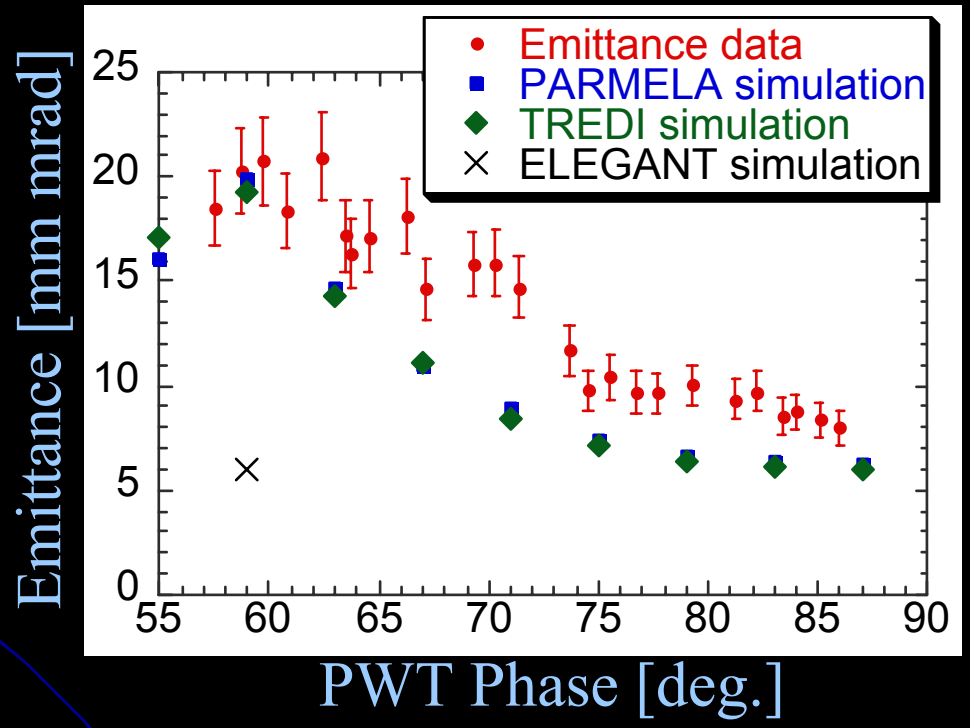
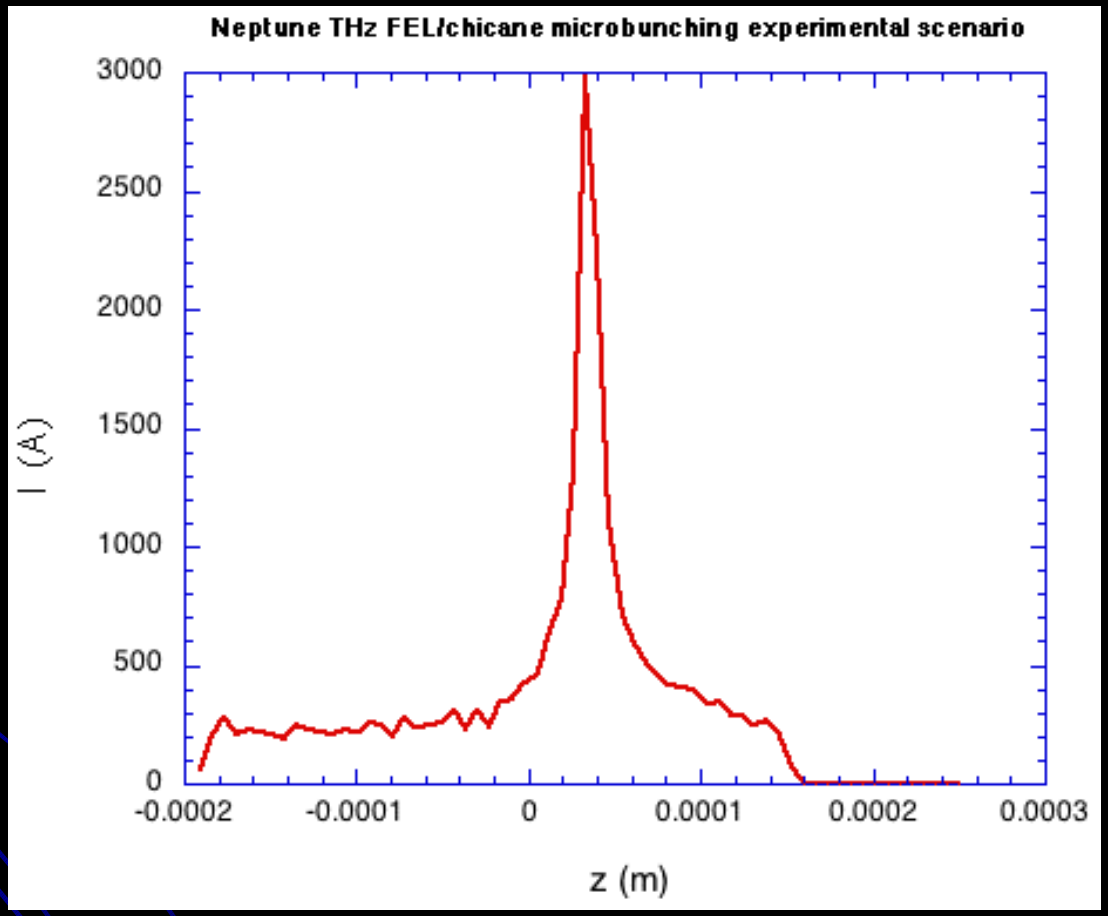
(Received 21 June 2002; published 14 August 2003)



- Simulation is difficult. Number of macro-particles is low because of time-intensive space-charge calculations.
- Sharp emittance increase when “fold over” begins is missing in simulations.

Scott Anderson Thesis defense

Line charge method (Courtesy of J. Rosenzweig)



9 MeV !!!

TREDI is a multi-purpose macroparticle 3D Monte Carlo

- Three dimensional effects in photo-injectors
  - Inhomogeneities of cathode quantum efficiency / Laser misalignments
  - Multipolar terms in accelerating fields
- "3-D" injector for high aspect ratio beam production
- Coherent radiation emission and space charge effects in bendings

FEATURES:

LW based 3D algorithm

SPC & CSR fields naturally included

- 15000 lines in C language
- Scalar & Parallel (MPI 2.0)
- Unix & Windows versions
- Tcl/Tk Gui (pre-processing)
- Mathematica & MathCad frontends
- Output format in NCSA HDF5 format

Available devices list:

✓Rf-guns

✓Linacs (TW & SW)

✓Solenoids

✓Bendings

✓Undulators

✓Quadrupoles

✓Field Maps

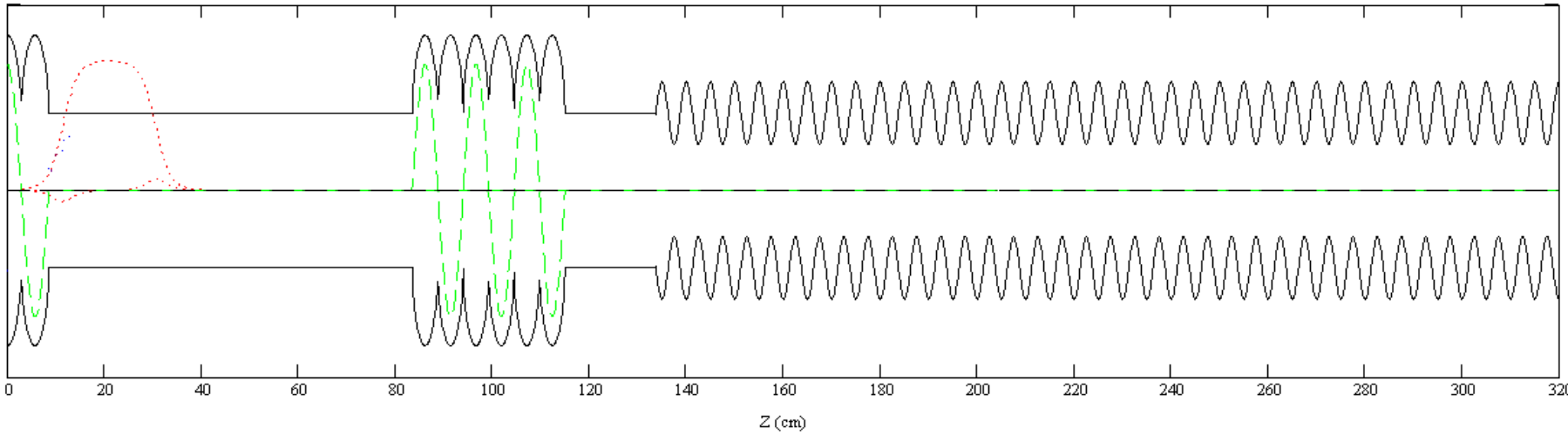
✓ Custom devices

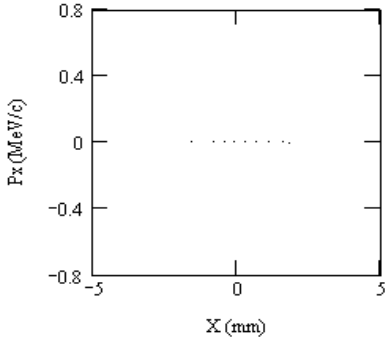
ELSEVIER Nuclear Instruments and Methods in Physics Research A 436 (1999) 443–444 RESEARCH Section A
www.elsevier.nl/locate/nima

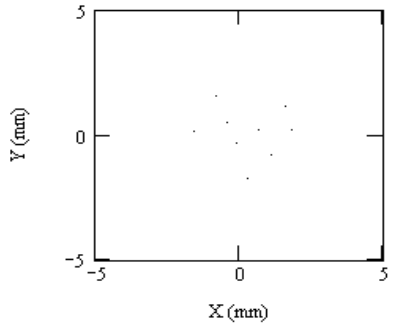
TREDI: fully 3D beam dynamics simulation of RF guns, bendings and FELs

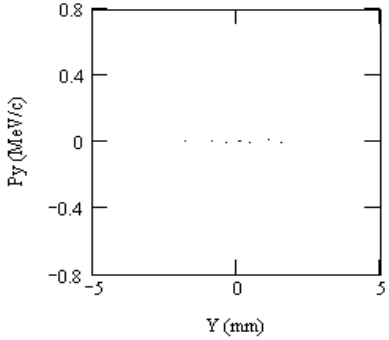
L. Giannessi*, P. Musumeci, M. Quattromini

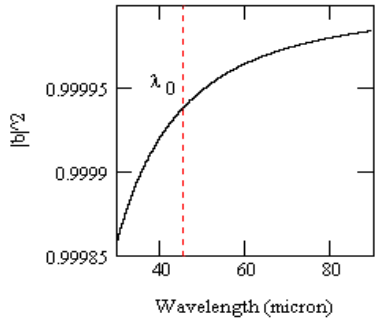
Virginia FEL Conference - TREDI exercise: GUN + solenoid + linac + FEL

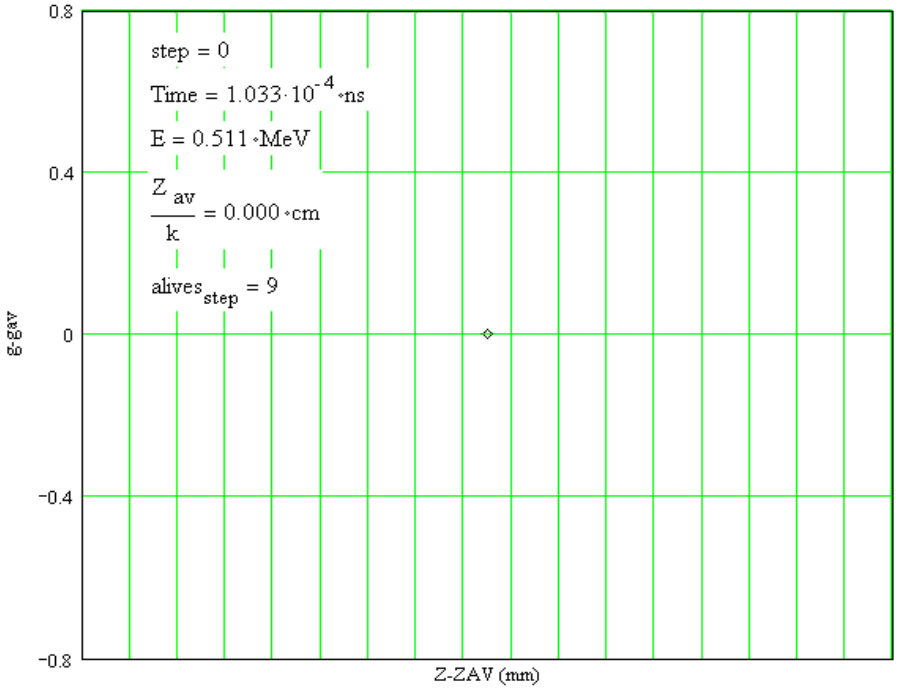












Conclusions

- ERL features asks “specialization” in models for space charge simulations
  - RF-Gun injector – Poisson eq. in the moving frame
  - CSR & Space charge effects in bends – Lienard Wiechert retarded pot.
  - CSR & Microbunching instability – Line charge method

- New ideas for efficiently combining together the various effects that may appear simultaneously are desirable.

- Some of the models are CPU consuming
  - Great improvement from Moore’s Law & parallel computing (ex. TREDI: 1996/300 particles, 2005/50000 particles)
  - EUROFEL – dedicated cluster with 50-60 nodes

- More codes benchmarking in other situations than RF-Photoinjectors is desirable

… other references related to TREDI

PHYSICAL REVIEW E VOLUME 58, NUMBER 3 SEPTEMBER 1998

Diamagnetic fields due to finite-dimension intense beams in high-gain free-electron lasers

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TREDI simulations for high-brilliance photoinjectors and magnetic chicanes

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EMITTANCE DILUITION DUE TO 3D PERTURBATIONS IN RF PHOTOINJECTORS

M. Quattromini, L. Giannessi, C. Ronsivalle, ENEA, C.R. Frascati (Roma), Italy

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S. Anderson, M. Borland, M. Ferrario, C. Limborg,
A. Kabel, P. Musumeci, C. Ronsivalle, J. Rosenzweig, L. Serafini