Calculations of Exclusive $A(e, e'p)B$ Processes off Few-Nucleon Systems by a Generalized Eikonal Approximation

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1. Introduction and Motivations

$A(e, e'p)B$ reaction can serve as a powerful tool investigate the nuclear ground-state wave function provided that the continuum final state could be correctly described (FSI, MEC, etc).

$$T_{fi} = \langle \Psi^f_{A,\mathbf{p}}(1, 2, ..., A) | \hat{J} | \Psi^0_A(1, 2, ..., A) \rangle \tag{1}$$

Low Energies Ψ^0_A and $\Psi^f_{A,\mathbf{p}} \Longrightarrow$ Solutions of $H\Psi = E\Psi$ with realistic interactions(Bochum, Pisa)

High Energies $\Psi^f_{A,\mathbf{p}}(1, 2, ..., A) = \hat{\mathcal{A}} \left[e^{i\mathbf{p}\cdot\mathbf{r}} \otimes \mathcal{S}\Psi^f_{A-1} \right]$ $\mathcal{S} \Rightarrow$ FSI Operator

Present contribution: realistic few-body wave functions and Generalized Eikonal Approximation (GEA) for the final state

Collaboration with **L. Kaptari**

2. The Final State Interaction in $A(e, e'p)B$ processes within the Generalized Eikonal Approximation (GEA).

- GLAUBER (Eikonal) approximation

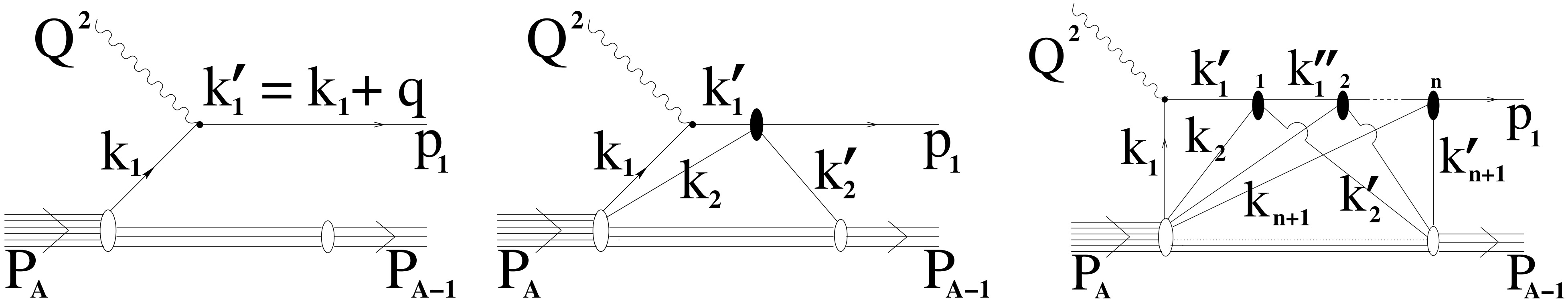
$$S \to S_G(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) = \prod_{j=2}^{A} \left[1 - \theta(z_j - z_1)\, \Gamma(\boldsymbol{b}_1 - \boldsymbol{b}_j)\right]$$

$$\Gamma(\boldsymbol{b}_1 - \boldsymbol{b}_j) = \frac{\sigma_{pN}}{4\pi B^2} \left(1 - i\, \alpha_{pN}\right) e^{-(\boldsymbol{b}_1 - \boldsymbol{b}_j)^2/2\,B}$$

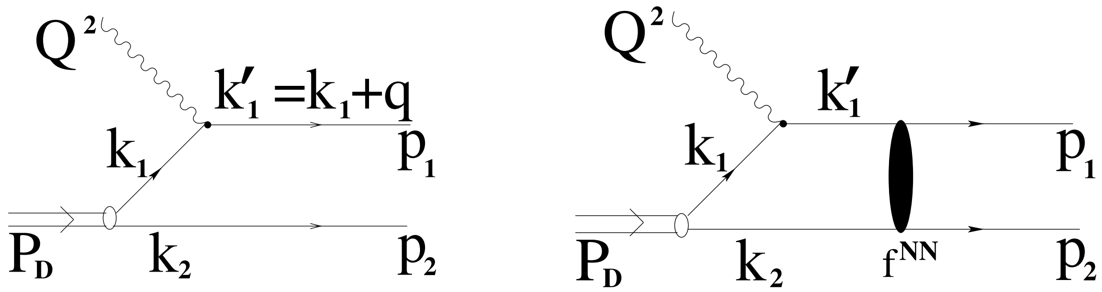
- Frozen approximation
- Glauber and Graphs (V. N. Gribov, Sov. Phys. JETP **30**(1970); L. Bertocchi, Nuovo Cimento **11A**(1972)); $p - A$ scattering

- L. L. Frankfurt, M. M. Sargsian, M. I. Strikman, Phys. Rev. **C56**(1997) Graphs and $(e, e'p)$ $\to$ frozen approximation removed$\to$ GENERALIZED EIKONAL APPROXIMATION (GEA)

GEA for $A(e,e'p)X$



a) b) c)



a) b)

The Feynman diagrams for the process $A(e,e'p)B$. f^{NN} denotes the elastic NN scattering amplitude

$\boxed{\text{The Cross Section}}$

$$\frac{d^6\sigma}{dE'd\Omega' d\mathbf{p}_m} = \mathcal{K}(Q^2, x, \mathbf{p}_m)\, \sigma^{eN}(\bar{Q}^2, \mathbf{p}_m) P_A{}^D(\mathbf{p}_m, E_m)$$

Distorted Spectral Function

$$P_A^D(\mathbf{p}_m, E_m) = \frac{1}{(2\pi)^3}\frac{1}{2J_A+1} \sum_f \sum_{\mathcal{M}_A, \mathcal{M}_{A-1},\, s_1} \times$$

$$\times \left| \sum_{n=0}^{A-1} \mathcal{T}_A^{(n)}(\mathcal{M}_A, \mathcal{M}_{A-1}, s_1) \right|^2 \times \delta\left(E_m - (E_{A-1}^f + E_{min})\right)$$

$\mathcal{T}_A^{(n)}$ - from Feynman diagram; n - order of rescattering.

$\boxed{\mathbf{A=2}}$

$$\mathcal{T}_D^{(0)}(\mathcal{M}_2, s_1, s_2) = \langle s_1, s_2 | \Psi_D^{\mathcal{M}_2}(\mathbf{k}_2)\rangle$$

and

$$\mathcal{T}_D^{(1)}(\mathcal{M}_2, s_1, s_2) = \int \frac{d^3k_2}{2E_{\mathbf{k}_2}(2\pi)^3} \frac{f^{NN}(\kappa_\perp)}{k_1'^2 - M_N^2 + i\epsilon} \langle s_1, s_2 | \Psi_D^{\mathcal{M}_2}(\mathbf{p}_2)\rangle$$

$\kappa = p_1 - k_1^{'} = k_2 - p_2$, momentum transfer in NN scattering

$$k_1^{\prime 2} - M_N^2 = (k_1 + q)^2 - M_N^2 \approx 2|\mathbf{q}|\,(\kappa_z + \Delta_z)\,; \qquad \Delta_z = \frac{q_0}{|\mathbf{q}|} E_m$$

Using

$$\frac{1}{\kappa_z + \Delta_z + i\epsilon} = -i \int \theta(z)\; \mathrm{e}^{i(\kappa_z + \Delta_z)\cdot z} dz$$

the cross section will be

$$\frac{d^6\sigma}{dE' d\Omega' d\mathbf{p}_m} = \mathcal{K}(Q^2, x, \mathbf{p}_m)\, \sigma^{eN}(\bar{Q}^2, \mathbf{p}_m)\, n_D^{FSI}(\mathbf{p}_m) \times$$
$$\times\, \delta(M_D + \nu - E_{\mathbf{p}_1} - E_{\mathbf{p}_m})$$

$\boxed{\text{The } \textit{distorted momentum distributions } n_D^{FSI}}$

$$n_D^{FSI}(\mathbf{p}_m) =$$
$$= \frac{1}{3}\frac{1}{(2\pi)^3} \sum_{\mathcal{M}_D, S_{23}} \left| \int d\mathbf{r} \chi_{S_{23}}^{\dagger} \Psi_D^{\mathcal{M}_2 \dagger}(\mathbf{r}) \mathcal{S}_{\Delta}^{FSI}(\mathbf{r})\, \exp(-i\mathbf{p}_m \mathbf{r}) \right|^2,$$

$S_{\Delta}^{FSI}(\mathbf{r})$describes the final state interaction between the hit nucleon and the spectator

$$\mathcal{S}_{\Delta}^{FSI}(\mathbf{r}) = 1 - \theta(z)\mathrm{e}^{i\Delta_z z}\Gamma(\mathbf{b})$$

with $\mathbf{r} = (\mathbf{b}, z)$.

$\boxed{\mathbf{A{=}3}}$

$\mathcal{S}_{\Delta}^{FSI}$describes the single and double rescattering of nucleon "1" with the spectators "2" and "3"

$$\mathcal{S}_{\Delta}^{FSI}(\rho, \mathbf{r}) = \mathcal{S}_{(1)}^{FSI}(\rho, \mathbf{r}) + \mathcal{S}_{(2)}^{FSI}(\rho, \mathbf{r})$$

with the single scattering contribution $\mathcal{S}_{(1)}^{FSI}$given by

$$\mathcal{S}_{(1)}^{FSI}(\rho, \mathbf{r}) = 1 - \sum_{i=2}^{3} \theta(z_i - z_1)\mathrm{e}^{i\Delta_z(z_i - z_1)}\Gamma(\mathbf{b}_1 - \mathbf{b}_i)$$

and the double scattering contribution by

$$
\begin{aligned}
&\mathcal{S}_{(2)}^{FSI}(\rho, \mathbf{r}) = \\
&= \Big[\theta(z_2 - z_1)\theta(z_3 - z_2)\mathrm{e}^{-i\Delta_3(z_2-z_1)}\mathrm{e}^{-i(\Delta_3-\Delta_z)(z_3-z_1)} + \\
&+ \theta(z_3 - z_1)\theta(z_2 - z_3)\mathrm{e}^{-i\Delta_2(z_3-z_1)}\mathrm{e}^{-i(\Delta_2-\Delta_z)(z_2-z_1)}\Big] \times \\
&\times \Gamma(\mathbf{b}_1 - \mathbf{b}_2)\Gamma(\mathbf{b}_1 - \mathbf{b}_3)
\end{aligned}
$$

where $\Delta_i = (q_0/|\mathbf{q}|)(E_{\mathbf{k}_i} - E_{\mathbf{k}_i'})$ and $\Delta_z = q_0/|\boldsymbol{q}|\, E_m$.

$$\Delta_z = 0 \;\; \rightarrow \text{Glauber } \mathcal{S}_G$$

$$\mathcal{S}_{\Delta}^{FSI} \rightarrow \mathcal{S}_G(\rho, \mathbf{r}) = \prod_{i=2}^{3} \left[1 - \theta(z_i - z_1)\, \Gamma(\mathbf{b}_i - \mathbf{b}_1)\right] \tag{2}$$

3. The exclusive quasi-elastic process $A(e,e'p)B$ off few nucleon systems

THEORETICAL INGREDIENTS

(CdA, L. P. Kaptari, *nucl-th 0407024*)

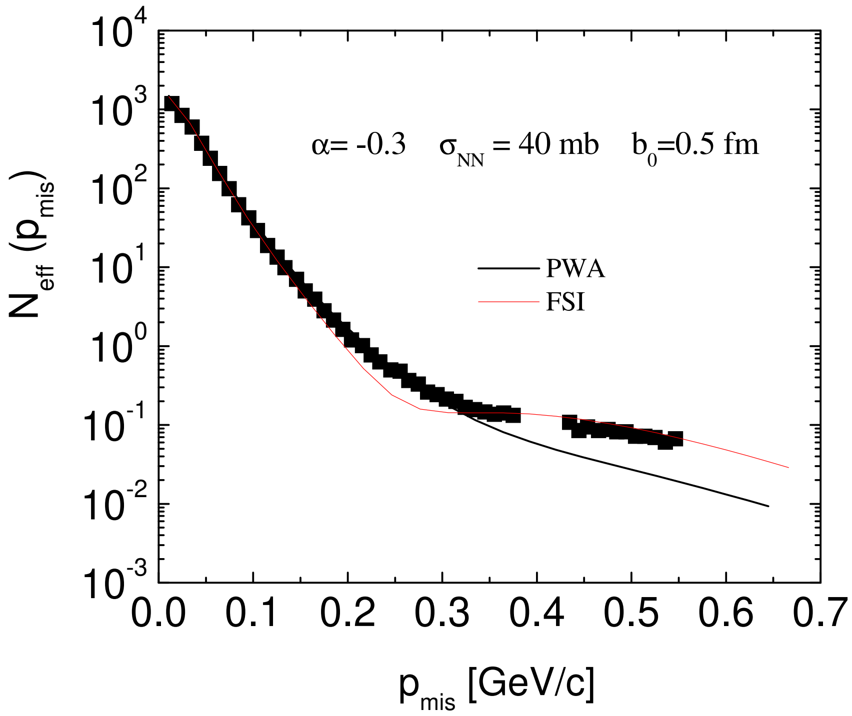
1. Realistic few-body wave functions (AV18 interactions)
2. Generalized Eikonal Approximation
3. Elastic NN scattering amplitude

$$\Gamma_{el}(\mathbf{b}) = \frac{\sigma_{tot}(1 - i\alpha)}{4\pi b_0^2} \exp(-b^2/2b_0^2) \tag{3}$$

σ_{tot}, α and b_0 from experimental world data.

No free parameters through the periodic table

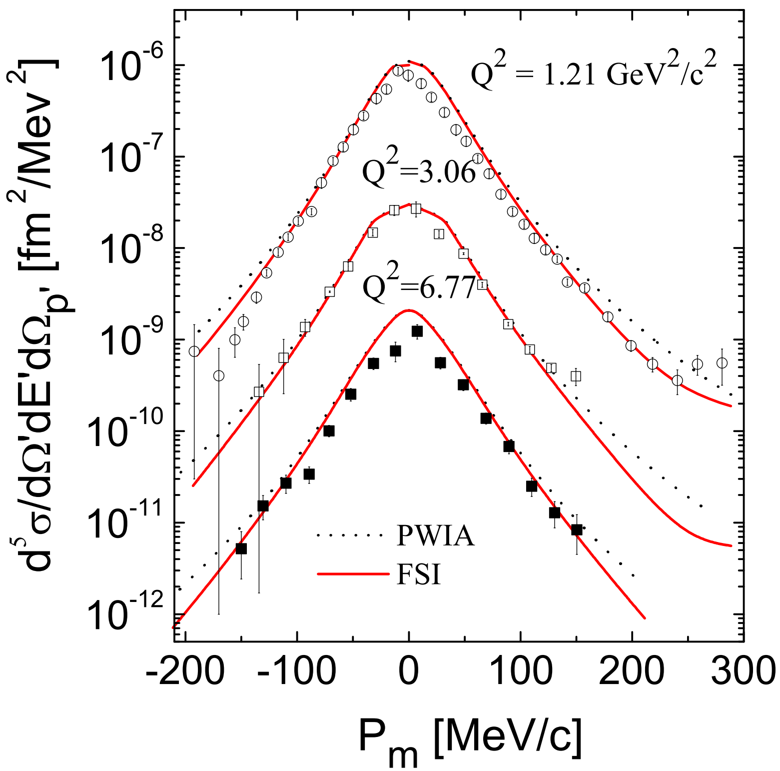
3.1 The $^2H(e,e'p)n$ Process



Jlab data (Ulmer *et al.* Phys. Rev. Lett. **89** (2002)) at *perpendicular* kinematics, with:

$$Q^2 \simeq 0.665 \ (GeV/c)^2, \quad |\boldsymbol{q}| \simeq 0.7 \ GeV/c, \quad x \simeq 0.96$$

$$N_{eff} = \frac{d^4\sigma^{exp}}{d\Omega' \, dE' \, d\Omega_{\boldsymbol{p}_m}} \left[f_{rec} \, \mathcal{K} \, \sigma_{cc1}^{eN} \right]^{-1}$$



SLAC data (Bulten *et al.* , Phys. Rev. Lett, **74** (1995)), at:

$$x \simeq 1$$

positive and negative values of p_m correspond to $\phi = \pi$ and $\phi = 0$, respectively.

3.2 The process $^3He(e,e'p)D(np)$

$$e +^3 He = \ e' \ + p' \ + (12) \qquad\qquad (12) \to D \quad or \quad (np);$$

The cross section is

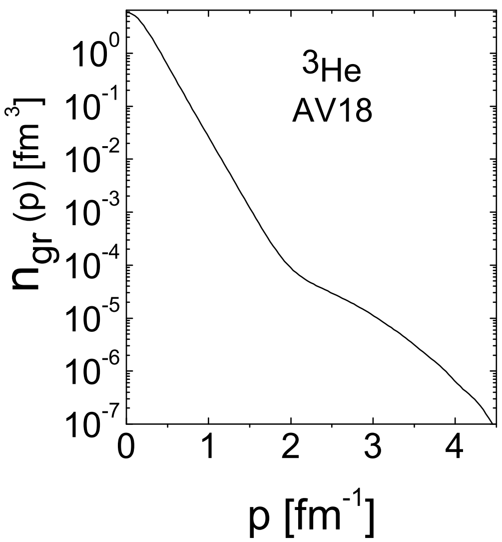
$$\frac{d^n\sigma}{dE'\,d\Omega'\,d\tau_f} = \mathcal{K}(\nu, Q^2, p_m)\,\sigma_{cc1}^{eN}(Q^2, \mathbf{p}_m)\,P_3^{FSI}(\mathbf{p}_m, E_m),$$

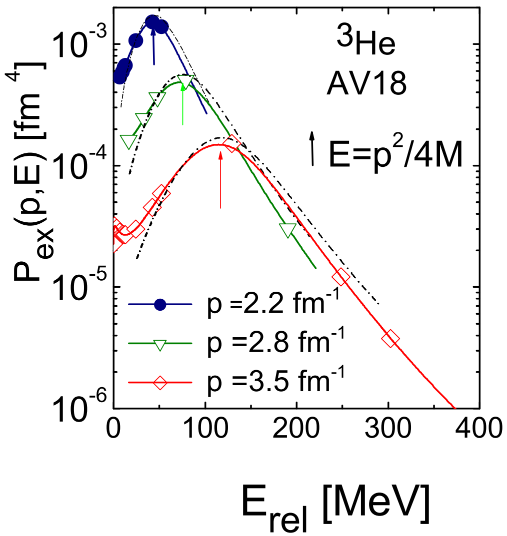
$d\tau_f{=}d\Omega_{p'}$ ($d\tau_f = d\Omega_{p'} dE_m$) for the two- (three-) break up.

Strategy of calculations

1) realistic ground state wf by the Pisa Group ($AV18$);
2) PWA - three plane waves in the continuum;
3) PWIA - (*Plane Wave Impulse Approximation*) plane wave for the hit proton, spectator pair interacting by the AV18 interaction;
4) FSI - All particles interact in the final state;

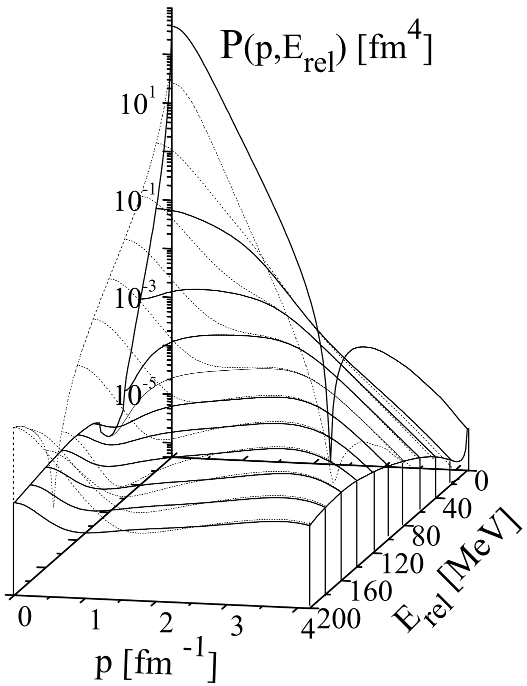
Proton Spectral Function





$$p \equiv |\boldsymbol{k}_1|$$
$$E_{rel} = E_m - E_{min}$$
$$= t^2/M_N$$

Neutron Spectral Function

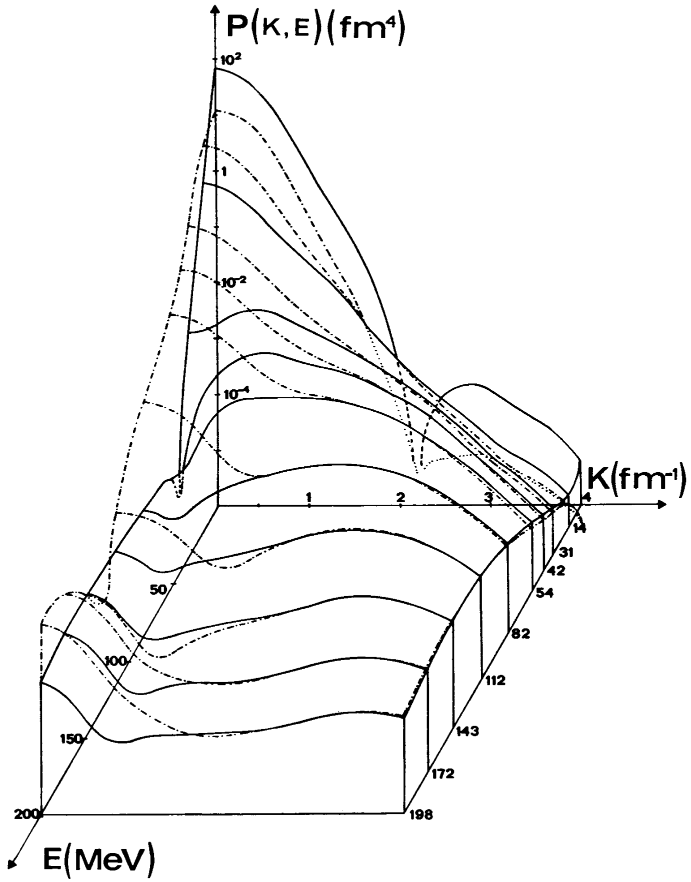


dotted curves: PWA

solid curves: PWIA

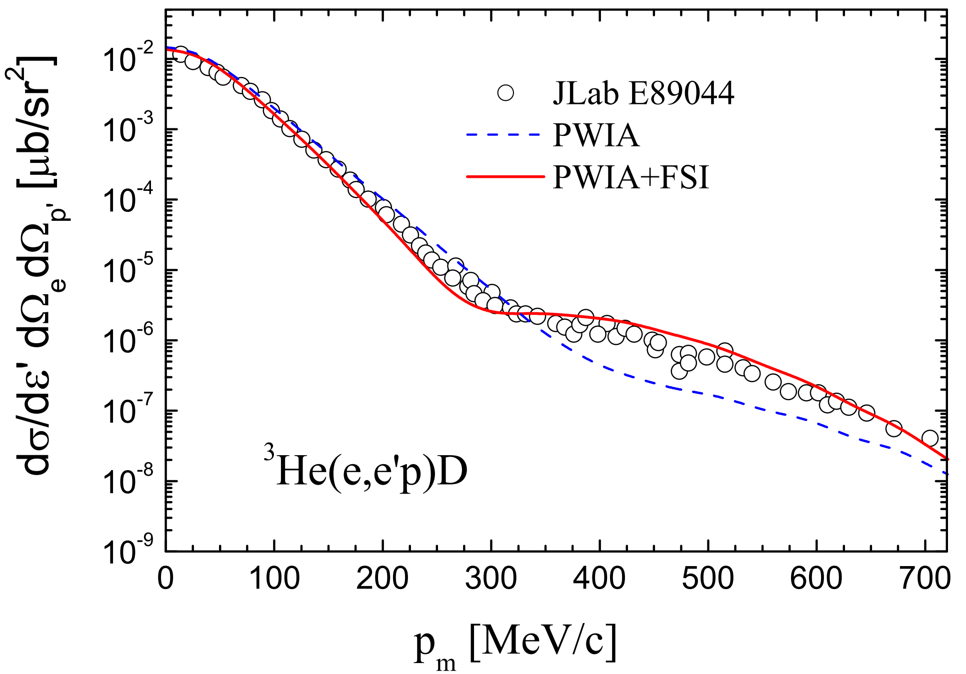
correlated bumps located at

$$E_{rel} = p^2/4M$$



The Spectral Function of ^3He. RSC potential
(C d A , E. Pace, G. Salme, Lecture Notes in Physics **86** (1978))

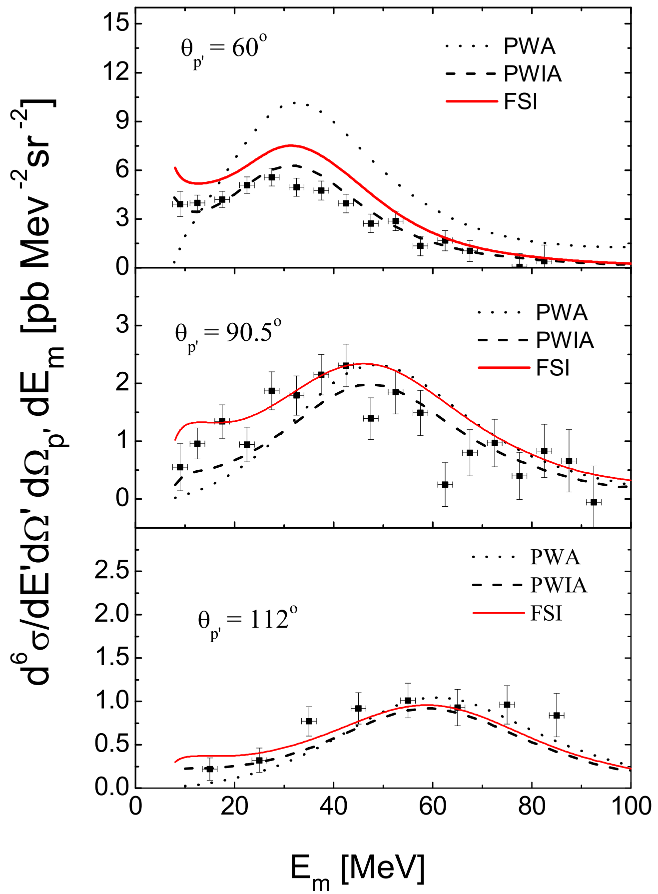
Results: 2bbu channel: $^3He(e,e'p)^2H$



JLab data from experiment *E-89-004* (M. M. Rvachev *et al.*, nucl-ex 0409005)

$$Q^2 \simeq 1.55 \ (GeV/c)^2 \qquad and \qquad x = 1$$

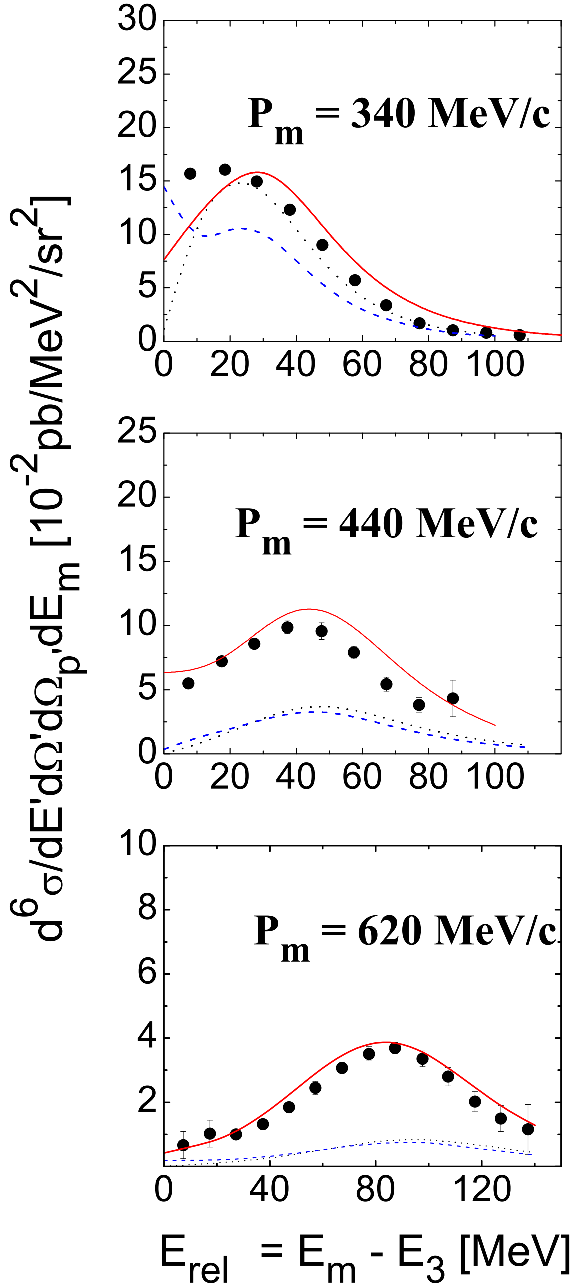
Results: 3bbu channel $^3He(e,e'p)(pn)$



Saclay data (C.Marchand *et al.*, Phys. Rev. Lett. **60**(1988)); $p' \equiv |\boldsymbol{p}_1|$.
Note that the data satisfy the relation:

$$E_m \simeq \frac{p_m^2}{4\,M_N}$$

CORRELATION BUMPS CLEARLY SEEN

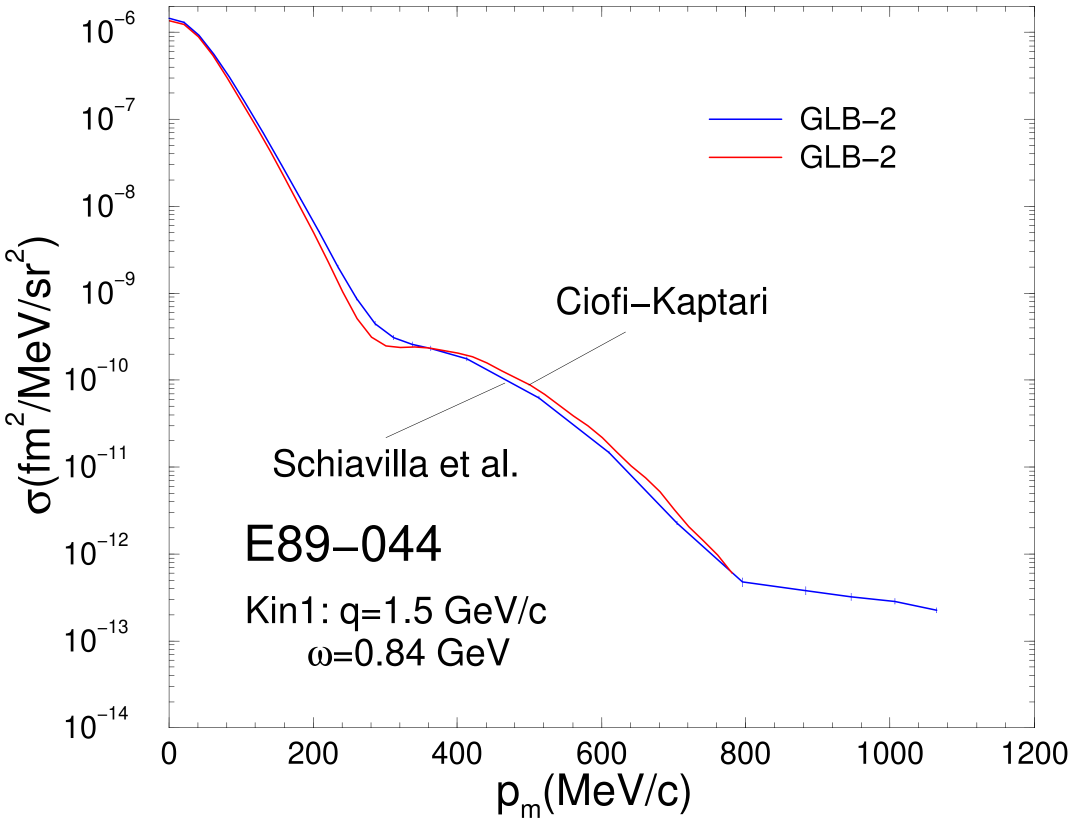


$$Q^2 = 1.55 \ (Gev/c)^2$$
$$x = 1$$
$$E_{rel} = E_2^f = E_m - E_3$$

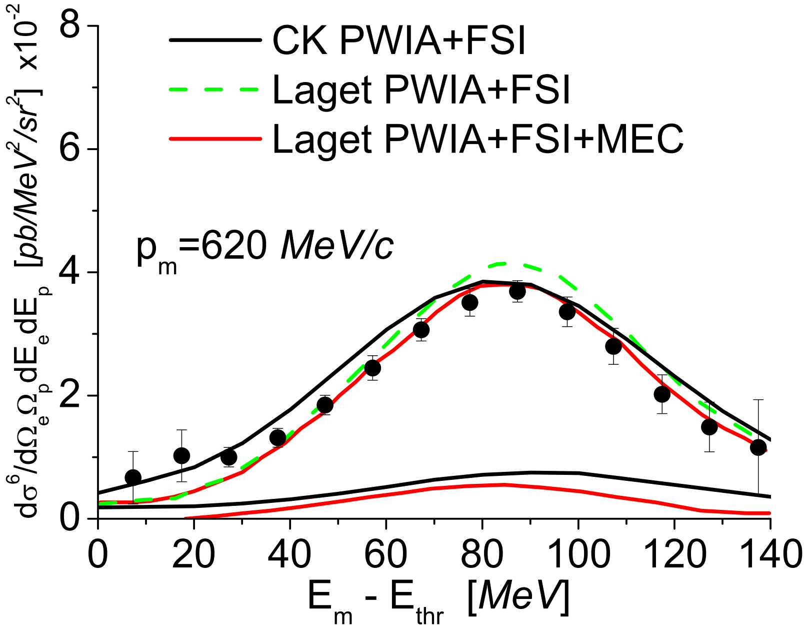
JLab data from experiment *E-89-004*
(F. Benmokhtar *et al.*, nucl-ex 0408015)

FSI in (n-p) pair- irrelevant;
FSI in n-(n-p) system- very large

4. Comparison with other approaches



R. Schiavilla *et al.*, *private communication*



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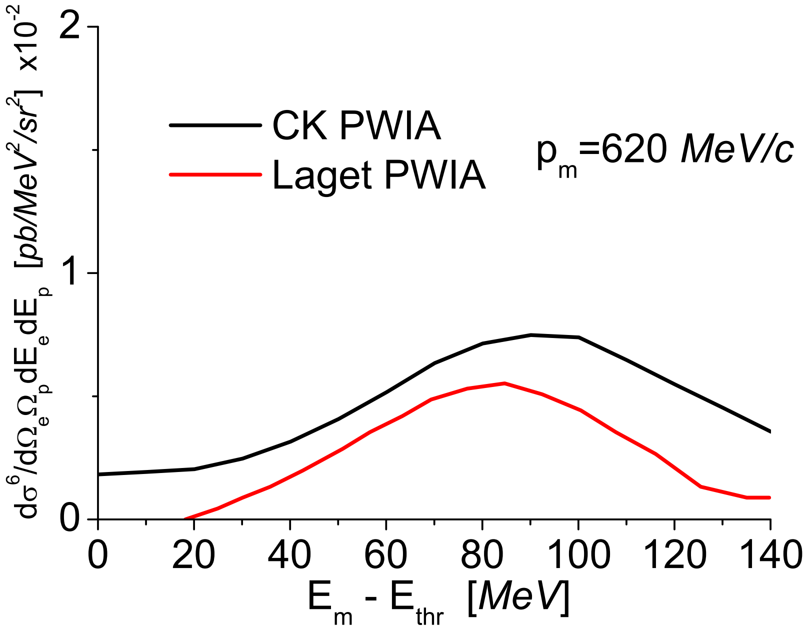

5. Conclusions

- Consistent treatment of ISC and FSI in $A(e, e'p)B$ processes off few-nucleon systems at high energies seems to be nowadays possible. Different approaches are in qualitative agreement although the origin of some still existing discrepancies should be clarified. Benchmark calculations are highly desirable.

- Existing experimental data and theoretical calculations show that in the region $|\mathbf{p}_{mis}| \leq 600 MeV/c$, $E_{mis} \leq 150 MeV$, proper kinematical conditions can be chosen where the effects from FSI, MEC, etc could be minimized and ISC could be investigated.

- The agreement of experimental data with calculations performed with realistic few-nucleon wave functions corresponding to realistic interactions is a satisfactory one.