Structure Functions at Finite Density

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Outline

- The Nambu–Jona-Lasinio (NJL) model:
  - Nucleon.
- Quark Distributions
  - Results.
- Finite Density Quark Distributions,
  - Results.
- Conclusion.

The NJL Model

- Lagrangian

$$\mathcal{L}_{NJL} = \overline{\psi}\,(i\,\partial\!\!\!/ - m)\,\psi + \mathcal{L}_I(4 - point),$$
$$\mathcal{L}_I(4 - point) = G\,\left(\overline{\psi}\Gamma\psi\right)^2.$$

  where Γ = Dirac, colour, isospin matrices.

- Shortcomings:
  - No explicit gluons,
  - Interaction is point-like $\Rightarrow$ non-renormalizable $\rightarrow$ Need explicit regularization.

Interaction Lagrangians

- Using Fierz transformation can decompose $\mathcal{L}_I$ into sum of qq interaction terms.

$$\mathcal{L}_{I,s} = G_s \left(\overline{\psi}\, \gamma_5\, C\, \tau_2\, \beta^A \overline{\psi}^T\right) \left(\psi^T\, C^{-1}\, \gamma_5\, \tau_2\, \beta^A \psi\right),$$

$$\mathcal{L}_{I,a} = G_a \left(\overline{\psi}\, \gamma_\mu\, C\, \vec{\tau}\tau_2\, \beta^A \overline{\psi}^T\right) \left(\psi^T C^{-1} \gamma_\mu\, \vec{\tau}\tau_2\, \beta^A \psi\right).$$

- Solving BS equation gives

$$\tau_s(q) = \frac{4iG_s}{1 + 2\, G_s\, \Pi_s(q^2)},$$

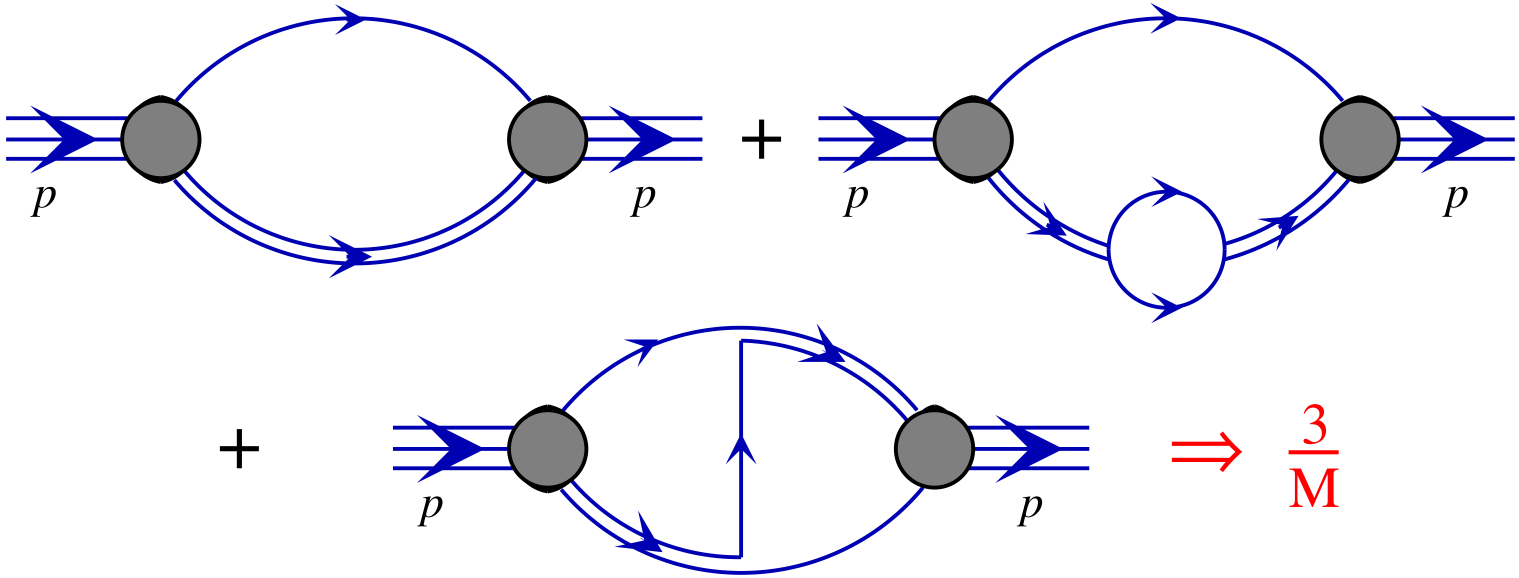
$$\tau_a^{\mu\nu}(q) = 4\, i\, G_a \left[g^{\mu\nu} - \frac{2G_a \Pi_a(q^2)}{1 + 2G_a\Pi_a(q^2)} \left(g^{\mu\nu} - \frac{q^\mu\, q^\nu}{q^2} \right) \right].$$

Faddeev Equation

- To determine the nucleon properties we must solve the Faddeev equation.

$$\Gamma_N^a = \frac{3}{M} K^{ab}\, \Gamma_N^b,$$

Static Approximation



$\Rightarrow \frac{3}{\mathrm{M}}$

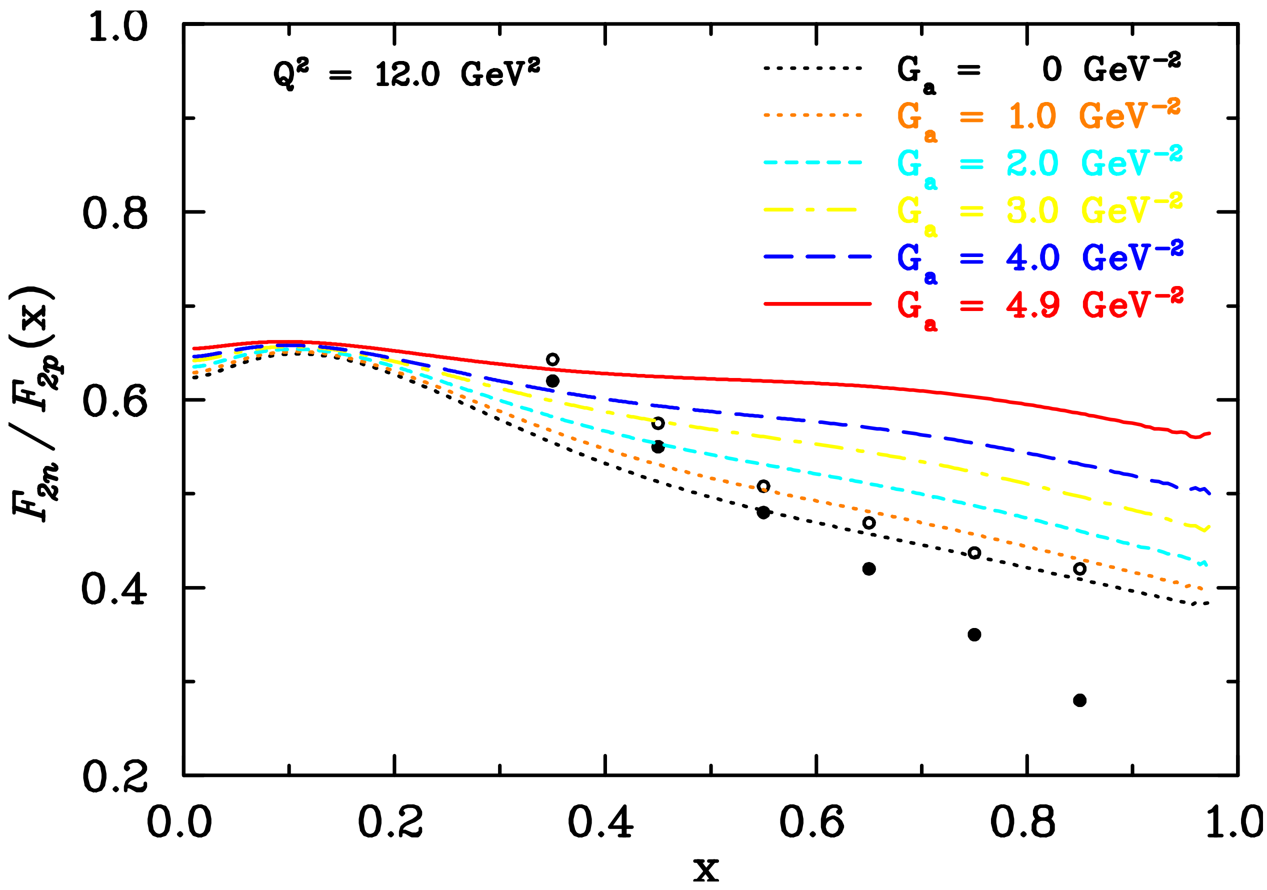
Model Parameters

- Free Parameters: Λ_{IR}, Λ_{UV}, M_0, G_π, G_s, G_a and G_ω.
- Determine as follows:
  - $\Lambda_{IR} = 200$ MeV, $M_0 = 400$ MeV.
  - $f_\pi = 93$ MeV and $m_\pi = 140$ MeV
    $\Rightarrow \quad \Lambda_{UV} = 638.5$ MeV and $G_\pi = 19.60$ GeV^{-2}.
  - Fadd. Eq. for Nucleon and $M_N = 940$ MeV $\Rightarrow G_s$.
  - G_ω fixed such that curve E_B/A v. ρ pass through $(\rho, E_B/A) = (0.17\,\text{fm}^{-3}, -15.7\text{ MeV})$

Model Parameters cont'd

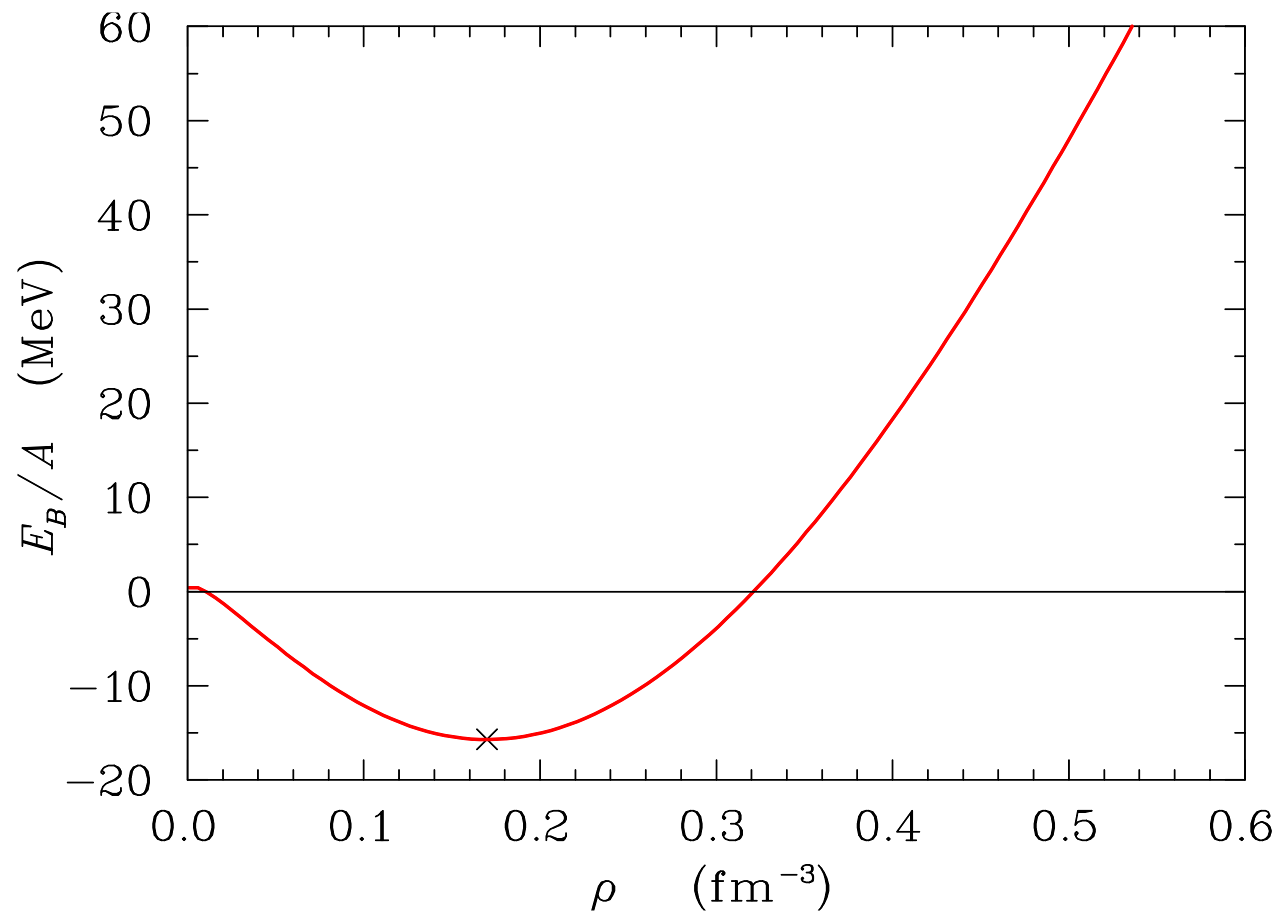
- This leaves G_a.
- Fadd. Eq. for $\Delta(1232)$ $\Rightarrow$ $G_a = 4.91$ GeV^{-2}.
  - To close to SU(6) limit, that is $u(x) \sim 2\,d(x)$.
- Choose G_a to reproduce F_{2n}/F_{2p}, $d(x)/u(x)$ and empirical saturation.

F_{2n}/F_{2p}

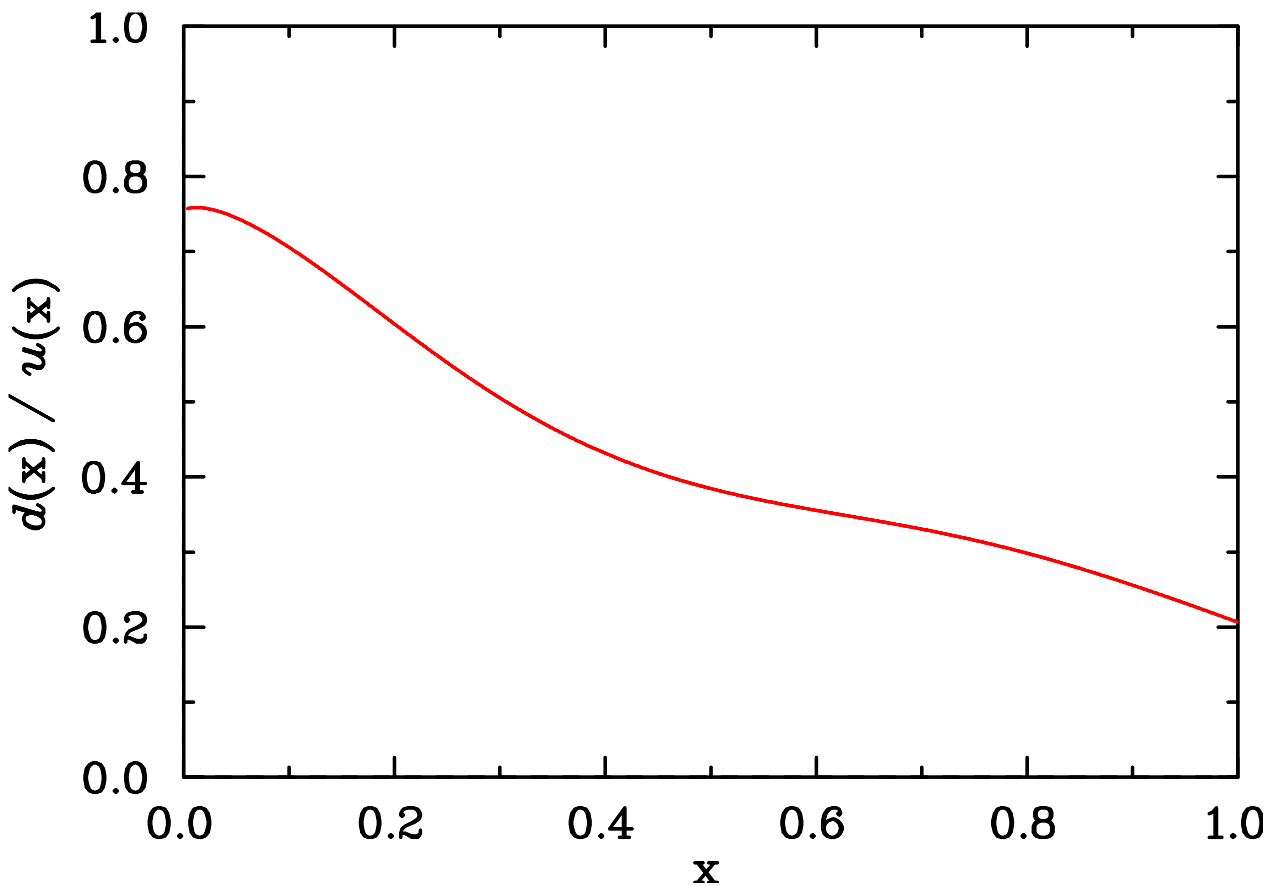


- *W. Melnitchouk, A.W. Thomas, Physics Letters B 377 (1996) 11-17.*

Saturation



$d(x)/u(x)$



Model Parameters Summary

- $G_a = 2.50$ GeV^{-2}.
- $G_s = 7.97$ GeV^{-2}.
- $1 + 2G_a\Pi_a(q^2 = M_s^2) = 0 \Rightarrow M_a = 990$ MeV.
- $1 + 2G_s\Pi_s(q^2 = M_s^2) = 0 \Rightarrow M_s = 658$ MeV.
- $G_\omega = 6.40$ GeV^{-2}.

The Quark Distributions $f(x)$ and $\Delta f(x)$.

- Formally

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{i\,x\,P^+\,\xi^+} \langle P, S|\overline{\psi}(0)\gamma^+\psi(\xi^-)|P, S\rangle_c,$$

$$\Delta f(x) = \int \frac{d\xi^-}{4\pi} e^{i\,x\,P^+\,\xi^+} \langle P, S|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi^-)|P, S\rangle_c.$$

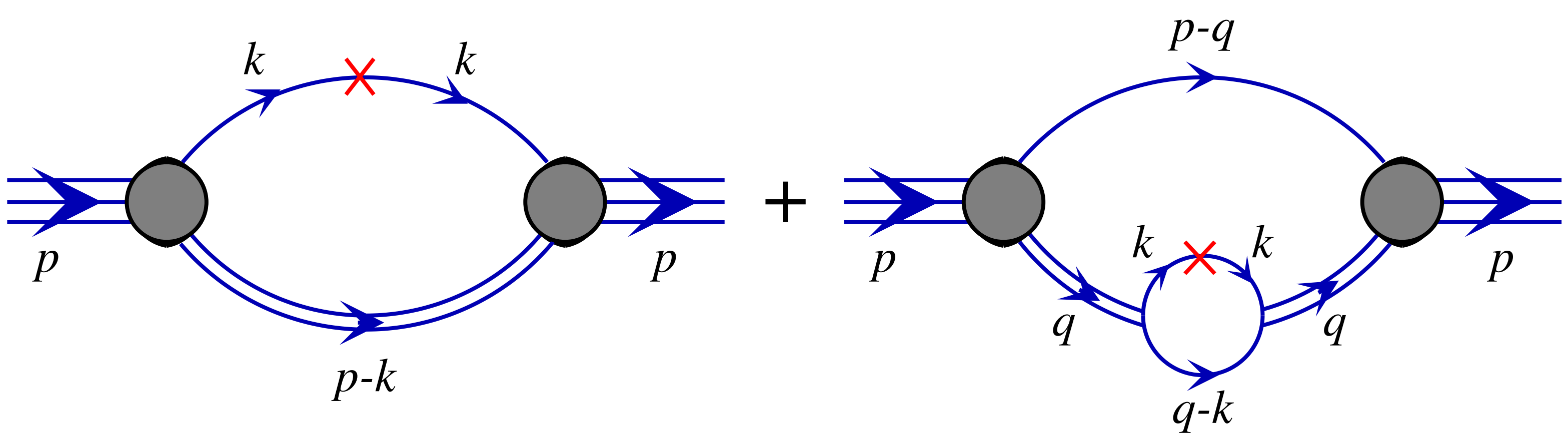
- Can show

$$f(x) = \frac{1}{2P^+} \int \frac{d^4k}{(2\pi)^4} \delta(x - \frac{k^+}{P^+}) \,\mathrm{Tr}\left[\gamma^+ M(P,k)\right],$$

$$\Delta f(x) = \frac{1}{2P^+} \int \frac{d^4k}{(2\pi)^4} \delta(x - \frac{k^+}{P^+}) \,\mathrm{Tr}\left[\gamma^+\gamma_5 M(P,k)\right].$$

$f(x)$ and $\Delta f(x)$

- $M(P,k)$ is the quark two-point function in the nucleon.
- Therefore $f(x)$ and $\Delta f(x)$ can be associated with a straight forward Feynman diagram calculation.



- $f(x) \to \mathsf{X} = \gamma^+ \, \delta(x - \frac{k^+}{P^+})$
- $\Delta f(x) \to \mathsf{X} = \gamma^+ \gamma_5 \, \delta(x - \frac{k^+}{P^+})$

Quark distributions in the Proton

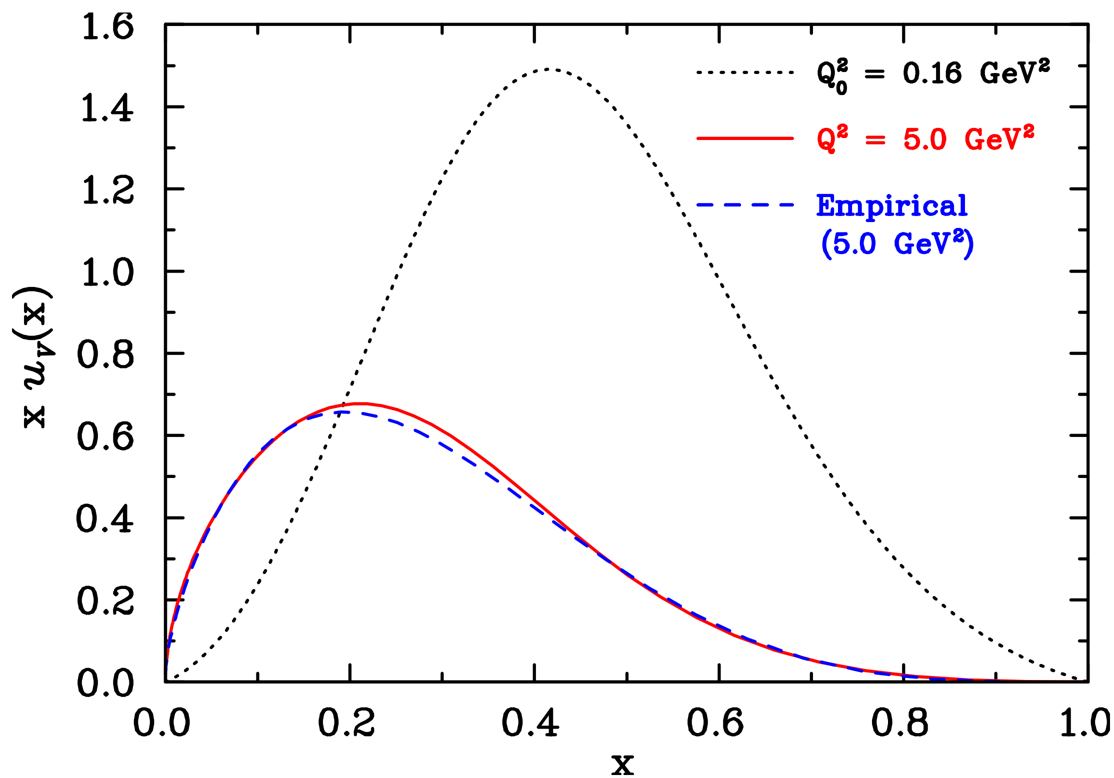
$$u_V(x) = f^s_{q/P}(x) + \frac{1}{2} f^s_{q(D)/P}(x) + \frac{1}{3} f^a_{q/P}(x) + \frac{5}{6} f^a_{q(D)/P}(x),$$

$$d_V(x) = \frac{1}{2} f^s_{q(D)/P}(x) + \frac{2}{3} f^a_{q/P}(x) + \frac{1}{6} f^a_{q(D)/P}(x),$$

$$\Delta u_V(x) = f^s_{q/P}(x) + \frac{1}{2} f^s_{q(D)/P}(x) + \frac{1}{3} f^a_{q/P}(x) + \frac{5}{6} f^a_{q(D)/P}(x) + \frac{1}{2\sqrt{3}} f^m_{q(D)/P}(x),$$

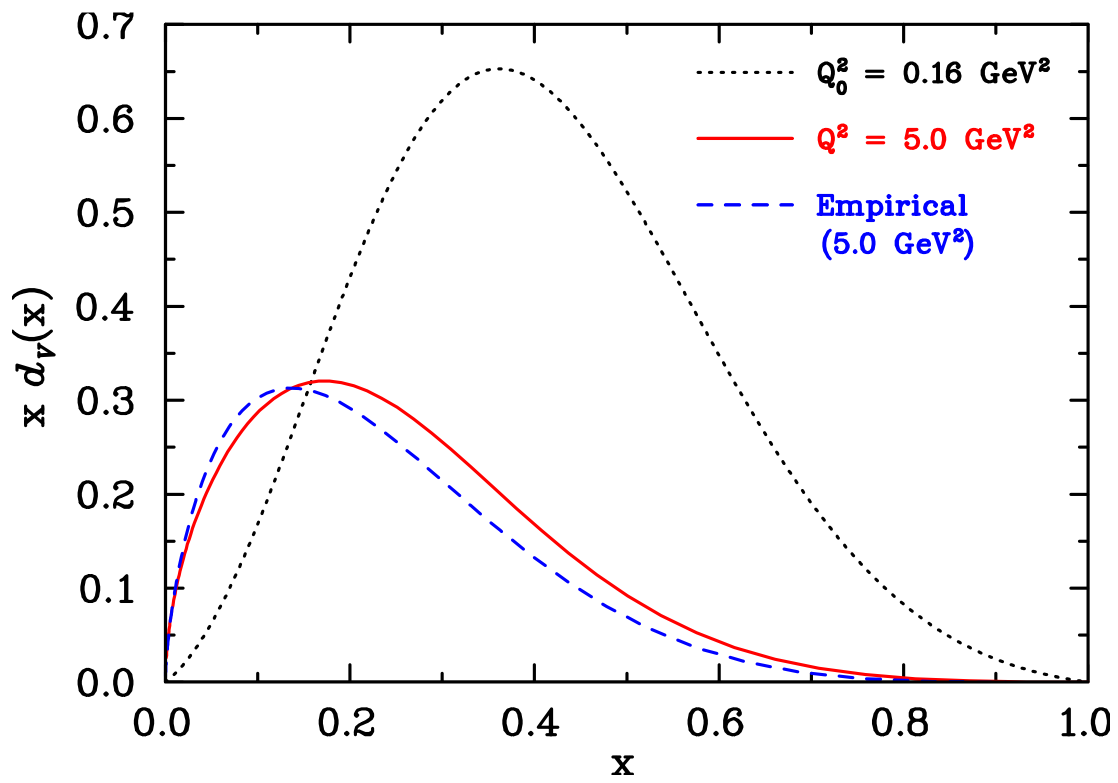
$$\Delta d_V(x) = \frac{1}{2} f^s_{q(D)/P}(x) + \frac{2}{3} f^a_{q/P}(x) + \frac{1}{6} f^a_{q(D)/P}(x) - \frac{1}{2\sqrt{3}} f^m_{q(D)/P}(x),$$

$u_V(x)$ distribution



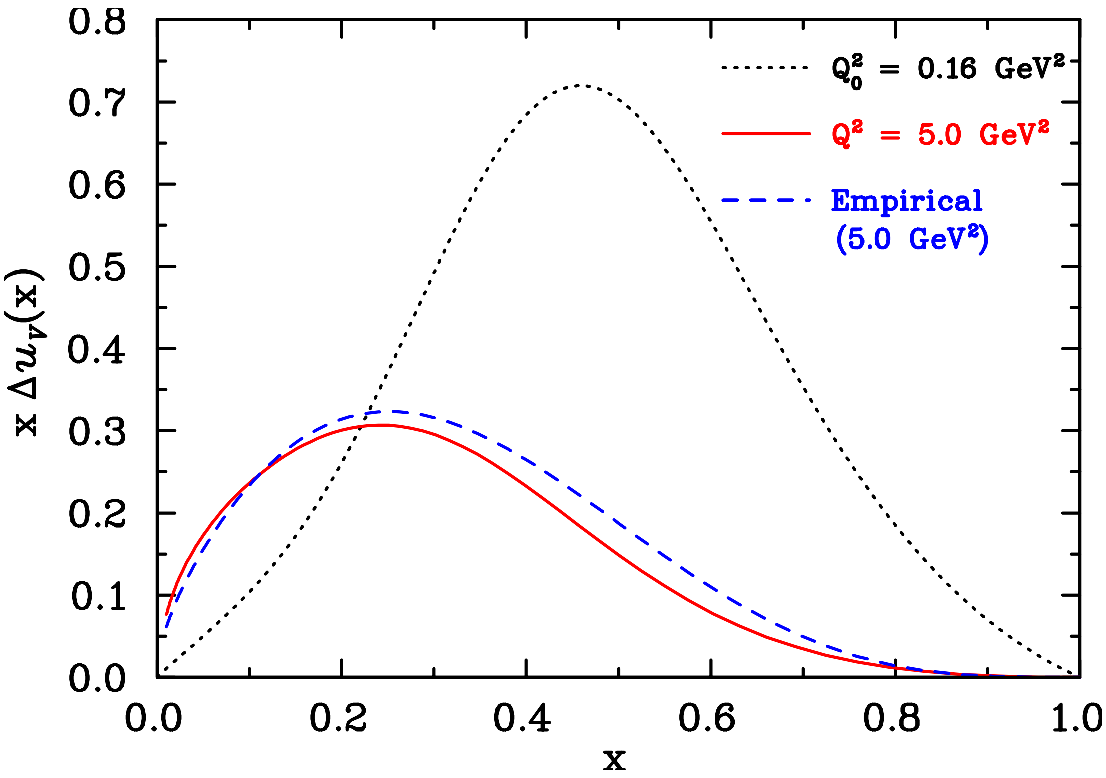
- *MRST, Phys. Lett. B* **531**, *216 (2002).*

$d_V(x)$ distribution



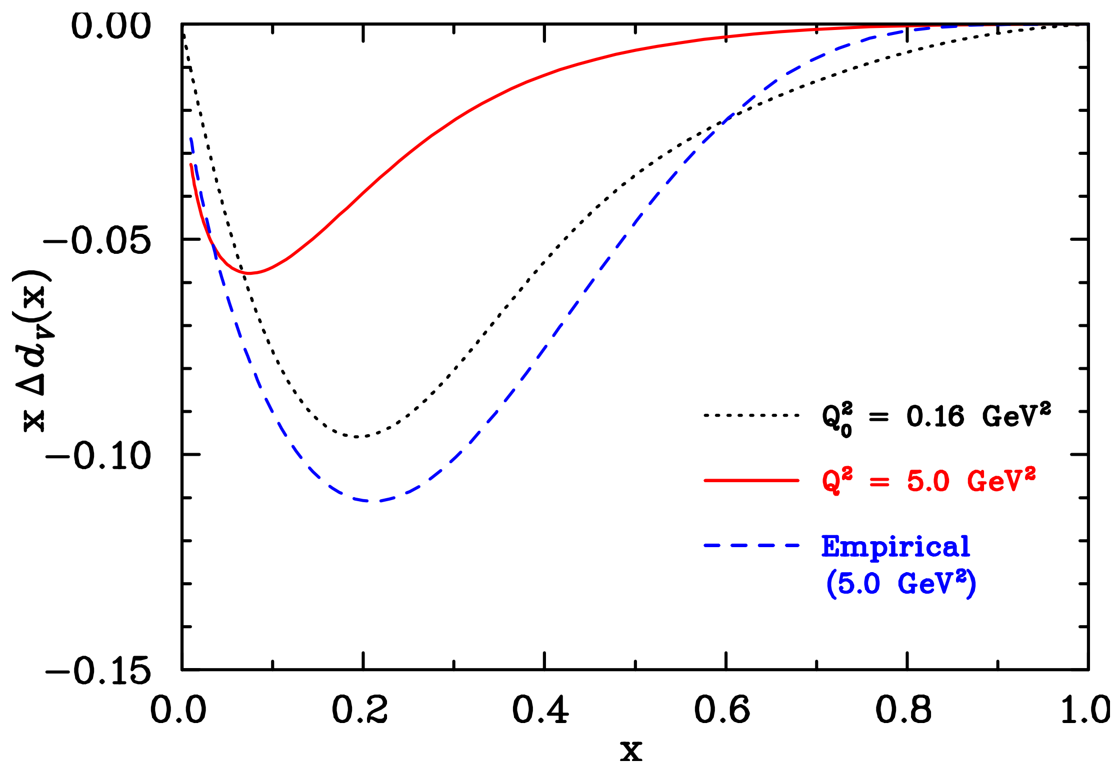
- *MRST, Phys. Lett. B* **531**, *216 (2002).*

$\Delta u_V(x)$ distribution



- *M. Hirai, S. Kumano and N. Saito, Phys. Rev. D* **69**, *054021 (2004).*

$\Delta d_V(x)$ distribution



- *M. Hirai, S. Kumano and N. Saito, Phys. Rev. D* **69**, *054021 (2004).*

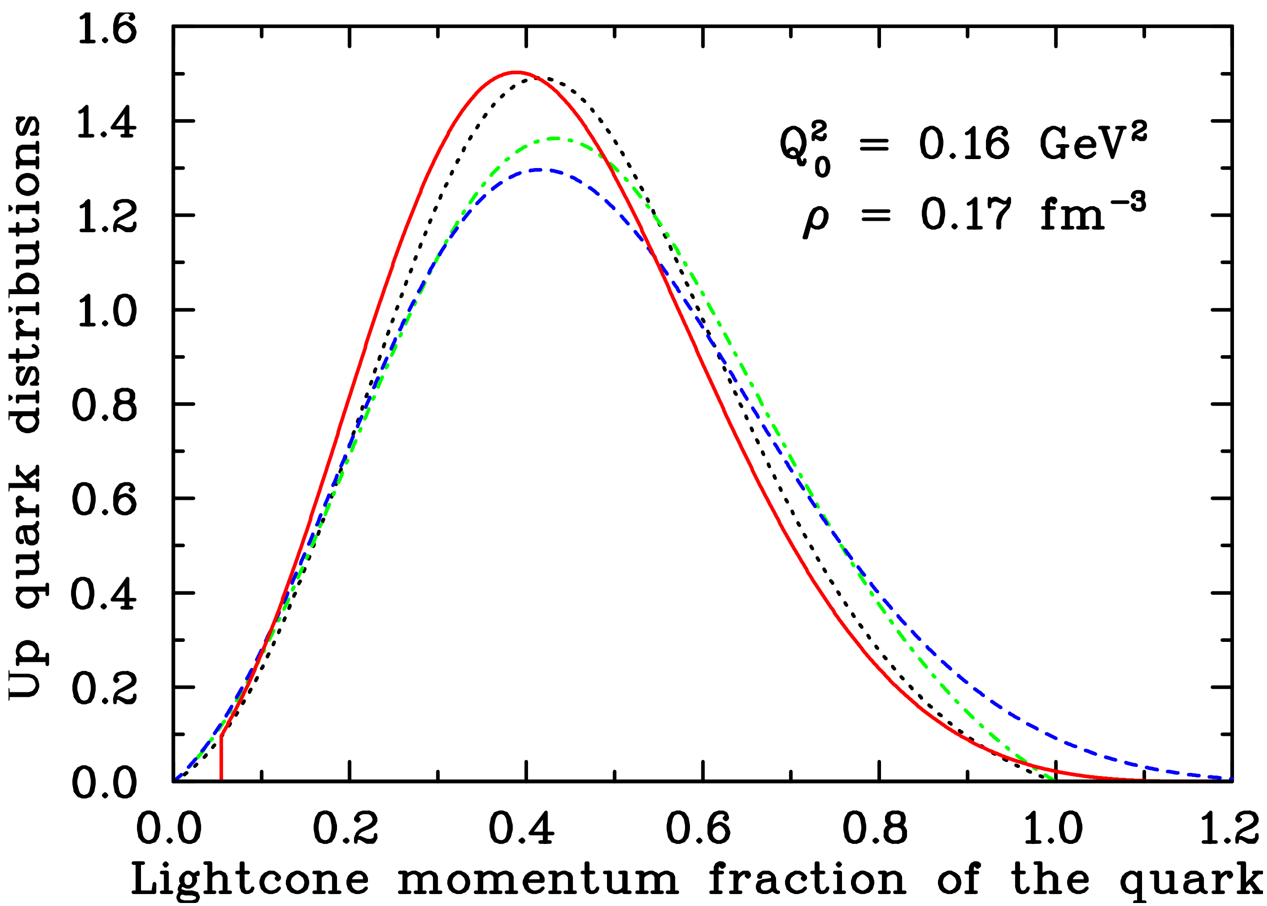
Finite Density Quark Distributions

- Scalar field: via effective masses
- Fermi motion: via convolution with smearing function $\Delta f_{N/A}(y_A)$, giving $\Delta f_{q/A0}(\tilde{x}_A)$.
- Vector field: via scale transformation

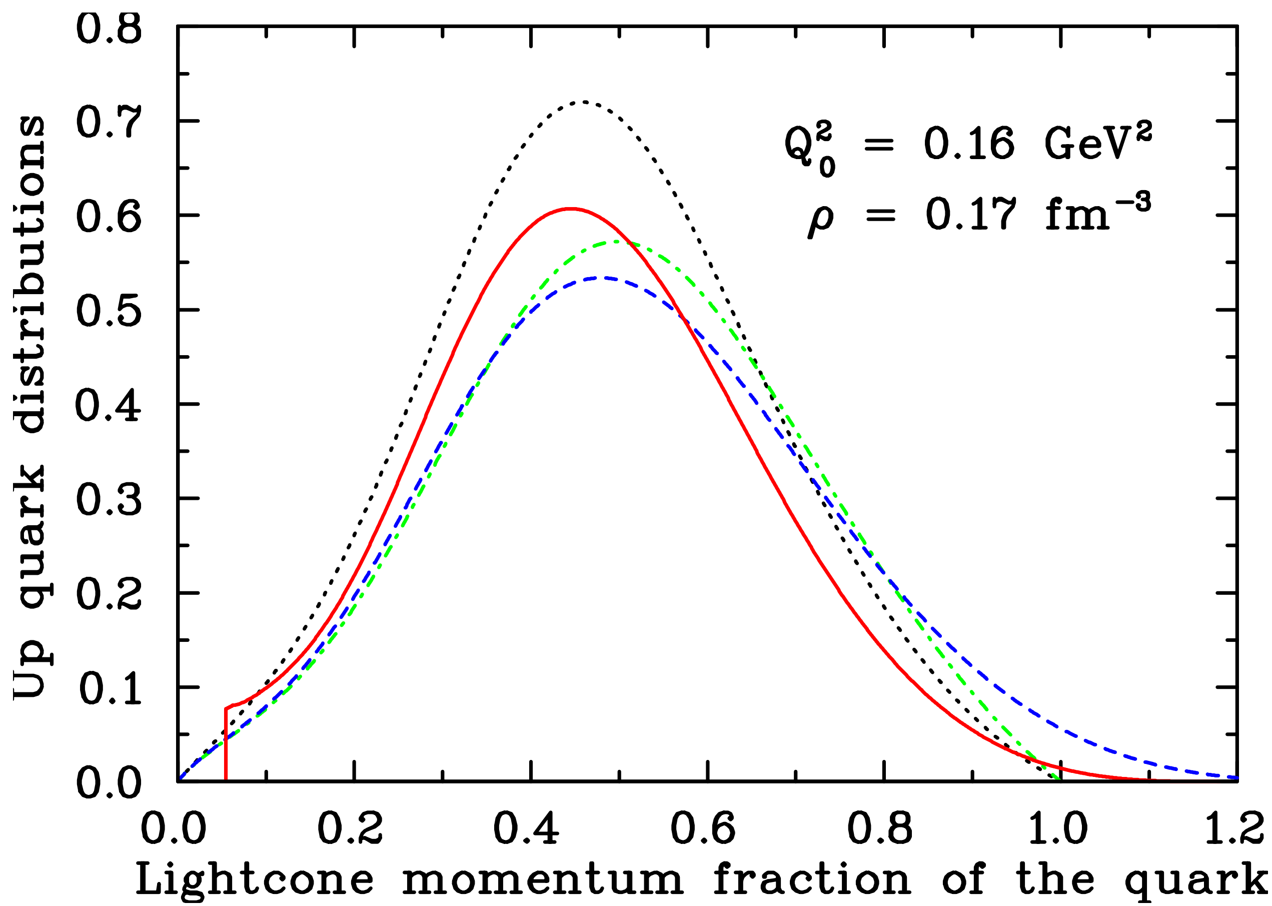
$$\Delta f_{q/A}\left(x_A\right) = \frac{\varepsilon_F}{E_F}\,\Delta f_{q/A0}\left(\tilde{x}_A = \frac{\varepsilon_F}{E_F} x_A - \frac{V_0}{E_F}\right),$$

where $\varepsilon_F = \sqrt{p_F^2 + M_N^2} + 3\,V_0 \equiv E_F + 3\,V_0$

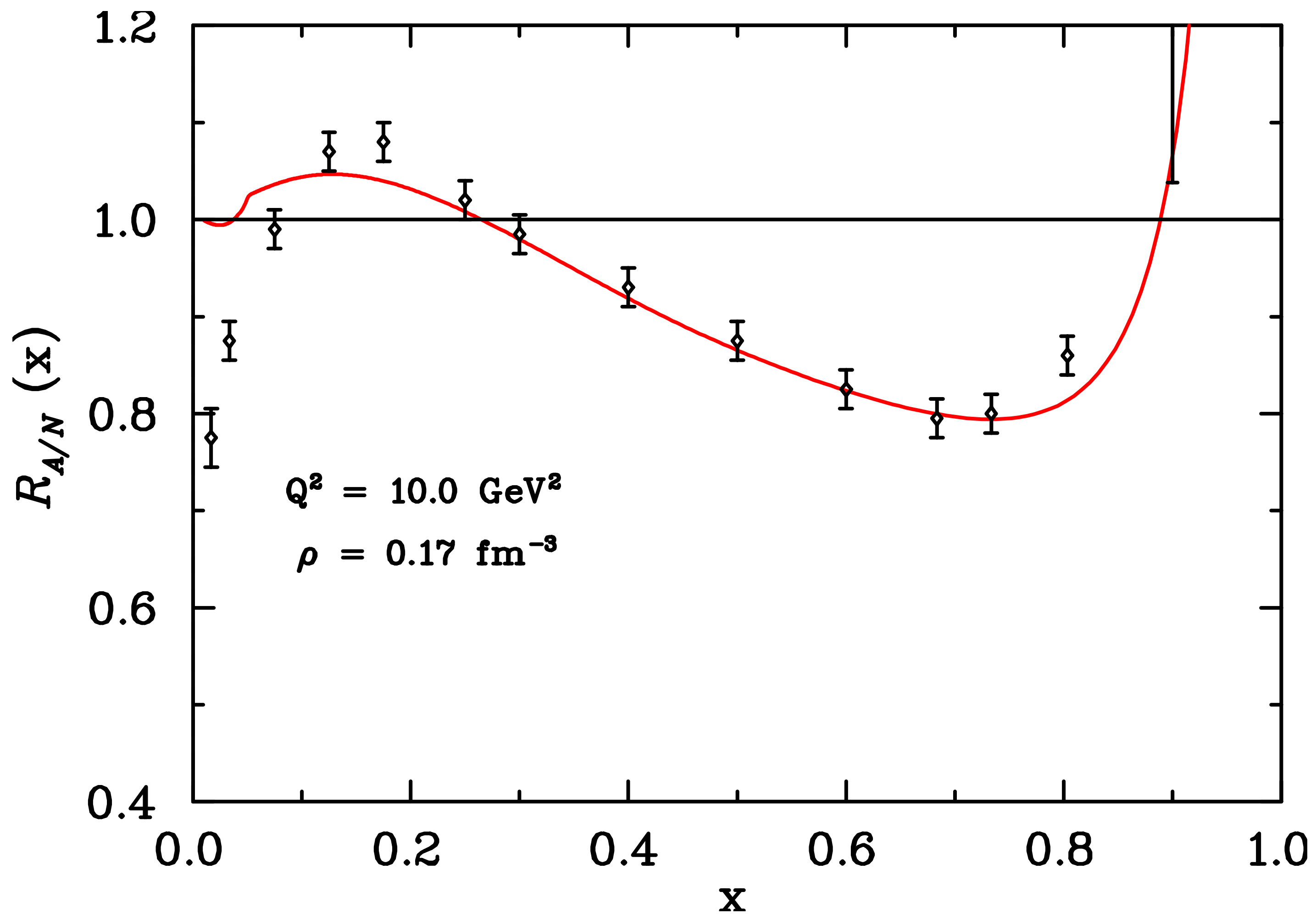
$u_V(x)$ distribution



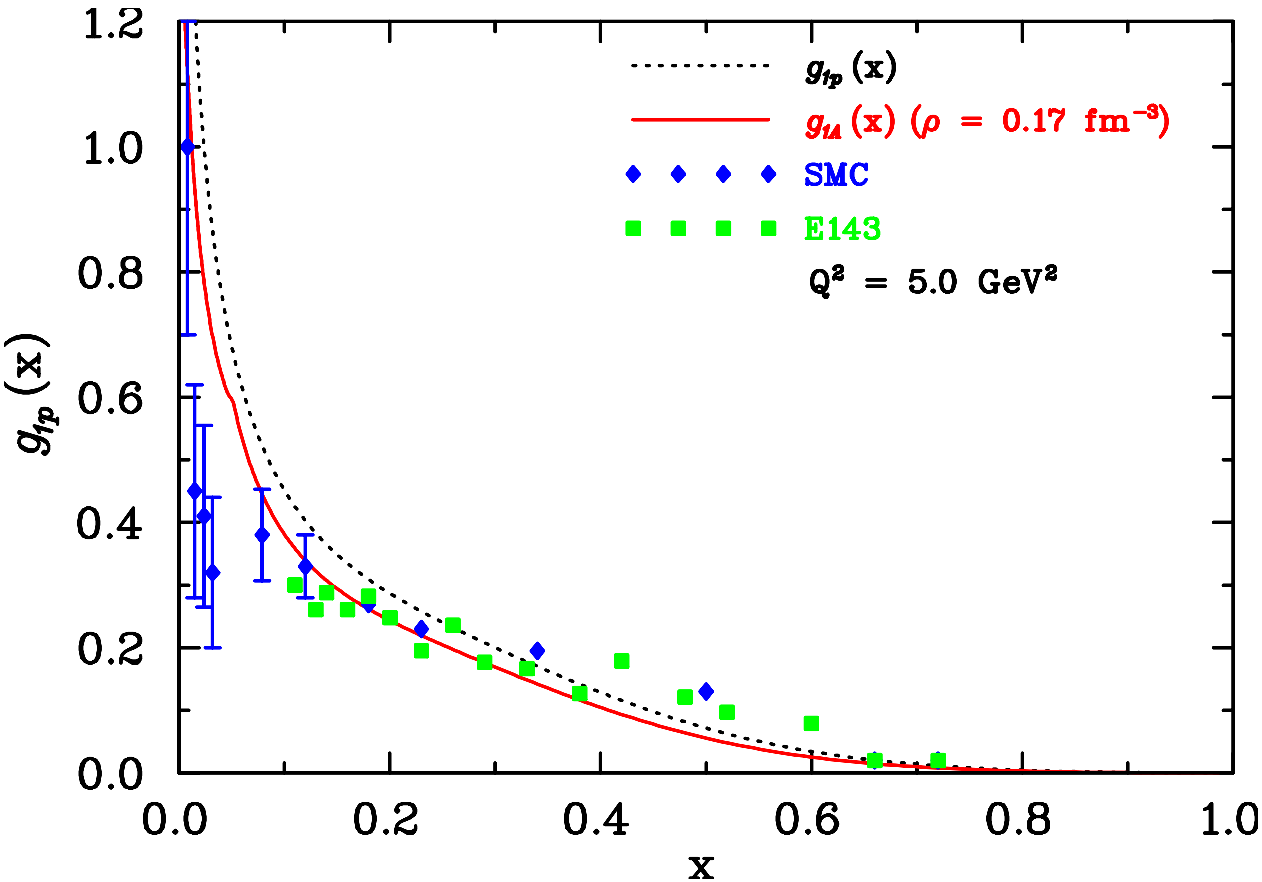
Δu_V distributions



Spin-Independent EMC effect

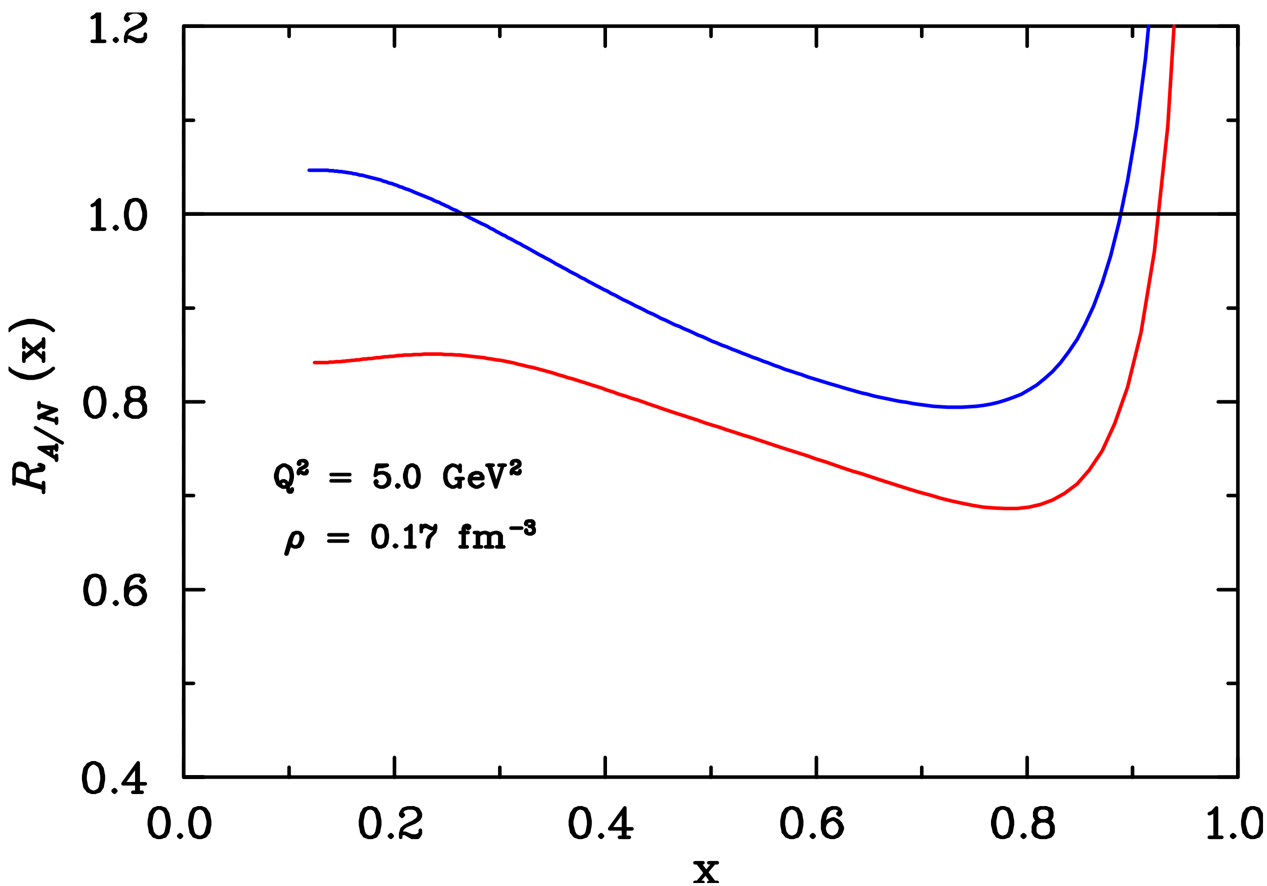


g_{1p} and g_{1A}



g_{1p} (x)
g_{1A} (x) (ρ = 0.17 fm^{-3})
SMC
E143
Q^2 = 5.0 GeV^2
g_{1p} (x)
x
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Spin-dependent EMC effect



Conclusions

- Using a modified NJL model we have:
  - Successfully described nucleon as a quark-diquark bound state.
  - Reproduced the empirical nuclear saturation.
  - Achieved good results for two of the leading-twist proton quark distributions.
  - Calculated medium modifications to these distributions.
- Achieved this with only 7 input parameters.
- We find: $g_A = 1.17$; Experiment: $g_A = 1.267 \pm 0.011$.
- In nuclear matter, we find $g_A = 0.94$.