Study of Bound Nucleons using Generalized Parton Distributions.

Swadhin Taneja
University of Virginia

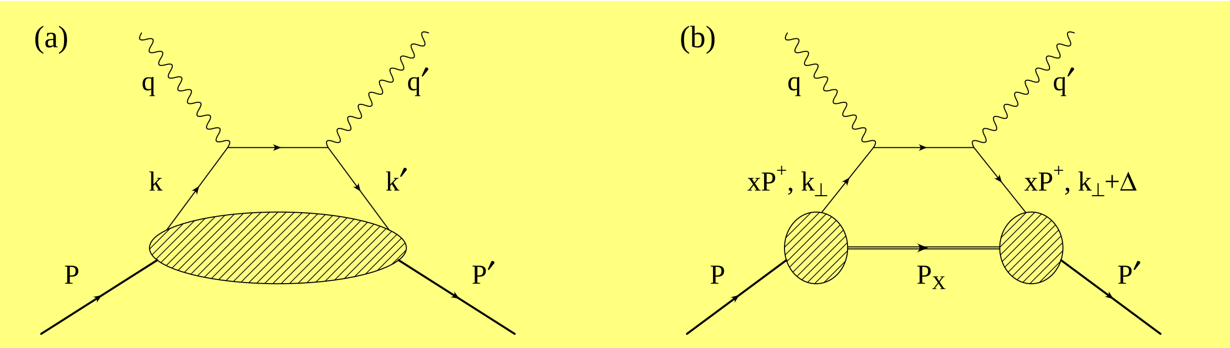
The Physics of Nuclei with 12GeV Electrons
Workshop
Jefferson Laboratory
November 1st - 5th, 2004

Outline

General aim: to study transverse degrees of freedom in hadronic structure using hard processes in nuclei.

- $\star$ Generalities.
- $\star$ Impact parameter space.
- $\star$ Exclusive Scattering in the Nuclear Medium.
- $\star$ Nuclear GPD model.
- $\star$ Conclusions and Outlook.

Generalities



- $\bar{P} = (P + P')/2$,
- $x = \bar{k}^+/\bar{P}^+$,
- $\xi = -\Delta^+/2\bar{P}^+$,
- $Q^2 = -q^2$,
- $t \equiv \Delta^2$, where $\Delta = P - P'$.
- H_q and E_q satisfy the following relations:

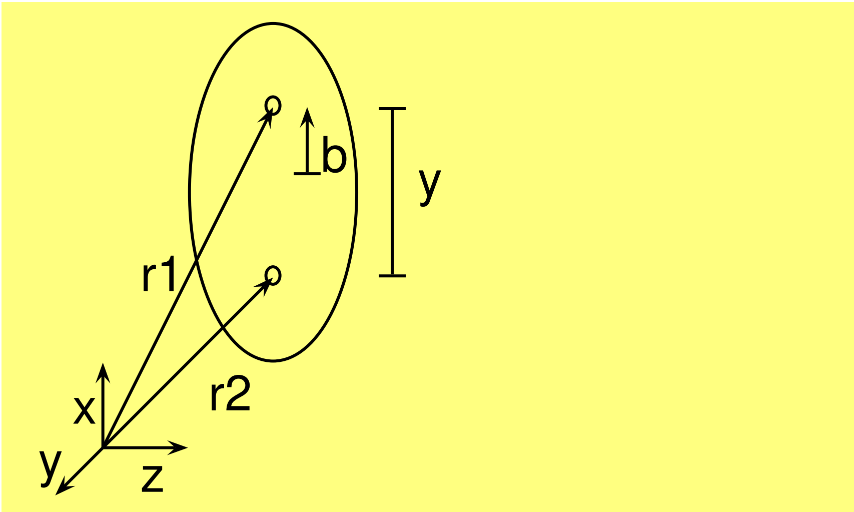
$$\int_{-1}^{1} dx H_q(x, \xi, t) = F_1^q(t),$$

$$\int_{-1}^{1} dx E_q(x, \xi, t) = F_2^q(t)$$

- For $x \geq 0$,

$$H_q(x, 0, 0) \equiv q(x)$$

Impact parameter space



- For the case of pure transverse momentum transfer,

$$q(x,\mathbf{b}) = \int \frac{\mathbf{d^2\Delta}}{\mathbf{(2\pi)^2}}\mathbf{H_q(x,-\Delta^2)e^{-ib.\Delta}}$$

- Mean square distance :

$$\langle\, \mathbf{b}^2(x)\,\rangle = \frac{\int d^2\mathbf{b}\; q(x,\mathbf{b})\,\mathbf{b}^2}{\int d^2\mathbf{b}\; q(x,\mathbf{b})}$$

- Mean square seperation :

  (Two component model)

$$\langle\, \mathbf{y}^2(x)\,\rangle = \frac{\langle\, \mathbf{b}^2(x)\,\rangle}{(1-x)^2}$$

- Radius :

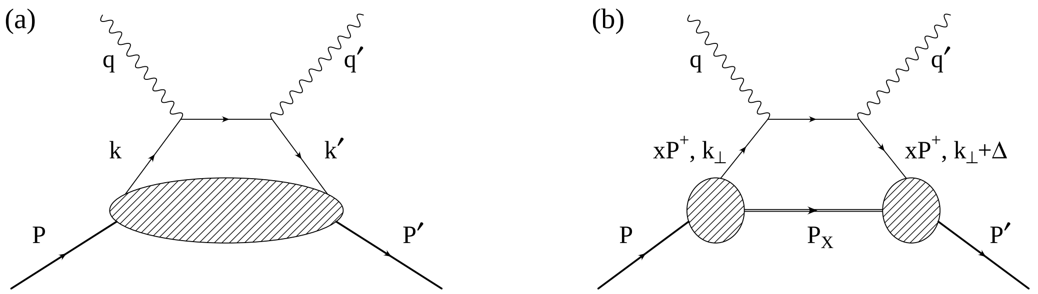
  S. Liuti & S.K. Taneja ,Phys. Rev. D 70, 074019

$$\boxed{\sqrt{\langle\, \mathbf{r}^2(x)\, \rangle} = MAX\left\{\sqrt{\langle\, \mathbf{b}^2\, \rangle}, \frac{x}{1-x}\sqrt{\langle\, \mathbf{b}^2\, \rangle}\right\}}$$

Model

S. Liuti & S.K. Taneja ,Phys. Rev. D 70, 074019 hep-ph/0405014

Consider a **two component** (quark-diquark) picture for the lowest Fock components of the proton.



Assumptions:

- Leading Twist $\Rightarrow$ Relativistic Impulse Approximation: final states are the scattered quark with 4-momentum $k' = k+q$, and diquark with 4-momentum P_X.

- Kinematics $\Rightarrow$ the struck quark is off-shell: $k^2 \neq m^2$, the diquark is on-shell with mass $P_X^2 = \mathrm{M_X}^2$.

- Dynamics $\Rightarrow$ introduce quark-diquark-nucleon vertex functions.

LC wave function: $\Phi(x, \mathbf{k}) = \dfrac{\mathcal{M}(k^2, P_X^2)}{D(x, \mathbf{k})}$

$$\left\{ \begin{array}{c} D(x, \mathbf{k}) = M_X^2\, x - \dfrac{\mathbf{k}^2}{1-x} \\ \mathcal{M} = g\, \dfrac{\Lambda^2 - m^2}{k^2 - \mathbf{\Lambda}^2} \end{array} \right.$$

DEFINITIONS IN TERMS OF LC W.F.:

$$\boxed{H(x,0,-\mathbf{\Delta}^2) = \int d^2\mathbf{k}\,\Phi^*(x,\mathbf{k})\Phi(x,\mathbf{k}+(1-x)\mathbf{\Delta})}$$

$$\nu W_2(x) = x\int d^2\mathbf{k}\ |\ \Phi(x,\mathbf{k})\ |^2$$

$$F_1(\Delta^2) = \int d^2\mathbf{k}\int_0^1 dx\,\Phi^*(x,\mathbf{k})\Phi(x,\mathbf{k}+(1-x)\mathbf{\Delta})$$

$$q(x,\mathbf{b}) = \mathcal{F}\left[H(x,\Delta)\right]$$

ROLE OF INTRINSIC $k_\perp$ $(\equiv \mathbf{k})$:

$$f(x,\mathbf{k}) \equiv |\ \Phi(x,\mathbf{k})\ |^2$$

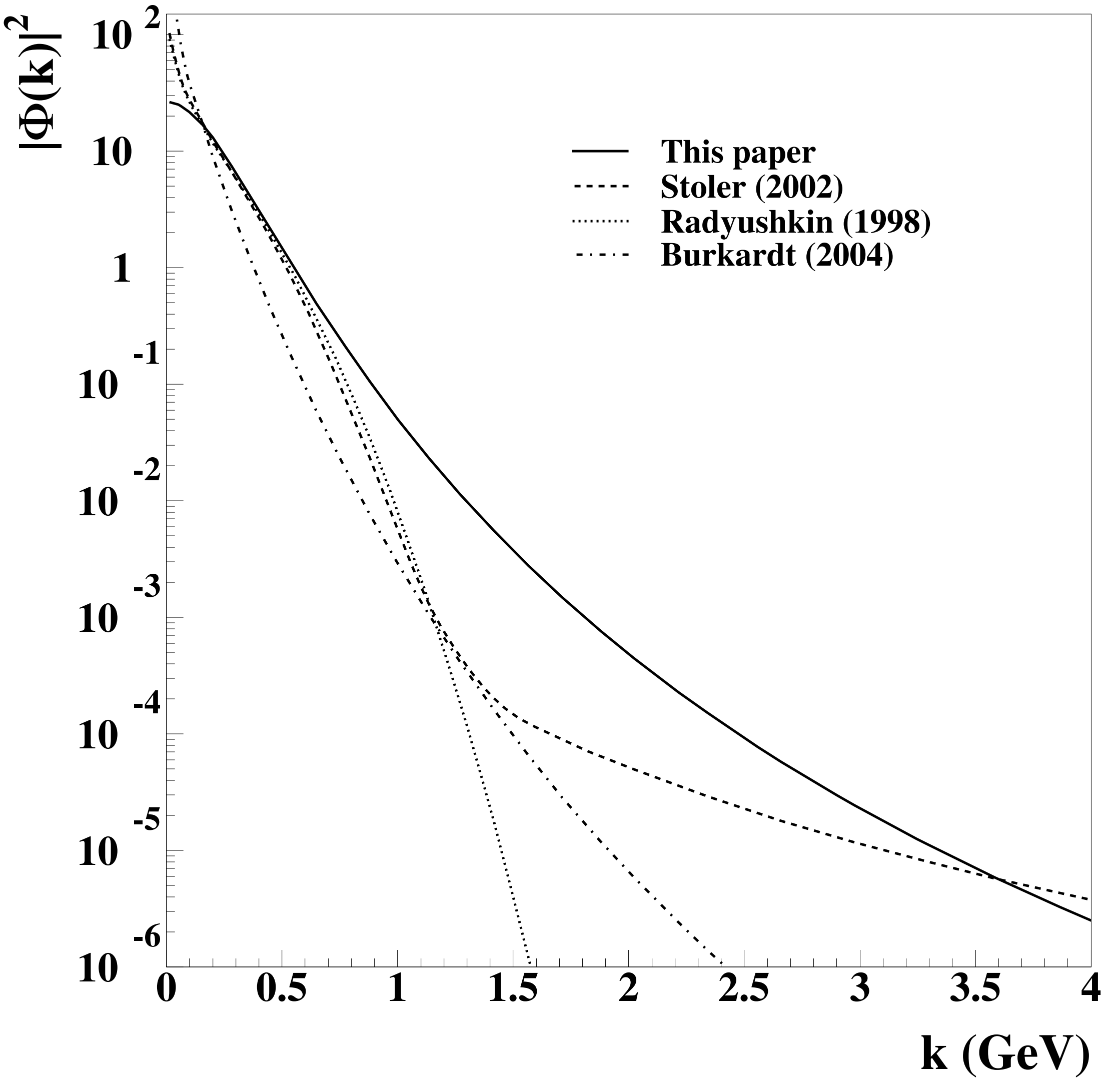
$$\boxed{f(x,\mathbf{k}) = \int d^2\mathbf{b}\int d^2\mathbf{b}'\,e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\,q(x,\mathbf{b},\mathbf{b}')}$$

unintegrated $k_\perp$ distribution.

$$q(x,\mathbf{b},\mathbf{b}') = \Psi^*(x,\mathbf{b}')\Psi(x,\mathbf{b}) \overset{\mathbf{b}\to\mathbf{b}'}{\equiv} q(x,\mathbf{b})$$

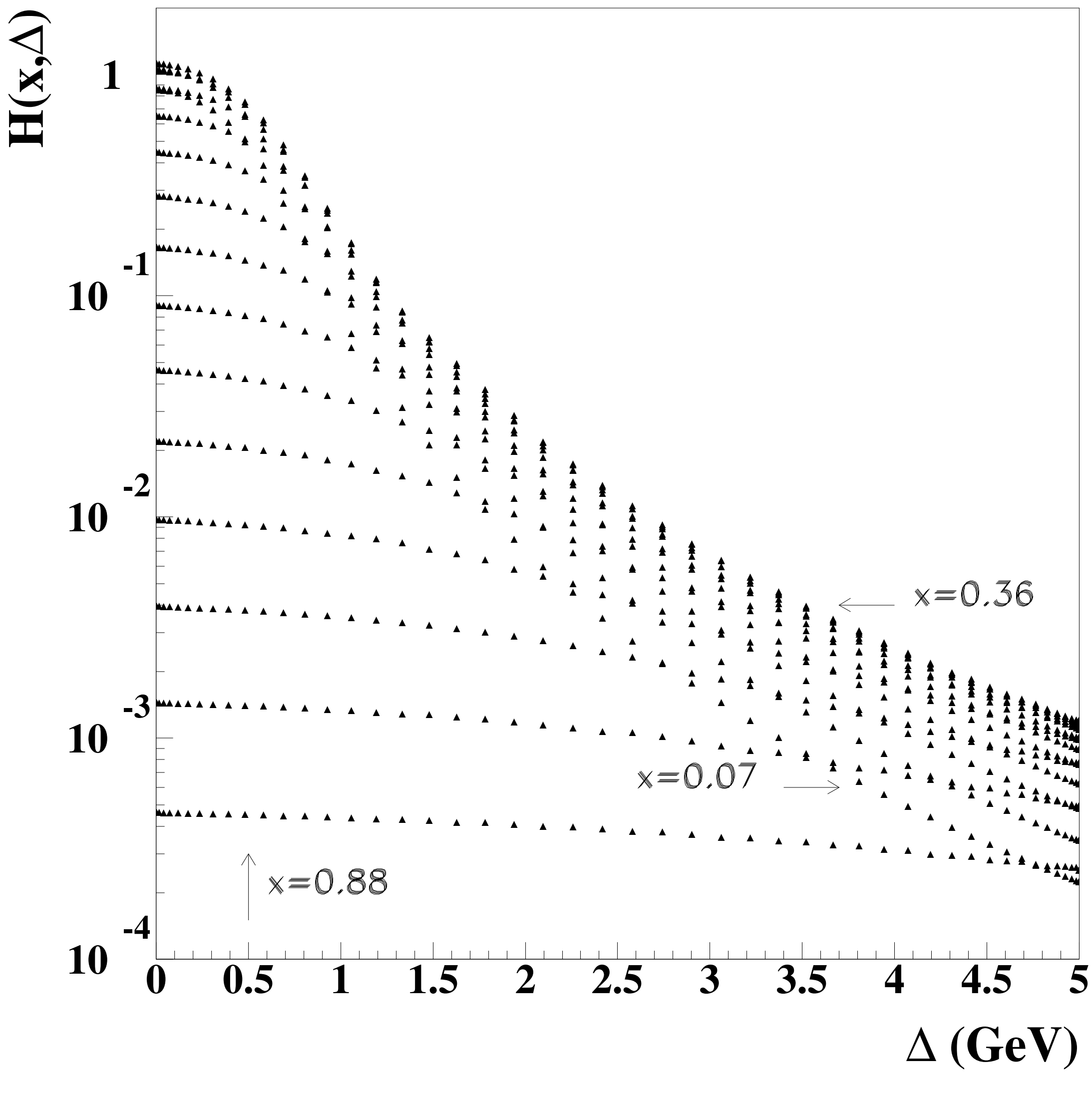
non-diagonal one-body transverse density.

Comparison with existing parametrizations



$$\Phi(\mathbf{k}) = \int dx\, \Phi(x, \mathbf{k})$$

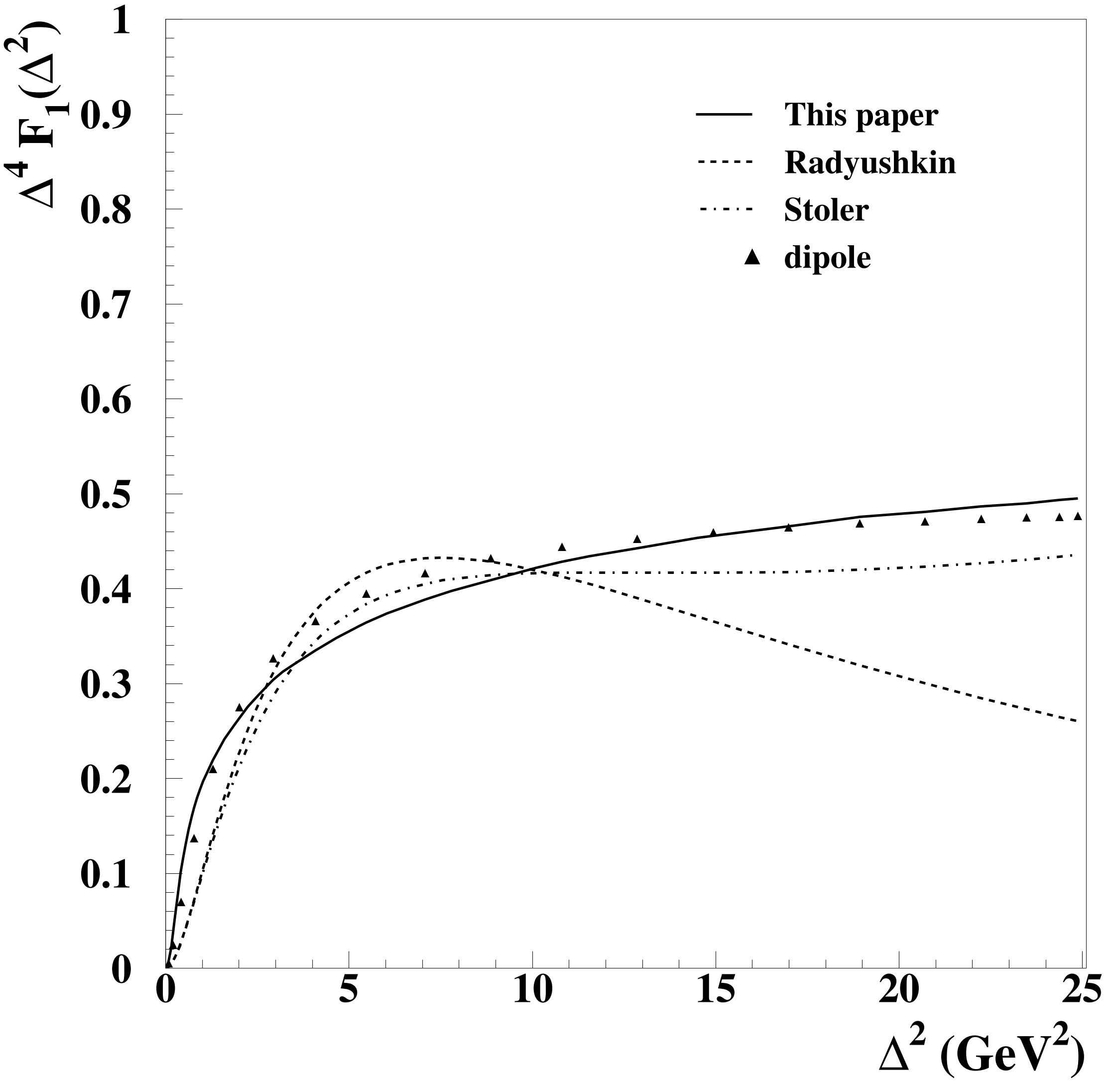
Generalized Parton Distribution



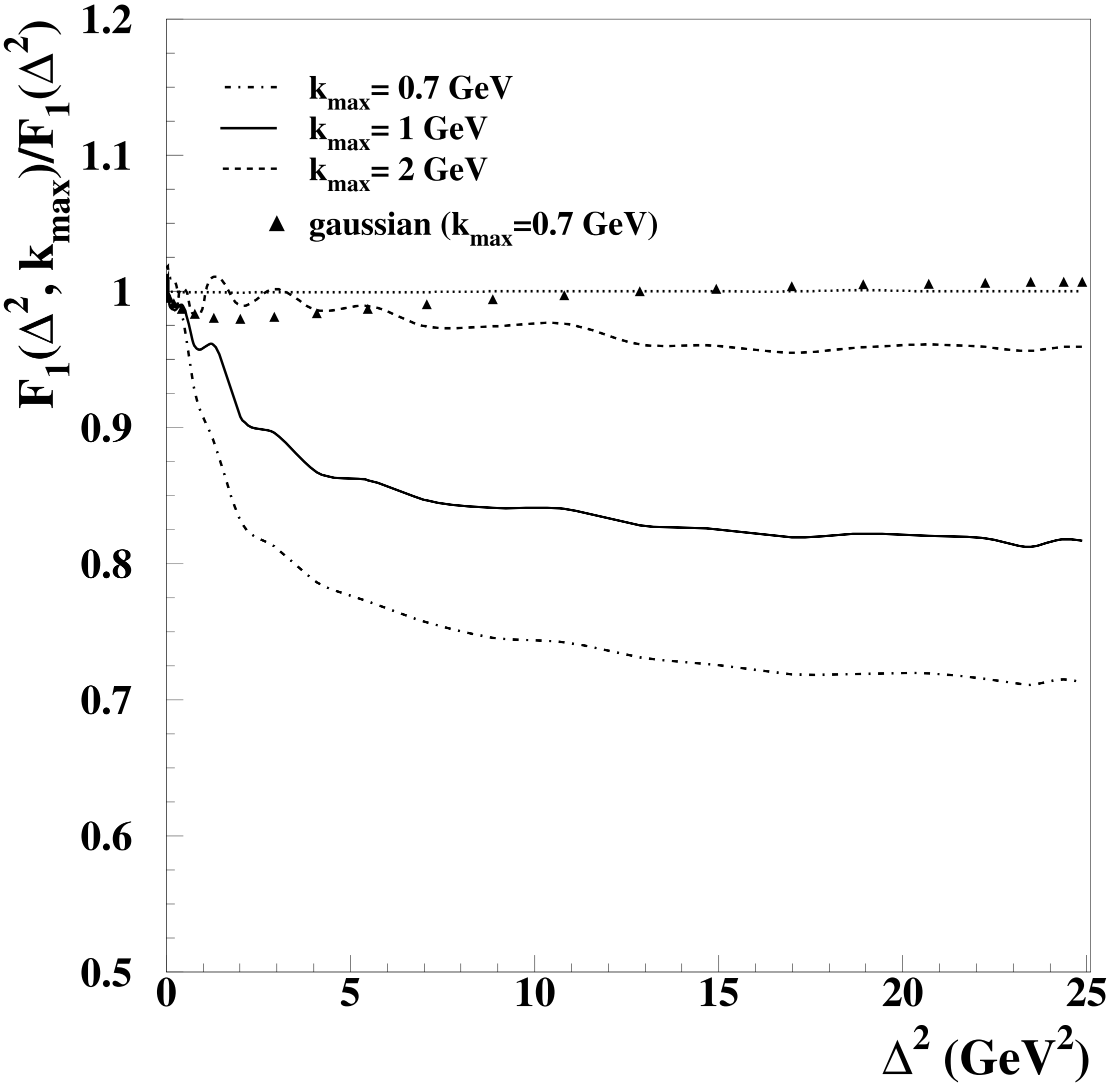
Parametrized as:

$$H(x, -\mathbf{\Delta}^2) = p_1(x)\frac{1}{(p_2(x) + p_3(x)\Delta + p_4(x)\Delta^2)^2}$$

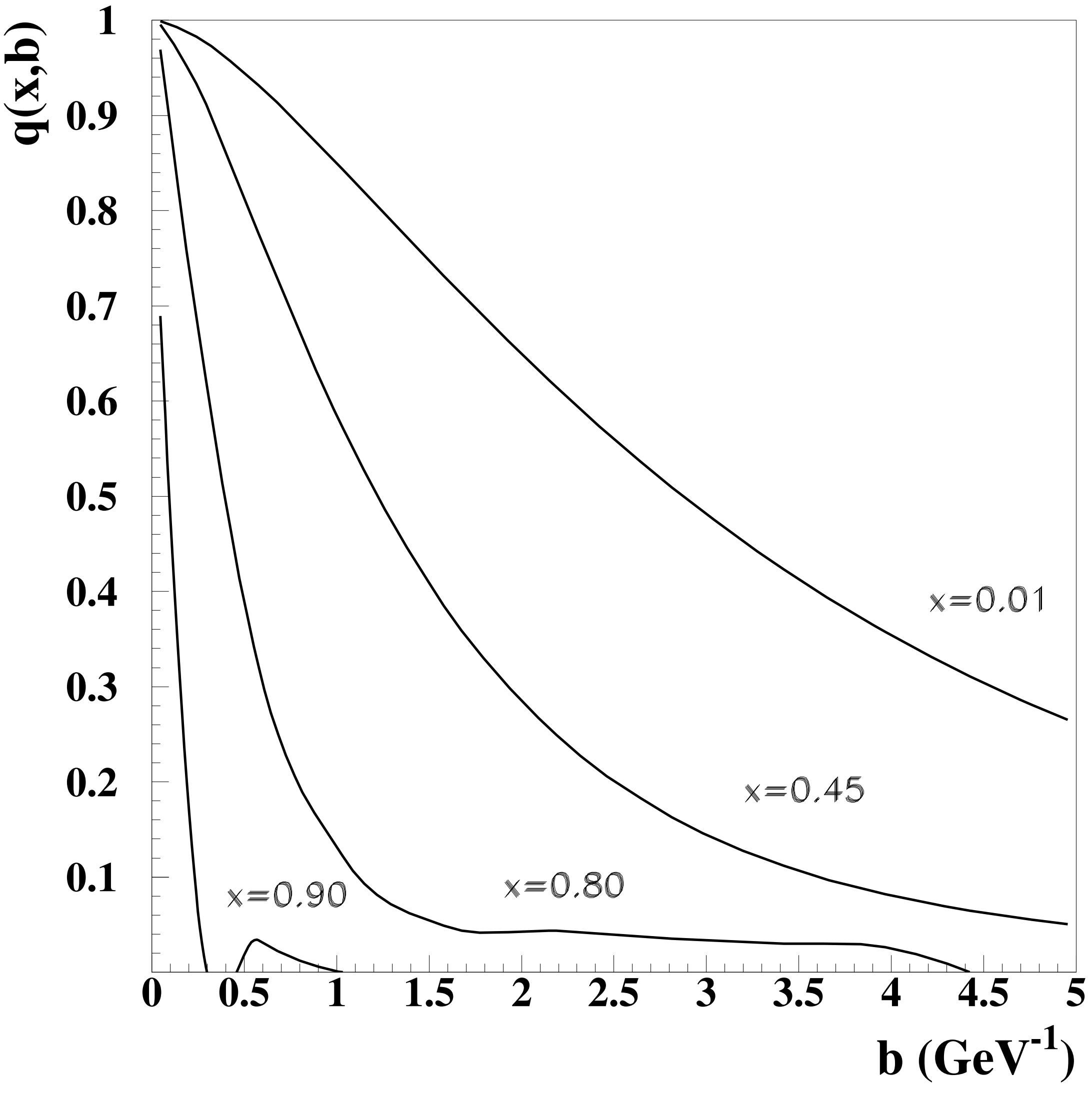
Dirac Form Factor



Dominant $k_\perp$ components in Form Factor

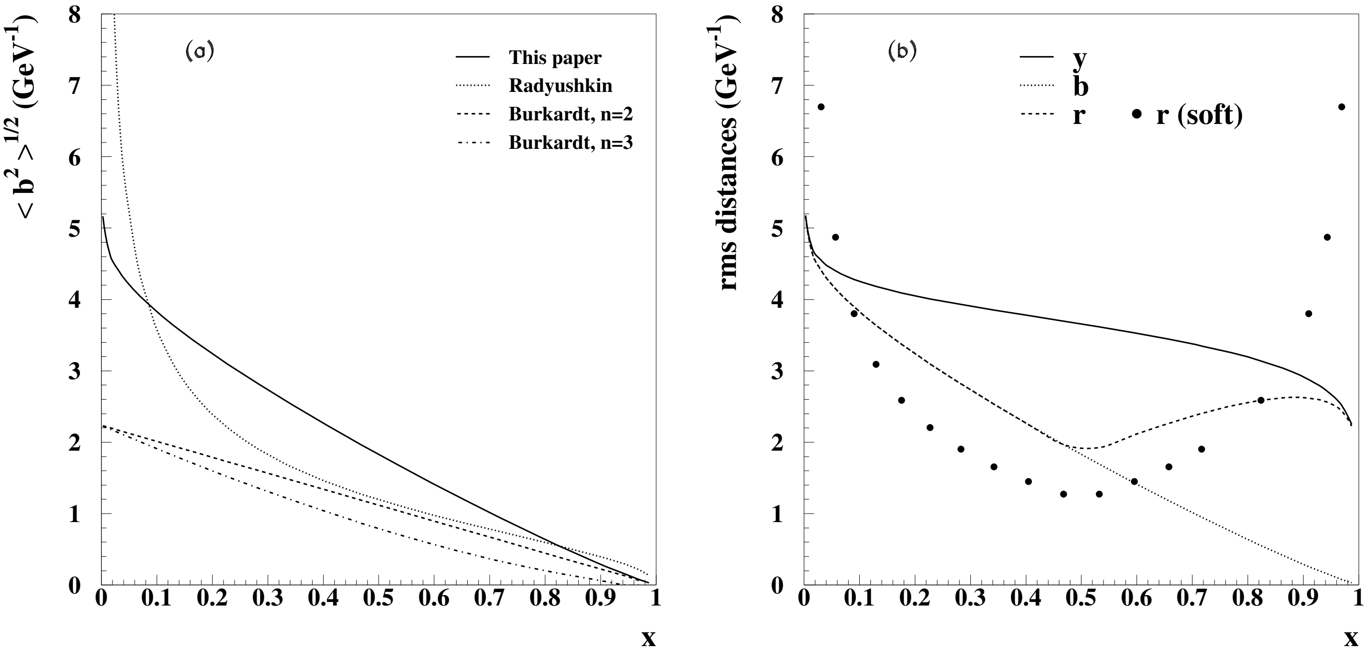


Transverse spatial dependence

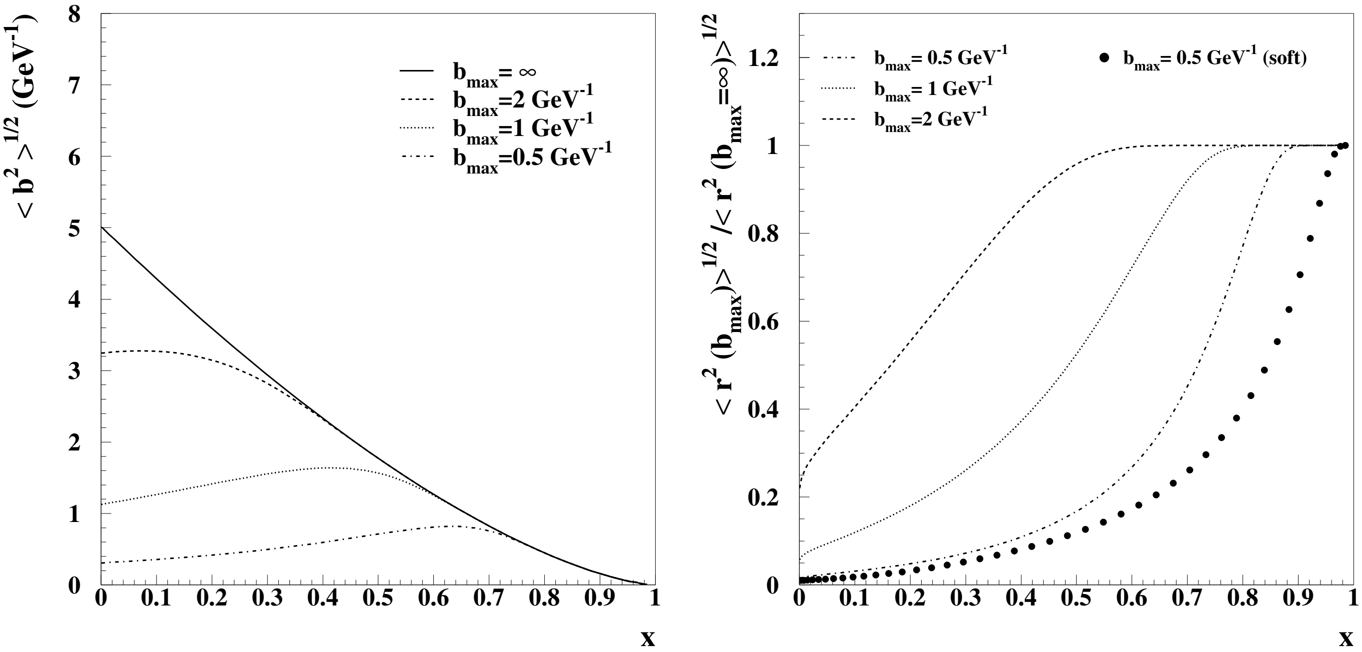


$q(x, \mathbf{b}) \rightarrow$ Hankel transform

Radius



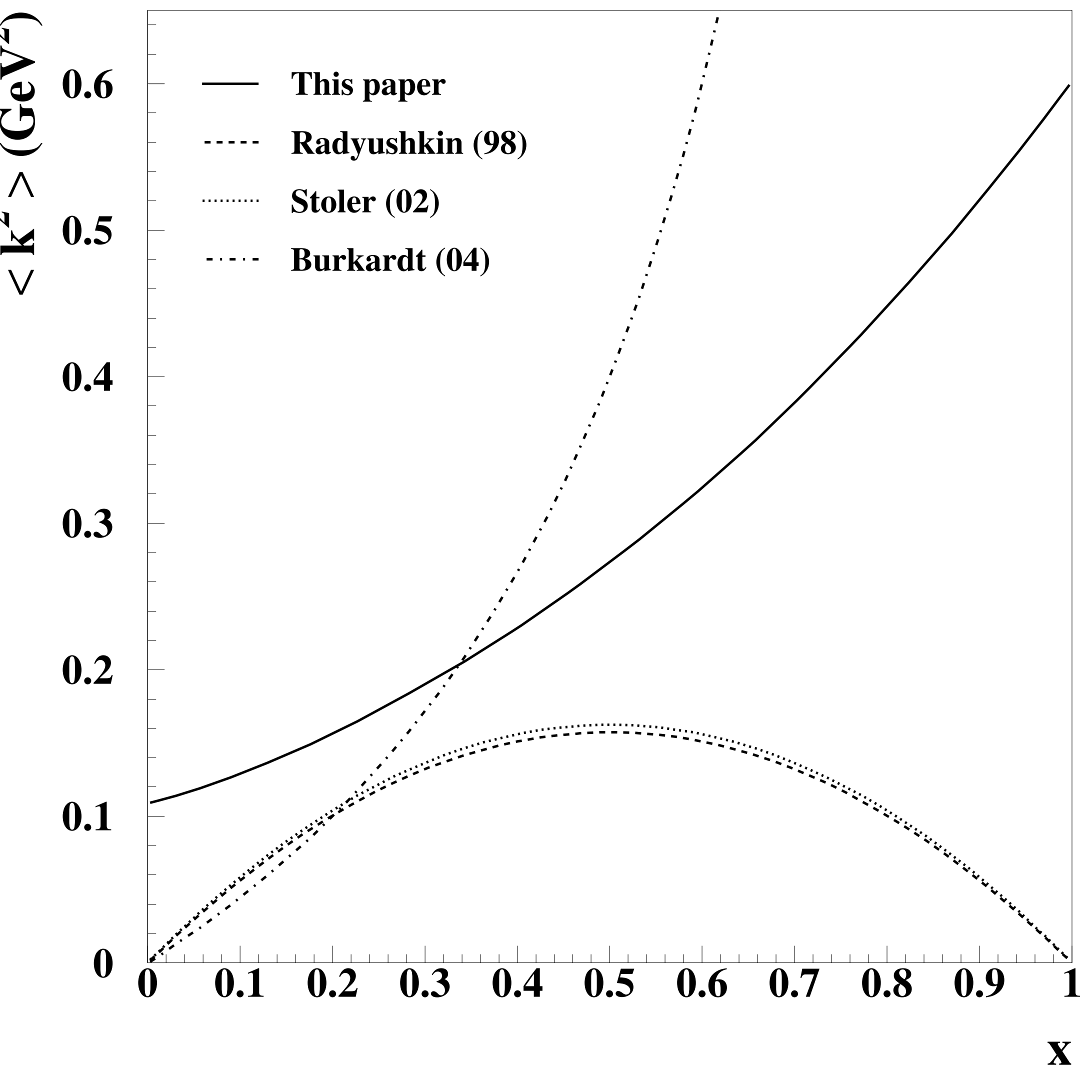
$$r_{rms} = MAX\left\{\sqrt{\langle b^2\rangle},\ \frac{x}{1-x}\sqrt{\langle b^2\rangle}\right\}$$



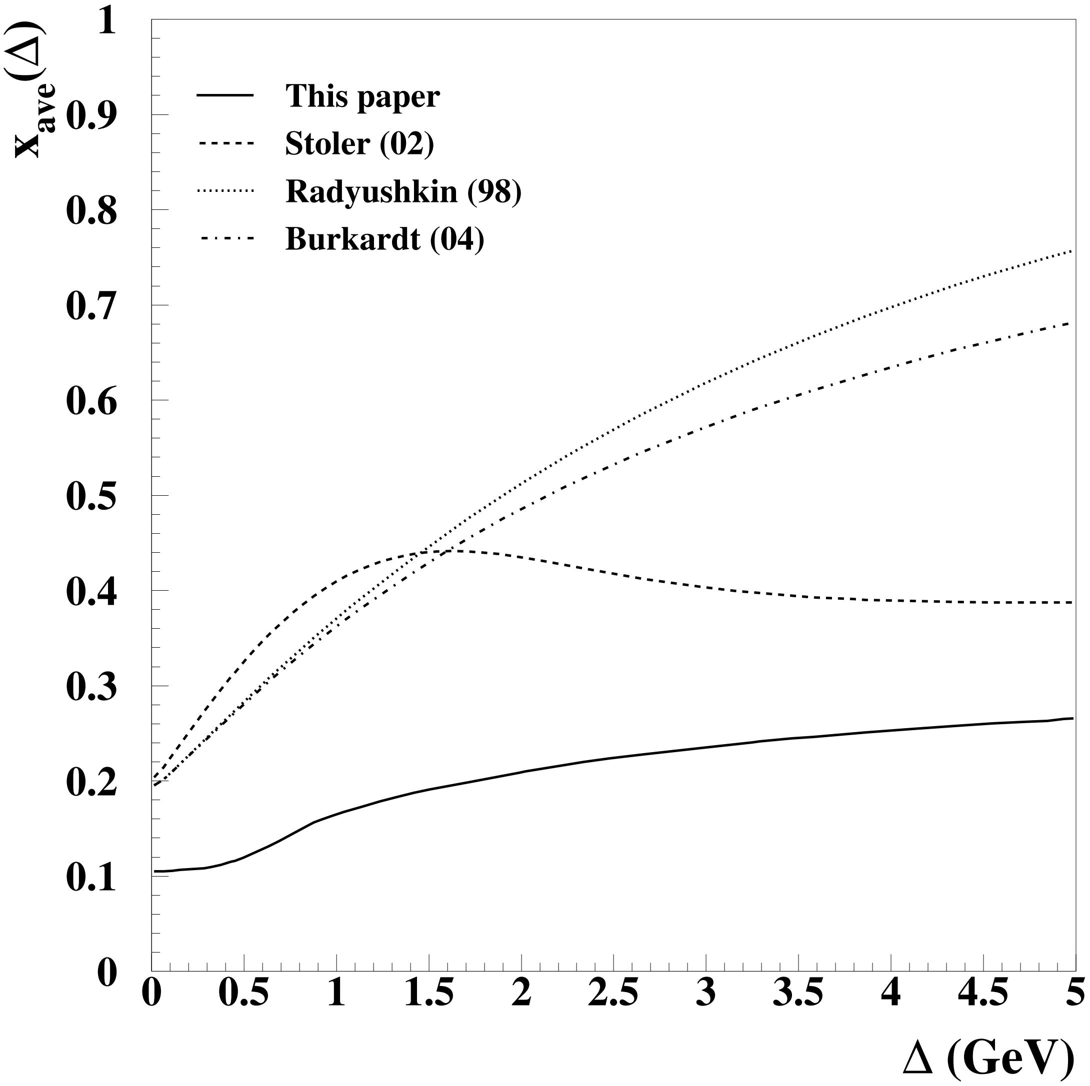
Contribution of different transverse distances to the rms radius $\langle r^2\rangle^{1/2}$.

$$\langle\, \mathbf{b}^2(x)\,\rangle = \frac{\int d^2\mathbf{b}\; q(x,\mathbf{b})\,\mathbf{b}^2}{\int d^2\mathbf{b}\; q(x,\mathbf{b})}$$

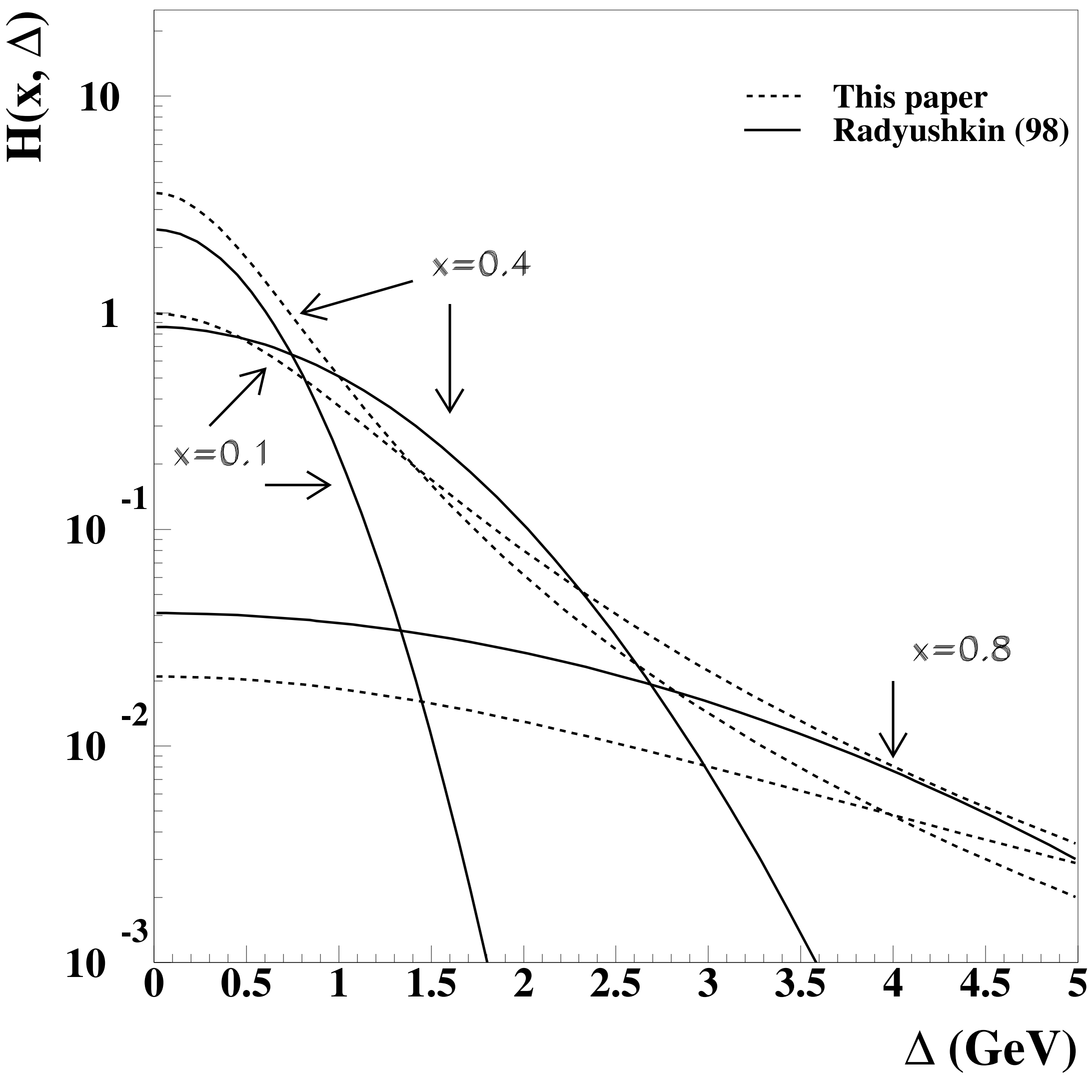
Dominant (transverse) $k_\perp$ components



Dominant (longitudinal) x components



$$x_{ave}(\Delta) = \frac{\int_0^1 dx\, x\, H(x,\Delta)}{\int_0^1 dx H(x,\Delta)}$$


H(x, Δ)
This paper
Radyushkin (98)
x=0.4
x=0.1
x=0.8
10
1
10^{-1}
10^{-2}
10^{-3}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Δ (GeV)

Color Transparency and GPDs

S. Liuti & S.K. Taneja ,Phys. Rev. D 70, 074019

$$T_A(Q^2) = \frac{\left[\int_0^1 dx H_A(x,\Delta)\right]^2}{\left[\int_0^1 dx H(x,\Delta)\right]^2}$$

$$H_A(x,\Delta) = \int_0^{b_{max}(A)} db\, b\, q(x,b) J_0(b\Delta)$$

$b_{max}(A)$ = size of the filter given by a square function

⇒ By assuming

$$q(x,b) = A(x) \exp(-\alpha(x)\, b)$$

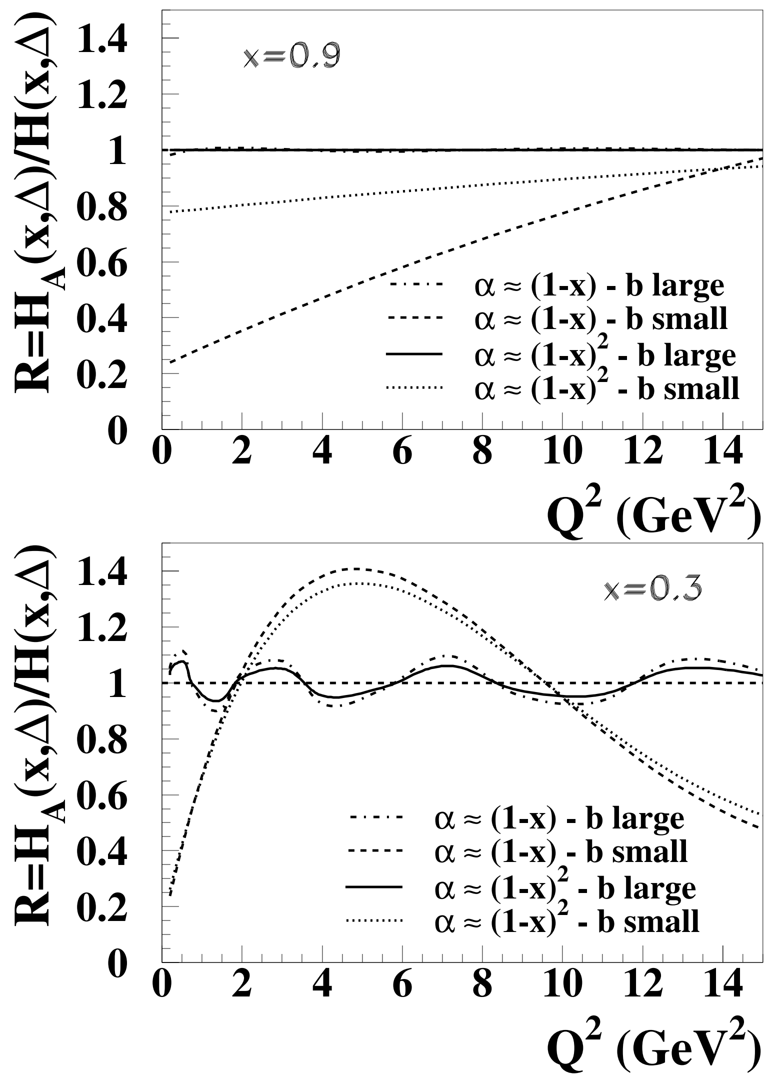
with

i) $\alpha \propto (1-x)/x$ (small $k_\perp$)

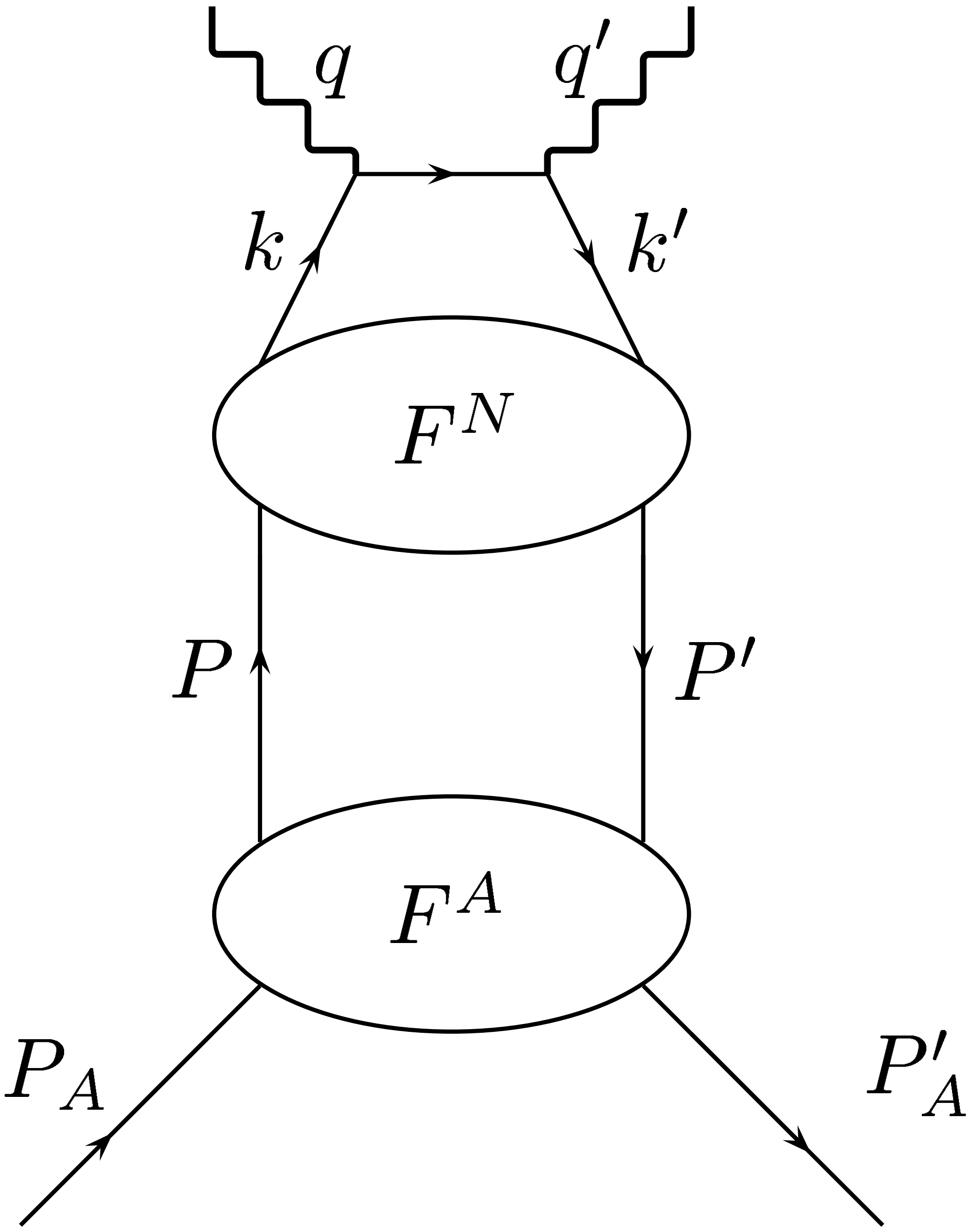
ii) $\alpha \propto (1-x)^2/x$ (large $k_\perp$)

we predict both damping and oscillations of the transparency ratio:

$$\boxed{R = 1 - \exp(-\alpha\, b_{max}) \left[\alpha\, b_{max} J_0(\Delta b_{max}) + \cos(\Delta b_{max})\right]}$$


x=0.9
R=H_A(x,Δ)/H(x,Δ)
α ≈ (1-x) - b large
α ≈ (1-x) - b small
α ≈ (1-x)^2 - b large
α ≈ (1-x)^2 - b small
Q^2 (GeV^2)
x=0.3
R=H_A(x,Δ)/H(x,Δ)
α ≈ (1-x) - b large
α ≈ (1-x) - b small
α ≈ (1-x)^2 - b large
α ≈ (1-x)^2 - b small
Q^2 (GeV^2)

Nuclear GPD



- From the four-momenta of the exclusive process in a nucleus we define the following sets of variables:

  $X_A = k^+/(P_A^+/A)$, $\zeta_A = -\Delta^+/(P_A^+/A)$,

  $X = k^+/P^+$, $\zeta = -\Delta^+/P^+$, $Z = P^+/(P_A^+/A)$

  where, $X = \frac{x+\xi}{1+\xi}$ and $\zeta = \frac{2\xi}{1+\xi}$

- Introducing a nuclear spectral function,ρ_A, and quark spectral function,ρ_N, the nuclear LC distribution function,F^A,and the nuclear GPD,H^A,(for small ζ) are written in terms of their nucleon counterparts,F^N and H^N, as :

Nuclear structure function

$$F^A(X_A,\zeta_A,t) = \int \frac{d^2P_\perp dZ}{2(2\pi)^3}\left(\frac{1}{(A-Z)}\right)\rho_A(Z,P^2,M^2_{A-1},\zeta_A,t)$$

$$\times\overline{F}^N(\tfrac{X_A}{Z},P^2,\zeta,t)$$

$$\overline{F}^N(P^2,X,\zeta,t) = \frac{2-\zeta}{\sqrt{1-\zeta}}\frac{\sqrt{(X-\zeta)X}}{1-X}\int\frac{d^2k_\perp}{(2\pi)^3}\rho_N(P^2,k^2,M^2_X,\zeta,t)$$

Nuclear GPD, H

$$H^A(X_A,\zeta_A,t) = \int \frac{d^2P_\perp dZ}{2(2\pi)^3}\left(\frac{1}{(A-Z)}\right)\rho_A(Z,P^2,M^2_{A-1},\zeta_A,t)$$

$$\times\sqrt{\tfrac{Z-\zeta_A}{Z}}H^N(\tfrac{X_A}{Z},P^2,\zeta,t)$$

$$H^N(P^2,X,\zeta,t) = \frac{2-\zeta}{1-\zeta}\frac{\sqrt{(X-\zeta)X}}{1-X}\int\frac{d^2k_\perp}{(2\pi)^3}\rho_N(P^2,k^2,M^2_X,\zeta,t)$$

Study of the Radius of a Bound Nucleon

ACCOUNT OF OFF-SHELLNESS IS VERY IMPORTANT (AND INTERESTING...!)

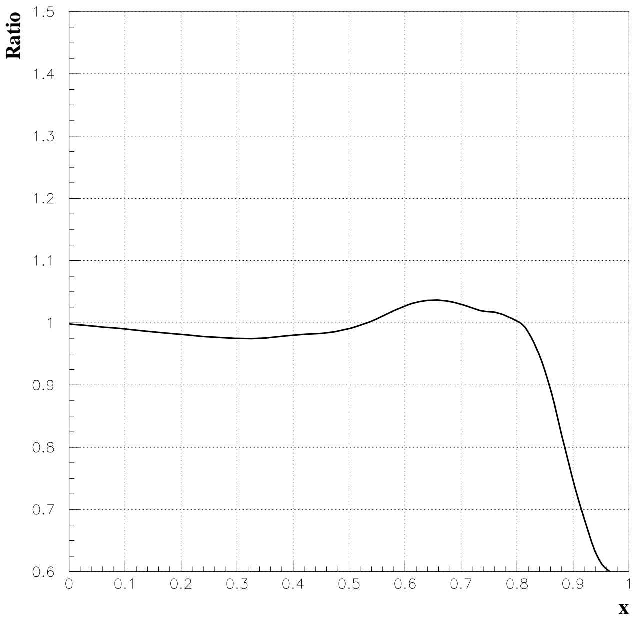
Naive Convolution:

$$\langle b_N^2(x)\rangle_{Bound} = \frac{\int_x^A dz\ \langle b_N^2(x/z)\rangle q_N(x/z) f_A(z)}{\int_x^A dz\ q_N(x/z) f_A(z)}$$

With account of off-shellness:

$$\langle b_N^2(x)\rangle_{Bound} = \left(\int d^2\mathbf{p}_\perp \int_x^A dz\ \langle b_N^2(x/z,p^2)\rangle q_N^{OFF}(x/z,p^2)\rho_A(z,p^2)\right) / \left(\int d^2\mathbf{p}_\perp \int_x^A dz\ q_N^{OFF}(x/z,p^2)\rho_A(z,p^2)\right) \quad (1)$$

Bound nucleon radius as a function of x



Overall modification is $\approx 1\%$

Conclusions and Outlook

- High $k_\perp$ components are responsible for decreasing the hadronic transverse size as x increases.

- This mechanism can be clearly tested by comparing different parametrizations.

- The interplay bewteen $\mathbf{k}_\perp$, x, $\mathbf{b}$ and $\boldsymbol{\Delta}$ determines the size of hadronic configurations $\Rightarrow$ another important role of GPDs.
  S. Liuti & S.K. Taneja ,Phys. Rev. D 70, 074019 hep-ph/0405014

- Yet, direct measurements of GPDs are a very slow process.

- Use CT phenomena, specifically Nuclear Filtering to extract information.

- Nuclear GPD and the size of bound nucleon is calculated.

- Experimental test of the above predictions could be the new direction for the 12 GeV upgrade.