Ways of introducing coupled channels

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- Meson Spectroscopy with the Crystal–Barrel Detector
  1. Data sets with different initial states but the same final state particles
  2. Data sets with particles belonging to the same isospin multiplet
  3. Data sets with different final states
- Baryon Spectroscopy with CB–ELSA and CLAS
  – Photoproduction of two–pion final states
- Comparing ways of introducing coupled channels
  – CMB model, K–matrix, Breit–Wigner models

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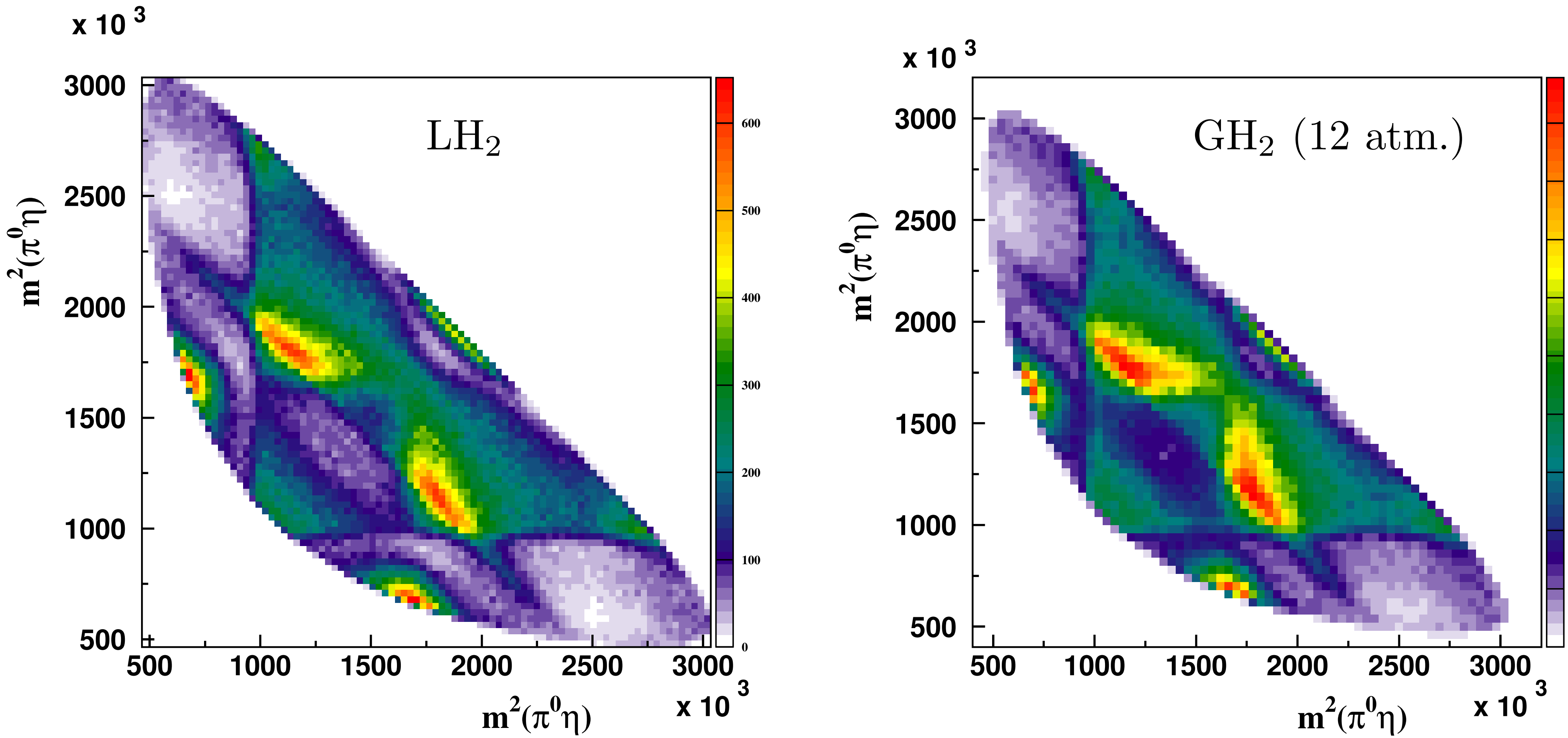
1) Different Initial States → Same Final State Particles



Example: $\bar{\mathrm{p}}\mathrm{p} \to \pi^0\,\pi^0\,\eta$

- Probability for S– and P–wave annihilation depends on target pressure
  ⇒ Dalitz plot changes with pressure

1) Different Initial States $\rightarrow$ Same Final State Particles

In single–channel analysis of LH_2 data set, P–wave annihilation was neglected

(Crystal Barrel Collaboration, C. Amsler et al., Phys. Lett. **B333** (1994) 277)

Why ? Inclusion leads to dramatic increase of number of parameters in the fit

$\Rightarrow$ often leads to unphysical solutions

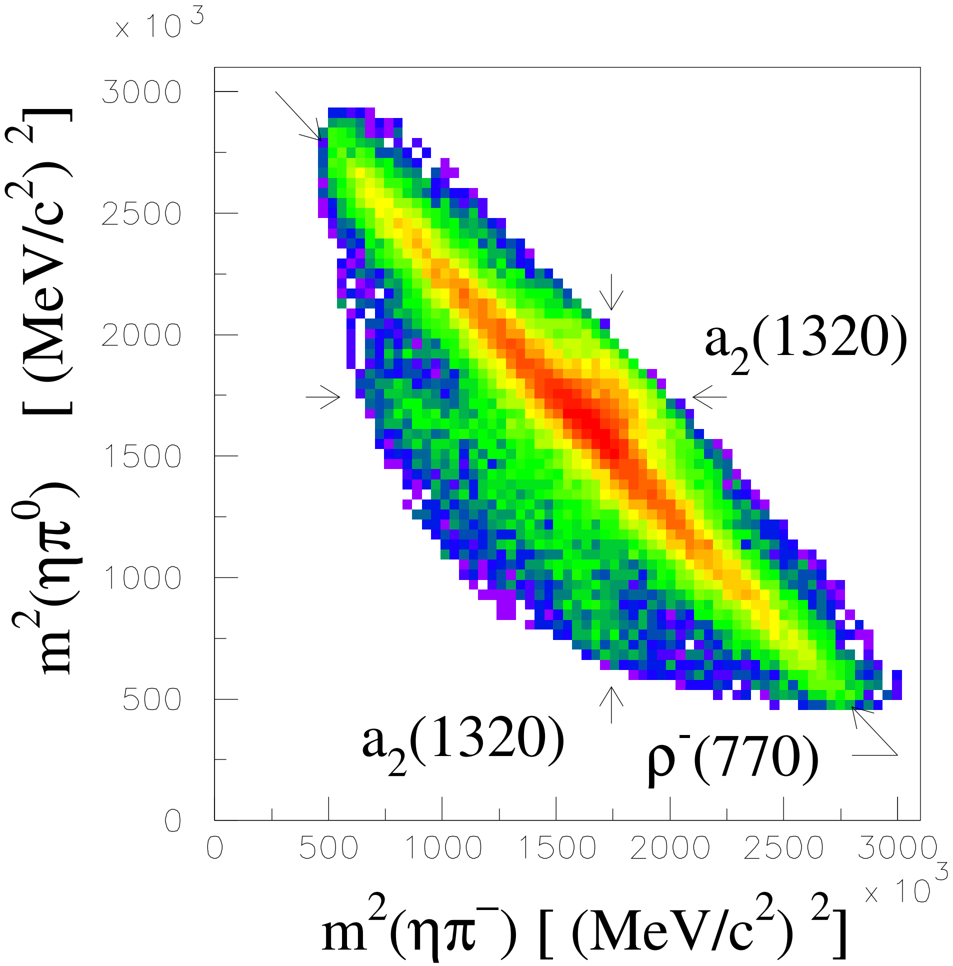
Solution: Adding gaseous hydrogen data yields additional information

$\Rightarrow$ Inclusion of P–wave annihilation possible

- Amplitudes for specific initial state remain the same for $LH_2 \rightarrow GH_2$
  $\Rightarrow$ Relative fraction of the different initial states changes
- Relative ratios for S– and P–wave annihilation taken from cascade models

Coupled–channel analysis supports evidence for the exotic state $\pi_1(1400)$ observed before in $\overline{\mathrm{p}}\mathrm{d} \rightarrow \pi^-\pi^0\eta\,\mathrm{p}_{\,\mathrm{spectator}}$

Observation of an exotic $J^{PC} = 1^{-+}$ state



Most prominent candidate for an exotic hybrid state: $\pi_1(1400)$

52567 events in the Dalitz plot

$$PWA \begin{cases} {}^3S_1 & \rho^-(770),\ \rho^-(1450),\ a_2(1320) \\ {}^1P_1 & a_0(980),\ a_0(1450) \end{cases}$$

$\Rightarrow$ confirmation of exotic state in analysis of $\bar{p}p \rightarrow \pi^0 \boxed{\pi^0\eta}$

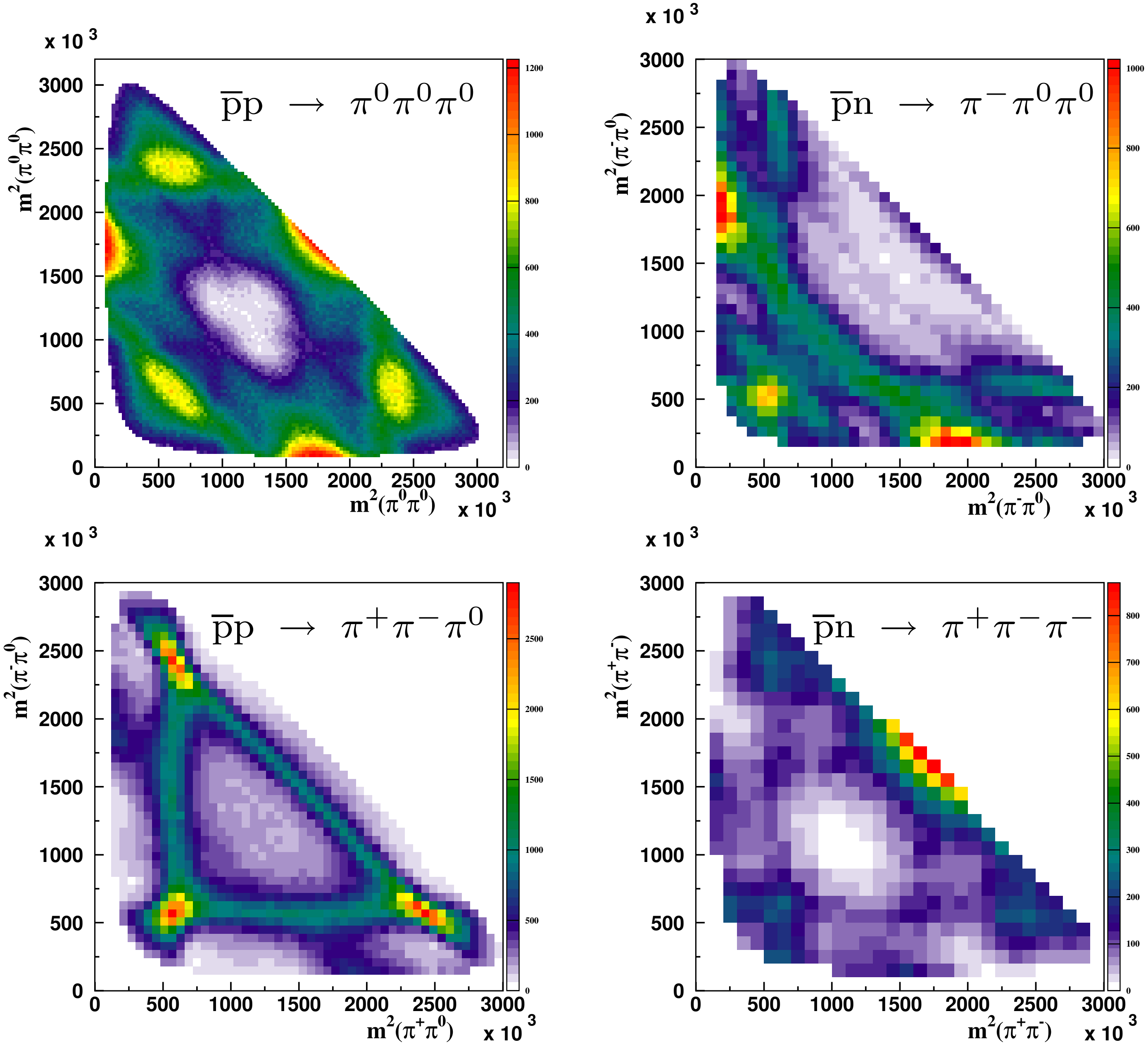
(PhD thesis Mario Herz, Bonn 1997)

Dalitz plot from: Abele et al., Phys.Lett. **B423** (1998) 175

Exotic $\eta\pi$ state in $\bar{p}d$ annihilation at rest into $\pi^-\pi^0\eta\, p_{\text{spectator}}$

in agreement with D.R. Thompson et al. (E852 collaboration)

2) Isospin Coupling Analyses: $\bar{\mathrm{p}}$N Annihilation into 3π



Additional constraints by combining data sets with particles belonging to the same isomultiplet

- isospin weights
- different interference effects

Similar to what Curtis pointed out for $\gamma p \rightarrow p\,4\pi$

- However, GlueX much more complicated!

2) Isospin Coupling Analyses: $\overline{\mathrm{p}}\mathrm{N}$ Annihilation into 3π

Three of these Dalitz plots have been part of a coupled–channel analysis (still in preliminary state):

- Restriction of $\overline{\mathrm{p}}\mathrm{n}$ to $I = 1$ initial states
  $\Rightarrow$ ${}^1S_0$, ${}^3P_1$, and ${}^3P_2$
- In $\overline{\mathrm{p}}\mathrm{p} \;\rightarrow\; \pi^+\pi^-\pi^0$, ${}^3S_1$ and ${}^1P_1$ also possible
- Amplitudes for annihilation can be coupled by the following isospin relations

$$A_{(\pi^+\pi^-)-S-wave} = -\sqrt{2} \cdot A_{(\pi^0\pi^0)-S-wave}$$

$$A_{(\pi^+\pi^-)-P-wave} = \quad +1 \cdot A_{(\pi^-\pi^0)-P-wave}$$

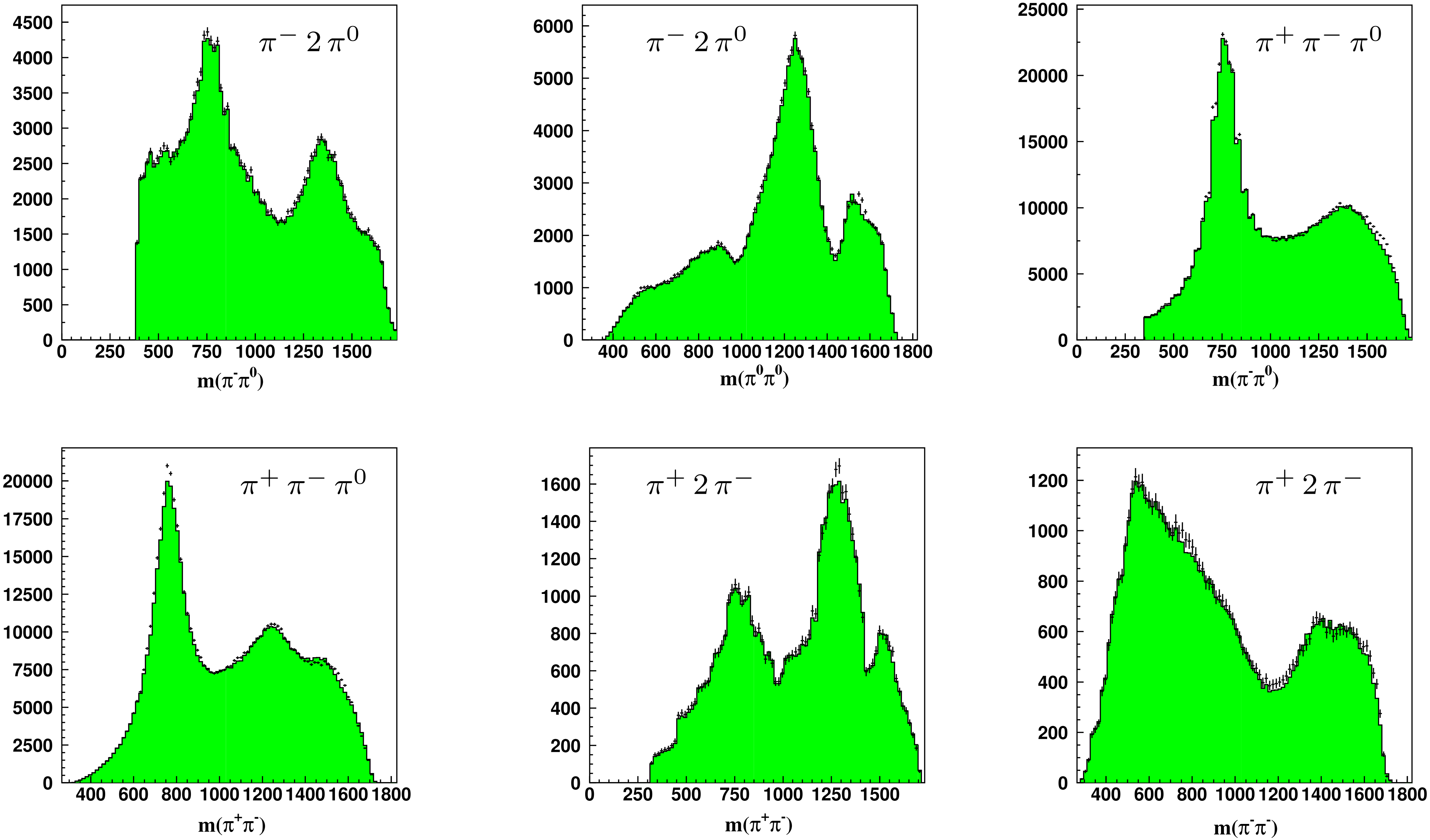
$$A_{(\pi^+\pi^-)-D-wave} = -\sqrt{2} \cdot A_{(\pi^0\pi^0)-D-wave}$$

- Overall scaling factor in order to account for different contributions from initial S– and P–wave annihilation in LH_2 and LD_2

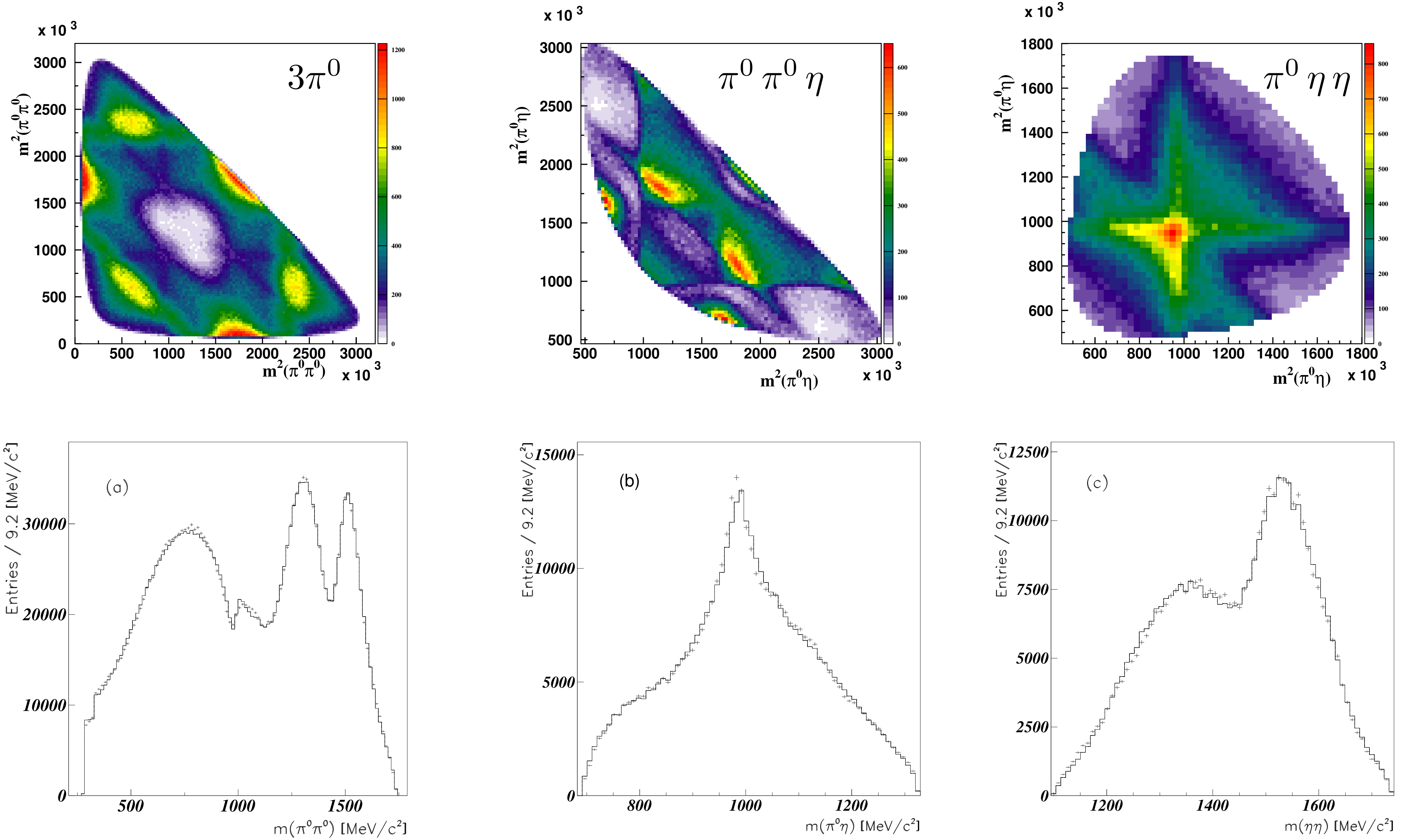
$\Rightarrow$ Data are reasonably well described!

(Parameters of the higher–mass vectors determined: $\rho(770)$, $\rho(1450)$, $\rho(1700)$)

2) Isospin Coupling Analyses: $\bar{p}N$ Annihilation into 3π



3) Different Final States



3) Different Final States

Analysis by Crystal Barrel including $\overline{\rm p}{\rm p} \to 3\,\pi^0$, $2\,\pi^0\,\eta$, $\pi^0\,2\,\eta$
+ CERN–Munich scattering data

(Crystal Barrel Collaboration, C. Amsler et al., Phys. Lett. **B355** (1995) 425)

- Investigation of resonances in different decay modes
  $\Rightarrow$ Production of resonance with certain spectator does not depend on decay
- Masses, widths, and production strengths of resonances must be the same in all different channels
- Measurement of resonance coupling to one channel automatically determines corresponding inelasticity in the other channels
- 0^{++} – wave described by using 3 x 3 K–matrix including couplings to the $\pi^0\pi^0$, $\eta\eta$, and $\rm K\overline{K}$ channel
  $\Rightarrow$ Latter parameterizes inelasticity into all other open channels because no corresponding data was included in the analysis
  $\Rightarrow$ Allowed determination of couplings of $\rm f_0(1500)$ to $\eta\,\eta$ relative to $\pi^0\,\pi^0$

The technique of Partial Wave Analysis

Measured intensity (incoherent sum over all possible $\overline{\mathrm{p}}\mathrm{N}$ initial states):

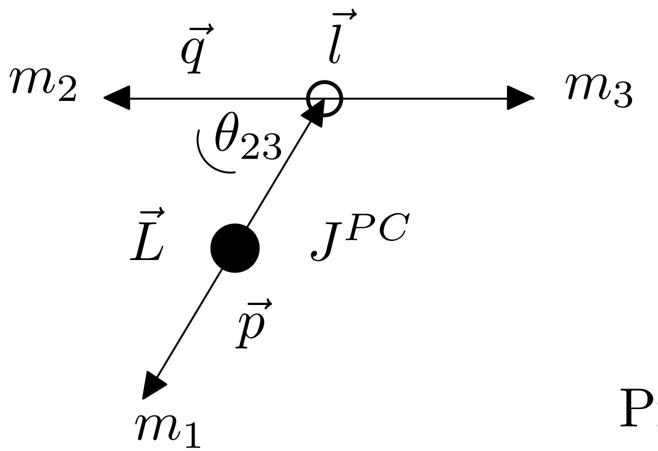
$$I = \sum_{J^{PC}(\overline{\mathrm{p}}\mathrm{N})} \underbrace{|A_{J^{PC}}|^2}$$

$B_L(p)$: centrifugal barrier of initial decay

$$A_{J^{PC}} = \sum_{a}^{\text{possible amplitude}} \left(\sum_{i}^{\text{combinations}} \mathrm{CG}_i \cdot H_{J^{PC},L,l}(\Theta) \cdot B_L(p) \cdot \hat{F}_l(q) \right)$$

Angular dependence in terms of helicity amplitudes

$$H_{\lambda_1\lambda_2,M}(\theta,\phi) = D^J_{\lambda M}(\theta,\phi) \sum_{ls} \alpha_{ls} \, \langle J\lambda | ls0\lambda \rangle \, \langle s\lambda | s_1 s_2 \lambda_1, -\lambda_2 \rangle$$



In simplest case, $\hat{F}$ given as Breit–Wigner function

$\Rightarrow$ Parametrization of $\hat{F}$ in K–matrix formalism

$$\hat{F} = \underbrace{(I - i\hat{K}\rho)^{-1}}_{\text{Propagation and decay}} \quad \underbrace{\hat{P}}_{\text{Production}}$$

Summary: Meson Spectroscopy

Procedure worked very well for Crystal Barrel data

⇒ Isobar model: series of successive two–body decays

Effects from

- direct three–body production
- t–channel exchange processes
- rescattering in the final state

are neglected!

⇒ One has to be careful concerning the interpretations of broad poles found in the PWA!

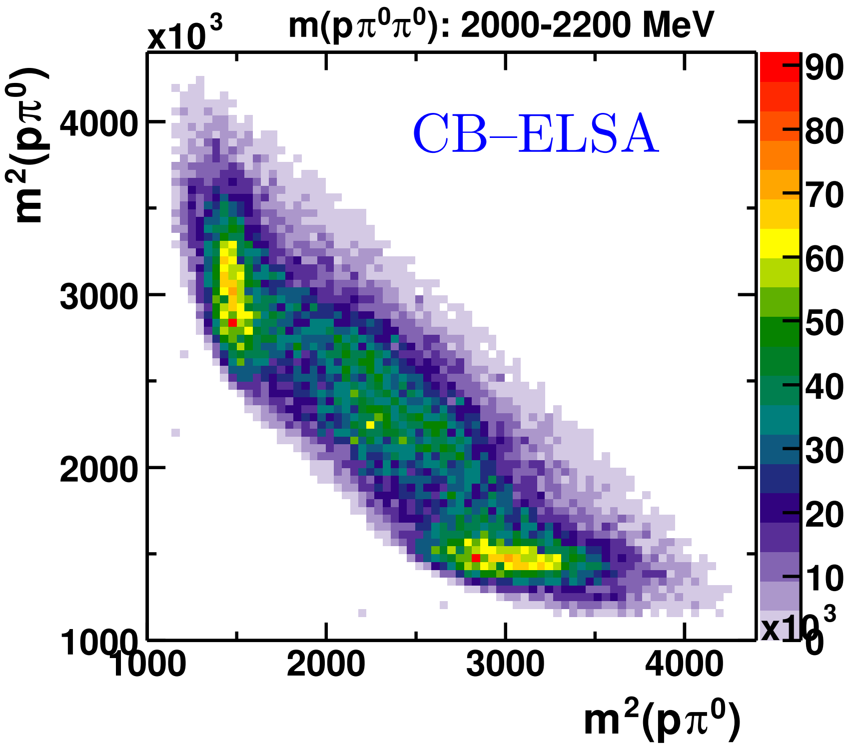
What's the applicability for the CB model?

- Radiative J/ψ decays and D decays, e.g. $D \rightarrow K\pi\pi$ (CLEO data) (✓)
- GlueX → would have to be extended! Much more complicated!
- What about baryons?

Investigation of $\pi\pi$ Photoproduction at ELSA and CLAS $\Rightarrow$ Complementary Detectors !

3–body final states are key for the discovery of *missing states* !

- Account for most of the cross section above $W \approx 1.7$ GeV
- 2–body final states largely explored $\Rightarrow$ No new states definitely found !
- Preliminary results: a) Polarization is key to unambiguous interpretation !



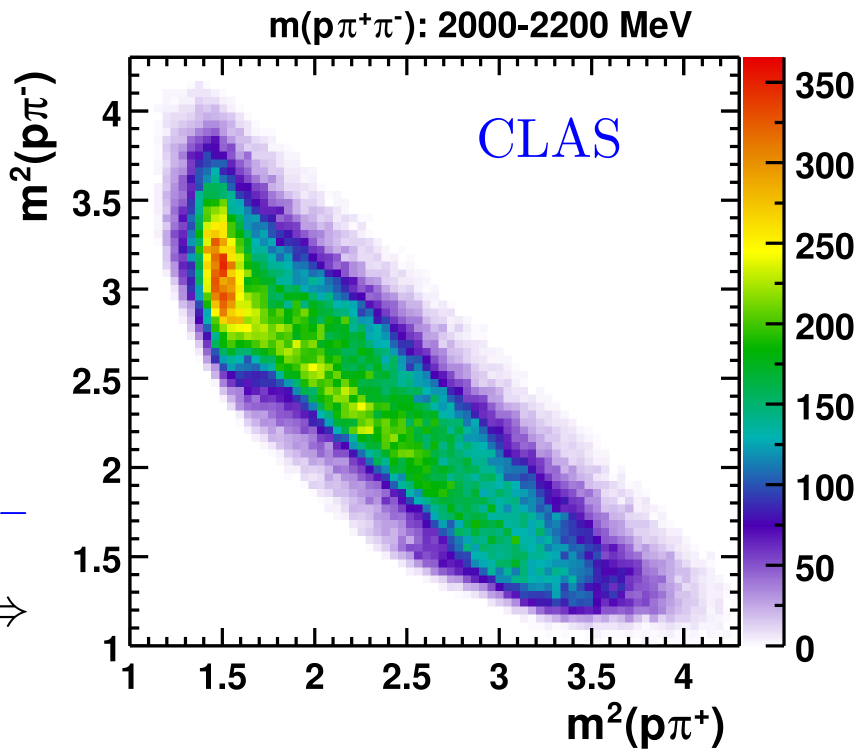
b)

$\gamma\, p \;\rightarrow\; p\, \pi^0\, \pi^0$

$\Leftarrow$

$\gamma\, p \;\rightarrow\; p\, \pi^+\, \pi^-$

$\Rightarrow$



$$\boxed{\begin{aligned}\gamma\mathbf{p} \longrightarrow\ & \Delta\pi^0 \\ & \Delta \longrightarrow \mathbf{p}\pi^0\end{aligned}}$$

Advantages of $\pi^0\pi^0$ photoproduction:

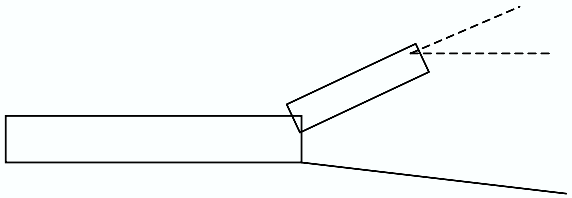
- No ρ - amplitudes
  - No diffractive ρ - production
  - Reduces the number of possible N^* decay amplitudes
- No direct $\gamma p \rightarrow \Delta^{++}\pi^-$ production (strong in $\gamma p \rightarrow \pi^+\pi^- p$)

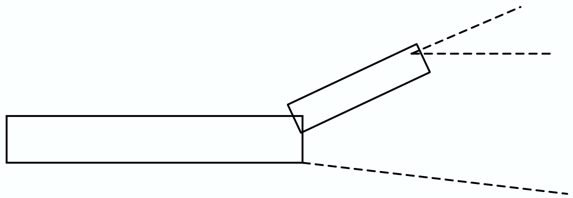
Advantages of $\gamma\, p \; \longrightarrow \; p\, \pi^+\, \pi^-$:

- Provides additional information on $N\rho$

$\Rightarrow$ Idea: Combined analysis of both channels !

Rescattering, resonance mixing

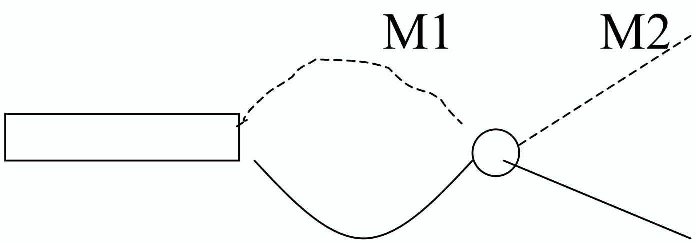


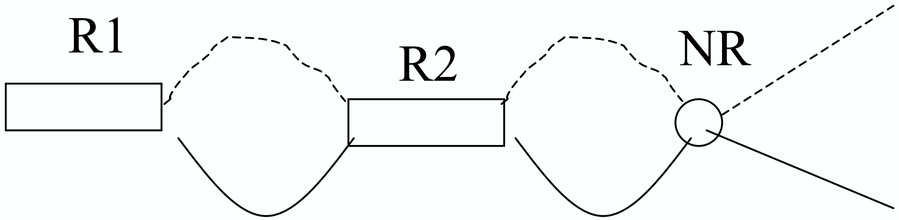


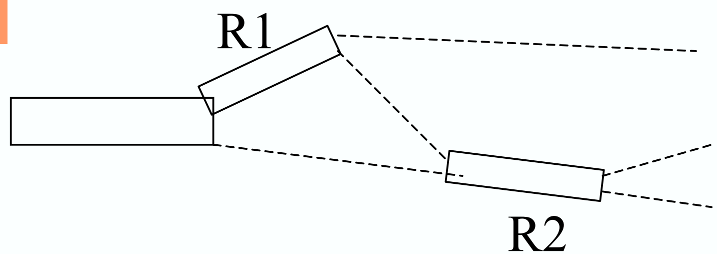
Baryon decay (BW) with resonant final state, e.g. N*→ρp

Meson decay (BW) with resonant final state, e.g. D→ρπ

Looks too simple, what about these?







Also possible, in fact likely, called rescattering and coupled channels effects. Can happen many times!

Definitions

Rescattering:
When different states (same Q, Jπ) couple to each other with similar strength, any propagation must be a quantum–mechanical mixture of all states!
(e.g. ρK $\rightarrow$ K$^*\pi$)

Likely to be important for D decays

Possibly important for D decays

Coupled Channel:
If different final states are possible (same Q, Jπ), they must be treated on common basis (matrix)
(e.g. D $\rightarrow$ K$^{*-}\pi^+$ $\rightarrow$ K$^-$ π^0 π^+ versus
D $\rightarrow$ K$^{*-}\pi^+$ $\rightarrow$ K^0_s π^- π^+)

How to analyse the data: Available Tools (models)

Two steps:

1. Partial wave decomposition (analysis):
   $\Rightarrow$ Helicity Formalism, Covariant Formalism (operator formalism), etc.
2. Determination of resonance properties

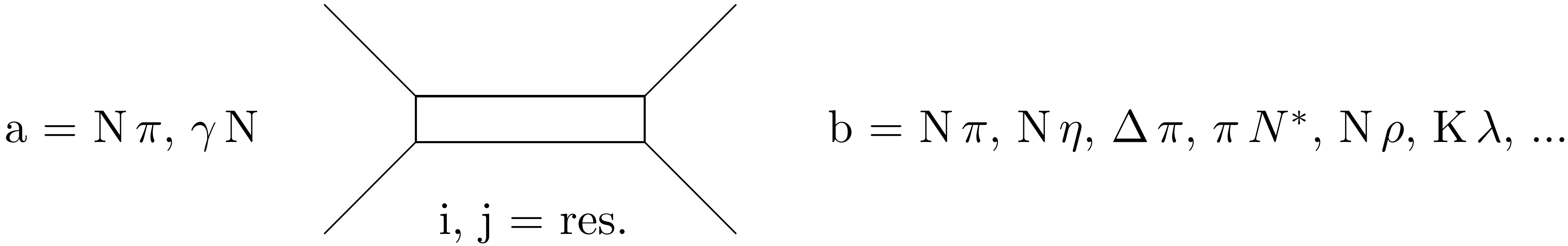
Model/Effect	Rescattering	Coupled Channel
Breit–Wigner	Rudimentary	None
K–matrix	Good	Very good
CMB	Excellent	Excellent

Is this important?

$\Rightarrow$ Rescattering and coupled–channel effects known to be important for baryons !

CMB model for baryons: *scattering amplitude*

$$T_{ab}^{CMB}(s) = \sum_{i,\,j} \sqrt{\rho_a(s)}\, f_a(s)\, \gamma_{a\,i}\, G_{i\,j}(s)\, \gamma_{j\,b}\, f_b \sqrt{\rho_b(s)}$$



- $\gamma_{a\,i}$ is (real) coupling between a (e.g. $\mathrm{N}\,\pi$) and i (e.g. $\mathrm{S}_{11}(1535)$)
- ρ_a is phase space for final state a
- f_a is a form factor (fixed, empirical)
- $\mathrm{G}_{\,i\,j}$ has all the action, full rescattering
- Every model has these elements in some form !

See Phys. Repts. 328, 181 (2000) for detailed discussion

CMB model for baryons: *scattering amplitude*

$$G_{ij} = G^0_{i\,j} + \sum_{k,\,l} G^0_{i\,k}\,\Sigma_{k\,l}\,G_{l\,j}$$

$$= G^0_{i\,j} + \sum_{k,\,l} G^0_{i\,k}\,\Sigma_{k\,l}\,G^0_{i\,j} + \sum_{k,\,l,\,m,\,n} G^0_{i\,k}\,\Sigma_{k\,l}\,G^0_{l\,m}\,\Sigma_{m\,n}\,G^0_{n\,j} + \ldots$$

$G^0_{i\,j} = \frac{\epsilon_i\,\delta_{ij}}{s - s_{0,i}}$

Bare pole,
$\epsilon_i = -1$ for resonance

$\Sigma_{k\,l} = \sum_c \gamma_{k\,c}\,\phi_c\,\gamma_{c\,l}$

Dyson equation
generates width !!

- Fitting constants s_0 and $\gamma_{i\,a}$
  $\Rightarrow$ 1 bare mass for every resonance and one coupling constant for each open channel (e.g. $N\pi$)
- For baryons, up to 9 constants per resonance (many)
- Unitarity for 2–body and quasi–2–body final states
- Analyticity through dispersion relations

Relationship of CMB to K–matrix model

$$T_{ab}^{CMB}(s) = \sum_{i,j} \sqrt{\rho_a(s)}\, f_a(s)\, \gamma_{a\,i}\, G_{i\,j}(s)\, \gamma_{j\,b}\, f_b \sqrt{\rho_b(s)}$$

$$K^{\,i}_{ab}(s) = \sqrt{\rho_a(s)}\, f_a(s)\, \gamma_{a\,i}\, G^0_{i\,j}(s)\, \gamma_{j\,b}\, f_b \sqrt{\rho_b(s)}$$

$$K_{a\,b} = \sum_{i\,\in\,res} K^i_{a\,b}\, K^{\text{nonres}}_{a\,b}$$

K$_{ab}$ is a real quantity,
condition for unitarity;
nonres is issue for N*

$$T_{a\,b} = \frac{K_{a\,b}}{1-iK_{a\,b}}$$

This does the rescattering,
well proven to include only on–shell pieces.
This cannot have full analytic description.

N.B. This is very similar to Chung, Klempt (Z. Phys. (1995))

Case study: S_{11} states near $N\eta$ threshold

Notation: $L_{2\,I,2\,J}$, where

L = orbital angular momentum as if $N\pi$ (e.g. S, P, D, etc.)

I = isospin of N^* resonance (I = 1/2, 3/2)

J = total angular momentum of N^* (J = 1/2, 3/2, 5/2, etc.)

For this reason, S_{11} is an $L = 0$ state with I = J = 1/2, πarity = –

- S_{11} refers to L = 0 in the $N\pi$ system
- P–wave excitation of 1 quark in the NRQM

2 states, $S_{11}(1535)$ and $S_{11}(1650)$, and each couples mostly to $N\,\pi$ and $N\,\eta$!

What's the nature of the $S_{11}(1535)$?

Why does it decay strongly into N η and $S_{11}(1650)$ does not ?

- Two states S_{11} have appreciable mixing ($\approx$ 30°)
  (N. Isgur and G. Karl, Phys. Lett. **72B** (1977) 109.)
- Phenomenological fit to baryon decays ($\approx$ 30°)
- Coupled ΣK-pη effect (Kaiser, Siegel and Weise)
  $\Rightarrow$ No genuine 3-quark resonance required
- Amplitude analysis (G. Hoehler)
  $\Rightarrow$ No pole is needed for N(1535)S_{11}
- Quark-diquark structure (Glozman and Riska)

$\Rightarrow$ Extraction of resonance properties important !

Interpretation of $\mathbf{S}_{11}(1535)$

FIT(1535)	Γ_{full}(MeV)	$\mathrm{BR}_{N\pi}$	$A_{1/2}$	comment
VPI(96)	105	0.31	60 ± 15	$N\pi \to N\pi$
Drechsel(99)	80	0.40	67	$\gamma p \to p\pi$
Krusche(97)	212	0.45	120 ± 20	$\gamma p \to p\eta$
Pitt–ANL(00)	126	0.34	87 ± 3	All
PDG	100 – 200	0.35 – 0.55	90 ± 30	Averaging

1. If we use $N\pi$ or $N\eta$ data, we get different answers!!
2. If we use coupled-channel model, we get intermediate result.

We expect rescattering and resonance interference to matter!

- Since $S_{11}(1535)$ ($\Gamma = 130$MeV) and $S_{11}(1650)$ ($\Gamma = 200$MeV) overlap, we must consider quantum mechanical interference
- Since $S_{11}(1535)$ decays roughly equally to $N\pi$ and $N\eta$, we must consider coupled–channel effects

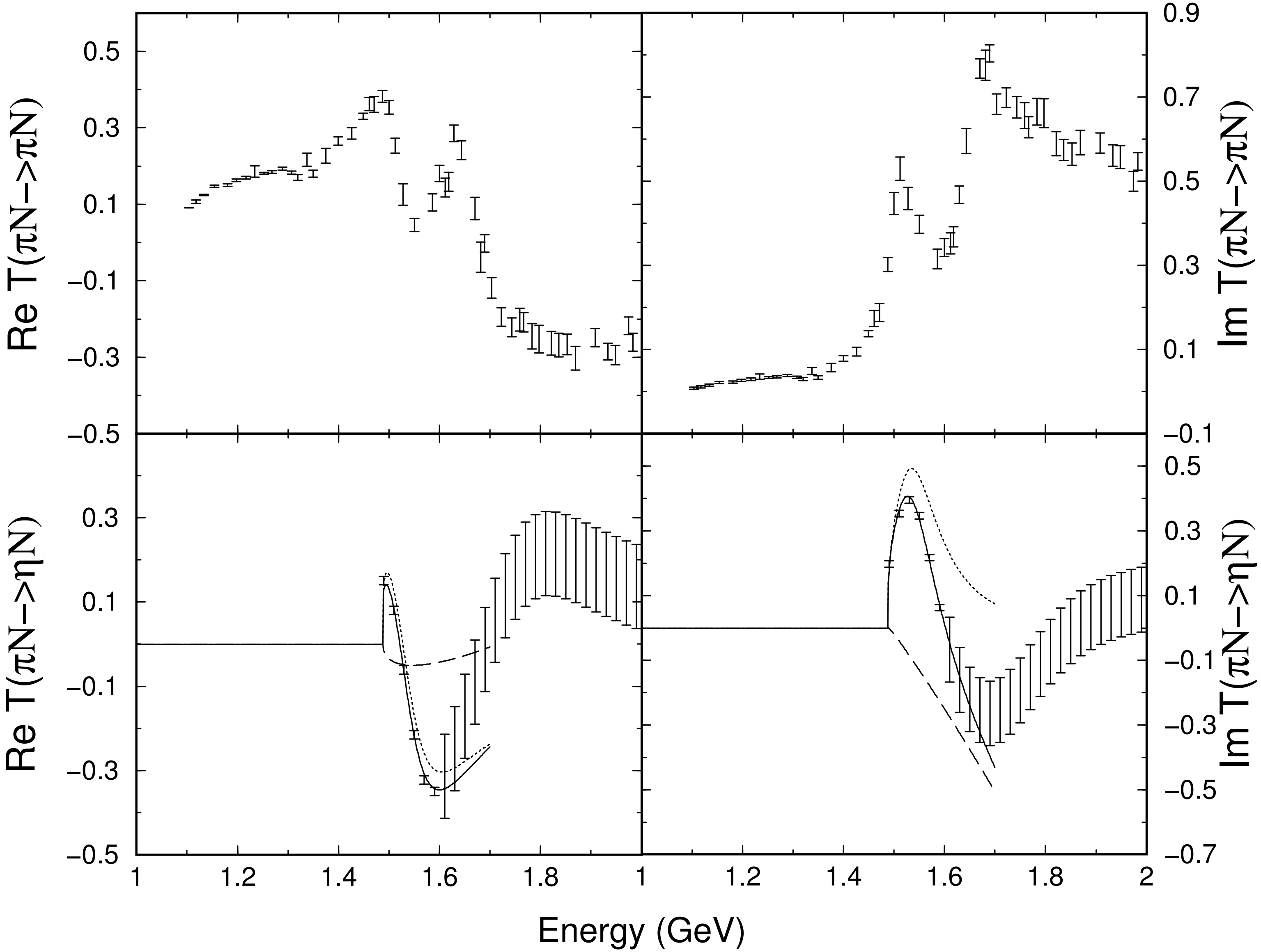
For this reason, design a little study that tests model dependence of
(a) Breit–Wigner versus (b) K–matrix versus (c) CMB model:

- Use identical data input
- Use models as close as possible to others

$\Rightarrow$ Work of Alvin Kiswandhi at FSU

Scattering amplitude 'data' (S_{11} only)

- Error bars are stat., syst.
- Traditionally unitless
- Argand plot is Im T(W) versus Re T(W)
- Sign of resonance: Peak in Im T(W) Zero in Re T(W)

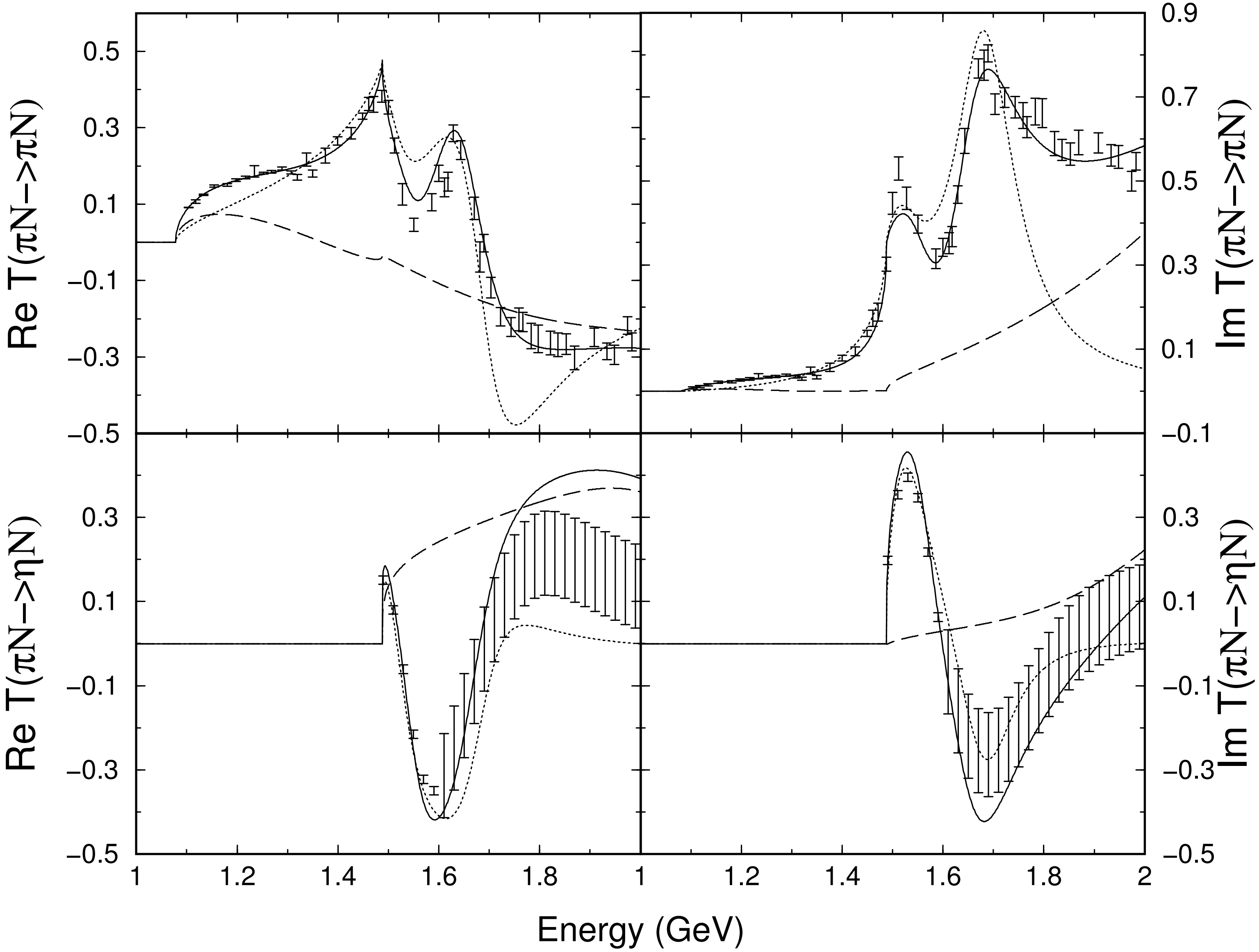


CMB model fit

Two–resonance, two–channel CMB model

- Fit not perfect missing channels $\Rightarrow$ e.g. $\Delta\pi$
- Non–res. important close to $N\pi$ threshold
- Errors of many parameters within errors of full model

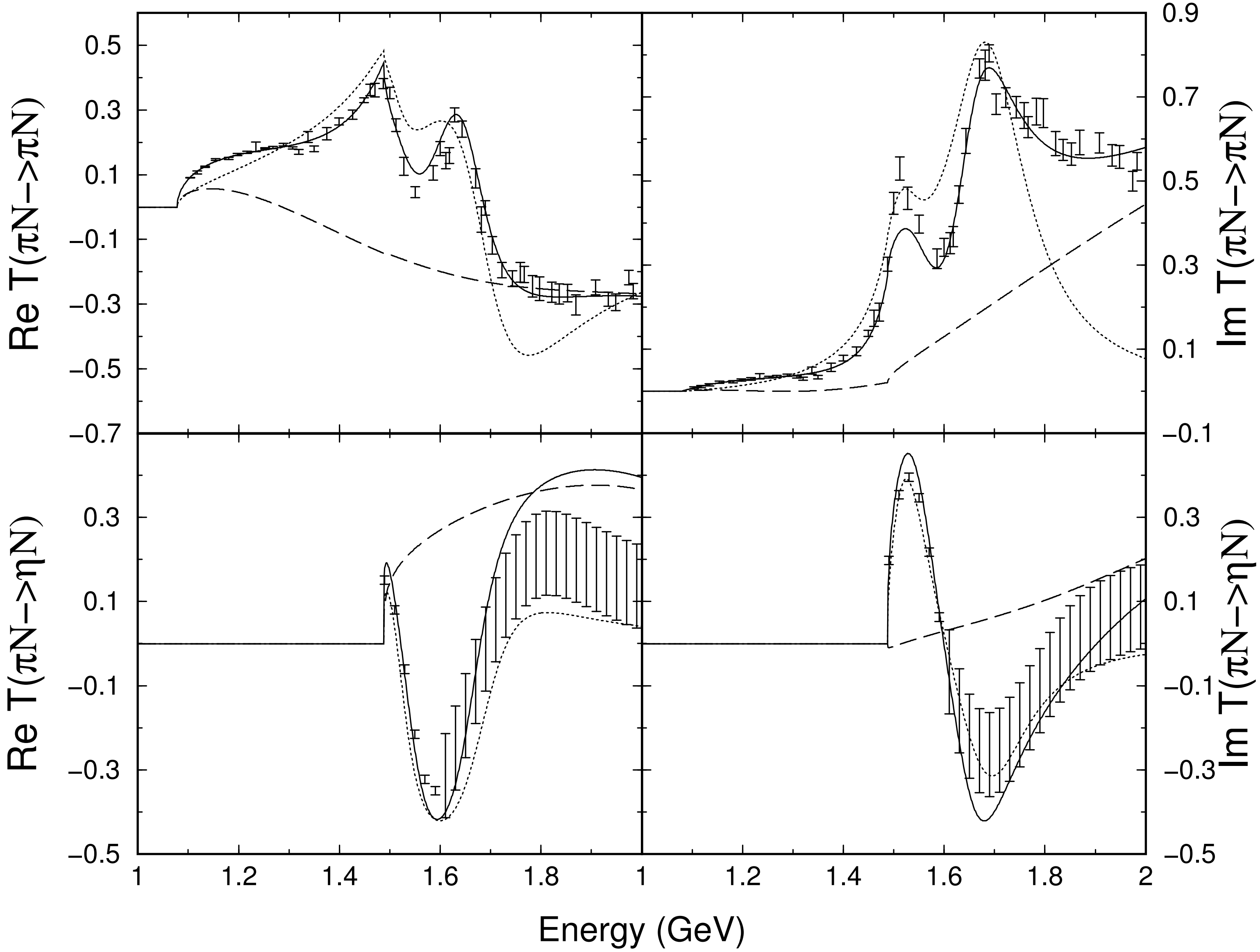
—	total amp.
· · ·	resonant amp.
- - -	non–resonant



K–matrix model fit

K–matrix model

- Fit very similar to CMB model
- At $W \approx 1.6$ GeV, difference in Im T($N\pi \rightarrow N\pi$)



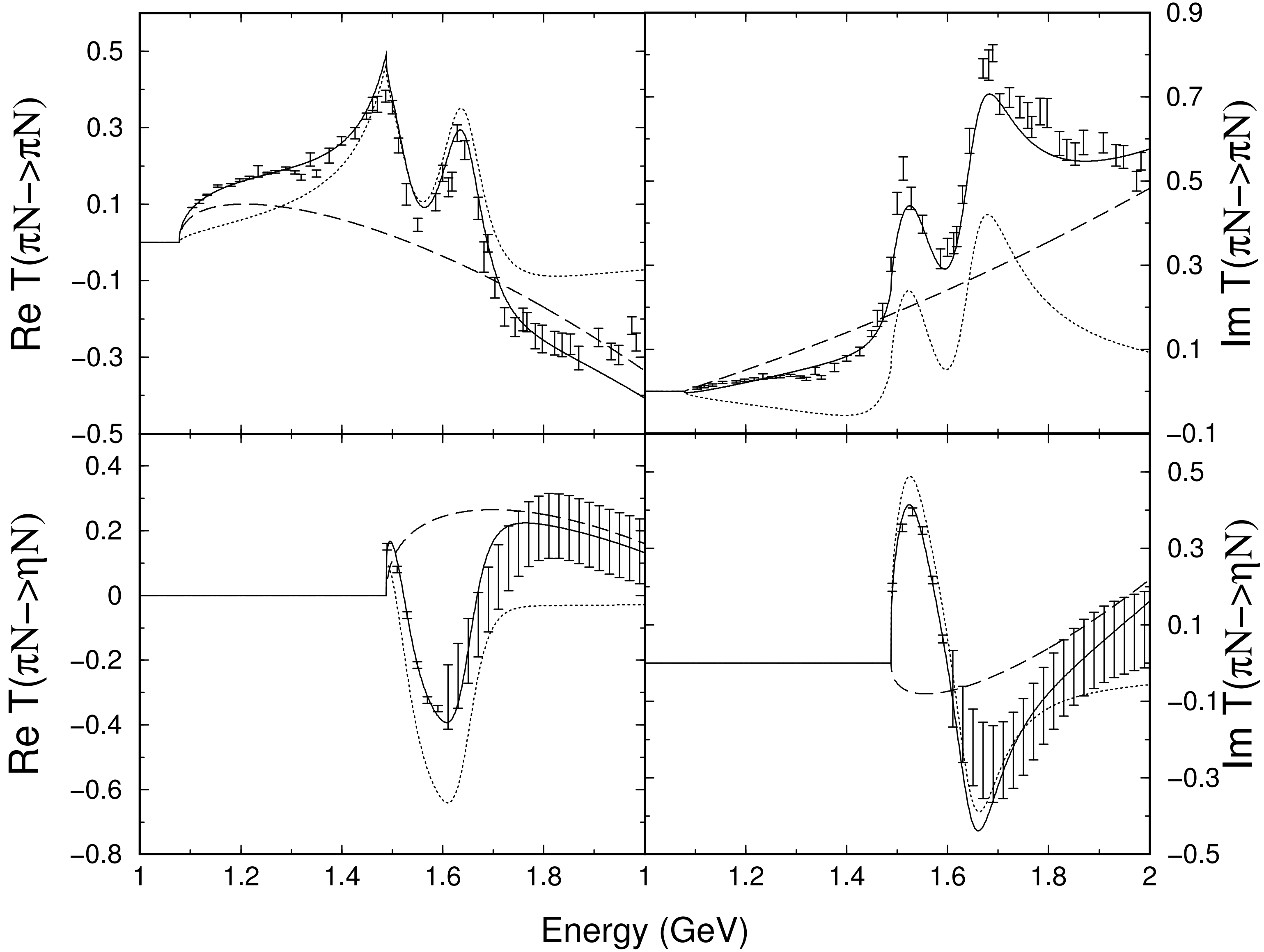
—	total amp.
· · ·	resonant amp.
- - -	non–resonant

Breit–Wigner model fit

Breit–Wigner model

- Extra phase necessary to get good fit
- Fits with BW best and worse
- Difference in res. versus nonres. with respect to CMB and K–matrix!

—	total amp.
· · ·	resonant amp.
- - -	non–resonant



Results

	model	CMB	K	BW	PDG
$S_{11}(1535)$	Mass [MeV]	1532 ± 2	1533 ± 1	1538 ± 2	1520 – 1555
	Width [MeV]	124 ± 6	119 ± 3	130 ± 6	100 – 200
	$BR_{N\pi}$ [%]	30 ± 2	33 ± 1	38 ± 1	35 – 55
$S_{11}(1650)$	Mass [MeV]	1685 ± 2	1682 ± 2	1647 ± 2	1640 – 1680
	Width [MeV]	168 ± 6	184 ± 5	109 ± 5	145 – 190
	$BR_{N\pi}$ [%]	79 ± 2	75 ± 1	51 ± 1	55 –90
	χ^2/N	3.8	3.7	5.0	

Errors shown come from Minuit

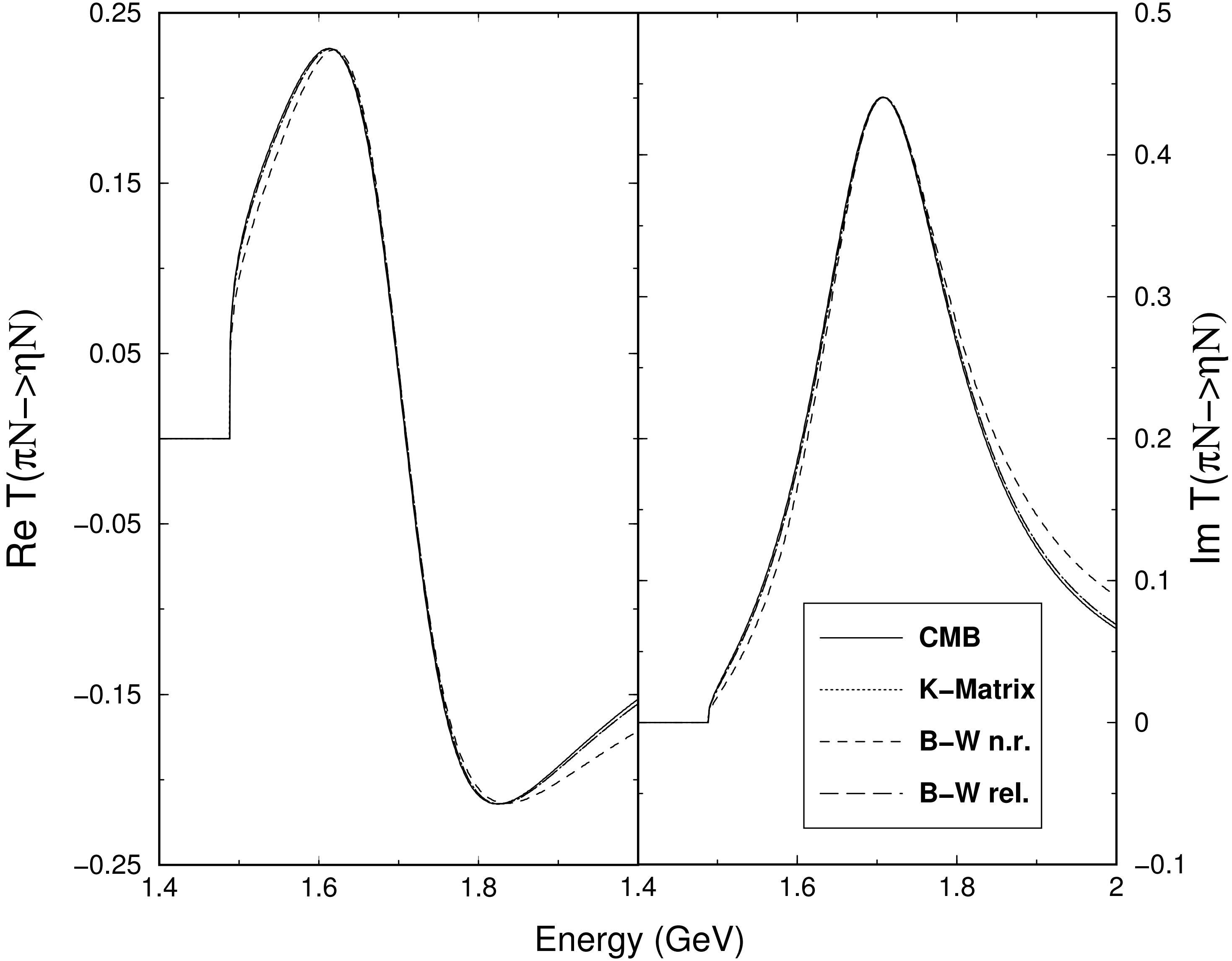
Interpretation and Conclusion

- Results not far from PDG for CMB and K–matrix
  $\Rightarrow$ Truncated model is ok
- Range of model results comparable to PDG error range
  $\Rightarrow$ PDG averages over various model results, i.e. it includes both statistical and systematic errors
- BW results are not close to CMB and K–matrix
  $\Rightarrow$ Lack of theoretical constraints is problem and requires ad–hoc parameters to fit the real data

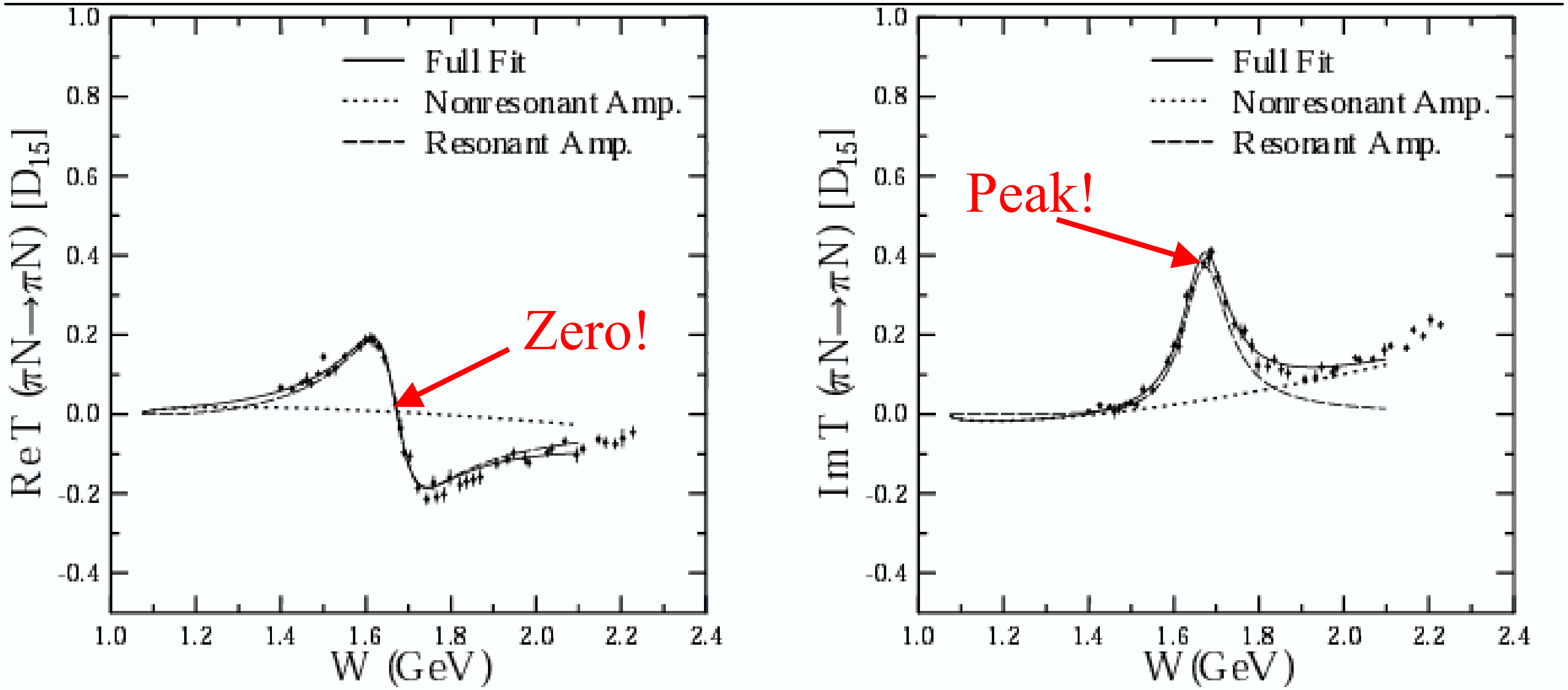
- CMB and K–matrix results 1–2 σ apart
  $\Rightarrow$ Is this large or small ?
- CMB model better constrained theoretically: Should it be preferred model ?
  $\Rightarrow$ Simplified dynamics of the K–matrix model has practical advantage
- How do we treat multi–particle final states ?

Basic resonance shapes are identical

- M = 1710 MeV
- Γ = 215 MeV



An isolated resonance – $D_{15}(1675)$



Very inelastic, peak in Im T at 0.4 (unitary bound is 1.0)
Low smooth background, no thresholds nearby
Strong signal in $\pi N \rightarrow \pi\Delta$ also