Transverse Momentum Dependent Parton Structure of Hadrons in the EIC Era

(A. Metz, Temple University)

- Lecture 1: Definition and Overview of TMDs
- Lecture 2: Observables for TMDs
- Lecture 3: Phenomenology of TMDs
- Lecture 4: TMDs: Special Topics I
- Lecture 5: TMDs: Special Topics II
- Lecture 6: TMDs: Opportunities at a new Electron-Ion Collider (EIC)

Some Literature

- Review Literature on TMDs (list not exhaustive)
  - Barone, Drago, Ratcliffe, *Transverse polarisation of quarks in hadrons*, hep-ph/0104283
  - D'Alesio, Murgia, *Azimuthal and single-spin asymmetries in hard scattering processes*, arXiv:0712.4328
  - Aidala, Bass, Hasch, Mallot, *The spin structure of the nucleon*, arXiv:1209.2803
  - Angeles-Martinez et al, *Transverse momentum dependent (TMD) parton distribution functions: status and prospects*, arXiv:1507.05267
  - Grosse Perdek., Yuan, *Transverse spin structure of the nucleon*, arXiv:1510.06783
  - Various articles on 3D parton structure of hadrons, Eur. Phys. J **A52** (2016)
  - Metz, Vossen, *Parton fragmentation functions*, arXiv:1607.02521

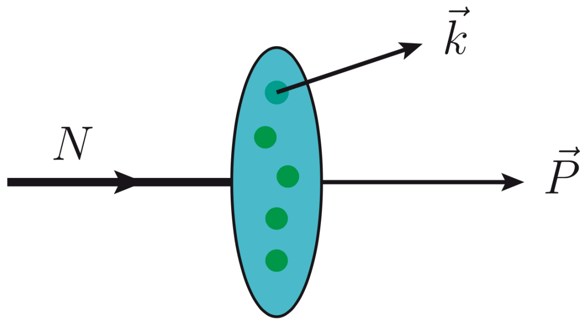
- Literature on TMDs at an EIC
  - Boer et al, Report on the joint BNL/INT/JLab Program on *Gluons and the quark sea at high energies: distributions, polarization, tomography*, arXiv:1108.1713
  - Accardi et al, *Electron Ion Collider: The next QCD Frontier — Understanding the glue that binds us all*, arXiv:1212.1701
  - 2015 Long Range Plan for Nuclear Science, *Reaching for the Horizon*

Lecture 1: Definition and Overview of TMDs

- Motivation
- The basic object in field theory: quark-quark correlator
- Integrated parton distribution functions (PDFs) of quarks
- Transverse momentum dependent (TMD) PDFs of quarks
- TMD fragmentation functions (FFs) of quarks
- TMD-PDFs of gluons

Why do we care about TMDs ?

- Partons do have (non-perturbative) transverse momentum



$$k = (k^+, k^-, \vec{k}_\perp) \qquad k_\perp \equiv |\vec{k}_\perp|$$

$$k^2 = 2k^+k^- - \vec{k}_\perp^{\,2}$$

$$k^+ = \frac{1}{\sqrt{2}}(k^0 + k^3) = xP^+ \qquad \text{(very) large}$$

$$k^- = \frac{1}{\sqrt{2}}(k^0 - k^3) = \frac{k^2 + \vec{k}_\perp^{\,2}}{2k^+} \qquad \text{(very) small}$$

- Hierarchy in TMD parton model (situation is different at small x)

$$k^+ \gg k_\perp \gg k^-$$

- Rough estimate based on uncertainty relation

$$\Delta k_\perp \sim 200\,\text{MeV} \;\rightarrow\; \text{confined motion}$$

- Interest in 3-D rather than just 1-D parton structure
- TMDs contain important information about (non-perturbative) QCD dynamics
- TMDs appear naturally in QCD description of many high-energy scattering processes
- TMDs can give rise to new phenomena (e.g., single-spin asymmetries)
- TMDs allow one to study interesting new nontrivial pQCD aspects:
  role of re-scattering of active partons, factorization, universality, evolution, ...
- etc.

- Research on TMDs is supported by funding agencies

Proposal for a Topical Collaboration in Nuclear Theory for the Coordinated Theoretical Approach to Transverse Momentum Dependent Hadron Structure in QCD

January 1, 2016 - December 31, 2020

The TMD Collaboration
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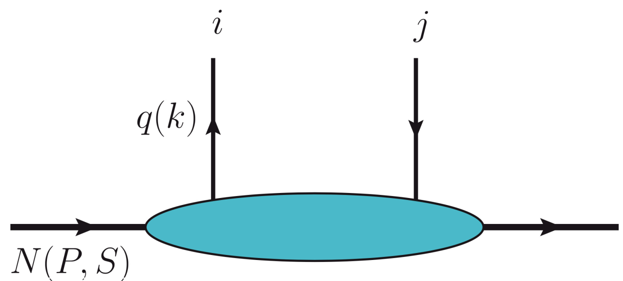
  - DOE-funded Topical Collaboration in Nuclear Theory
  - 22 senior investigators, 13 institutions
  - support for 2 tenure-track faculty members
  - junior investigators
  - affiliated members

(Unintegrated) Quark-Quark Correlator

1. Definition of qq-correlator (suppressing some subtleties)
   - Graphical representation of $\Phi^q_{ij}(k, P, S)$



   - Appears in QCD description of many processes (before kinematical approximations)
   - Field-theoretic definition

$$\Phi^q_{ij}(k, P, S) = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \langle P, S|\bar{\psi}^q_j(-\tfrac{z}{2})\, \mathcal{W}[-\tfrac{z}{2}, \tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})|P, S\rangle$$

   – spin 4-vector (can be obtained from boosting rest-frame spin vector)

$$S = \left(\frac{\Lambda P^+}{M}, -\frac{\Lambda P^-}{M}, \vec{S}_\perp\right) \qquad S^2 = -\Lambda^2 - \vec{S}^2_\perp = -1 \qquad P\cdot S = 0$$

   – gauge-link (Wilson line) ensures color gauge invariance of correlator

$$\mathcal{W}[z_i, z_f] = P\exp\left(-ig\int_{z_i}^{z_f} dz_\mu\, A^\mu_a(z)\, T_a\right)$$

2. General form of qq-correlator

- Constraints

$$\Phi^\dagger(k,P,S) = \gamma_0\,\Phi(k,P,S)\,\gamma_0 \qquad \text{[hermiticity]}$$

$$\Phi(k,P,S) = \gamma_0\,\Phi(\bar{k},\bar{P},-\bar{S})\,\gamma_0 \qquad \text{[parity]}$$

$$\Phi^*(k,P,S) = (-i\gamma_5 C)\,\Phi(\bar{k},\bar{P},-\bar{S})\,(-i\gamma_5 C) \qquad \text{[time-reversal]}$$

  - $\bar{k}^\mu = (k^0, -\vec{k})$, etc.
  - $C = i\gamma^2\gamma_0 \qquad -i\gamma_5 C = i\gamma^1\gamma^3$
  - T-reversal constraint applies in this form only if $\mathcal{W}$ neglected

- General form of Φ (neglecting S and direction of $\mathcal{W}$)

$$\Phi(k,P,S) = M\,A_1 + \not{P}\,A_2 + \not{k}\,A_3 + i\,\frac{[\not{P},\not{k}]}{2M}\,A_4 \qquad (*)$$

$$A_i(k^2, k\cdot P) \text{ are real (due to hermiticity)}$$

- Problems
  - Problem 1: Derive hermiticity and parity constraints (use: $\mathcal{P}\psi(z)\mathcal{P}^\dagger = \gamma_0\psi(\bar{z})$)
  - Problem 2: Show that $(*)$ is compatible with hermiticity and parity constraints
  - Problem 3: Show that last term in $(*)$ not allowed if T-reversal constr. is applied

3. Correlators for TMDs and PDFs

- qq-correlator for TMDs: integrate upon k^-

$$\Phi^q_{ij}(x,\vec{k}_\perp,P,S) = \int dk^-\,\Phi^q_{ij}(k,P,S)$$

$$= \int \frac{dz^-\,d^2\vec{z}_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle P,S|\bar{\psi}^q_j(-\tfrac{z}{2})\,\mathcal{W}_{\rm TMD}[-\tfrac{z}{2},\tfrac{z}{2}]\,\psi^q_i(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=0}$$

we have used

$$\int dk^-\,e^{ik\cdot z} = 2\pi\,\delta(z^+)\exp(ik^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp)$$

- qq-correlator for PDFs: integrate (also) upon $\vec{k}_\perp$

$$\Phi^q_{ij}(x,P,S) = \int d^2\vec{k}_\perp\,\Phi^q_{ij}(x,\vec{k}_\perp,P,S)$$

$$= \int \frac{dz^-}{2\pi}\,e^{ik\cdot z}\,\langle P,S|\bar{\psi}^q_j(-\tfrac{z}{2})\,\mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\,\psi^q_i(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- Structure of $\Phi(k,P,S)$ also determines structure of TMD and PDF correlators

4. Decomposition of $q q$-correlator into contributions of different twist

- Convenient defintion

$$\Phi^{q\,[\Gamma]}(k,P,S) \equiv \frac{1}{2}\,\mathrm{Tr}\big[\Phi^q(k,P,S)\Gamma\big] = \frac{1}{2}\,\Phi^q_{ij}(k,P,S)\Gamma_{ji}$$

$$= \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\,e^{ik\cdot z}\,\langle P,S|\bar{\psi}^q(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\,\psi^q(\tfrac{z}{2})|P,S\rangle$$

- Expansion of Φ^q in basis of Dirac matrices

$$\Phi^q = \frac{1}{2}\,\Phi^{q\,[\gamma^+]}\,\gamma^- - \frac{1}{2}\,\Phi^{q\,[\gamma^+\gamma_5]}\,\gamma^-\gamma_5 + \frac{1}{2}\,\Phi^{q\,[i\sigma^{i+}\gamma_5]}\,i\sigma^{i-}\gamma_5 + \dots \qquad (*)$$

- Leading (working) twist if Γ carries one plus-index (for nucleon with large P^+)

- Leading-twist terms dominate in observables

- Problem 4: Check whether $(*)$ is correct

PDFs of Quarks

- Forward qq-correlator

$$\Phi^q_{ij}(x,P,S) = \int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\, \langle P,S|\bar\psi^q_j(-\tfrac{z}{2})\, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=\vec z_\perp=0}$$

- Definition of leading-twist quark PDFs

$$\begin{aligned} f_1^q(x) &= \Phi^{q\,[\gamma^+]}(x,P,S) \\ &= \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle P,S|\bar\psi^q(-\tfrac{z}{2})\, \gamma^+\, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=\vec z_\perp=0} \\ \Lambda\, g_1^q(x) &= \Phi^{q\,[\gamma^+\gamma_5]}(x,P,S) \\ S_\perp^i\, h_1^q(x) &= \Phi^{q\,[i\sigma^{i+}\gamma_5]}(x,P,S) \end{aligned}$$

- Interpretation as number densities using light-front quantization
- Density interpretation spoiled by QCD effects (radiative corrections)
- Dependence on renormalization scale (μ) has been suppressed

- More on polarization dependence
  - quarks with definite helicity/chirality

$$\psi_\lambda = P_\lambda \, \psi \qquad P_\lambda = \frac{1}{2}\,(1 + \lambda\gamma_5) \qquad \lambda = \pm 1$$

$$\bar{\psi}_\lambda = \bar{\psi}\, P_{-\lambda} \qquad P_\lambda^2 = P_\lambda \qquad\qquad (*)$$

  - Problem 5: Verify the two relations in $(*)$
  - rewriting of operator for unpolarized distribution f_1

$$\begin{aligned} \bar{\psi}\,\gamma^+\,\psi &= \bar{\psi}\,\gamma^+ \left[\frac{1}{2}\,(1+\gamma_5) + \frac{1}{2}\,(1-\gamma_5)\right]\psi \\ &= \bar{\psi}\,\frac{1}{2}\,(1-\gamma_5)\,\gamma^+\,\frac{1}{2}\,(1+\gamma_5)\,\psi + \bar{\psi}\,\frac{1}{2}\,(1+\gamma_5)\,\gamma^+\,\frac{1}{2}\,(1-\gamma_5)\,\psi \\ &= \bar{\psi}_+\,\gamma^+\,\psi_+ + \bar{\psi}_-\,\gamma^+\,\psi_- \end{aligned}$$

$\rightarrow$ f_1 describes sum of two densities

  - rewriting of operator for helicity distribution g_1

$$\bar{\psi}\,\gamma^+\gamma_5\,\psi = \bar{\psi}_+\,\gamma^+\,\psi_+ - \bar{\psi}_-\,\gamma^+\,\psi_-$$

$\rightarrow$ g_1 describes difference of two densities (can become negative)

– h_1 describes difference of two densities for transverse quark polarization

$$\begin{aligned} i\sigma^{i+}\gamma_5 &= \gamma^+\gamma^i\gamma_5 \\ P_i &= \frac{1}{2}(1 \pm \gamma^i\gamma_5) \quad \text{projects on states with transverse polarization} \end{aligned}$$

- g_1 and h_1 can also be interpreted as strength of spin-spin correlations

$$\lambda\, \Phi^{q\,[\gamma^+\gamma_5]}(x, P, S) = \lambda\, \Lambda\, g_1^q(x)$$

$$s_\perp^i\, \Phi^{q\,[i\sigma^{i+}\gamma_5]}(x, P, S) = \vec{s}_\perp \cdot \vec{S}_\perp\, h_1^q(x)$$

- Overview

quark polarization

	U	L	T
U	f_1^q		
L		g_1^q	
T			h_1^q

– polarization of nucleon and quark "aligned"

– h_1^q is chiral-odd
$\rightarrow$ decouples from many processes

TMD-PDFs of Quarks

- TMD qq-correlator

$$\Phi^q_{ij}(x, \vec{k}_\perp, P, S) = \int \frac{dz^- \, d^2\vec{z}_\perp}{(2\pi)^3} \, e^{ik\cdot z} \, \langle P, S | \bar{\psi}^q_j(-\tfrac{z}{2}) \, \mathcal{W}_{\mathrm{TMD}}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q_i(\tfrac{z}{2}) | P, S \rangle \Big|_{z^+=0}$$

- Definition of leading-twist quark TMD-PDFs (in Amsterdam notation)
  (Mulders, Tangerman, 1995 / Boer, Mulders, 1997 / Bacchetta et al, 2006)

$$\Phi^{q\,[\gamma^+]}(x, \vec{k}_\perp) = f_1^q - \frac{\varepsilon_\perp^{ij} \, k_\perp^i \, S_\perp^j}{M} \, f_{1T}^{\perp q}$$

$$\lambda \, \Phi^{q\,[\gamma^+\gamma_5]}(x, \vec{k}_\perp) = \lambda \, \Lambda \, g_1^q + \frac{\lambda \, \vec{k}_\perp \cdot \vec{S}_\perp}{M} \, g_{1T}^q$$

$$s_\perp^i \, \Phi^{q\,[i\sigma^{i+}\gamma_5]}(x, \vec{k}_\perp) = \vec{s}_\perp \cdot \vec{S}_\perp \, h_1^q + \frac{\Lambda \, \vec{k}_\perp \cdot \vec{s}_\perp}{M} \, h_{1L}^{\perp q} - \frac{\varepsilon_\perp^{ij} \, k_\perp^i \, s_\perp^j}{M} \, h_1^{\perp q} + \frac{1}{2M^2} \Big(2 \, \vec{k}_\perp \cdot \vec{s}_\perp \; \vec{k}_\perp \cdot \vec{S}_\perp - \vec{k}_\perp^{\,2} \, \vec{s}_\perp \cdot \vec{S}_\perp \Big) \, h_{1T}^{\perp q}$$

  - TMDs depend on x and $\vec{k}_\perp^{\,2}$ $\rightarrow$ forward limit is readily recovered
  - improved definition nedeed (rapidity divergence, double counting in factorization)
  - dependence on two scales $(\mu \, , \; \zeta)$ has been suppressed

- TMD-PDFs of quarks have names

$f_{1T}^{\perp q}$: Sivers function (Sivers, 1989)

$h_1^{\perp q}$: Boer-Mulders function (Boer, Mulders, 1997)

g_{1T}^{q} $h_{1L}^{\perp q}$: worm-gear functions (polarization of hadron and quark perpendicular)

$h_{1T}^{\perp q}$: pretzelosity (quadrupole pattern of pre-factor)

- Sivers function: a closer look
  - density of unpolarized quarks in (transversely) polarized nucleon

$$\Phi^{q\,[\gamma^+]}(x, \vec{k}_\perp, P, S) = f_1^q(x, \vec{k}_\perp^{\,2}) - \frac{(\vec{k}_\perp \times \vec{S}_\perp) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_\perp^{\,2})$$

  - $f_{1T}^{\perp}$ describes difference of two densities for transverse nucleon polarization
  - $f_{1T}^{\perp}$ can generate transverse single-spin asymmetries (SSAs) in scattering processes
  - observed large transverse SSAs were motivation for Sivers to explore this effect

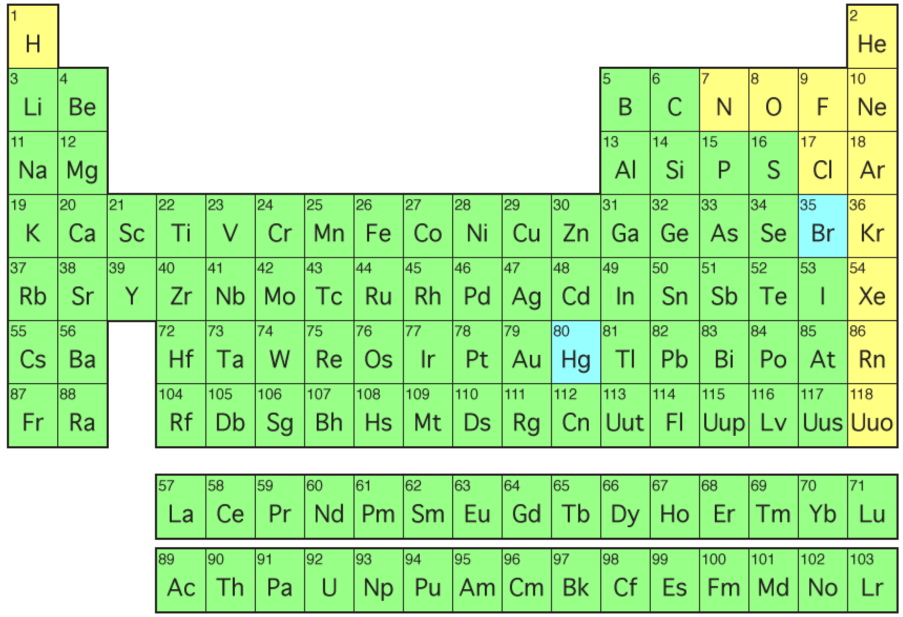
- Overview

quark polarization

	U	L	T
U	f_1^q		$h_1^{\perp q}$
L		g_1^q	$h_{1L}^{\perp q}$
T	$f_{1T}^{\perp q}$	g_{1T}^q	h_1^q $h_{1T}^{\perp q}$

- 4 TMDs associated with dipole structure: $f_{1T}^{\perp q}$ $h_1^{\perp q}$ g_{1T}^q $h_{1L}^{\perp q}$
- 1 TMD associated with quadrupole structure: $h_{1T}^{\perp q}$
- 2 (naïve) time-reversal odd (T-odd) TMD-PDFs: $f_{1T}^{\perp q}$ $h_1^{\perp q}$
  (T-reversal forbids the coresponding correlations unless there is non-trivial imaginary part at amplitude level)
- no effect for U/L and L/U polarizations due to parity invariance

- “Stamp collecting”? ... maybe ... but we are in good company
  - periodic table of elements

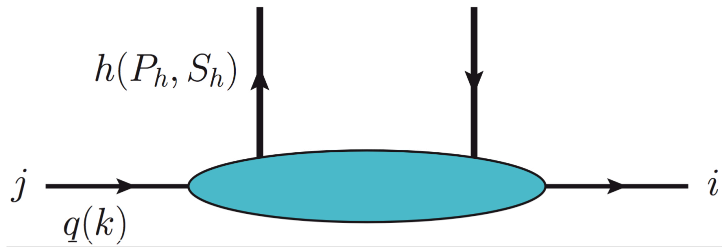


    don’t forget the isotopes ...

  - (supersymmetric) extensions of the Standard Model
  - materials science
  - etc.

TMD-FFs of Quarks

- Graphical representation of qq-correlator $\Delta^q_{ij}(k, P_h, S_h)$ for fragmentation



- Field-theoretic definition of unintegrated qq-correlator for fragmentation into hadron h

$$\Delta^{h/q}_{ij}(k, P_h, S_h) \;=\; \sum_X \int \frac{d^4 z}{(2\pi)^4}\, e^{ik \cdot z}\, \langle 0|\, \mathcal{W}[-\tfrac{z}{2}, \tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})\, |P_h, S_h, X\rangle \\ \times \langle P_h, S_h, X|\bar{\psi}^q_j(-\tfrac{z}{2})|0\rangle$$

  - qq-correlator for TMD-FFs obtained by integration upon small light-cone momentum of quark
  - definition of TMD-FFs basically analogous to case of TMD-PDFs

- Overview (in Amsterdam notation)

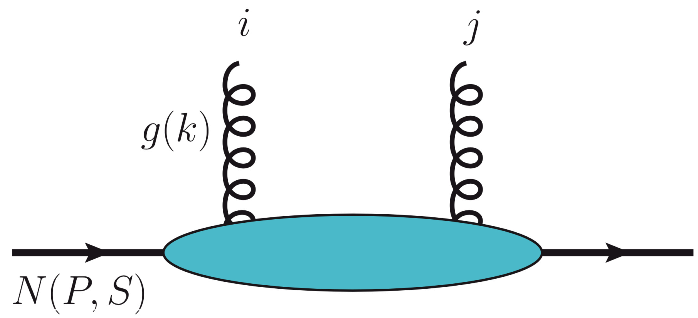
quark polarization

	U	L	T
U	D_1^q		$H_1^{\perp q}$
L		G_1^q	$H_{1L}^{\perp q}$
T	$D_{1T}^{\perp q}$	G_{1T}^q	$H_1^q \quad H_{1T}^{\perp q}$

  - interpretation like for TMD-PDFs, but role of parton and hadron interchanged
  - only two functions matter for unpolarized hadrons: $D_1^q \quad H_1^{\perp q}$
  - $H_1^{\perp q}$: Collins (fragmentation) function (Collins, 1992)
  - $D_{1T}^{\perp q}$: Sivers-type / polarizing fragmentation function
  - 2 (naïve) time-reversal odd (T-odd) TMD-FFs: $D_{1T}^{\perp q} \quad H_1^{\perp q}$

TMD-PDFs of Gluons

- Graphical representation of gg-correlator $\Phi^{g\,[ij]}(k, P, S)$



- Field-theoretic definition of unintegrated gg-correlator (leading twist only)

$$\Phi^{g\,[ij]}(k, P, S) = \int \frac{d^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle P, S| F_a^{+j}(-\tfrac{z}{2})\, \mathcal{W}_{ab}[-\tfrac{z}{2}, \tfrac{z}{2}]\, F_b^{+i}(\tfrac{z}{2}) |P, S\rangle$$

  - (gauge-invariant) operator contains components of gluon field strength tensor $F_a^{\mu\nu}$, with maximum amount of plus-indices (leading twist)
  - different combinations of indices $i,\, j$ describe densities for unpolarized, circularly polarized and linearly polarized gluons

- Overview (in Amsterdam notation) (Mulders, Rodrigues, 2000 / Meissner, Metz, Goeke, 2007)

gluon polarization

	U	Circ	Lin
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g \quad h_{1T}^{\perp g}$

  - only two functions matter for unpolarized target: $f_1^g \quad h_1^{\perp g}$
  - $f_{1T}^{\perp g}$: gluon Sivers function
  - $h_1^{\perp g}$: sometimes called gluon Boer-Mulders function
  - 4 (naïve) time-reversal odd (T-odd) TMD-PDFs: $f_{1T}^{\perp g} \quad h_{1L}^{\perp g} \quad h_1^g \quad h_{1T}^{\perp g}$
  - also 8 leading-twist TMD-FFs for gluons (Mulders, Rodrigues, 2000)

Lecture 2: Observables for TMDs

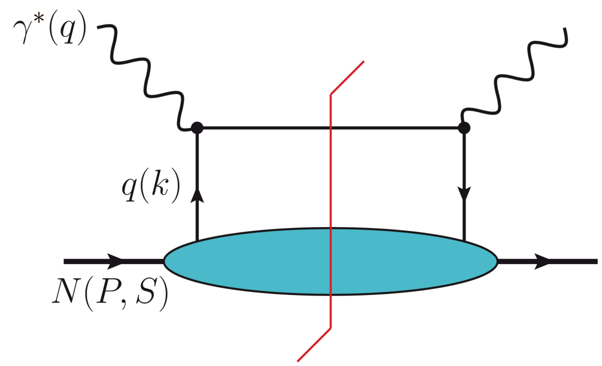
- Inclusive DIS with qq-correlator
- Semi-inclusive DIS
  - kinematics
  - model-independent form of cross section
  - (generalized) parton model and TMDs
- Other processes
- Special case: direct sensitivity to transverse parton momenta
- Some elements of TMD factorization

Inclusive DIS with qq-Correlator

- Process

$$\ell(l, \lambda_\ell) + N(P,S) \to \ell(l', \lambda'_\ell) + X$$

- Handbag diagram



- Structure of cross section

$$d\sigma \sim L_{\mu\nu}\, W^{\mu\nu}$$

- Leptonic tensor

$$\begin{aligned} L^{\mu\nu} &= \Big[\bar{u}(l', \lambda'_\ell)\,\gamma^\nu\,u(l, \lambda_\ell)\Big]\Big[\bar{u}(l', \lambda'_\ell)\,\gamma^\mu\,u(l, \lambda_\ell)\Big]^* \\ &= 2\Big(l^\mu l'^\nu + l'^\mu l^\nu - l\cdot l'\, g^{\mu\nu}\Big) + 2\,i\,\lambda_\ell\,\varepsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma \qquad (*) \end{aligned}$$

  – Problem 6: Derive the polarization-independent part of $(*)$

- Hadronic tensor (neglect m_q from start; show only dependence on k in qq-correlator)

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k \, \mathrm{Tr}\left[\Phi^q(k)\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)$$

$$= \sum_q e_q^2 \int d^4k \, \Phi^{q\,[\gamma^+]}(k)\,\frac{1}{2}\,\mathrm{Tr}\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big) + \ldots$$

$$= \sum_q \frac{e_q^2}{2} \int d^4k \, \Phi^{q\,[\gamma^+]}(k)\,\mathrm{Tr}\left(\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)\right)\Big|_{k^-=\vec{k}_\perp=0} + \ldots$$

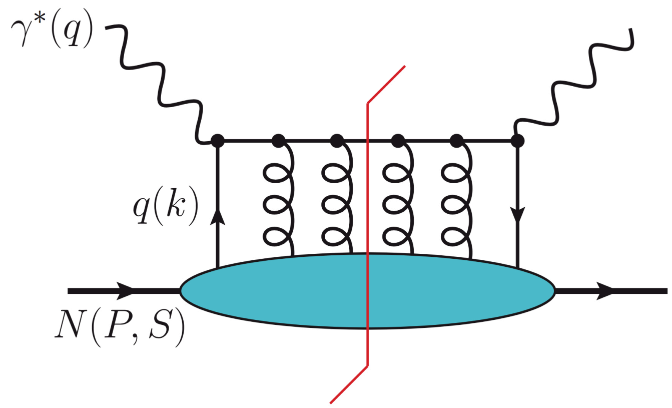
$$= \sum_q \frac{e_q^2}{2} \int dk^+ \, \Phi^{q\,[\gamma^+]}(x)\,\mathrm{Tr}\left(\left[\gamma^-\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\delta\big((k+q)^2\big)\right)\Big|_{k^-=\vec{k}_\perp=0}$$

$$= \sum_q \frac{e_q^2}{2} \int \frac{dk^+}{k^+} \, \underbrace{\Phi^{q\,[\gamma^+]}(x)}_{f_1^q(x)}\,\mathrm{Tr}\left(\left[\not{k}\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\underbrace{\delta\big((k+q)^2\big)}_{\frac{x_B}{Q^2}\,\delta(x-x_B)}\right)\Big|_{k^-=\vec{k}_\perp=0} + \ldots$$

$$= \frac{1}{Q^2}\sum_q \frac{e_q^2}{2} f_1^q(x_B)\,\mathrm{Tr}\left(\left[\not{k}\,\gamma^\mu\,(\not{k}+\not{q})\,\gamma^\nu\right]\right)\Big|_{k^+=x_B P^+,\; k^-=\vec{k}_\perp=0} \qquad (*)$$

  - note the equivalence to calculation in "simple" parton model
  - Problem 7: Repeat the steps leading to $(*)$
  - missing: gauge invariance of correlator/PDFs

- Handbag diagram including re-scattering of quark



  - including re-scattering graphs (to all orders) renders gauge-invariant correlator

$$\Phi^q_{ij}(x,P,S) = \int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\, \langle P,S|\bar{\psi}^q_j(-\tfrac{z}{2})\, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q_i(\tfrac{z}{2})|P,S\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

  - in other words: $\mathcal{W}_{PDF}$ generated by final-state interaction (FSI) of active quark
  - path of Wilson line is straight line (consequence of keeping leading order terms only)

Semi-Inclusive DIS

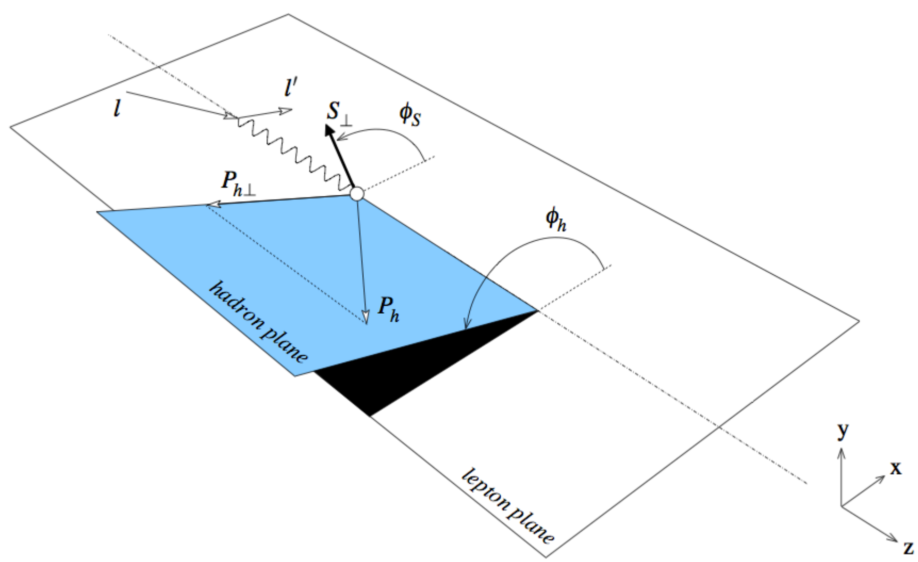
- Process

$$\ell(l,\lambda_\ell) + N(P,S) \to \ell(l',\lambda'_\ell) + h(P_h,S_h) + X$$

- 6 independent kinematical variables

$$x_{\rm B} = \frac{Q^2}{2P\cdot q} \qquad Q^2 \qquad \phi_S \qquad z_h = \frac{P\cdot P_h}{P\cdot q} \qquad P_{h\perp} = |\vec{P}_{h\perp}| \qquad \phi_h$$

$$y = \frac{P\cdot q}{P\cdot l} \approx \frac{Q^2}{x_{\rm B}\, S} \quad \text{not independent}$$



(figure from Bacchetta et al, hep-ph/0611265)

- Model-independent form of cross section (in notation of hep-ph/0611265)

$$\frac{d\sigma}{dx_{\mathrm{B}}\,dy\,d\phi_S\,dz_h\,d\phi_h\,dP_{h\perp}^2} \sim \Big\{ \big(1-y+\tfrac{1}{2}y^2\big)\,F_{UU,T} + (1-y)\,\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \Lambda\,(1-y)\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h} + \lambda_\ell\,\Lambda\,y\big(1-\tfrac{1}{2}y\big)\,F_{LL}$$

$$+ |\vec{S}_\perp|\big(1-y+\tfrac{1}{2}y^2\big)\,\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$$

$$+ |\vec{S}_\perp|\big(1-y\big)\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ |\vec{S}_\perp|\big(1-y\big)\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

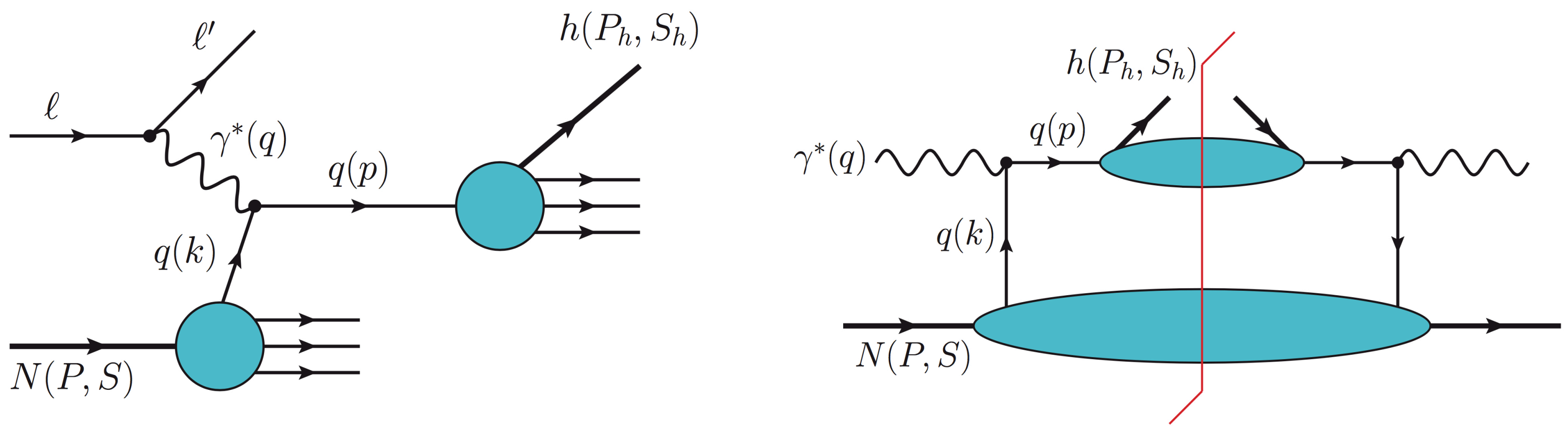
$$+ \lambda_\ell\,|\vec{S}_\perp|\,y\big(1-\tfrac{1}{2}y\big)\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \text{10 additional terms}\Big\}$$

  - structure functions depend on 4 variables:

$$F_i = F_i(x_{\mathrm{B}}, z_h, P_{h\perp}^2, Q^2)$$

  - at low $P_{h\perp}$, leading contribution to $d\sigma$ given by 8 listed structure functions

- Feynman diagram at tree level



$$d\sigma \sim d\hat{\sigma}_{pert} \otimes \Phi \otimes \Delta$$

- factorization formula depends on kinematical situation:
  1. cross section integrated upon $P_{h\perp}$
  2. cross section differential in $P_{h\perp}$, and $P_{h\perp} \sim Q$
  3. cross section differential in $P_{h\perp}$, and $P_{h\perp} \ll Q$ $\rightarrow$ realm of TMDs

- full description of differential cross section from small to large $P_{h\perp}$ still area of active research (see, e.g., Collins et al, arXiv:1605.00671)

- Hadronic tensor at tree level

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^4k\, d^4p\, \delta^{(4)}(k+q-p)\,\mathrm{Tr}\left[\Phi^q(k)\,\gamma^\mu\,\Delta^q(p)\,\gamma^\nu\right]$$

  - consider P^+ and P_h^- large, as well as $k^+ = xP^+$ and $P_h^- = zp^-$
  - consider frame with $\vec{P}_{h\perp} = 0$, and small $\vec{q}_\perp \neq 0$
  - neglect small light-cone components of parton momenta k^- and p^+ in delta-function (approximation for TMD parton model)

$$W^{\mu\nu} \;\sim\; \frac{2\,x_{\rm B}\,z_h}{Q^2} \sum_q e_q^2 \int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)$$
$$\times\,\mathrm{Tr}\left[\Phi^q(x_{\rm B},\vec{k}_\perp)\,\gamma^\mu\,\Delta^q(z_h,\vec{p}_\perp)\,\gamma^\nu\right] \qquad (*)$$

  - express Φ^q and Δ^q in terms of TMD-PDFs and TMD-FFs, respectively
  - contract with leptonic tensor $L^{\mu\nu}$
  - compare with model-independent form of cross section to find tree-level results for structure functions F_i
  - Problem 8: Check whether $(*)$ is correct

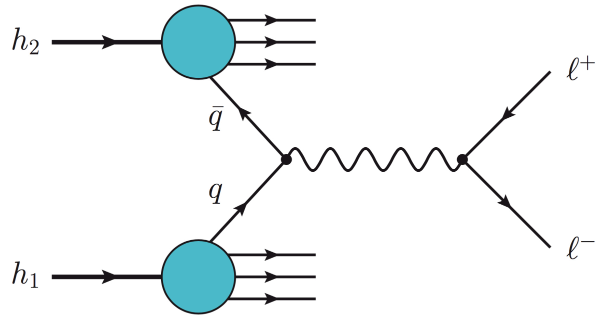
- Structure functions at tree level (e.g., hep-ph/0611265)

$$
\begin{aligned}
F_{UU,T} &= x_{\rm B} \sum_q e_q^2 \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1^q(x_{\rm B}, \vec{k}_\perp^{\,2}) \, D_1^q(z_h, \vec{p}_\perp^{\,2}) \\
F_{UU}^{\cos 2\phi_h} &\sim h_1^\perp \otimes H_1^\perp \\
F_{UL}^{\sin 2\phi_h} &\sim h_{1L}^\perp \otimes H_1^\perp \\
F_{LL} &\sim g_1 \otimes D_1 \\
F_{UT,T}^{\sin(\phi_h-\phi_S)} &\sim f_{1T}^\perp \otimes D_1 \qquad \text{[Sivers effect]} \\
F_{UT}^{\sin(\phi_h+\phi_S)} &\sim h_1 \otimes H_1^\perp \qquad \text{[Collins effect]} \\
F_{UT}^{\sin(3\phi_h-\phi_S)} &\sim h_{1T}^\perp \otimes H_1^\perp \\
F_{LT}^{\cos(\phi_h-\phi_S)} &\sim g_{1T} \otimes D_1
\end{aligned}
$$

  - transverse parton momenta of TMD-PDFs and TMD-FFs are convoluted
  - except for $F_{UU,T}$ expressions are symbolic; in most cases convolutions contain additional powers of transverse parton momenta
  - all 8 TMD-PDFs can be studied
  - all 8 structure functions have been measured

Other Processes

1. Drell-Yan process: $h_1 + h_2 \to \ell^+ + \ell^- + X$
   - Feynman diagram at tree level



   - Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

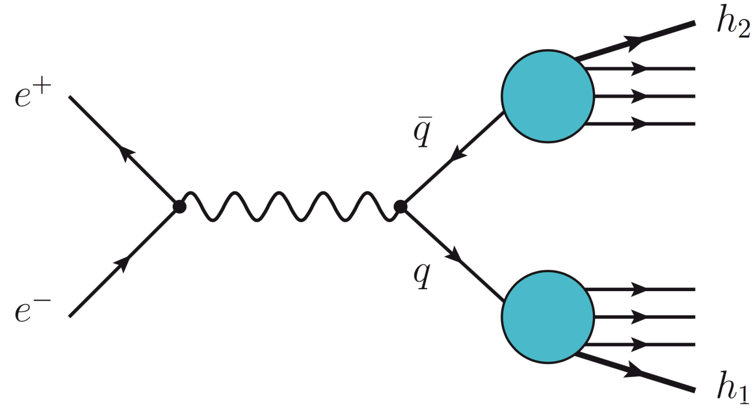
$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{k}_{a\perp}\, d^2\vec{k}_{b\perp}\, \delta^{(2)}(\vec{k}_{a\perp} + \vec{k}_{b\perp} - \vec{q}_\perp)$$
$$\times \operatorname{Tr}\left[\Phi^q(x_a, \vec{k}_{a\perp})\, \gamma^\mu\, \Phi^{\bar{q}}(x_b, \vec{k}_{b\perp})\, \gamma^\nu\right]$$

     – sensitivity to "product" of two TMD-PDFs
     – transverse parton momenta are convoluted
     – longitudinal momentum fractions x_a and x_b fixed by kinematics of reaction

   - 48 structure functions; cross checks possible for various TMDs
     (Arnold, Metz, Schlegel, 2008)

2. Electron-positron annihilation: $e^+ + e^- \to h_1 + h_2 + X$

- Feynman diagram at tree level



- Hadronic tensor at tree level, for low $\vec{q}_\perp$ of gauge boson

$$W^{\mu\nu} \sim \sum_q e_q^2 \int d^2\vec{p}_{a\perp}\, d^2\vec{p}_{b\perp}\, \delta^{(2)}(\vec{p}_{a\perp} + \vec{p}_{b\perp} - \vec{q}_\perp)$$

$$\times \operatorname{Tr}\left[\Delta^q(z_a, \vec{p}_{a\perp})\, \gamma^\mu\, \Delta^{\bar{q}}(z_b, \vec{p}_{b\perp})\, \gamma^\nu\right]$$

  - sensitivity to "product" of two TMD-FFs
  - transverse parton momenta are convoluted
  - longitudinal momentum fractions z_a and z_b fixed by kinematics of reaction

3. Some additional processes

(a) $\ell\, N \to \ell\, \text{jet}\, \text{jet}\, X$ $\quad$ $\ell\, N \to \ell\, J/\psi\, X$

(b) $p\, p \to \gamma\, \gamma\, X$ $\quad$ $p\, p \to \gamma\, \text{jet}\, X$ $\quad$ $p\, p \to \text{jet}\, \text{jet}\, X$

(c) $p\, p \to (h\, \text{jet})\, X$

(d) $p\, p \to J/\psi\, X$ $\quad$ $p\, p \to \eta_c\, X$ $\quad$ $p\, p \to \text{Higgs}\, X$

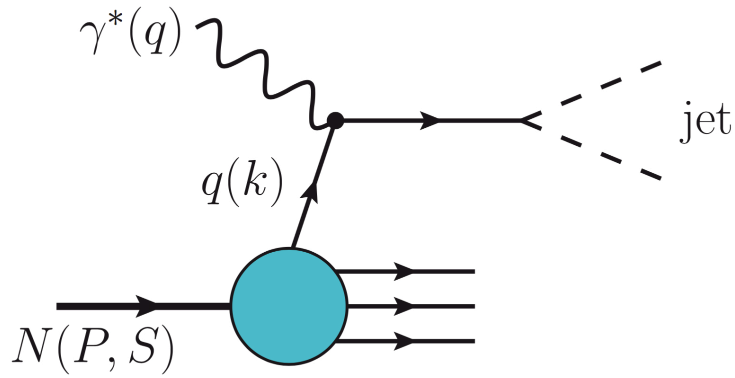
(e) $p\, A$-collisions

(f) etc.

- Very rich phenomenology
- Status of TMD factorization for additional processes:
  - holds in some cases (according to current knowledge)
  - breaks down in some cases
  - unclear in some cases (further studies needed) $\longrightarrow$ active research area

Direct Sensitivity to Transverse Parton Momenta

1. Jet production in DIS: $\ell + N \to \ell + \mathrm{jet} + X$
   - Feynman diagram at tree level



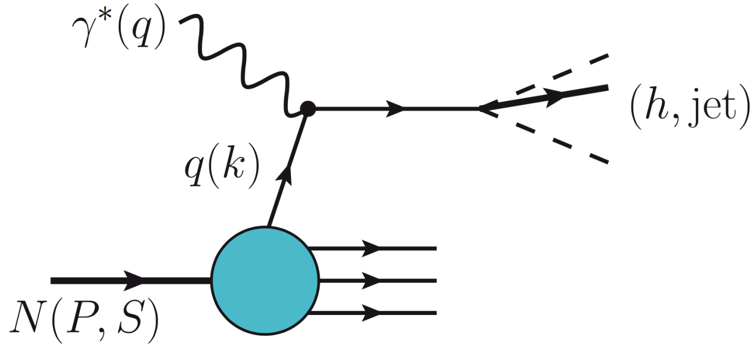
   - Main features:
     - transverse momentum $\vec{k}_\perp$ determined by jet transverse momentum $\vec{P}_{j\perp}$
     - same model-independent structure of cross section as for semi-inclusive DIS
     - example: unpolarized structure function

$$F_{UU,T} = x_{\mathrm{B}} \sum_q e_q^2 \, f_1^q(x_{\mathrm{B}}, P_{j\perp}^2)$$

     - important challenge: measurement of $\vec{P}_{j\perp}$ has uncertainty (on the order of typical intrinsic transverse parton momenta)

2. Measurement of hadrons inside jets, e.g.: $\ell + N \to \ell + (h\,\text{jet}) + X$

- Feynman diagram at tree level

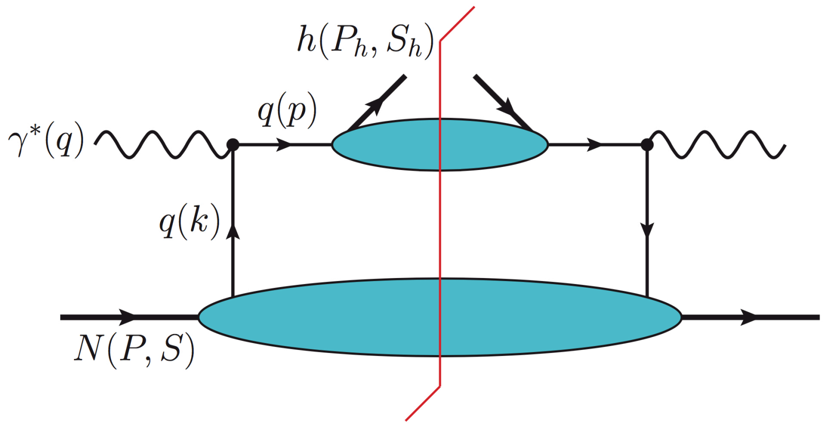


- Main features
  - particularly suitable to study TMD-FFs
  - jet can have large transverse momentum
  - has been exploited in proton-proton collisions at RHIC (for Collins effect)
    (STAR, arXiv:1708.07080)

Elements of TMD Factorization

1. Semi-inclusive DIS at tree level (Ralston, Soper, 1979)

- Feynman diagram for cross section



  – generalized parton model
  – no gluon radiation

- Expression for unpolarized structure function (suppressing flavor labels)

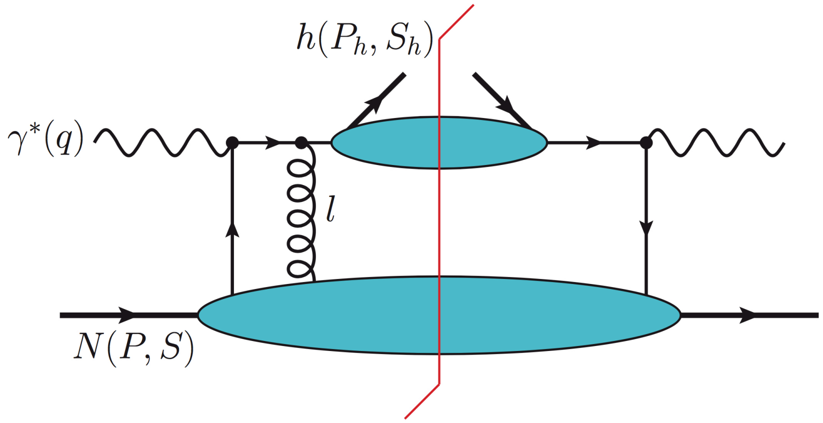
$$F_{UU,T} \sim \underbrace{H_{LO}}_{\text{from } \gamma^* q \to q} \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1(x_{\rm B}, \vec{k}_\perp^{\,2}) \, D_1(z_h, \vec{p}_\perp^{\,2})$$

- TMD-PDFs and TMD-FFs not gauge invariant

$$f_1(x, \vec{k}_\perp^{\,2}) = \int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle P, S|\bar{\psi}(-\tfrac{z}{2}) \, \gamma^+ \, \psi_{(\frac{z}{2})}|P, S\rangle\Big|_{z^+=0}$$

2. Semi-inclusive DIS at tree level, but gauge invariant
(... / Belitsky, Ji, Yuan, 2002 / Boer, Mulders, Pijlman, 2003 / ...)

- Sample Feynman diagram for cross section



  – leading region of loop momentum

$$l \sim Q(1, \lambda^2, \lambda) \quad \lambda \sim m/Q \text{ small}$$

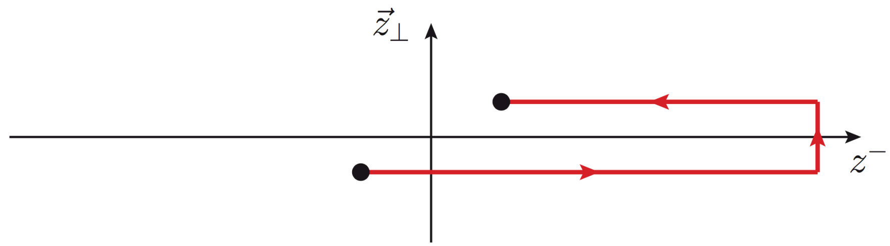
- Expression for unpolarized structure function

$$F_{UU,T} \sim H_{LO} \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \, f_1(x_{\rm B}, \vec{k}_\perp^{\,2}) \, D_1(z_h, \vec{p}_\perp^{\,2})$$

- TMD-PDFs and TMD-FFs gauge invariant (include Wilson line)

- Complications: rapidity divergences, Wilson line self energies
  → under control, but requires 1st modification of definition of TMDs
  (Collins, Soper, 1981 / Collins, Hautmann, 2000 / Cherednikov, Stefanis, 2007 / Collins 2011 / Echevarria, Idilbi, Schimemi, 2011 / ...)

- Path of Wilson line for TMD-PDFs



  - TMD factorization leads to staple-like Wilson line
  - in semi-inclusive DIS, Wilson line for TMD-PDFs is future-pointing (FSI of active parton)
  - Wilson line reduces to straight line upon integration over $\vec{k}_\perp$

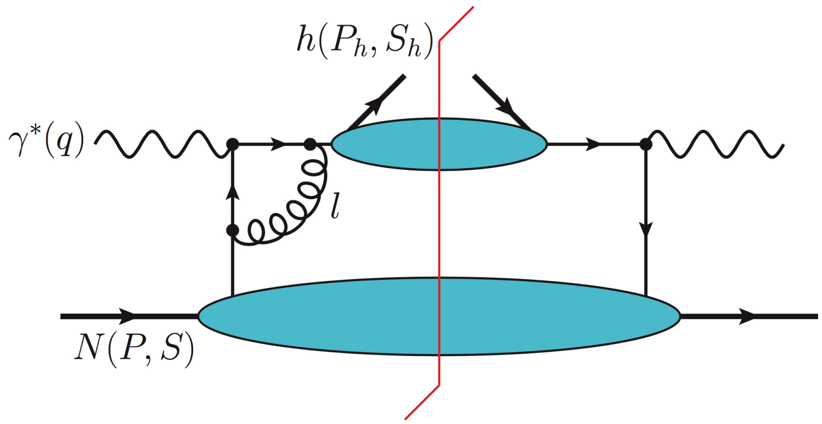
- In semi-inclusive DIS, Wilson line for TMD-FFs, *a priori*, is past-pointing (ISI of active parton)

- More complicated path of Wilson lines were found for purely hadronic reactions $\rightarrow$ "generalized TMD factorization" (Bomhof, Mulders Pijlman, 2004 / ...)

3. Semi-inclusive DIS beyond tree level

(Collins, Soper, 1981 / Collins, Soper, Sterman, 1985 / Ji, Ma, Yuan, 2004 / Collins, Metz, 2004 / ...)

- Sample Feynman diagram for cross section



– leading regions of loop momentum

$$l \sim Q(1, \lambda^2, \lambda) \quad [N\text{-collinear}]$$
$$l \sim Q(\lambda^2, 1, \lambda) \quad [h\text{-collinear}]$$
$$l \sim Q(\lambda, \lambda, \lambda) \quad [\text{soft}]$$
$$l \sim Q(1, 1, 1) \quad [\text{hard}]$$

- Expression for unpolarized structure function

$$F_{UU,T} \sim H_{NLO} \int d^2\vec{k}_\perp \, d^2\vec{p}_\perp \, d^2\vec{l}_\perp \, \delta^{(2)}(\vec{k}_\perp + \vec{q}_\perp + \vec{l}_\perp - \vec{p}_\perp) \times f_{1\,\mathrm{sub}}(x, \vec{k}_\perp^{\,2}) \, D_{1\,\mathrm{sub}}(z, \vec{p}_\perp^{\,2}) \, S(\vec{l}_\perp)$$

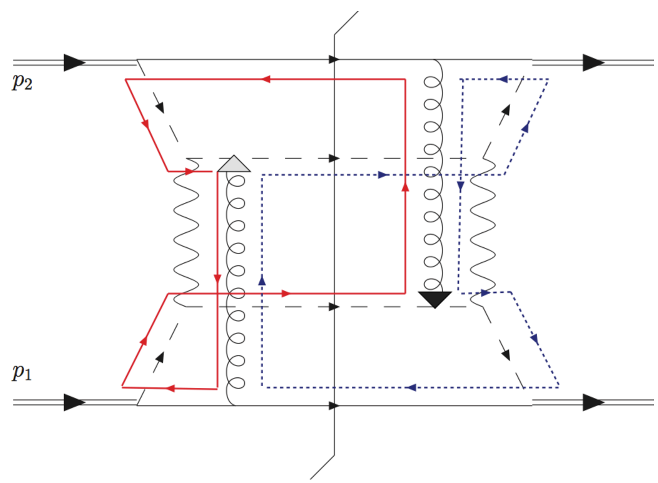
- Avoid double counting by subtraction formalism $\rightarrow$ modified definitions of TMDs
- Further development: absorb soft gluon effects in TMDs after Fourier transform to b_T-space (Collins, *Foundations of Perturbative QCD*, 2011)

4. Breakdown of TMD factorization

- Sample process: $p\,p \to \mathrm{jet}\,\mathrm{jet}\,X$

- Originally thought to obey generalized TMD factorization
  $\longrightarrow$ definition of TMDs depends on partonic subprocess
  (Bomhof, Mulders, Pijlman, 2004 / ... / Collins, Qiu, 2007 / Collins, 2007)

- But, even generalized TMD factorization breaks down (Rogers, Mulders, arXiv:1001.2977)



  – complicated color flow does not allow one to define two individual TMDs (color-entanglement)

  – specific to non-Abelian gauge theory

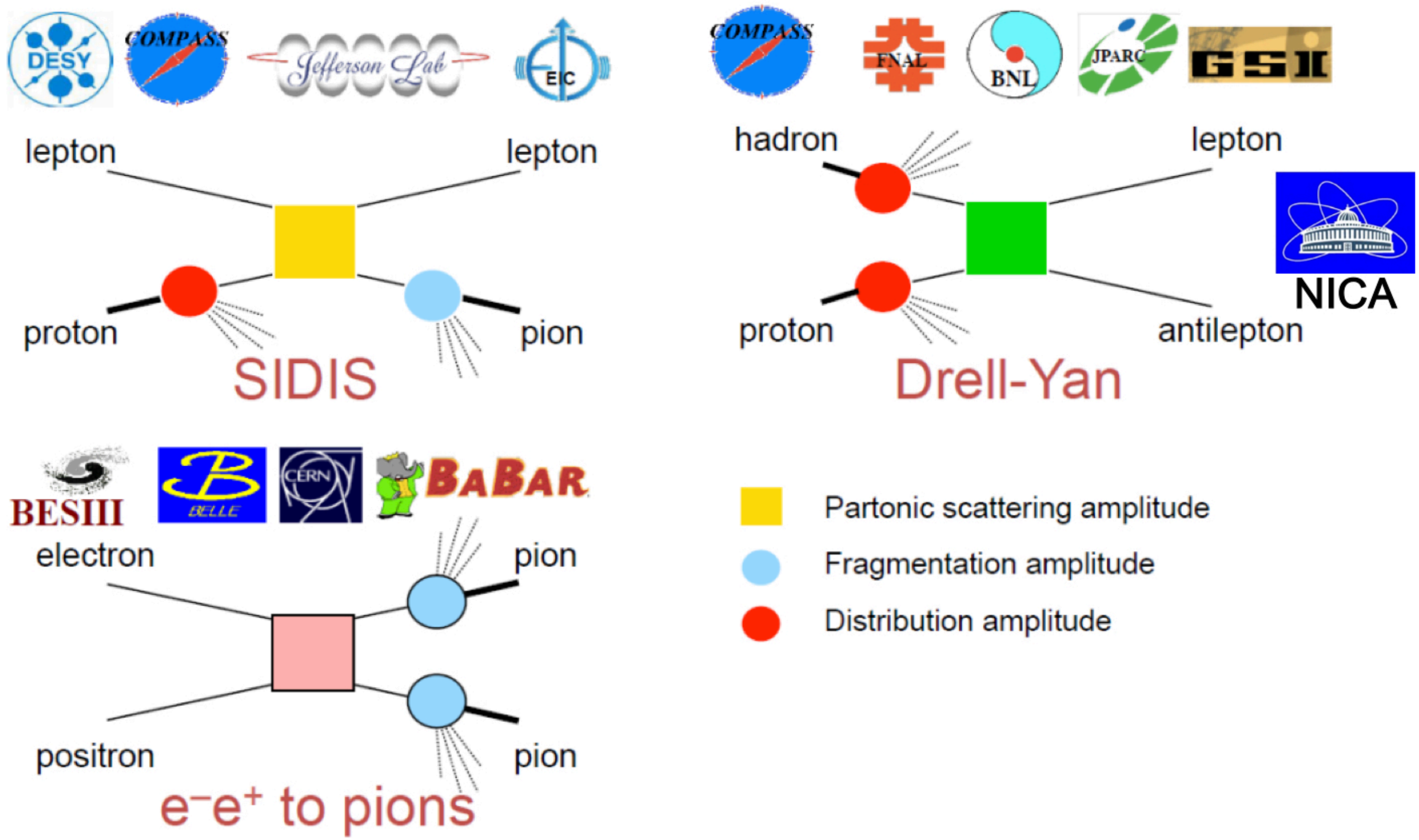
  – simplification occurs if "hybrid approach" can be justified (neglect transverse parton momenta in one of the incoming hadrons)

Lecture 3: Phenomenology of TMDs

- Experimental data on TMD observables
- Sivers function from experiment
- Collins function and transversity from experiement
- Unpolarized TMDs from experiment
- TMDs in lattice QCD
- TMDs in models
- Model-independent constraints
  - momentum sum rules
  - positivity bounds (Bacchetta, Boglione, Henneman, Mulders, hep-ph/9912490)
  - results from large-N_c limit (Pobylitsa, hep-ph/0301236)

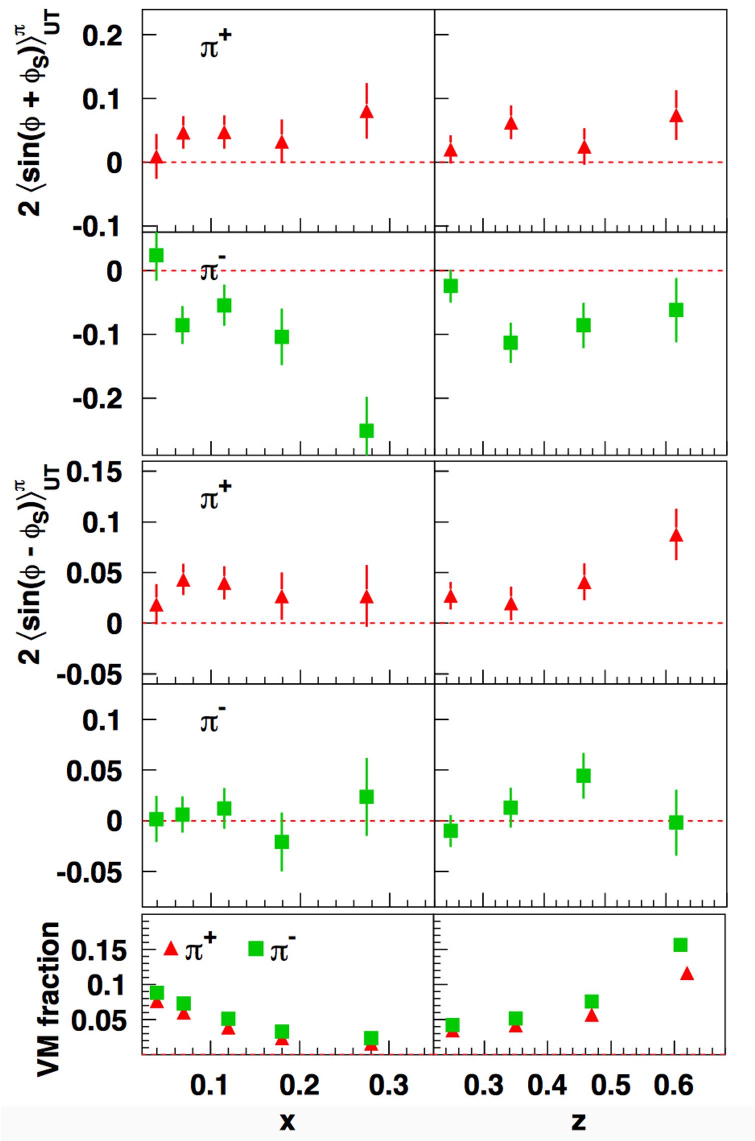
Experimental Data on TMD Observables

1. Overview of experimental facilities (→ much more in lectures by others)



(from M. Grosse Perdekamp, talk at TMD Summer School, 2017)

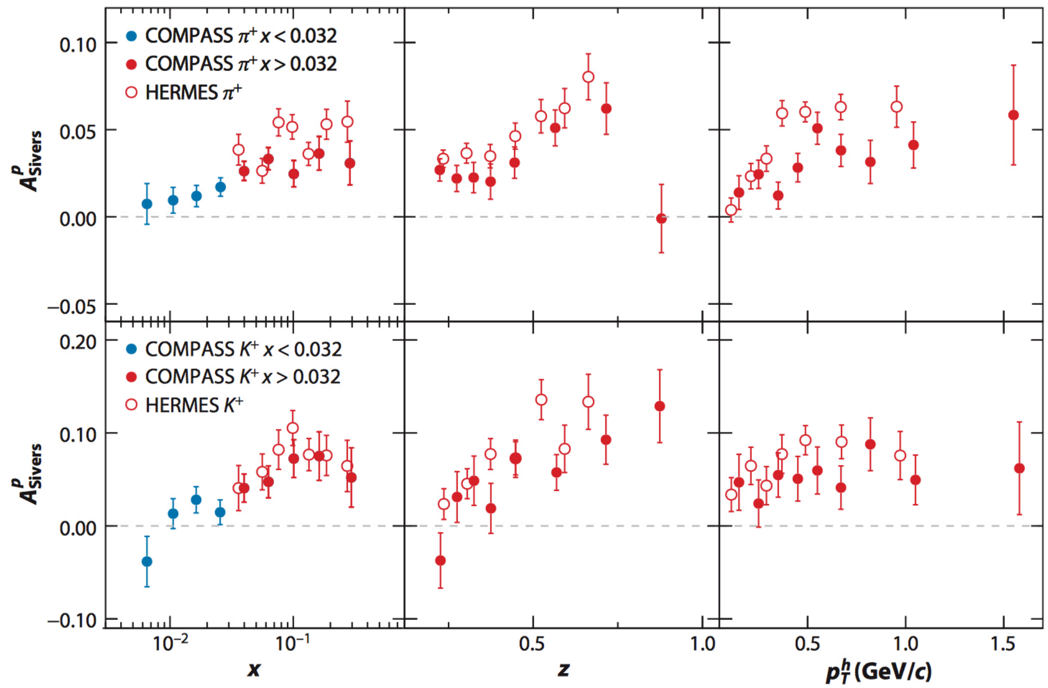
2. Pioneering steps for Sivers and Collins effects (HERMES, hep-ph/0408013)



- first measurement of Sivers asymmetry, essentially $F_{UT,T}^{\sin(\phi_h-\phi_S)}/F_{UU,T}$, in SIDIS
- first measurement of Collins asymmetry, essentially $F_{UT}^{\sin(\phi_h+\phi_S)}/F_{UU,T}$, in SIDIS
- hydrogen target
- detection of charged pions
- nonzero Sivers effect for π^+
- nonzero Collins effect for π^+ and π^-
- in the meantime, many more data from COMPASS, HERMES, JLab, with hydrogen, deuteron and ^{3}He targets

3. More data on Sivers effect in semi-inclusive DIS

Comparison of data from COMPASS and HERMES



(compilation from Grosse Perdekamp, Yuan, arXiv:1510.06783)

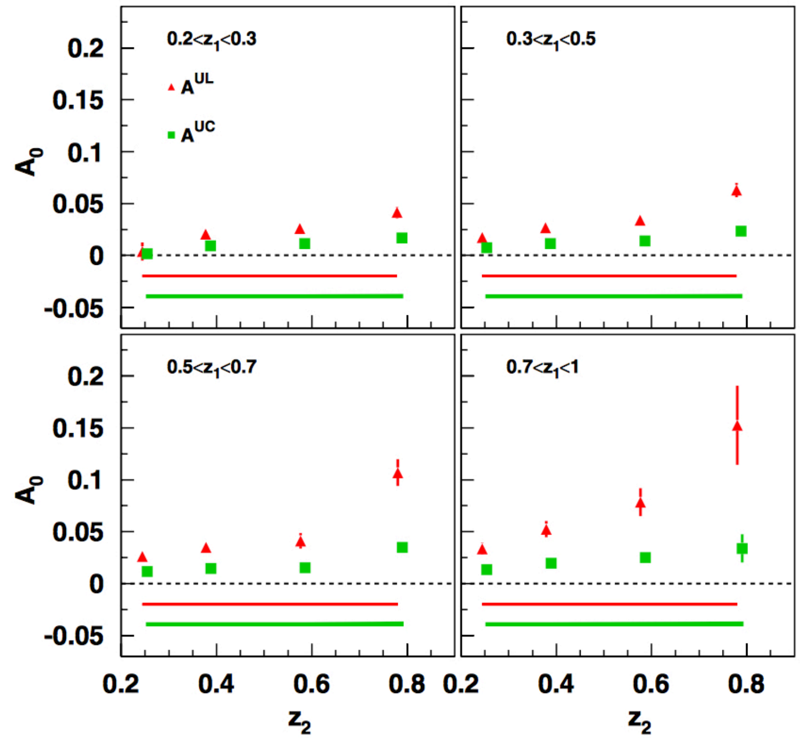
- measurements for π^+ and K^+
- results from COMPASS and HERMES largely agree
- robust nonzero effects
- asymmetry for K^+ at least as large as for π^+

4. (Double) Collins effect in electron-positron annihilation: $e^+e^- \to h_1 h_2 X$

- Azimuthal modulation due to Collins effect (Boer, Jakob, Mulders, hep-ph/9702281)

$$d\sigma \sim \sum_q e_q^2 \left[D_1^{h_1/q} \otimes D_1^{h_2/\bar{q}} + \cos(2\phi)\, H_1^{\perp\, h_1/q} \otimes H_1^{\perp h_2/\bar{q}} \right]$$

- Sample data (Belle, arXiv:0805.2975)

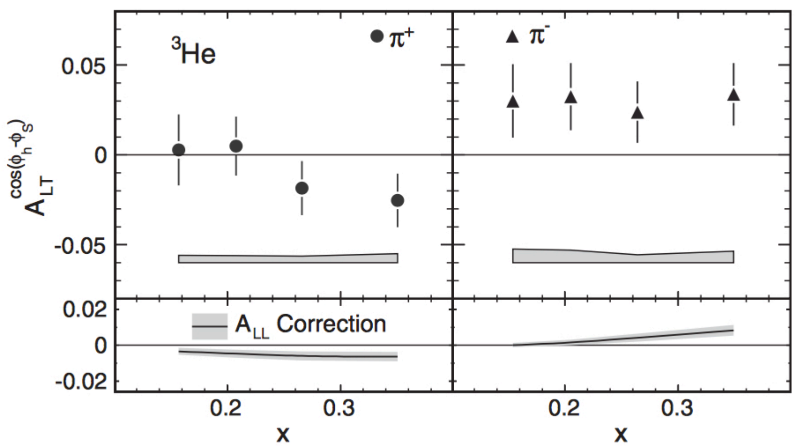


  – effect is remarkably large, given results from SIDIS and square of $H_1^\perp / D_1$
  – most of the data points have a very small error

- More data available, also from BaBar and BES

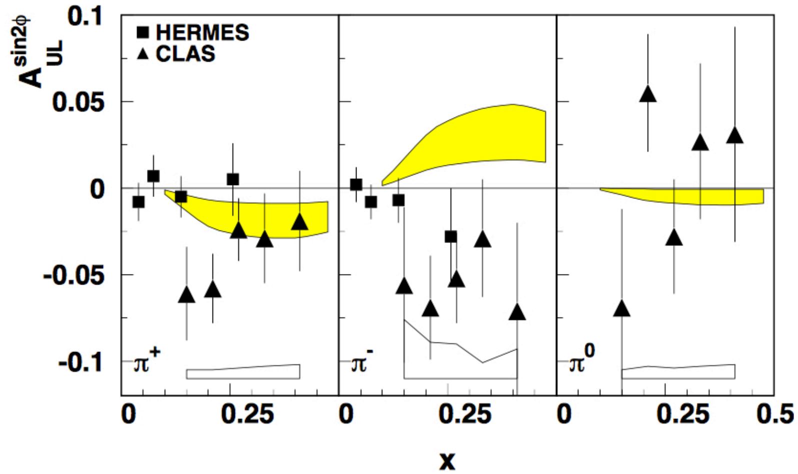
5. Addressing worm gear functions in semi-inclusive DIS

$$A_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$$



(JLab Hall A, arXiv:1108.0489)

$$A_{UL}^{\sin 2\phi_h} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$$



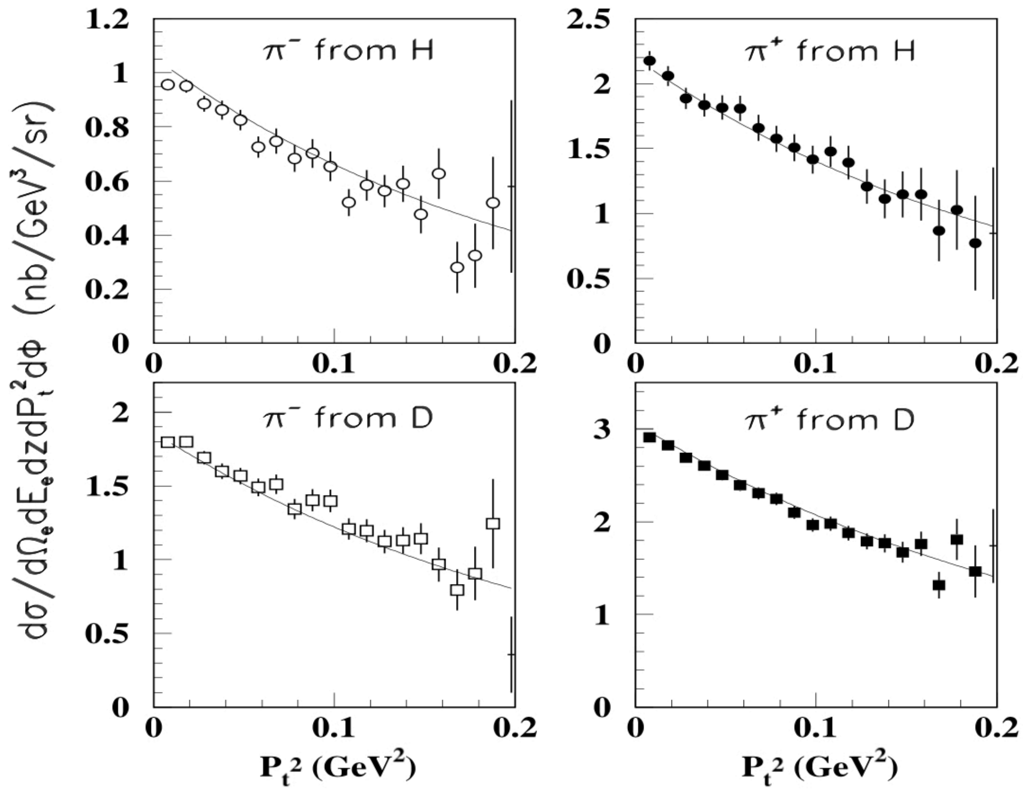
(JLab Hall B, arXiv:1003.4549)

- Data for ^{3}He allow one to extract information for neutron
- More data available on both asymmetries from COMPASS and HERMES

6. Addressing transverse momentum dependence of TMDs

- Transverse momentum dependence of SIDIS cross sections / asymmetries

Unpolarized cross section σ_{UU} (JLab Hall C, arXiv:0709.3020)

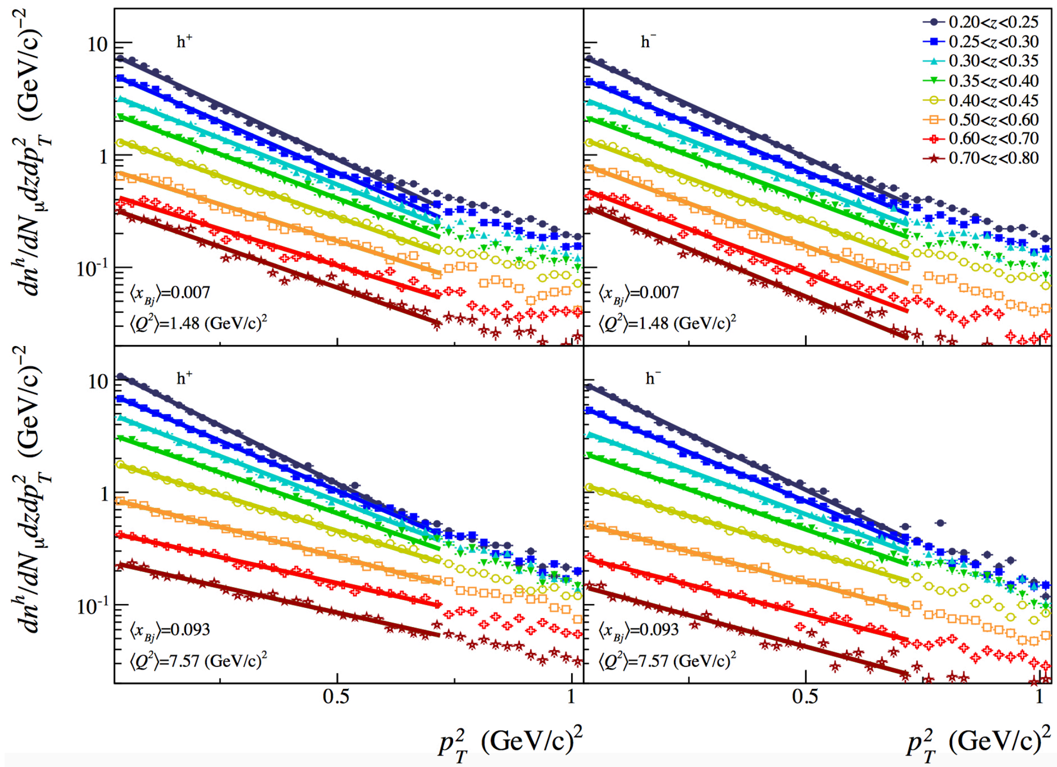


- data can help constrain unpolarized TMD-PDFs and TMD-FFs
- also, data available on transverse momentum dependence of $A_{LL} \sim F_{LL}/F_{UU,T}$
  $\rightarrow$ information on $g_{1L}^q(x, \vec{k}_\perp^{\,2})$ (JLab Hall B, arXiv:1003.4549)

- Transverse momentum dependence of SIDIS multiplicities

$$M^h(x_B, Q^2, z_h, P_{h\perp}^2)\Big|_{\text{COMPASS}} = \frac{d^4\sigma_{\text{SIDIS}}(x_B, Q^2, z_h, P_{h\perp}^2)}{dx_B\, dQ^2\, dz_h\, dP_{h\perp}^2} \Bigg/ \frac{d^2\sigma_{\text{DIS}}(x_B, Q^2)}{dx_B\, dQ^2}$$

(COMPASS, arXiv:1305.7317)



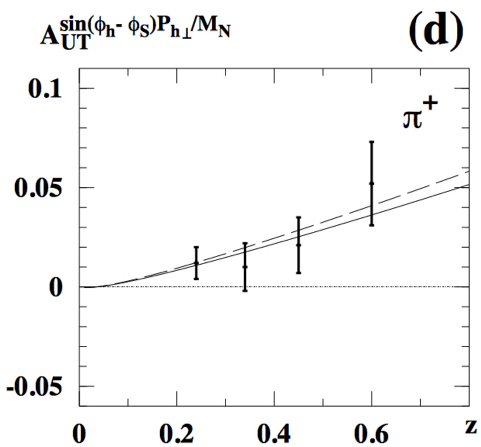
  - data are quite accurate, and they cover considerable range of $P_{h\perp}$
  - more data available (see also, e.g., HERMES, arXiv:1212.5407)
  - currently, multiplicities provide the strongest constraints on unpolarized TMDs

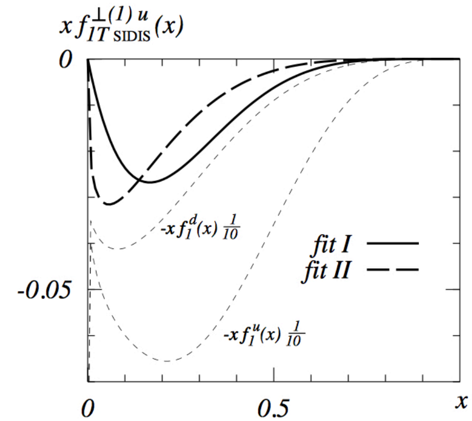
Sivers function from Experiment

1. Exploratory attempt with very first data (Efremov et al, hep-ph/0412353)
   - preliminary data on Sivers asymmetry weighted with $P_{h\perp}/M$
   - observable sensitive to particular moment:

$$f_{1T}^{\perp(1)}(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M^2} f_{1T}^\perp(x, \vec{k}_\perp^{\,2})$$

   - numerical results





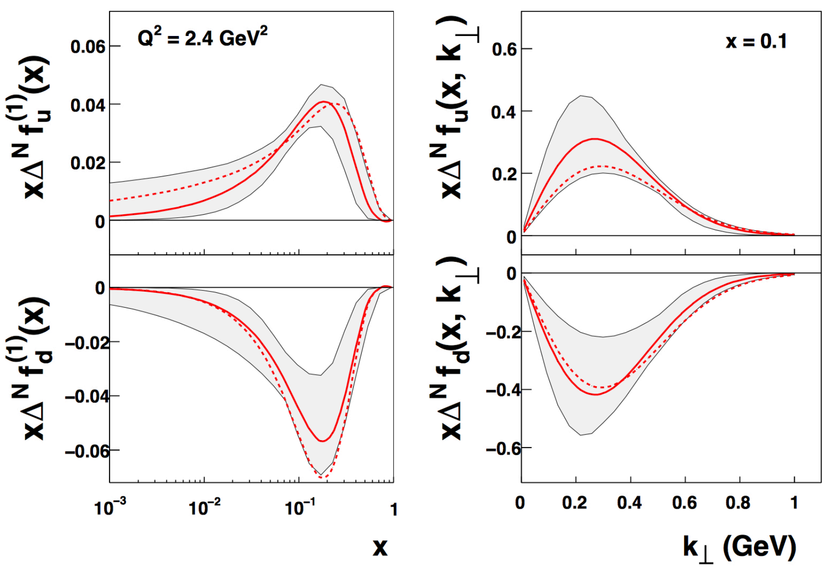
   - fit used large-N_c prediction: $f_{1T}^{\perp u} = -f_{1T}^{\perp d} + \mathcal{O}(\frac{1}{N_c})$ (Pobylitsa, hep-ph/0301236)
   - Sivers function at most 10% of unpolarized PDF
   - general features still hold for modern fits

2. More sophisticated extraction (Anselmino et al, arXiv:0805.2677)

- important ingredient: Gaussian shape (for all involved TMDs)

$$f_1^q(x, \vec{k}_\perp^{\,2}; \mu^2) = f_1^q(x; \mu^2) \frac{e^{-\vec{k}_\perp^{\,2}/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \rightarrow \int d^2\vec{k}_\perp \, f_1^q(x, \vec{k}_\perp^{\,2}; \mu^2) = f_1^q(x; \mu^2)$$

- numerical results



  – figure shows $k_\perp$-moment of

$$\Delta^N f_q(x, k_\perp) = -\frac{2\,k_\perp}{M} f_1^q(x, \vec{k}_\perp^{\,2})$$

  – fit includes also $\bar{u}$, $\bar{d}$, s, $\bar{s}$
  – 11 free parameters
  – DGLAP evolution used
  – error bands based on errors of data
  – at present, systematic error of TMD extractions hard to quantify

- some recent analyses use TMD evolution (Aybat, Prokudin, Rogers, 2011 / Anselmino et al, 2012 / Sun, Yuan, 2013 / Echevarria et al, 2014 / ...)

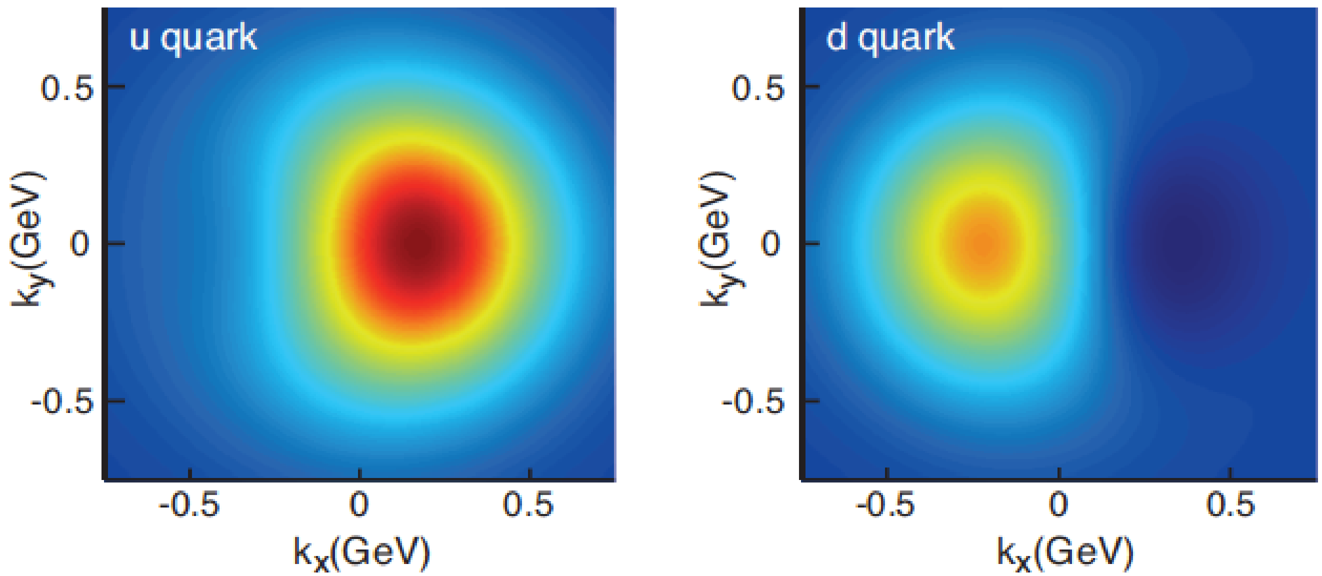
3. Unpolarized quarks in transversely polarized nucleon

- Recall density of unpolarized quarks:

$$\Phi^{q\,[\gamma^+]}(x, \vec{k}_\perp, P, S) = f_1^q(x, \vec{k}_\perp^{\,2}) - \frac{(\vec{k}_\perp \times \vec{S}_\perp) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_\perp^{\,2})$$

- Visualization

$$\Phi^{q\,[\gamma^+]}(x, \vec{k}_\perp, P, S) \;\; \text{at} \;\; x = 0.1$$



(from arXiv:1212.1701, based on Anselmino et al, arXiv:1012.3565)

- Sivers effect generates distorted distribution of unpolarized quarks
- Such 3-D imaging of the nucleon now possible (plots based on data!)

Collins function and Transversity from Experiment

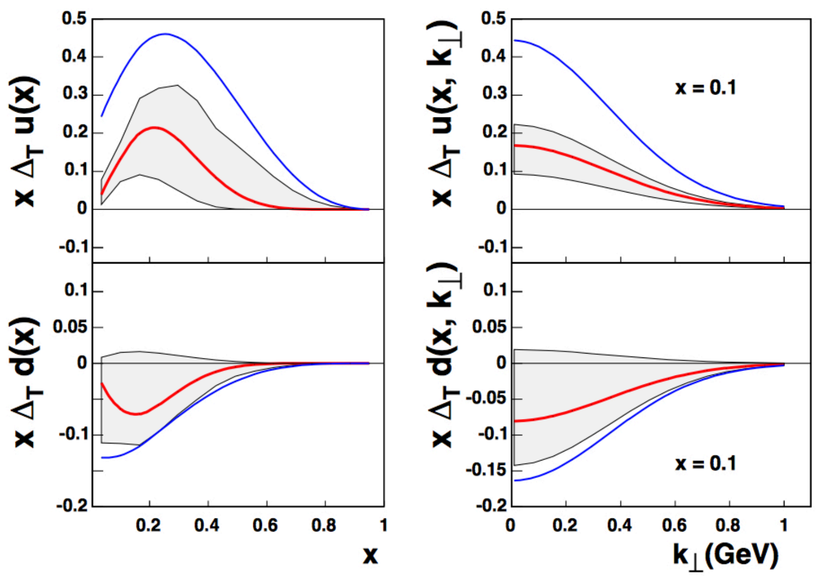
1. General strategy: use observables in semi-inclusive DIS and $e^+e^- \to h_1 h_2 X$

$$d\sigma\big|_{\ell N \to \ell h X} \sim \sum_q e_q^2 \left[f_1^q \otimes D_1^{h/q} + \sin(\phi_h + \phi_S)\, h_1^q \otimes H_1^{\perp h/q} \right]$$

$$d\sigma\big|_{e^+e^- \to h_1 h_2 X} \sim \sum_q e_q^2 \left[D_1^{h_1/q} \otimes D_1^{h_2/\bar{q}} + \cos(2\phi)\, H_1^{\perp\, h_1/q} \otimes H_1^{\perp h_2/\bar{q}} \right]$$

- combined analysis gives access to both $H_1^{\perp q}$ and h_1^q

2. Pioneering analysis (Anselmino et al, hep-ph/0701006)

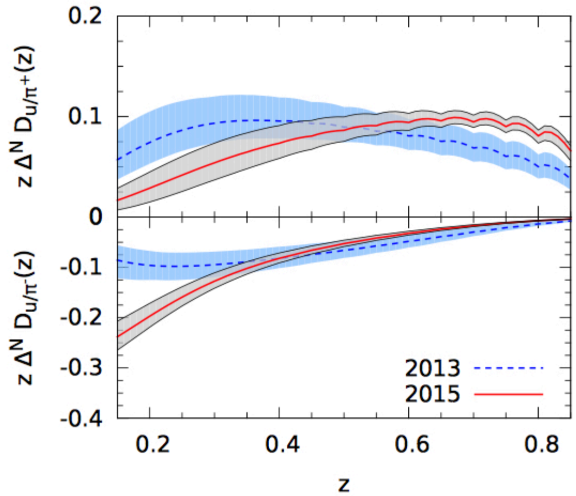


- figure shows $\Delta_T\, q = h_1^q$
- first-ever extraction of transversity
- fit for h_1^d saturates Soffer positivity bound (blue line) (Soffer, hep-ph/9409254)

$$h_1^q \leq \frac{1}{2}\,(f_1^q + g_1^q)$$

3. More recent analyses

- Extraction of Collins function (no TMD evolution) (Anselmino et al, arXiv:1510.05389)



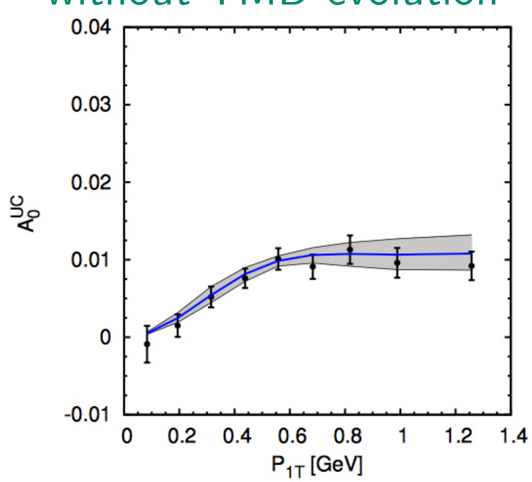
  – figure shows $p_\perp$-moment of

$$\Delta^N D_{h/q} = \frac{2\,p_\perp}{M_h}\,H_1^{h/q}$$

  – opposite sign for favored and disfavored Collins function

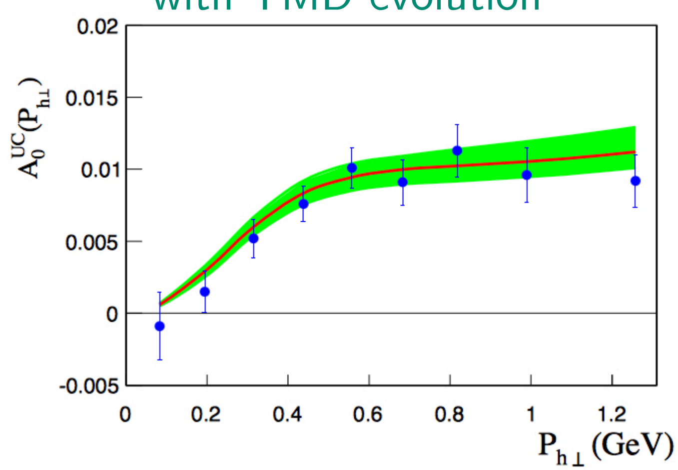
- Description of data (azimuthal asymmetry in $e^+e^- \to h_1 h_2 X$)

without TMD evolution



(Anselmino et al, arXiv:1510.05389)

with TMD evolution

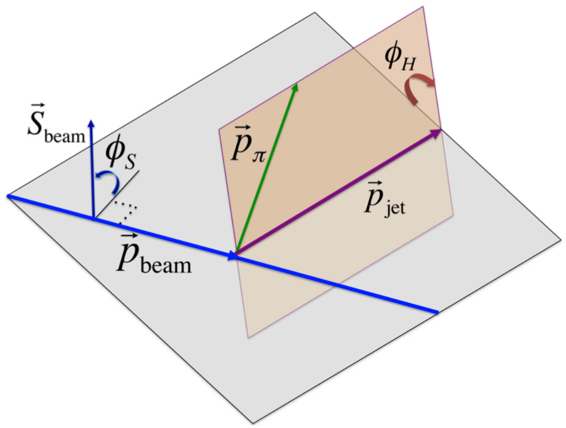


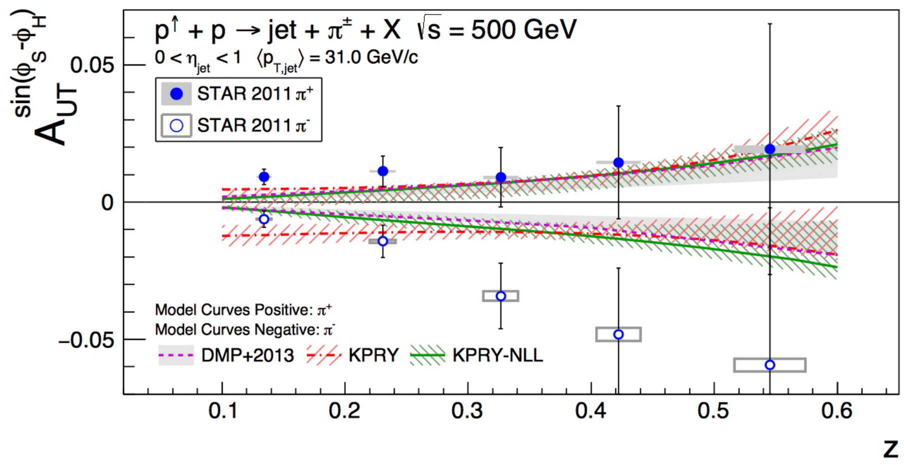
(Kang et al, arXiv:1505.05589)

  – current phenomenology of Collins effect does not require use of TMD evolution

4. Collins effect in $p^{\uparrow}p \to (h\,\mathrm{jet})X$

- Measurement at RHIC (STAR, arXiv:1708.07080)





  - data largely agree with calculation with TMD evolution (Kang et al, arXiv:1707.00913)
  - data largely agree with calculation without TMD evolution (D'Alesio, Murgia, Pisano, arXiv:1707.00914)
  - role of TMD evolution unclear for this observable
  - but, data compatible with TMD factorization and universality of Collins effect

Unpolarized TMDs from Experiment

1. First attempts using Gaussian model
   - Transverse width from Cahn effect
     - azimuthal dependence of unpolarized SIDIS cross section

$$d\sigma\big|_{\ell N \to \ell h X} \sim F_{UU} + \cos\phi_h\, F_{UU}^{\cos\phi_h} + \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \ldots$$

     - at small $P_{h\perp}$, $F_{UU}^{\cos\phi} \sim 1/Q$
     - Cahn effect: higher-twist effect due to transverse parton motion (Cahn, 1978)

$$F_{UU}^{\cos\phi_h} \;\sim\; \sum_q e_q^2\, f_1^q(x_\mathrm{B})\, D_1^{h/q}(z_h)\, \frac{\langle k_\perp^2\rangle\, z_h\, |\vec{P}_{h\perp}|}{\langle P_{h\perp}^2\rangle Q}\, \frac{e^{-\vec{P}_{h\perp}^2/\langle P_{h\perp}^2\rangle}}{\pi\langle P_{h\perp}^2\rangle}$$

$$\langle P_{h\perp}^2\rangle = z^2\langle k_\perp^2\rangle + \langle P_{hT}^2\rangle \;\text{ with }\; P_{hT} = z\, p_\perp$$

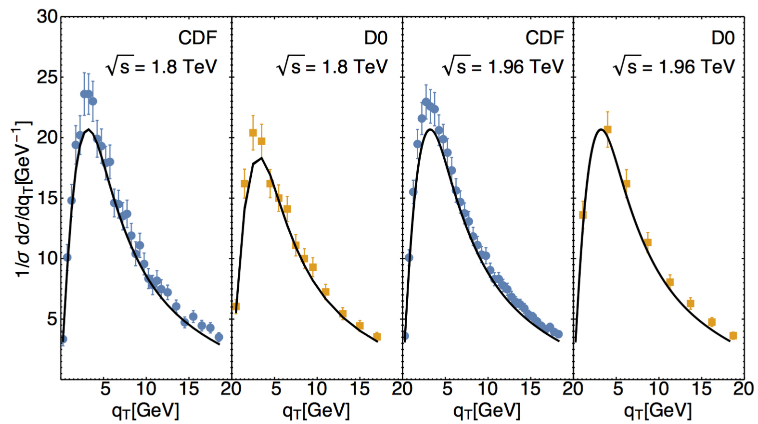
     - extraction of transverse widths (Anselmino et al, hep-ph/0501196)

$$\langle k_\perp^2\rangle = 0.25\,\mathrm{GeV}^2 \qquad \langle P_{hT}^2\rangle = 0.20\,\mathrm{GeV}^2$$

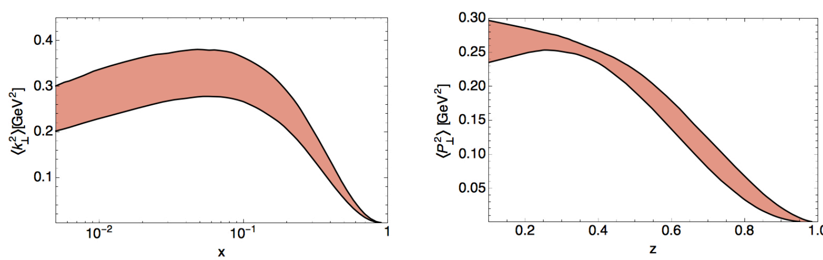
     - role of (other) higher-twist effects and radiative corrections ? (Bacchetta et al, 2008)
   - Similar results obtained from transverse momentum dependence of (twist-2) $F_{UU,T}$ in semi-inclusive DIS (Teckentrup, Schweitzer, Metz, 2010)

2. Recent analyses

- Fit from Pavia group (Bacchetta et al, arXiv:1703.10157)
  - data from SIDIS, Drell-Yan and Z-boson production ($>$ 8000 data points)
  - TMD evolution included
  - Z-boson production at Tevatron



  - widths for $k_\perp^2$ and P_{hT}^2

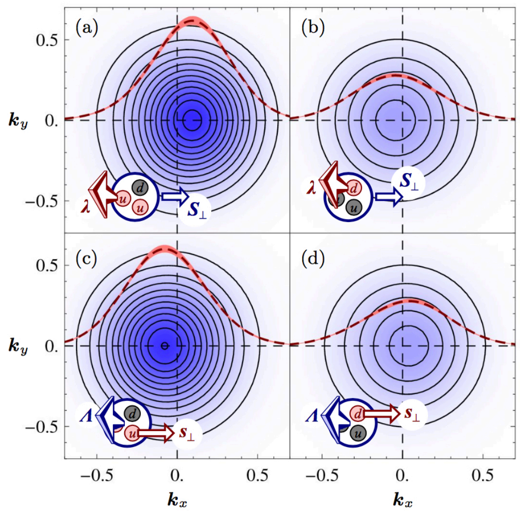


  - overall, fit quite successful, but normalization adjusted to some data sets
    $\rightarrow$ issue with (general) formalism and/or data sets ?
- Other related work exists that includes a large set of data
  (see, e.g., Scimemi, Vladimirov, arXiv:1706.01473)

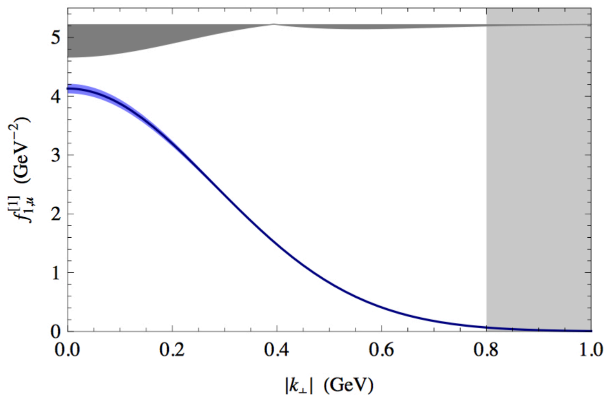
TMDs in Lattice QCD

- Pioneering results (Musch et al, arXiv:0908.1283, arXiv:1011.1213)

  worm gear functions g_{1T}^{q} and $h_{1L}^{\perp q}$



  unpolarized TMD-PDF f_1^q



  - important ingredients: $m_\pi \approx 500\,\mathrm{MeV}$; straight link connecting two quark fields; effects integrated upon x
  - g_{1T}^{q}, $h_{1L}^{\perp q}$ have opposite sign for u-quarks and d-quarks, and relative to each other
  - for small $k_\perp$, lattice data compatible with Gaussian shape of TMDs

- More results available
  - Sivers and Boer-Mulders effects for nucleon (Musch et al, arXiv:1111.4249)
  - Boer-Mulders effect for pion (Engelhardt et al, arXiv:1506.07826)

TMDs in Models

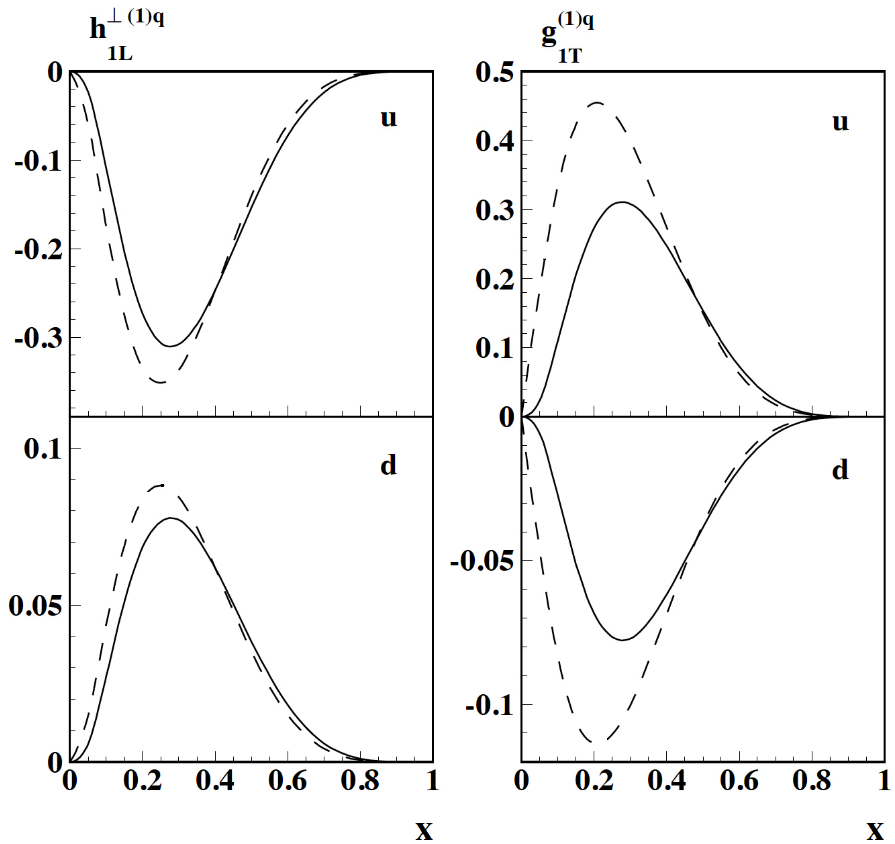
- Many calculations of TMDs in different models available:
  spectator models; bag model; chiral quark soliton model; covariant parton model; constituent quark models; etc.

- General philosophy for spectator models (Jakob, Mulders, Rodrigues, hep-ph/9704335)
  - typical diagrams

  - scalar and (axial) vector diquark as spectator
  - often phenomenological nucleon-quark-diquark vertex used
  - comprehensive studies exist (see e.g., Gamberg, Goldstein, Schlegel, arXiv:0708.0324 / Bacchetta, Conti, Radici, arXiv:0807.0323 / ...)

- Particular result in chiral quark soliton model (Schweitzer, Strikman, Weiss, arXiv:1210.1267)
  - TMD distributions (much) wider for anti-quarks than for valence quarks

- Particular result in constituent quark model (Pasquini, Cazzaniga, Boffi, arXiv:0806.2298)
  - worm gear functions g_{1T}^{q} and $h_{1L}^{\perp q}$



  - same general pattern later seen in lattice QCD

- Generally, model calculations often allow one to get rough estimate and intuition

Momentum Sum Rules for TMDs

1. Burkardt sum rule for Sivers function and implication
   - Average transverse momentum of quark in transversely polarized nucleon

$$\langle k_\perp^i(x)\rangle = \int d^2\vec{k}_\perp\, k_\perp^i\, \Phi^{[\gamma^+]}(x,\vec{k}_\perp,P,S)$$

$$= -M\varepsilon_\perp^{ij}\,S_\perp^j\,f_{1T}^{\perp(1)}(x) \;\; \text{with} \;\; f_{1T}^{\perp(1)}(x) = \int d^2\vec{k}_\perp\,\frac{\vec{k}_\perp^{\,2}}{2M^2}\,f_{1T}^\perp(x,\vec{k}_\perp^{\,2})$$

   - Sum rule (Burkardt, hep-ph/0402014)

$$\sum_a \int_0^1 dx\,\langle k_\perp^{i,a}(x)\rangle = 0 \;\;\rightarrow\;\; \sum_a \int_0^1 dx\, f_{1T}^{\perp(1)\,a}(x) = 0$$

   - Implication (Efremov et al, hep-ph/0412353)
     - use $f_{1T}^{\perp u} \approx -f_{1T}^{\perp d}$, and small contribution to sum rule from anti-quarks (from large-N_c analysis and phenomenolgy)
     - one then finds

$$\int_0^1 dx\, f_{1T}^{\perp(1)\,g}(x) \;\; \text{small}$$

     - also indication from phenomenology that gluon Sivers function small (e.g., Anselmino et al, hep-ph/0608211)

2. Schäfer-Teryaev sum rule for Collins function and implication

- Average transverse momentum of hadron in transversely polarized quark

$$\langle P_{hT}^{i}(z)\rangle \;\sim\; \varepsilon_{\perp}^{ij}\, s_{\perp}^{j}\; H_1^{\perp(1)\,h/q}(z)$$

$$H_1^{\perp(1)\,h/q}(z) \;=\; z^2 \int d^2\vec{p}_\perp\, \frac{\vec{p}_\perp^{\,2}}{2M_h^2}\, H_1^{\perp\, h/q}(z,\vec{p}_\perp^{\,2})$$

- Sum rule (Schäfer, Teryaev, hep-ph/9908412 / Meissner, Metz, Pitonyak, arXiv:1002.4393)

$$\sum_h \sum_{S_h} \int_0^1 dz\, \langle P_T^{i,h}(z)\rangle = 0 \;\;\rightarrow\;\; \sum_h \sum_{S_h} \int_0^1 dz\, z M_h\, H_1^{\perp(1)\,h/q}(z) = 0$$

- Implication
  - consider fragmentation into (charged) pions only
  - definition of favored and disfavored FFs

$$H_1^{\perp\,\mathrm{fav}} = H_1^{\perp\,\pi^+/u} = H_1^{\perp\,\pi^-/d} \qquad\qquad H_1^{\perp\,\mathrm{dis}} = H_1^{\perp\,\pi^-/u} = H_1^{\perp\,\pi^+/d}$$

  - one then finds

$$\int_0^1 dz\, z\, H_1^{\perp(1)\,\mathrm{fav}}(z) \approx - \int_0^1 dz\, z\, H_1^{\perp(1)\,\mathrm{dis}}(z)$$

  - in reasonable agreement with information from experiment

Lecture 4: TMDs: Special Topics I

- Some remarks on the evolution of TMDs

- Process dependence of the Sivers function
  - some history of the Sivers function
  - more on process dependence of the Sivers function
  - phenomenology of process dependence of the Sivers function

- Universality of TMD-FFs

- Transverse SSAs in processes like $p^{\uparrow} + p \to h + X$
  - sample data
  - why transverse polarization ?
  - necessary ingredients for transverse SSAs
  - A_N in $p^{\uparrow} p \to hX$ in the generalized parton model (GPM)
  - A_N in $p^{\uparrow} p \to hX$ in collinear twist-3 factorization

Some Remarks on the Evolution of TMDs

- Further reading, for instance:
  - Collins, *Foundations of perturbative QCD*, 2011
  - Rogers, arXiv:1509.04766

- In full glory QCD, TMDs depend on two (auxiliary) scales; e.g.:

$$f_1 = f_1(x, \vec{k}_\perp^{\,2}; \mu, \zeta)$$

  - μ: due to UV divergence
  - ζ: due to rapidity divergence (various prescriptions for regulating this divergence)

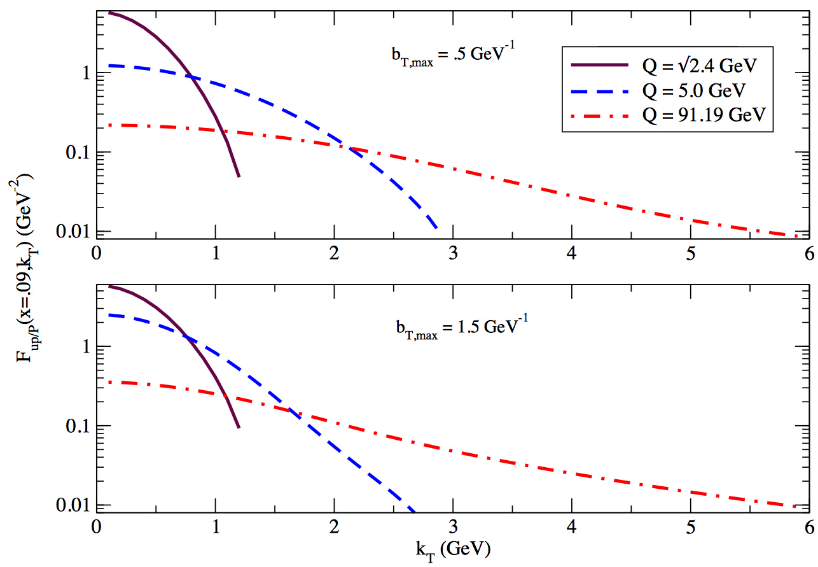
- Rapidity divergence complicates the relation between TMDs and PDFs
  - after regulating rapidity divergence one has

$$f_1(x, \mu) \neq \int d^2\vec{k}_\perp \, f_1(x, \vec{k}_\perp^{\,2}; \mu, \zeta)$$

  - rapidity divergence cancels for PDF $f_1(x, \mu)$ (between real and virtual diagrams) (e.g., Collins, hep-ph/0304122)

- Evolution of TMDs
  - μ-dependence: differs from DGLAP evolution
  - ζ-dependence: governed by Collins-Soper evolution equation (Collins, Soper, 1981)
  - TMD evolution has also dependence on non-perturbative input
    $\rightarrow$ at present, considerable uncertainties

- Numerical study (Aybat, Rogers, arXiv:1101.5057)

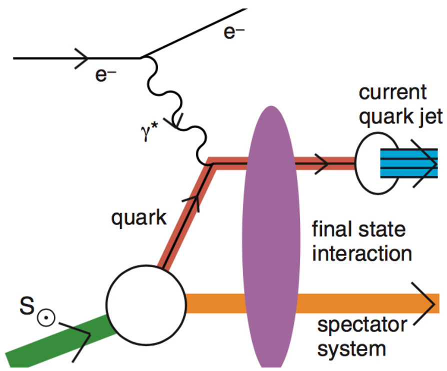


  - evolution broadens TMDs
  - evolution expected to dilute effects like Sivers asymmetry, but present data not conclusive in that regard

- At present, implication of TMD evolution still largely uncertain for many observables and data sets

Process Dependence of the Sivers Function

1. Some history of the Sivers function
   - In 1989, Dennis Sivers suggested the function (correlation)
   - In 1992, John Collins argued that $f_{1T}^{\perp} = 0$ due to T-reversal invariance
   - Model calculation of transverse SSA in DIS (Brodsky, Hwang, Schmidt, hep-ph/0201296)



     - spectator system modeled by scalar diquark
     - FSI modeled by single photon exchange
     - nonzero transverse target SSA A_{UT}
     - A_{UT} given by interference of lowest-order graph and (imaginary part of) box graph

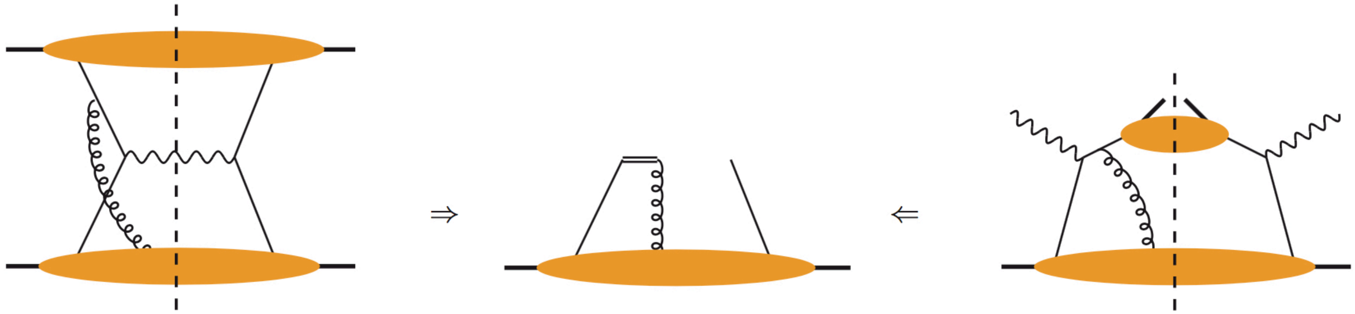
   - Interpretation of BHS calculation, correction, prediction (Collins, hep-ph/0204004)
     - nonzero A_{UT} of BHS can be described in TMD factorization using $f_{1T}^{\perp}$
     - if $\mathcal{W}_{TMD}$ taken into account, T-reversal does not forbid existence of $f_{1T}^{\perp}$
     - T-reversal rather predicts process dependence:

$$f_{1T}^{\perp}\big|_{\text{DIS}} = -\, f_{1T}^{\perp}\big|_{\text{DY}} \qquad\qquad h_{1}^{\perp}\big|_{\text{DIS}} = -\, h_{1}^{\perp}\big|_{\text{DY}}$$

   - These developments were very important for entire field of TMDs
   - Citations of 1989 Sivers paper: ~ 100 before 2002, ~ 1000 today

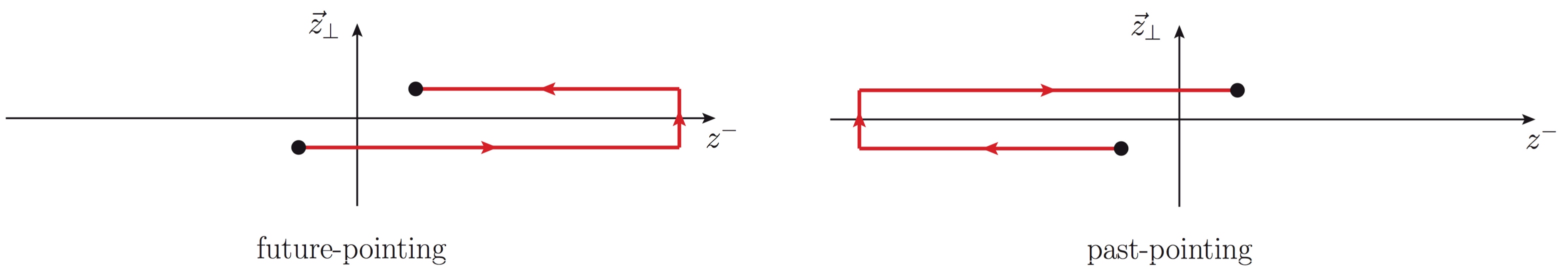
2. More on process dependence of the Sivers function

- "Box graph" and TMD factorization



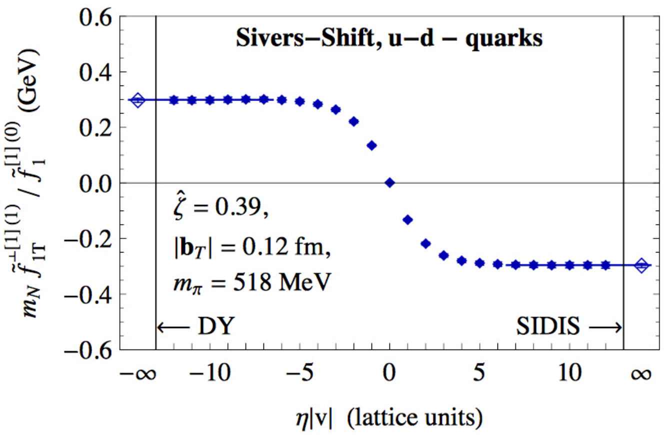
(figure from Diehl, arXiv:1512.01328)

- Gauge link structure in semi-inclusive DIS and Drell-Yan (FSI vs ISI)



- T-reversal allows one to relate definitions in two processes (Collins, hep-ph/0204004)
  - T-even TMDs are universal
  - T-odd TMDs change sign between SIDIS and DY
  - breakdown of universality, but in well-defined way
  - strictly speaking, T-odd TMDs not exclusively property of nucleon

- Sign reversal in lattice QCD (Musch et al, arXiv:1111.4249)



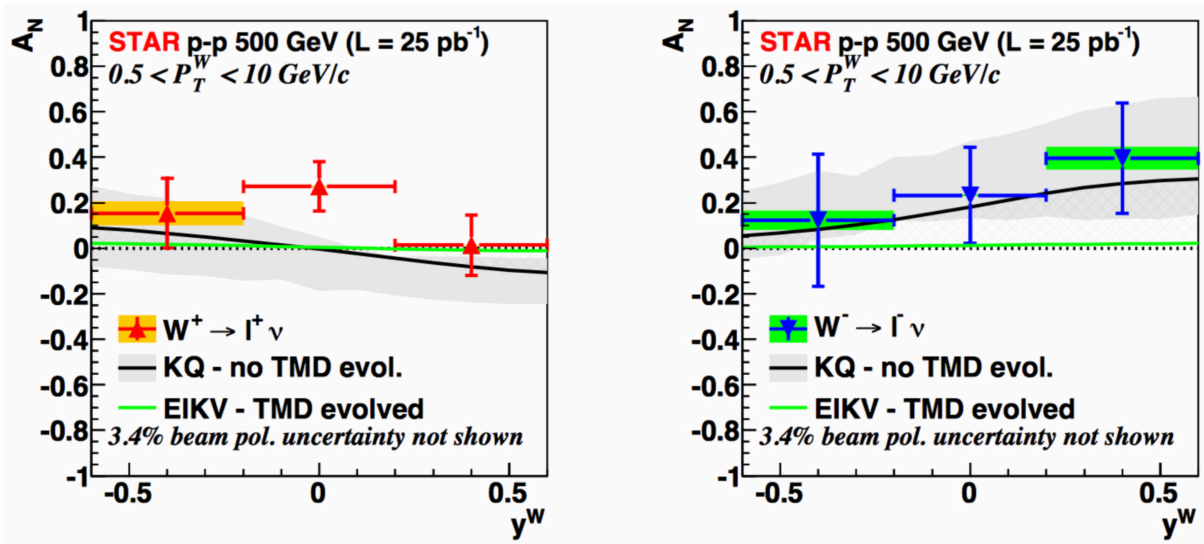
  - calculation for staple-like gauge links with finite length
  - results saturate for distances of about 0.4 fm

- What if sign reversal of $f_{1T}^{\perp}$ not confirmed by experiment ?
  - would not imply that QCD is wrong
  - would imply that SSAs not understood in QCD
  - problem with TMD-factorization
  - implication on resummation of large transverse momentum logarithms
  - implication on many calculations for, e.g., observables at the LHC
  - problem with collinear twist-3 factorization

- Experimental check of process dependence of $f_{1T}^{\perp}$ is crucial
  (DOE Hadron Physics Performance Milestone, HP13: *Test unique QCD predictions for relations between single transverse spin phenomena in p-p scattering and those observed in deep-inelastic lepton scattering*)

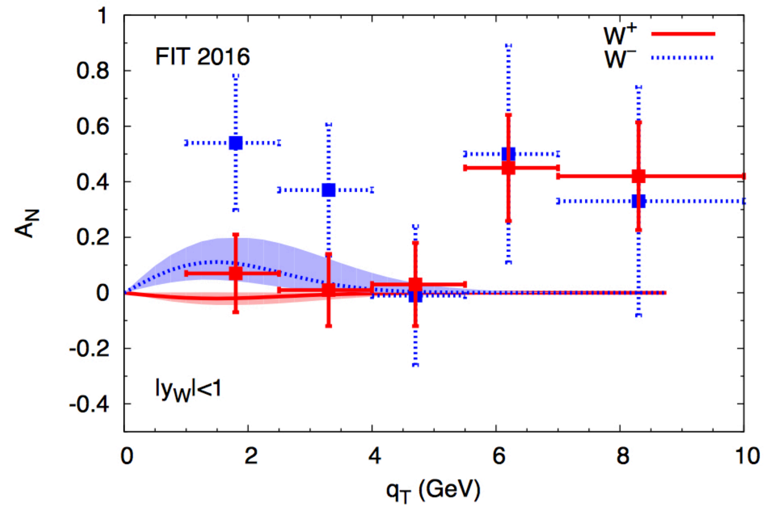
3. Phenomenology of process dependence of the Sivers function

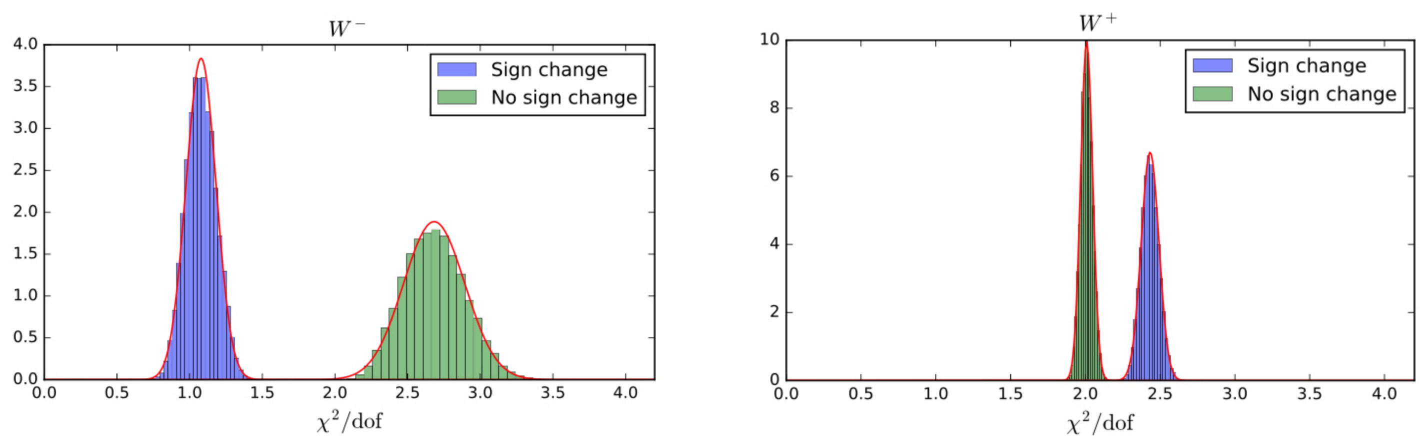
- Measurement of Sivers asymmetry in $p^{\uparrow}p \to W^{\pm}/Z^0\,X$ at RHIC (STAR, arXiv:1511.06003)



  - relevant scale is mass of heavy gauge bosons
  - long evolution from measurements of Sivers effect in semi-inclusive DIS
  - calculations with and without TMD evolution lead to very different results for asymmetry
  - such measurements can help constrain the (TMD) evolution
  - based on Kang-Qiu (KQ) calculation without evolution one could conclude verification of sign reversal

– another calculation without TMD evolution (Anselmino et al, arXiv:1612.06413)

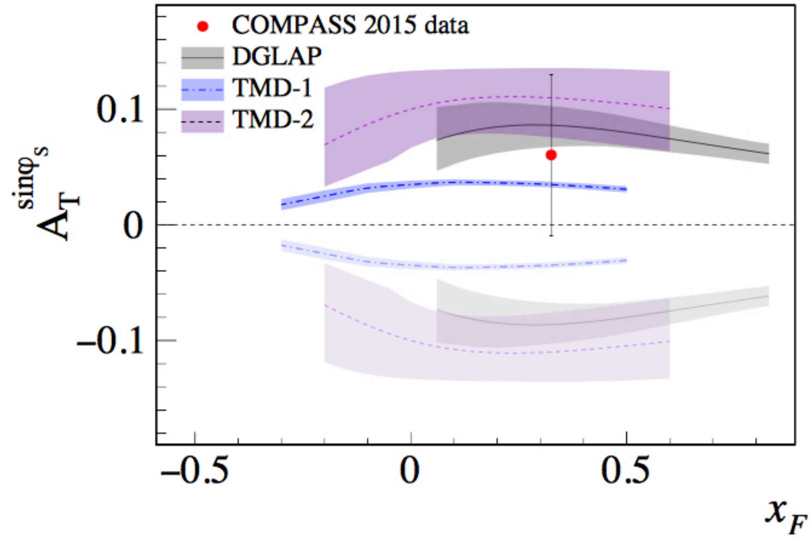




∗ according to this calculation, STAR measurement less conclusive

∗ generally, large uncertainties of Sivers functions for antiquarks imply large uncertainties for asymmetry

- Measurement of Sivers asymmetry in $\pi^- p^\uparrow \to \mu^+ \mu^- X$ at COMPASS
  (COMPASS, arXiv:1704.00488)



  – scale of measurement: $4.3\,\mathrm{GeV}^2 \leq m_{\mu\mu}^2 \leq 8.5\,\mathrm{GeV}^2$
  – data point favors sign reversal of Sivers fucntion

- Other work on process dependence of Sivers effect
  – simultaneous study of transverse SSAs in inclusive and in semi-inclusive DIS
    (Metz et al, arXiv:1209.3138)
  – study of transverse SSA A_N in $p^\uparrow p \to \mathrm{jet}\,X$
    (Gamberg, Kang, Prokudin, arXiv:1302.3213)

- Overall, strong indication from phenomenology that Sivers effect depends on process

Universality of TMD-FFs

- TMD-FFs in $\ell N \to \ell h X$ and $e^+e^- \to h_1 h_2 X$, $a\ priori$, have different Wilson lines (ISI vs FSI)

- TMD-FFs in two processes cannot be related by means of T-reversal $\to$ universality ?

- However, one-loop calculation in spectator model reveals universality of two T-odd fragmentation functions $D_{1T}^{\perp}$, $H_1^{\perp}$ (Metz, hep-ph/0209054)

- TMD-FFs not sensitive to direction of Wilson lines (based on kinematics) (Collins, Metz, hep-ph/0408249)

- All later studies confirmed universality of (T-odd) TMD-FFs (Yuan, 2007, 2008 / Gamberg, Mukherjee, Mulders, 2008, 2010 / Meissner, Metz, 2008 / Yuan, Zhou, 2009)

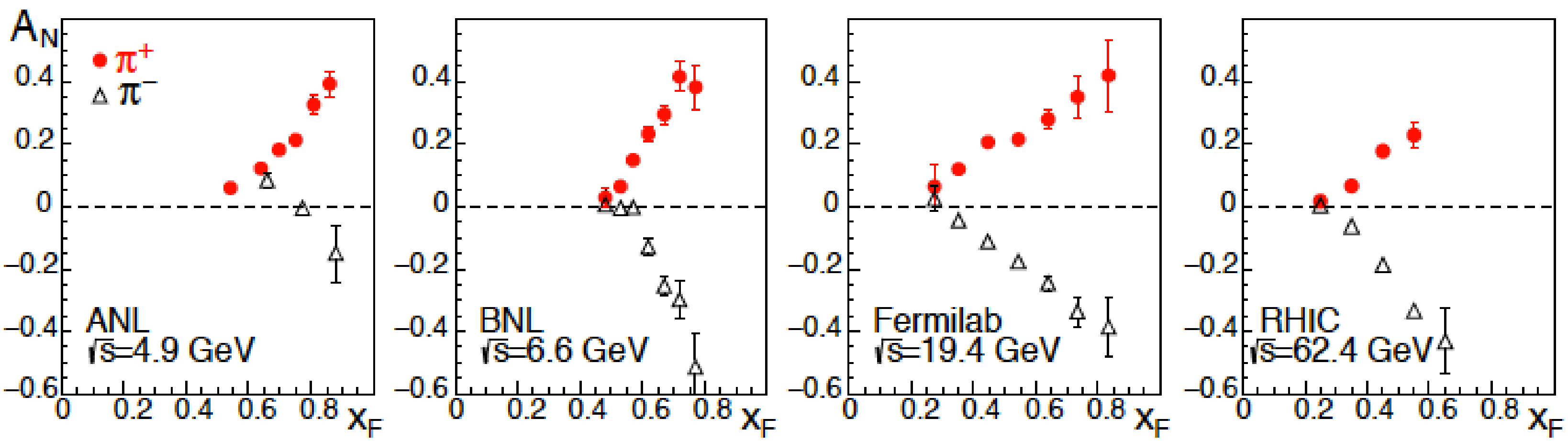
- For instance, universality of Collins function $H_1^{\perp}$ is crucial ingredient for first extraction of transversity distribution h_1 (Anselmino et al, hep-ph/0701006)

Transverse SSAs in Processes like $p^\uparrow + p \to h + X$

1. Sample data
   - Observable

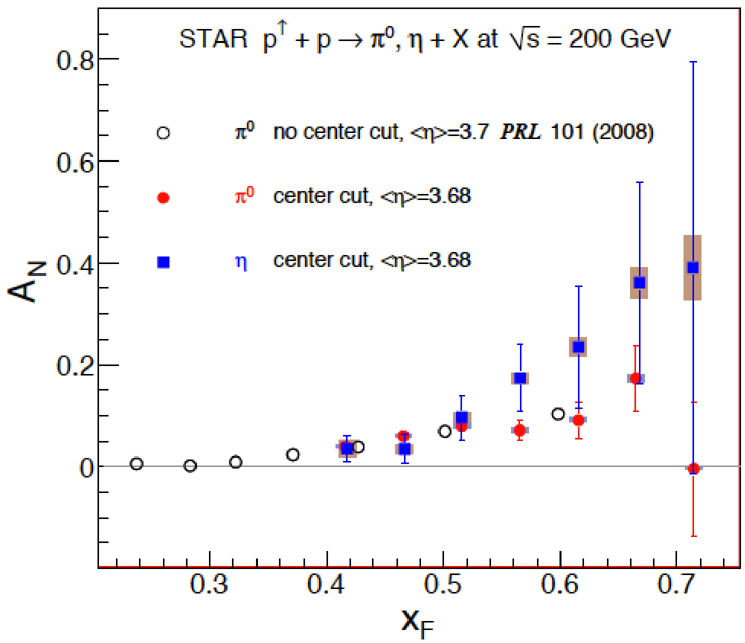
$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \sim \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

   - Charged pions: sample data $(x_F = 2P_{hL}/\sqrt{s})$

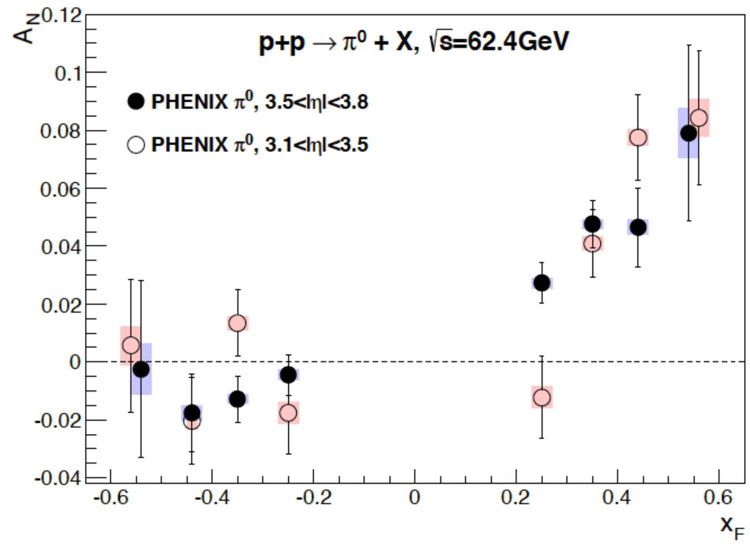


(figure from Aidala, et al, arXiv:1209.2803)

- Neutral pions: sample data



(STAR, 2012) $\sqrt{s} = 200\ \mathrm{GeV}$



PHENIX, 2013 $\sqrt{s} = 62.4\ \mathrm{GeV}$

- General features
  - very striking effects at large x_F
  - A_N survives at large $\sqrt{s}$
  - $A_N^{\pi^0}$ systematically smaller than $A_N^{\pi^\pm}$
  - A_N cannot be explained in collinear parton model (Kane, Pumplin, Repko, 1978)
  - data on transverse SSAs represent 40-year old puzzle

2. Why transverse polarization ?

- Consider following process

$$p(P_a, S_a) + p(P_b) \to h(P_h) + X$$

- Parity-conserving single-spin correlation

$$\varepsilon_{\mu\nu\rho\sigma}\, P_a^\mu\, P_b^\nu\, P_h^\rho\, S_a^\sigma \sim \vec{S}_a \cdot (\vec{P}_a \times \vec{P}_h) \qquad (*)$$

  - only transverse vector $\vec{S}_{a\perp}$ enters
  - SSA necessarily transverse $(A_{UT}\,,\;\; A_N)$

- Parity-conserving longitudinal single-spin correlations
  - if more particles detected, parity-conserving longitudinal SSAs allowed; example: $F_{UL}^{\sin 2\phi_h}$ in $\ell N \to \ell h X$

- Problem 9: Consider center-of-mass frame of two protons and verify $(*)$

3. Necessary ingredients for transverse SSAs

- Consider, for instance, elastic pion-proton scattering $(\pi\, p^{\uparrow} \to \pi\, p)$

- Amplitude in Pauli-space

$$\mathcal{M} = \chi_f^{\dagger}\, (A + i\, \vec{\sigma} \cdot \vec{B})\, \chi_i$$

- Transverse SSA (for polarization along y-direction)

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{\mathrm{Tr}\,(A + i\,\vec{\sigma}\cdot\vec{B})\,\sigma_y\,(A^* - i\,\vec{\sigma}\cdot\vec{B}^*)}{\mathrm{Tr}\,(A + i\,\vec{\sigma}\cdot\vec{B})\,(A^* - i\,\vec{\sigma}\cdot\vec{B}^*)}$$

$$= \frac{2\,\mathrm{Im}(AB_y^* + B_x B_z^*)}{|A|^2 + |B_x|^2 + |B_y|^2 + |B_z|^2} \qquad (*)$$

  - General result: A_N nonzero only if

    (i) interference of two different amplitudes

    (ii) at least one of these amplitudes has imaginary part

- Problem 10: Check whether $(*)$ is correct

4. A_N in $p^\uparrow p \to hX$ in the generalized parton model (GPM)

- Assumes TMD factorization for unpolarized and polarized cross sections
  - already used about 40 years ago (Feynman, Field, Fox, 1978)
  - generic structure of unpolarized cross section

$$d\sigma = H \otimes \Phi(x_a, \vec{k}_{a\perp}) \otimes \Phi(x_b, \vec{k}_{b\perp}) \otimes \Delta(z, \vec{k}_{c\perp})$$

- Main advantages
  - decent description of twist-2 unpolarized cross section at leading order
  - can mimic effects of higher-order corrections of collinear treatment
  - contains certain kinematical higher twist effects that may be important
  - provides "simple" explanation of nonzero A_N through

    (i) Sivers effect (Sivers, 1989)

    (ii) Collins effect

- Main drawbacks
  - presently, no derivation of TMD factorization for this process
  - (arbitrary) infrared cutoff for k_T integrations needed
  - does not take into account physics of ISI/FSI for Sivers effect

- Many detailed phenomenological studies available

  (e.g., Anselmino, Boglione, Murgia, hep-ph/9503290 / ... / Anselmino et al, arXiv:1304.7691)

- Flavor structure of A_N (use: no antiquarks, dominance of $qg \to qg$ channel)
  (i) Sivers effect

$$d\sigma^{\uparrow}_{\rm Siv}(\pi^+) \;\sim\; f_{1T}^{\perp u} \otimes f_1^g \otimes D_1^{\rm fav} + f_{1T}^{\perp d} \otimes f_1^g \otimes D_1^{\rm dis}$$

$$d\sigma^{\uparrow}_{\rm Siv}(\pi^-) \;\sim\; f_{1T}^{\perp d} \otimes f_1^g \otimes D_1^{\rm fav} + f_{1T}^{\perp u} \otimes f_1^g \otimes D_1^{\rm dis}$$

  - contribution from gluon fragmentation (not shown) largely cancels
  - can explain reversed sign for $A_N(\pi^+)$ and $A_N(\pi^-)$
  - partial cancellation between favored and disfavored fragmentation

  (ii) Collins effect

$$d\sigma^{\uparrow}_{\rm Col}(\pi^+) \;\sim\; h_1^u \otimes f_1^g \otimes H_1^{\perp,\rm fav} + h_1^d \otimes f_1^g \otimes H_1^{\perp,\rm dis}$$

$$d\sigma^{\uparrow}_{\rm Col}(\pi^-) \;\sim\; h_1^d \otimes f_1^g \otimes H_1^{\perp,\rm fav} + h_1^u \otimes f_1^g \otimes H_1^{\perp,\rm dis}$$

  - h_1^u and h_1^d have opposite signs
  - can explain reversed sign for $A_N(\pi^+)$ and $A_N(\pi^-)$
  - contributions from favored and disfavored fragmentation have same sign
  - can be larger than Sivers contribution

5. A_N in $p^{\uparrow}p \to hX$ in collinear twist-3 factorization

- Estimate in (twist-2) parton model (Kane, Pumplin, Repko, 1978)

$$A_N \sim \alpha_s \frac{m_q}{P_{h\perp}} \qquad \text{Note: } A_N \not\sim \alpha_s \frac{m_q}{\sqrt{s}}$$

  - α_s due to NLO graphs needed for imaginary part
  - this transverse spin effect proportional to mass of polarized particle
  - calculation clearly reveals subleading-twist (twist-3) nature of A_N

- Collinear twist-3 factorization in full glory ($P_{h\perp}$ is the only scale)
  (Ellis, Furmanski, Petronzio, 1983 / Efremov, Teryaev, 1983, 1984 / Qiu, Sterman, 1991, 1998 / Koike et al, 2000, ... / etc.)
  - Generic structure of polarized cross section

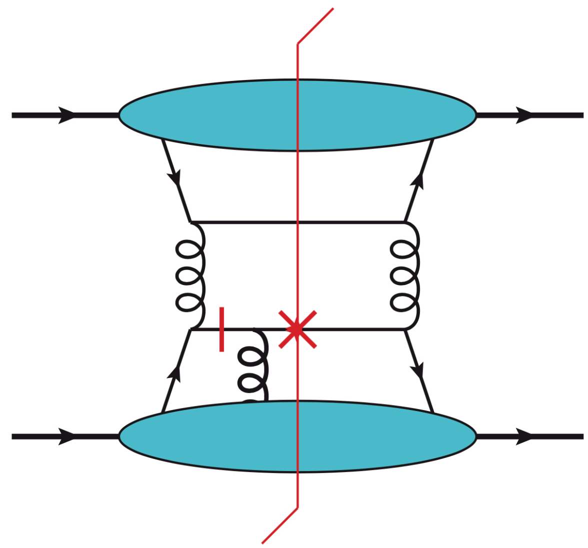
$$\begin{aligned} d\sigma^{\uparrow} &= H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} && \to \text{Sivers-type} \\ &+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} && \to \text{Boer-Mulders-type} \\ &+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)} && \to \text{``Collins-type''} \end{aligned}$$

- Sivers-type contribution
  - contribution from Qiu-Sterman function T_F (Qiu, Sterman,1991)

$$\int \frac{dz_1^- dz_2^-}{4\pi} e^{ixP^+ z_1^-} \langle P, S | \bar{\psi}^q(0)\, \gamma^+\, g\, F^{+i}(z_2^-)\, \psi^q(z_1^-) | P, S \rangle = \varepsilon_\perp^{ij}\, S_\perp^j\, T_F^q(x,x)$$

vanishing gluon momentum $\rightarrow$ soft gluon pole matrix element

  - sample diagram for $qq \rightarrow qq$ channel



* quark propagator goes on-shell for vanishing gluon momentum
* provides required imaginary part
* attach extra gluon in all possible ways and consider all graphs and channels
* contributions from both ISI and FSI

– generic structure of $d\sigma^{\uparrow}_{\rm Siv}$

$$d\sigma^{\uparrow}_{\rm Siv} \sim \sum_i \sum_{a,b,c} H^i \otimes T_F^a(x_a, x_a) \otimes f_1^b \otimes D_1^c \qquad \rightarrow \quad \text{SGPs}$$

$$+ \sum_i \sum_{a,b,c} \tilde{H}^i \otimes \left(T_F^a(0, x_a) + \tilde{T}_F^a(0, x_a)\right) \otimes f_1^b \otimes D_1^c \quad \rightarrow \quad \text{SFPs}$$

* soft gluon pole (SGP) contribution has relation to TMD approach
* soft fermion pole (SFP) contribution has no relation to TMD approach
* SFP matrix elements may be small (Kang et al, 2010 / Braun et al, 2011)
* H^i and $\tilde{H}^i$ contain also physics of ISI/FSI (in contrast to GPM)

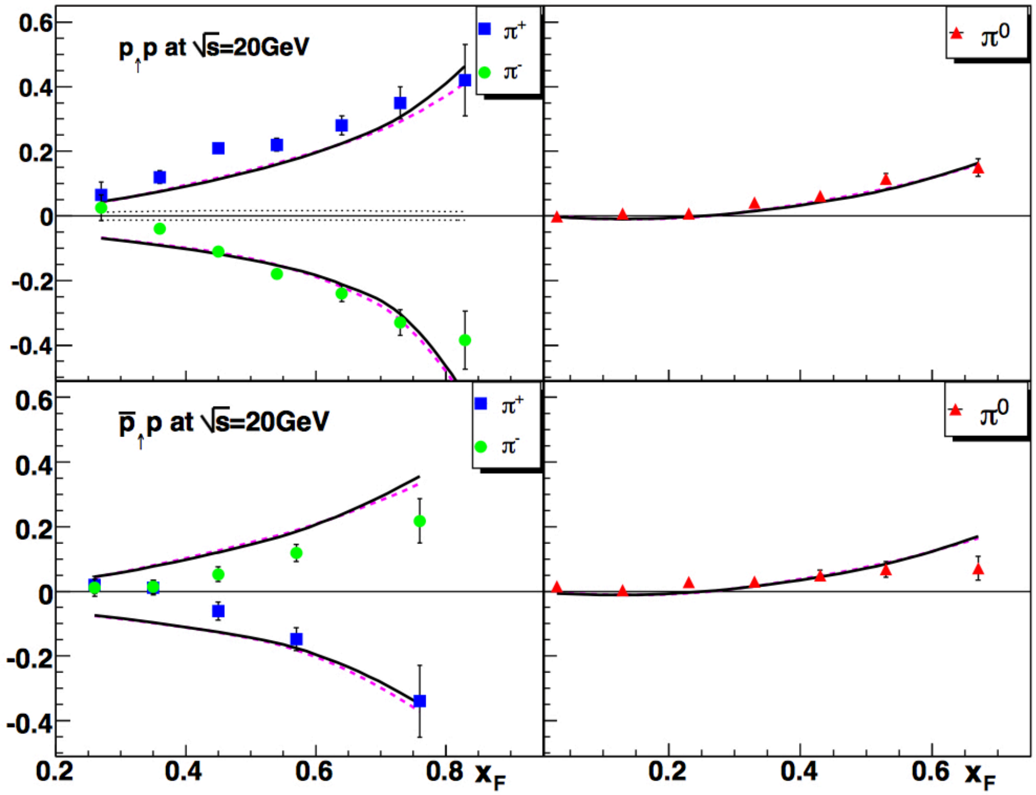
– relation between T_F and $f_{1T}^{\perp}$ (Boer, Mulders, Pijlman, hep-ph/0303034)

$$T_F^q(x, x) = -\int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^{\,2}}{M} \, f_{1T}^{\perp q}(x, \vec{k}_\perp^{\,2})\Big|_{\rm DIS} = 2\, \varepsilon_\perp^{ij} \, S_\perp^j \, \langle k_\perp^{i,q}(x) \rangle$$

* provides very intuitive interpretation of T_F
* relation between $A_{\rm SIDIS}^{\rm Siv}$ in SIDIS and A_N in $p^{\uparrow} p \rightarrow h\, X$ possible
* flavor structure of A_N like in TMD approach
* because of ISI/FSI, magnitude and sign of A_N may differ from TMD approach

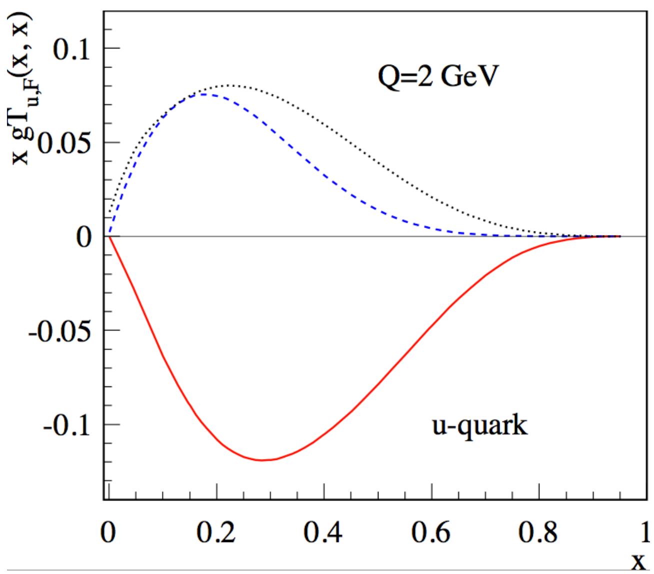
– successful phenomenology using (soft-gluon pole function) $T_F(x, x)$

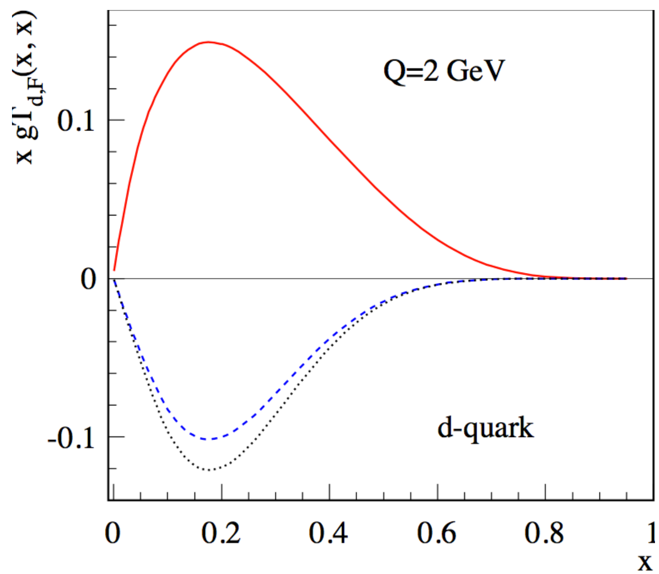
* fit of A_N for pion and kaon production in proton-proton collisions
  (Kouvaris et al, hep-ph/0609238)



* later on, general results confirmed, and studies extended (SFPs included)
  (Kanazawa, Koike, arXiv:1005.1468, arXiv:1104.0117)

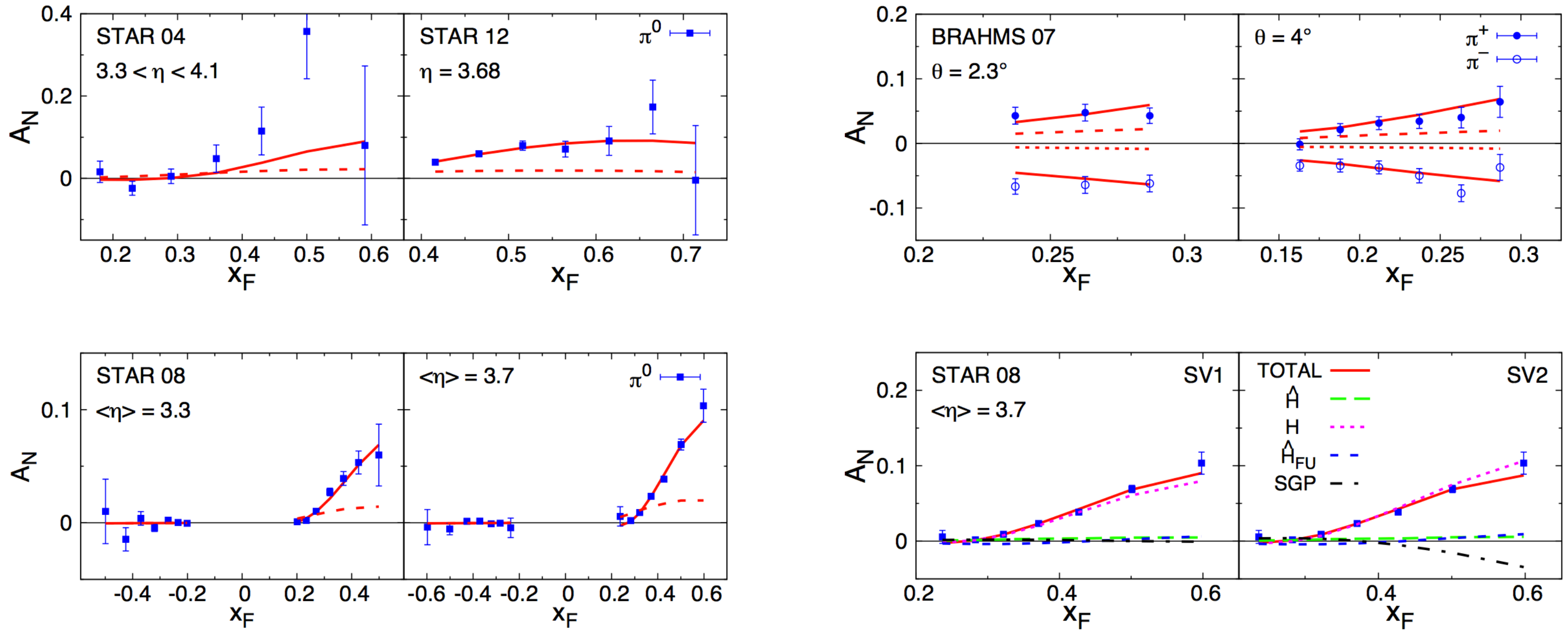
- Sivers-type contribution and sign-mismatch problem (Kang, et al, arXiv:1103.1591)
  - assume A_N in $p^{\uparrow}p \to hX$ is dominated by Sivers-type contribution
  - $T_F(x,x)$ can be extracted from different sources (direct vs Sivers input)





  - striking sign mismatch !
  - which sign for $T_F(x,x)$ is correct ?
  - model calculation suggests sign coming from Sivers input (Braun et al, 2011)
  - one may doubt dominance of Sivers-type contribution in A_N
  - Boer-Mulders type contribution to $d\sigma^{\uparrow}$ small (Koike, Kanazawa, hep-ph/0007272)
  - can large A_N in $p^{\uparrow}p \to hX$ be caused by "Collins-type" contribution ?

- Twist-3 fragmentation contribution to A_N in $p^{\uparrow}p \to hX$
  - analytical results available (Metz, Pitonyak, arXiv:1212.5037)
  - fit of data from RHIC (Kanazawa et al, arXiv:1404.1033)



  - simultaneous description of A_N, and $A_{\rm SIDIS}^{\rm Siv}$, $A_{\rm SIDIS}^{\rm Col}$, $A_{e^+e^-}^{\cos(2\phi)}$ possible
  - A_N in $p^{\uparrow}p \to hX$ may be largely caused by (higher-twist) fragmentation effect
  - independent information on relevant 3-parton fragmentation correlator needed
  - recent update of numerics; general conclusions unchanged (Gamberg et al, 1701.09170)

Lecture 5: TMDs: Special Topics II

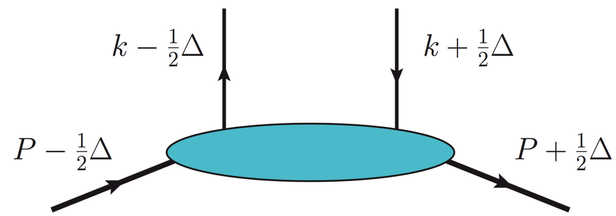
- Some remarks on GPDs
  (→ lectures by Daria Sokhan)

- Nontrivial relations between TMDs and GPDs
  - relation between $f_{1T}^{\perp}$ and the GPD $\mathcal{E}$
  - more nontrivial relations between TMDs and GPDs

- Generalized TMDs (GTMDs)
  - definition of GTMDs
  - GTMDs as "mother distributions"
  - GTMDs and Wigner functions
  - GTMDs and orbital angular momentum

Some Remarks on GPDs

- Appear in QCD-description of hard exclusive reactions (DVCS, HEMP)
- Graphical representation of GPD correlator, and kinematics in symmetric frame



$$P = \frac{p+p'}{2} \qquad \Delta = p' - p$$

- GPD-correlator for unpolarized quarks (spin dependence of hadrons suppressed)

$$F^{q\,[\gamma^+]} = \int \frac{dz^-}{4\pi} e^{ik\cdot z} \langle p' \,|\, \bar\psi^q(-\tfrac{z}{2})\, \gamma^+ \, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2},\tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=\vec z_\perp=0}$$

$$= \frac{1}{2P^+}\,\bar u(p') \left(\gamma^+ \, H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\, E^q(x,\xi,t) \right) u(p)$$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

- (Eight) leading twist quark GPDs for

$$\bar\psi\,\gamma^+\,\psi \qquad \bar\psi\,\gamma^+\gamma_5\,\psi \qquad \bar\psi\, i\sigma^{i+}\gamma_5\,\psi$$

- Relation to forward PDFs and form factors (crucial for modeling)

$$H^q(x,0,0) = f_1^q(x) \qquad \int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t)$$

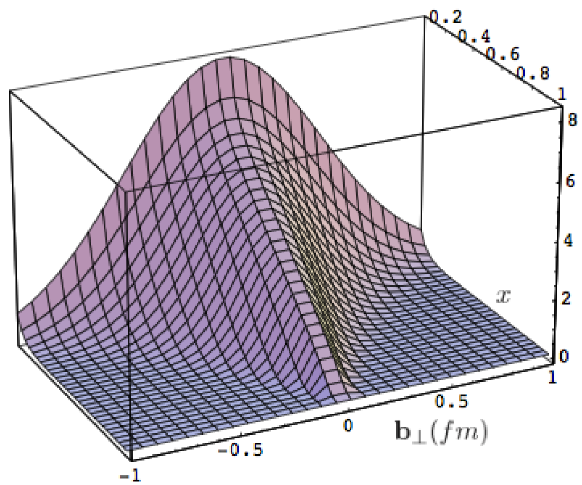
- Impact parameter representation $(\xi = 0) \to$ density interpretation
(Burkardt, hep-ph/0005108, hep-ph/0207047 / Pire, Ralston, hep-ph/0110075 / Diehl, hep-ph/0205208)

$$\begin{aligned}\mathcal{F}^{q\,[\gamma^+]}(x,\vec{b}_\perp,S) &= \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\,\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, F^{q\,[\gamma^+]}(x,\vec{\Delta}_\perp,S) \\ &= \mathcal{H}^q(x,\vec{b}_\perp^{\,2}) + \frac{\varepsilon_\perp^{ij}\, b_\perp^i\, S_\perp^j}{M}\, \frac{\partial}{\partial\vec{b}_\perp^{\,2}}\, \mathcal{E}^q(x,\vec{b}_T^{\,2})\end{aligned}$$

  - 3-D structure in $(x,\vec{b}_\perp)$-space ("spatial" imaging)
  - $\vec{b}_\perp$ relative to transverse center of longitudinal momentum $\sum_i p_i^+\,\vec{b}_{\perp i} \,/\, \sum_i p_i^+$
  - term containing $\mathcal{E}^q$ generates dipole pattern
  $\to$ (numerically large) distortion of $\mathcal{F}^{q\,[\gamma^+]}(x,\vec{b}_\perp,S)$

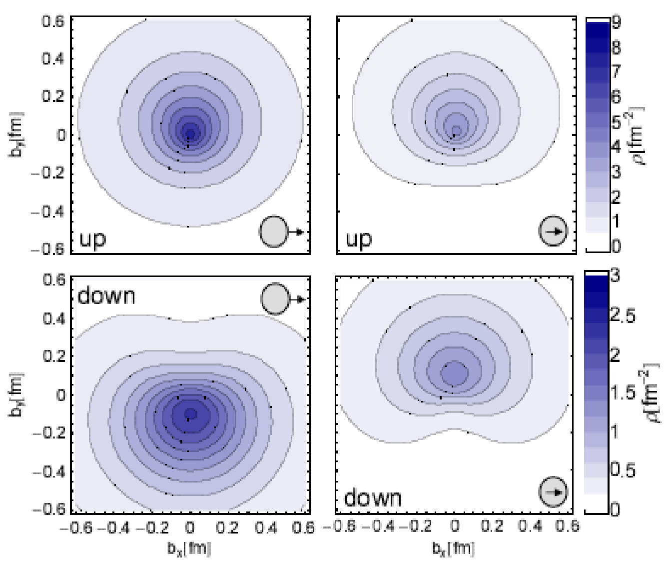
- GPDs in impact parameter space: sample plots

  (i) no polarization: toy model for GPD (Burkardt, hep-ph/0207047)



  - $b_\perp$ distribution gets narrow at large x
  - general pattern agrees with phenomenology

  (ii) with transverse polarization (nucleon and quark) (QCDSF-UKQCD, hep-lat/0612032)



  left: unpolarized quarks in transversely polarized target

  right: transversely polarized quarks in unpolarized target

  - distortion stronger for down quarks
  - distortion stronger for transv. pol. quarks in unpol. nucleon
  - similar results in models and GPD parameterizations

Nontrivial Relations between TMDs and GPDs

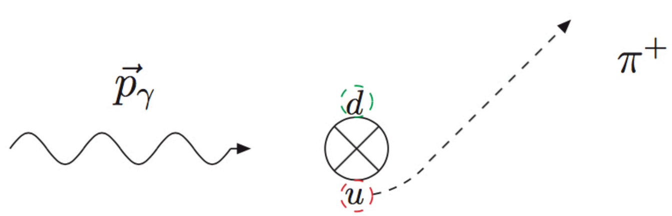
1. Relation between $f_{1T}^{\perp}$ and the GPD $\mathcal{E}$
   - Example of trivial relation between TMDs and GPDs

$$\int d^2\vec{k}_\perp \, f_1^q(x, \vec{k}_\perp^{\,2}) = f_1^q(x) = H^q(x,0,0)$$

   - Sivers effect and "chromodynamic lensing" (Burkardt, hep-ph/0302144)
     - flavor dipole moment

$$\begin{aligned} d^{q,i} &= \int dx \int d^2\vec{b}_\perp \, b_\perp^i \, \mathcal{F}^{q\,[\gamma^+]}(x, \vec{b}_\perp, S) \\ &= -\frac{\varepsilon_\perp^{ij} S_\perp^j}{2M} \int dx \, E^q(x,0,0) = -\frac{\varepsilon_\perp^{ij} S_\perp^j}{2M} \kappa^q \quad \rightarrow \quad |d^q| \approx 0.2\,\mathrm{fm} \ \ (\mathrm{large}) \end{aligned}$$

     - connection with Sivers effect



* assume that γ^* "sees" $b_\perp$-distribution
* assume that FSI of struck quark is attractive
* allows one to determine sign of Sivers effect for $\pi^\pm$ in semi-inclusive DIS

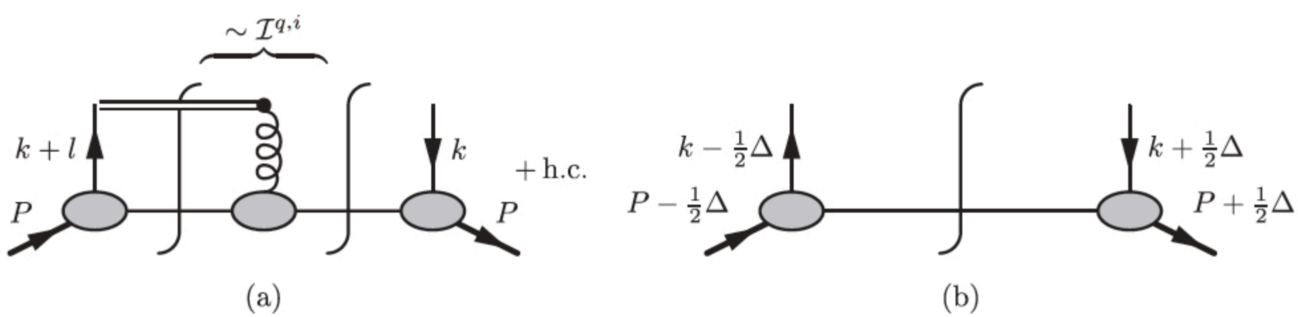
- Relation between $f_{1T}^{\perp}$ and the GPD $\mathcal{E}$ in spectator model
  - rationale underlying "chromodynamic lensing", *a priori*, not obvious: why should QCD description of SIDIS be related to GPD-correlator in $b_\perp$-space ?
  - but, quantitative relation in scalar diquark model (Burkardt, Hwang, hep-ph/0309072)

$$\langle k_\perp^{q,i}(x)\rangle_{UT} = -\int d^2\vec{k}_\perp\, k_\perp^i\, \frac{\varepsilon_\perp^{jk}\, k_\perp^j\, S_\perp^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2}) \qquad \text{(model-independent)}$$

$$= \int d^2\vec{b}_\perp\, \underbrace{\mathcal{I}^{q,i}(x,\vec{b}_\perp)}_{\text{lensing function}}\, \frac{\varepsilon_T^{jk}\, b_\perp^j\, S_\perp^k}{M}\, \Big(\mathcal{E}^q(x,\vec{b}_\perp^{\,2})\Big)' \qquad \text{(model-dependent)}$$

  - Interpretation of model result

Sivers effect = Distortion ⊗ FSI



- Sign reversal of Sivers function and "chromodynamic lensing"
  - "chromodynamic lensing" compatible with sign reversal

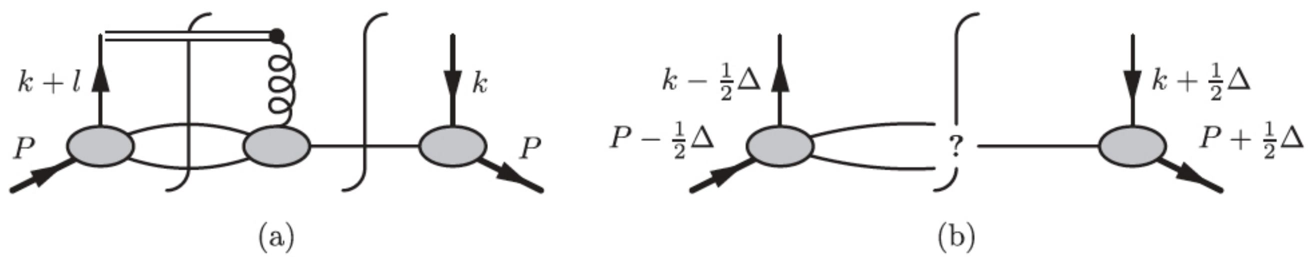
$$f_{1T}^{\perp}\big|_{\rm DIS} = -\, f_{1T}^{\perp}\big|_{\rm DY}$$

  - in model, lensing function $\mathcal{I}^{q,i}(x,\vec{b}_\perp)$ reverses sign
  - in "lensing picture", sign reversal due to difference between attractive FSI and repulsive ISI (often used to explain sign reversal to general audience)
  - in "lensing picture", Sivers effect factorizes into universal property of hadron (distortion) and non-universal contribution

- No model-independent factorization of Sivers effect

Sivers effect $\neq$ Distortion $\otimes$ FSI

  - is obvious, e.g., from model calculations (Meissner, Metz, Goeke, hep-ph/0703176)



  - numerical impact of "factorization breaking" contributions presently not known

2. More nontrivial relations between TMDs and GPDs

- Comparing correlators for TMDs and GPDs (Diehl, Hägler, hep-ph/0504175)

$$\Phi^{q\,[\gamma^+]}(x,\vec{k}_\perp,S) \;=\; f_1^q(x,\vec{k}_\perp^{\,2}) - \frac{\varepsilon_\perp^{ij}\,k_\perp^i\,S_\perp^j}{M}\,f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2})$$

$$\mathcal{F}^{q\,[\gamma^+]}(x,\vec{b}_\perp,S) \;=\; \mathcal{H}^q(x,\vec{b}_\perp^{\,2}) + \frac{\epsilon_\perp^{ij}\,b_\perp^i\,S_\perp^j}{M}\left(\mathcal{E}^q(x,\vec{b}_\perp^{\,2})\right)'$$

  – comparison allows one to find “analogy”

$$f_{1T}^{\perp q} \;\leftrightarrow\; -\left(\mathcal{E}^q\right)'$$

  – comparison can be extended to other quark and gluon distributions

  – no relation for GPDs $\tilde{E},\ \tilde{E}_T$ (drop out for $\xi = 0$) and TMDs $g_{1T},\ h_{1L}^\perp$

- Results from comparing correlators (Meissner, Metz, Goeke, hep-ph/0703176)

  (i) (trivial) relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \qquad\qquad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^2}{2M^2}\, h_{1T}^{\perp q}\right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^2}{M^2}\, \Delta\tilde{\mathcal{H}}_T^q\right)$$

  (ii) relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g}\right)' \qquad\qquad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q\right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^2}{2M^2}\, h_{1T}^{\perp g}\right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^2}{M^2}\, \Delta\tilde{\mathcal{H}}_T^g\right)'$$

  (iii) relations of third type

$$h_{1T}^{\perp q} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^q\right)'' \qquad\qquad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g\right)''$$

  (iv) relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g\right)'''$$

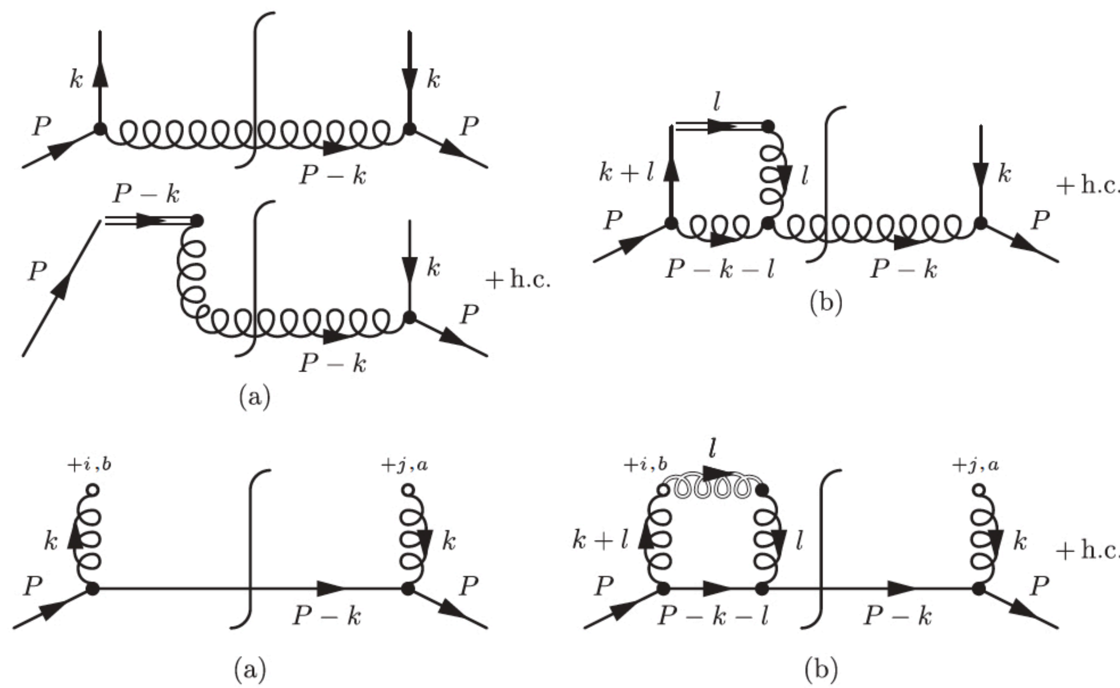
- Some consequences
  - relation for Boer-Mulders function $h_1^{\perp q}$ expected to match with the one for $f_{1T}^{\perp q}$ (Burkardt, hep-ph/0505189 / Meissner, Metz, Goeke, hep-ph/0703176)

$$
\begin{aligned}
\langle k_\perp^{q,i}(x)\rangle_{UT} &= -\int d^2\vec{k}_\perp\, k_\perp^i \,\frac{\varepsilon_\perp^{jk}\, k_\perp^j\, S_\perp^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_\perp^{\,2}) \\
&= \int d^2\vec{b}_\perp\, \mathcal{I}^{q,i}(x,\vec{b}_\perp)\,\frac{\varepsilon_T^{jk}\, b_\perp^j\, S_\perp^k}{M}\left(\mathcal{E}^q(x,\vec{b}_\perp^{\,2})\right)'
\end{aligned}
$$

$$
\begin{aligned}
\langle k_\perp^{q,i}(x)\rangle_{TU}^{j} &= -\int d^2\vec{k}_\perp\, k_\perp^i \,\frac{\varepsilon_\perp^{kj}\, k_\perp^k}{M}\, h_1^{\perp q}(x,\vec{k}_\perp^{\,2}) \\
&= \int d^2\vec{b}_\perp\, \mathcal{I}^{q,i}(x,\vec{b}_\perp)\,\frac{\varepsilon_\perp^{kj}\, b_\perp^k}{M}\left(\mathcal{E}_T^q(x,\vec{b}_\perp^{\,2}) + 2\tilde{\mathcal{H}}_T^q(x,\vec{b}_\perp^{\,2})\right)'
\end{aligned}
$$

    * distortion of quark densities governed by "anomalous magnetic moments"
      (i) unpol. quarks: $\kappa^u = 1.7 \quad \kappa^d = -2.0$ (experiment)
      (ii) transv. pol. quarks: $\kappa_T^u = 3.0 \quad \kappa_T^d = 1.9$ (QCDSF-UKQCD, hep-lat/0612032)
    * current phenomenology of $f_{1T}^{\perp q}$ and $h_1^{\perp q}$ compatible with these numbers
  - relation for $h_{1T}^{\perp q}$ expected to be different

- Model results continued (Meissner, Metz, Goeke, hep-ph/0703176)
  - scalar diquark model of the nucleon
    * allows one to study relations for quarks and (scalar) diquarks
  - quark target model in QCD



    * allows one to study relations for quarks and gluons

– moments of TMDs and GPDs

$$X^{(n)}(x) = \int d^2\vec{k}_\perp \left(\frac{\vec{k}_\perp^{\,2}}{2M^2}\right)^n X(x, \vec{k}_\perp^{\,2})$$

$$Y^{(n)}(x) = \frac{1}{2M^2}\int d^2\vec{\Delta}_\perp \left(\frac{\vec{\Delta}_\perp^{\,2}}{2M^2}\right)^{n-1} Y\Big(x, 0, -\frac{\vec{\Delta}_\perp^{\,2}}{(1-x)^2}\Big)$$

– relations of second type

$$f_{1T}^{\perp q\,(n)}(x) = H_2(n)\,\frac{1}{1-x}\,E^{q\,(n)}(x) \qquad (0 \le n \le 1)$$

* $H_2(n)$ depends on model
* formula holds for all the relations of second type
* it is not necessary to go to $b_\perp$ space in order to establish relations
* particular cases

$$f_{1T}^{\perp q\,(0)}(x) = \frac{\pi\, e_q\, e_s}{48(1-x)}\,E^q(x,0,0) \qquad \text{(Lu, Schmidt, hep-ph/0611158)}$$

$$f_{1T}^{\perp q\,(1)}(x) = \frac{e_q\, e_s}{4(2\pi)^2\,(1-x)}\,E^{q\,(1)}(x)$$

– relations of third type

$$h_{1T}^{\perp q\,(n)}(x) = H_3(n)\,\frac{1}{(1-x)^2}\,\tilde{H}_T^{q\,(n)}(x) \qquad\qquad (0 \leq n \leq 1)$$

* $H_3(n)$ is the same in both models
* formula holds for all the relations of third type
* particular case

$$h_{1T}^{\perp q\,(0)}(x) = \int d^2\vec{k}_\perp\, h_{1T}^{\perp q}(x, \vec{k}_\perp^{\,2}) = \frac{3}{(1-x)^2}\,\tilde{H}_T^q(x,0,0)$$

* same relation, but with pre-factor 2 on r.h.s., found in other model
  (Pasquini, Cazzaniga, Boffi, arXiv:0806.2298)

– relation of fourth type
* trivially satisfied in quark target model because

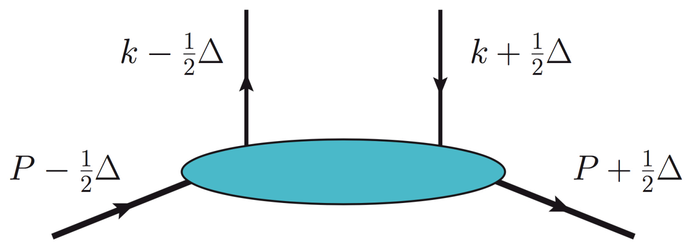
$$h_{1T}^{\perp g} = \tilde{\mathcal{H}}_T^g = 0$$

- All nontrivial relation between TMDs and GPDs are model-dependent
  (Meissner, Metz, Schlegel, arXiv:0906.5323)

Generalized TMDs

1. Definition of GTMDs

- Graphical representation of GTMD correlator, and kinematics in symmetric frame



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GTMD correlator: definition (through traces)

$$W^{q\,[\Gamma]} = \int \frac{dz^-\, d^2\vec{z}_\perp}{2\,(2\pi)^3}\, e^{ik\cdot z}\, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}_{\mathrm{TMD}}[-\tfrac{z}{2}, \tfrac{z}{2}]\, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=0}$$

– $W^{q\,[\Gamma]}$ parameterized through GTMDs $X^q(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad \vec{k}_\perp \qquad \vec{\Delta}_\perp = \vec{p}\,'_\perp - \vec{p}_\perp$$

– note that $\Delta^2 = t\,(\xi, \vec{\Delta}_\perp^2)$

- Leading-twist chiral-even quark GTMDs (Meissner, Metz, Schlegel, arXiv:0906.5323)

$$W^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p')\left[F_{1,1} + \frac{i\sigma^{i+}\,k_\perp^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\,\Delta_\perp^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}\,k_\perp^i\,\Delta_\perp^j}{M^2}\,F_{1,4}\right]u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M}\,\bar{u}(p')\left[-\frac{i\varepsilon_\perp^{ij}\,k_\perp^i\,\Delta_\perp^j}{M^2}\,G_{1,1} + \frac{i\sigma^{i+}\,\gamma_5\,k_\perp^i}{P^+}\,G_{1,2} + \frac{i\sigma^{i+}\,\gamma_5\,\Delta_\perp^i}{P^+}\,G_{1,3} + i\sigma^{+-}\,\gamma_5\,G_{1,4}\right]u(p)$$

- General results
  - 16 leading-twist GTMDs for quarks (Meissner, Metz, Schlegel, arXiv:0906.5323)
  - 16 leading-twist GTMDs for gluons (Lorcé, Pasquini, arXiv:1307.4497)
  - GTMDs have real and imaginary part

2. GTMDs as "mother distributions"

- GTMD-correlator

$$W^{q\,[\Gamma]} = \int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=0}$$

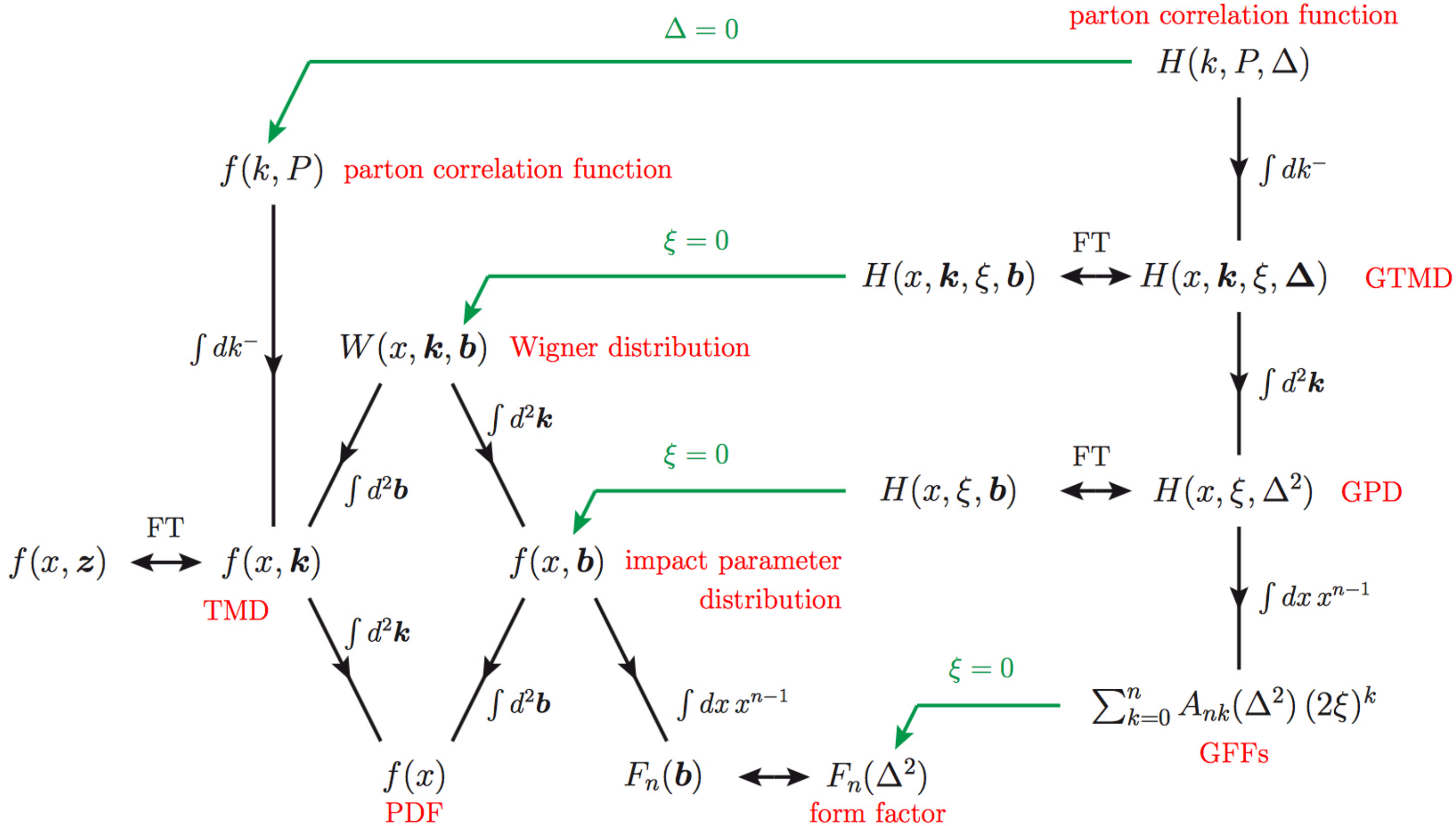
- Projection onto TMDs and GPDs

$$\Phi^{q\,[\Gamma]} = \int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle p \,|\, \bar{\psi}^q(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=0} = W^{q\,[\Gamma]} \Big|_{\Delta=0}$$

$$F^{q\,[\Gamma]} = \int \frac{dz^-}{4\pi} \, e^{ik\cdot z} \, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\rm PDF}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=\vec{z}_\perp=0} = \int d^2\vec{k}_\perp \, W^{q\,[\Gamma]}$$

  - all TMDs and GPDs are projections of GTMDs
  - GTMDs contain genuine new physics
  - GTMDs describe the most general two-parton structure of hadrons

- Overview of objects characterizing the parton structure of hadrons



(figure from Diehl, arXiv:1512.01328)

  - hardly any studies on describing processes with k^- dependent correlators (parton correlation functions)
  - mapping out GTMDs may be an ultimate goal of parton structure studies

3. GTMDs and Wigner functions

- Wigner quasi-probability distribution in QM (calculable from wave function)

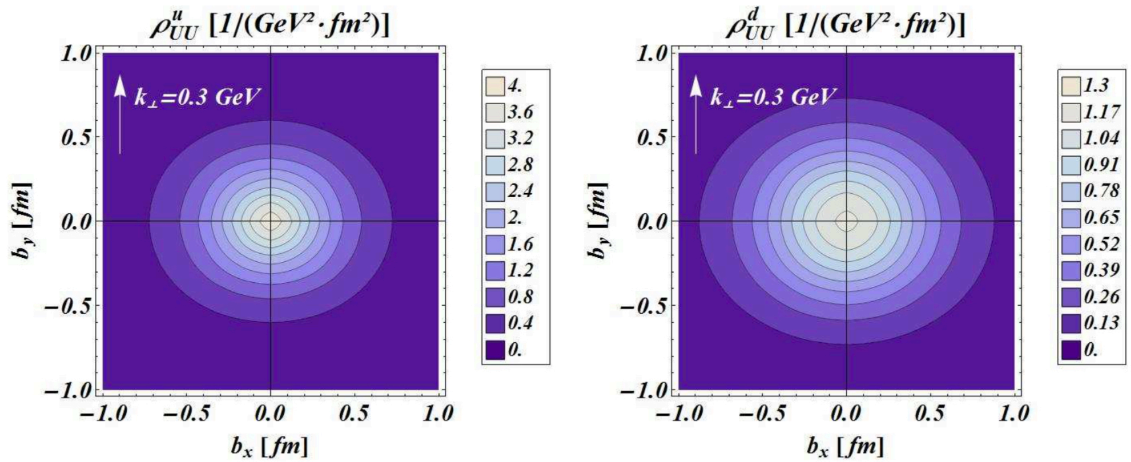
$$
\begin{aligned}
|\psi(x)|^2 &= \int dp\, \mathcal{W}(x,p) \\
|\psi(p)|^2 &= \int dx\, \mathcal{W}(x,p) \\
\langle O(x,p)\rangle &= \int dx\, dp\, O(x,p)\, \mathcal{W}(x,p)
\end{aligned}
$$

- Analogy: Wigner distributions for 3-D imaging of hadrons

(Belitsky, Ji, Yuan, hep-ph/0307383 / Lorcé, Pasquini, Vanderhaeghen, arXiv:1102.4704)

$$
\begin{aligned}
\mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) &= \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-i\,\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, W^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{\Delta}_\perp)\Big|_{\xi=0} \\
\mathcal{F}^{q\,[\Gamma]}(x,\vec{b}_\perp) &= \int d^2\vec{k}_\perp\, \mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) \\
\Phi^{q\,[\Gamma]}(x,\vec{k}_\perp) &= \int d^2\vec{b}_\perp\, \mathcal{W}^{q\,[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp) \\
\langle O(x,\vec{k}_\perp,\vec{b}_\perp)\rangle &= \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, O(x,\vec{k}_\perp,\vec{b}_\perp)\, \mathcal{W}^{q[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp)
\end{aligned}
$$

- Numerical example of Wigner distributions (Lorcé, Pasquini, arXiv:1106.0139)



  – figures show $\mathcal{W}_U^{q\,[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$ (unpolarized quarks and unpolarized nucleon), integrated upon x, for fixed $\vec{k}_\perp$
  – results in light-cone constituent quark model
  – wider distribution for down quarks (known also from form factor studies)
  – distortion due to dependence on $\vec{k}_\perp \cdot \vec{b}_\perp$
  – top-bottom symmetry since $\mathcal{W}_U^{q\,[\gamma^+]}$ is even function of $\vec{k}_\perp \cdot \vec{b}_\perp$
  – overall results in line with intuition from confinement

- Wigner functions can become negative
  – probability interpretation, in general, does not work
  – appropriate smearing of Wigner distribution may be remedy for problem (e.g., Husimi distribution) (Hagiwara, Hatta, arXiv:1412.4591)

4. GTMDs and orbital angular momentum

- Parton OAM in longitudinally polarized nucleon (Lorcé, Pasquini, arXiv:1106.0139 / Hatta, arXiv:1111.3547 / Hägler, Mukherjee, Schäfer, hep-ph/0310136)

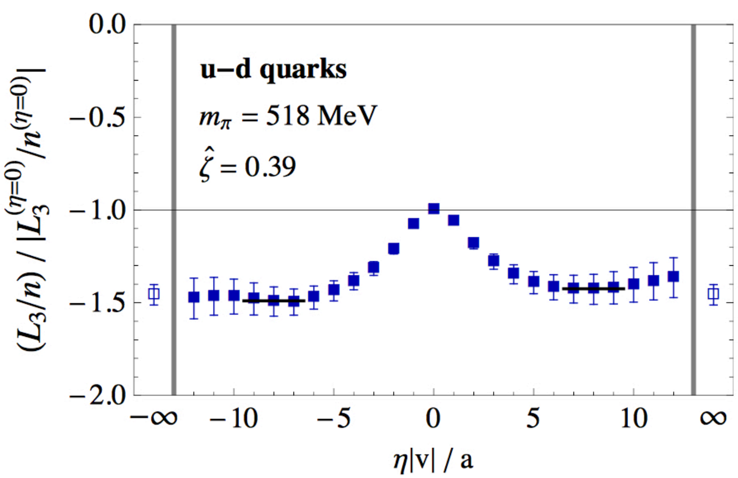
$$L_z^q = \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, (\vec{b}_\perp \times \vec{k}_\perp)_z\, \mathcal{W}_L^{q\,[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= -\int dx\, d^2\vec{k}_\perp\, \frac{\vec{k}_\perp^{\,2}}{M^2}\, F_{1,4}^q(x, \vec{k}_\perp^{\,2})\Big|_{\Delta=0}$$

  – intuitive definition of OAM

  – same equation for both $L_{\rm JM}$ (staple-like link) and $L_{\rm Ji}$ (straight link) (Ji, Xiong, Yuan, arXiv:1202.2843)

  – equation holds for gluons as well

- Exploratory calculation of $L_{\rm JM}^q$ in lattice QCD (Engelhardt, arXiv:1701.01536)



  – figure shows essentially $L_{\rm JM}/L_{\rm Ji}$

  – large numerical difference between $L_{\rm JM}$ and $L_{\rm Ji}$

  – entire development leading to this result is milestone in spin physics

Lecture 6: TMDs: Opportunities at a new EIC

- Limitations of existing data / facilities

- Jefferson Lab, 12 GeV upgrade

- (Potential) future electron-ion collider(s)
  ($\rightarrow$ lectures by Yulia Furletova)

- Addressing the gluon Sivers function

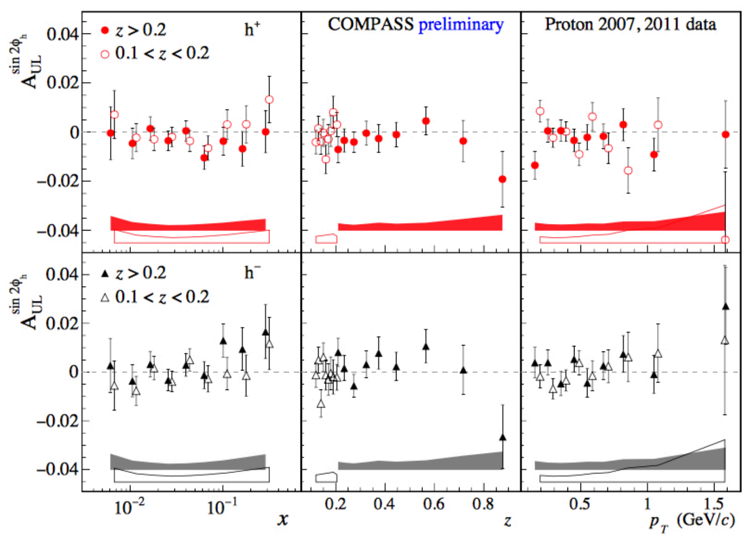
- Addressing the density of linearly polarized gluons

- Addressing GTMDs / Wigner functions

Limitations of Existing Data / Facilities

Existing data / facilities typically suffer from one or more of the following:

- lack of data precision (due to lack of machine luminosity)



(COMPASS, arXix:1801.01488)

  - sample data for

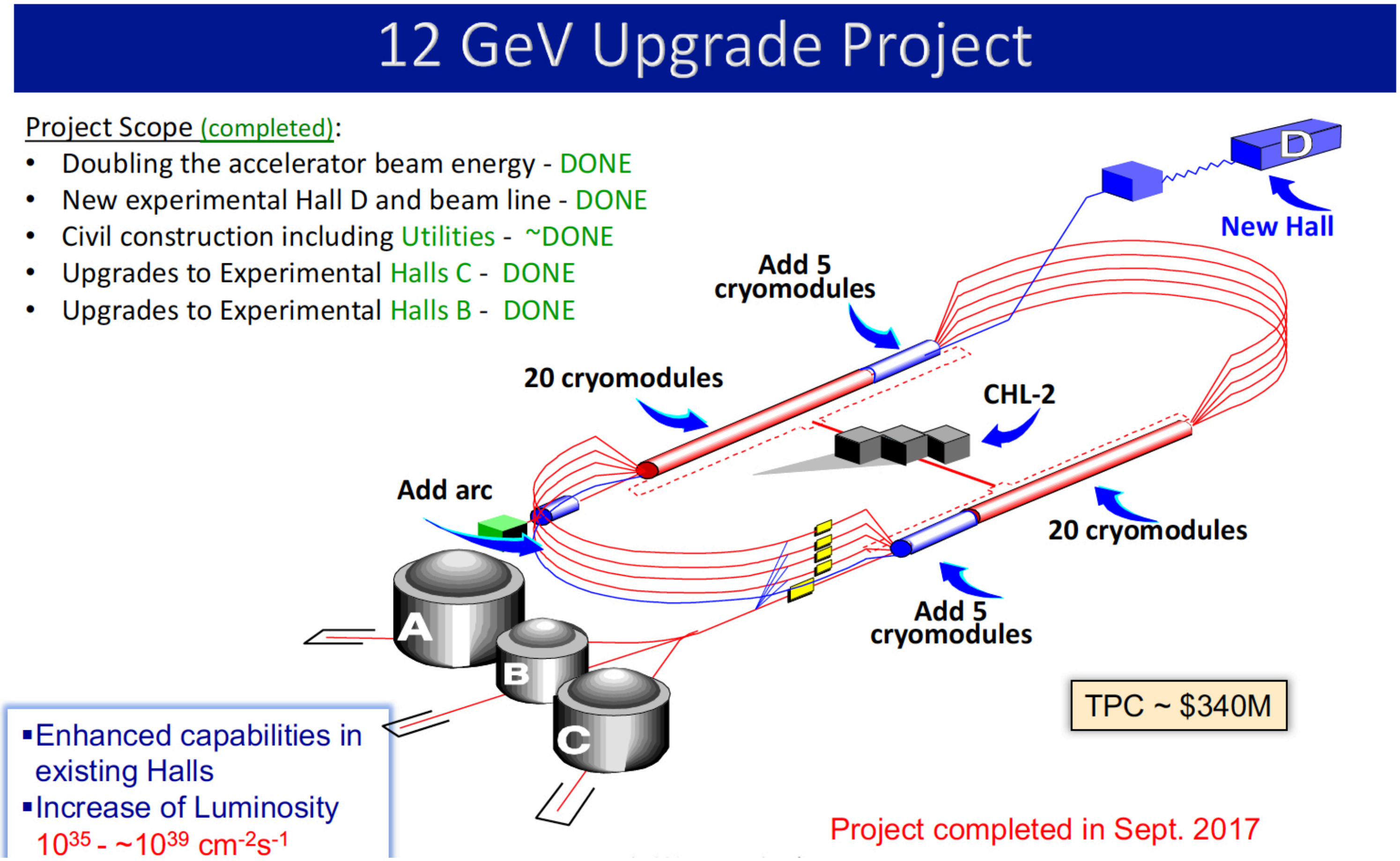
$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$$

  - models predict small effect
  - data basically only allow conclusion that effect compatible with zero
  - also, often only 1-D binning for SIDIS observables

- lack of kinematical coverage — in particular, range in Q^2 and x
  - higher-twist contributions, sea quarks and gluons, parton saturation, etc. ?
- lack of polarization
- lack of particle species, e.g., heavy nuclei
  - parton saturation, etc. ?
- limited detector capabilities

Jefferson Lab, 12 GeV Upgrade

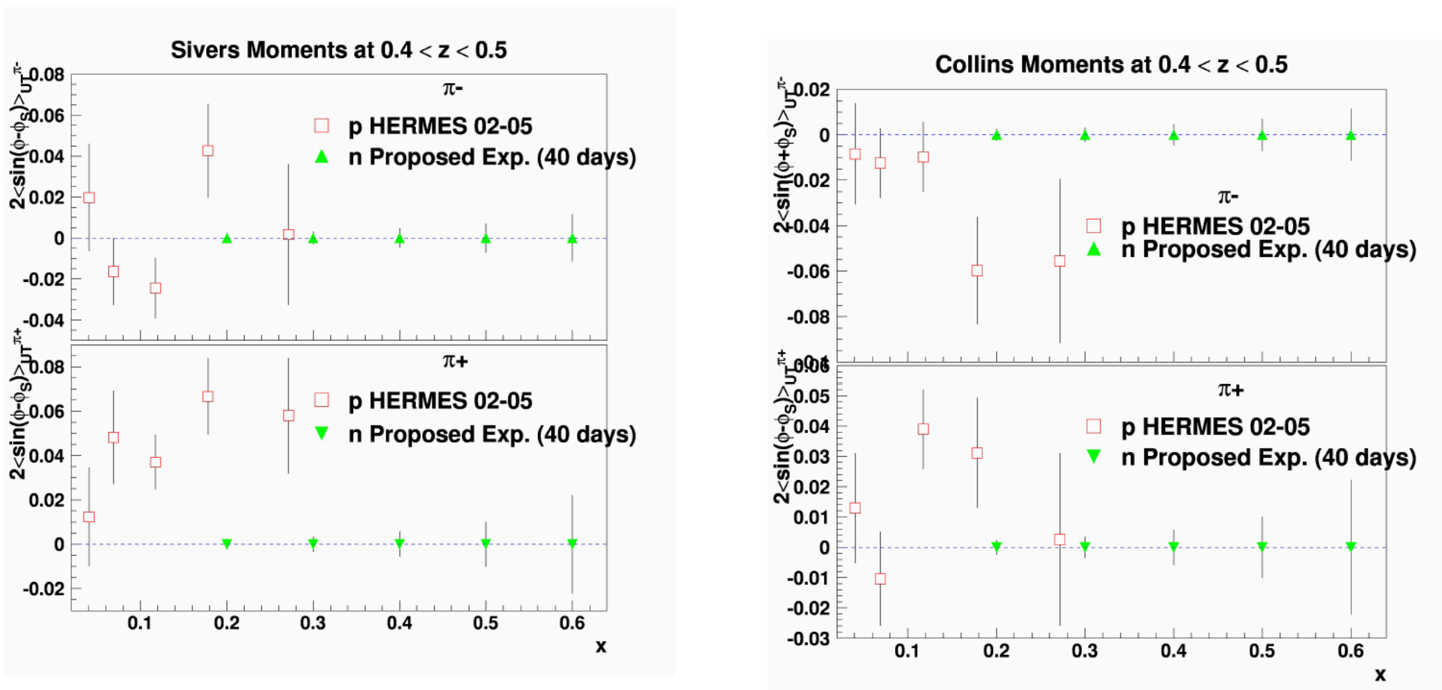
- Overview



(from Z.-E. Meziani, talk at Transversity 2017)

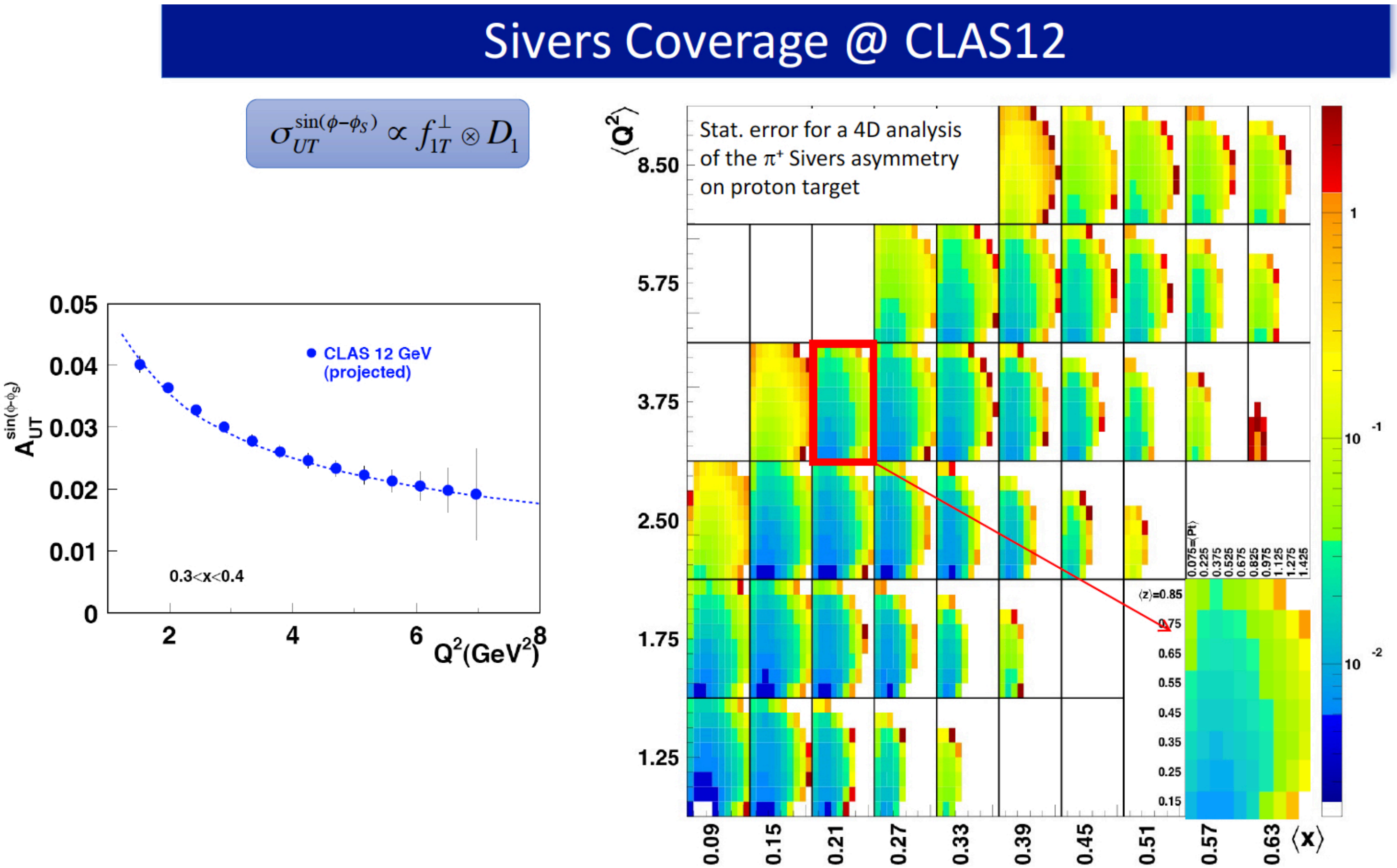
- Projections for Sivers and Collins asymmetries in Hall A

SBS projections for Sivers and Collins Moments



(from Z.-E. Meziani, talk at Transversity 2017)

- Kinematical coverage and projections for Sivers asymmetry in Hall B



(from Z.-E. Meziani, talk at Transversity 2017)

(Potential) Future Electron-Ion Collider(s)

- Overview

Past | Possible Future

	HERA @ DESY	LHeC @ CERN	HIAF @ CAS	ENC @ GSI	JLEIC @ JLab	eRHIC @ BNL
$\sqrt{s}$ [GeV]	320	800 - 1300	12 - 65	14	20 - 140	78 - 145
proton x_{min}	1 x 10^{-5}	5 x 10^{-7}	7 x 10^{-3} - 3 x 10^{-4}	5 x 10^{-3}	1 x 10^{-4}	5 x 10^{-5}
ion	p	p to Pb	p to U	p to ~^{40}Ca	p to Pb	p to U
polarization	-	-	p, d, ^{3}He	p, d	p, d, ^{3}He (^{6}Li)	p, ^{3}He
L [$cm^{-2}s^{-1}$]	2 x 10^{31}	10^{34}	10^{32-33} - 10^{35}	10^{32}	10^{33-34}	10^{33}
Interaction Points	2	1 (?)	1	1	2+	1-2
Year	1992 - 2007	post ALICE	2019 - 2030	upgrade to FAIR	post 12 GeV	2025

High-Energy Physics | Nuclear Physics

(from E. Sichterman, talk at POETIC 8, 2018)

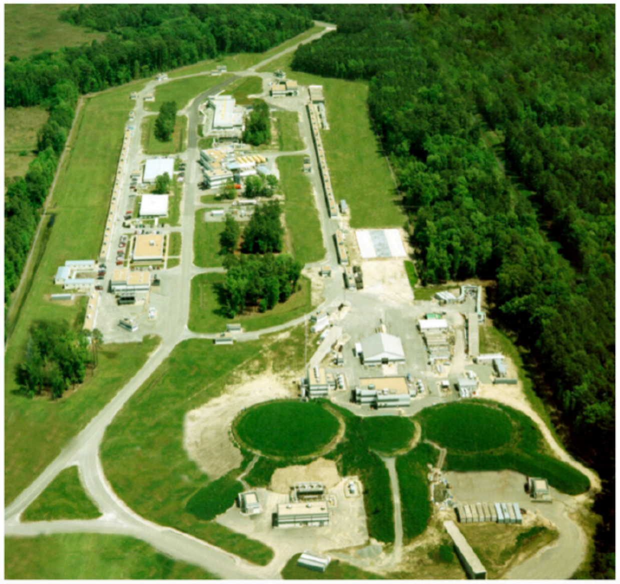
- Two initiatives for future EIC in the US

eRHIC:

- upgrade to existing RHIC hadron beam,

JLEIC:

- upgrade to existing CEBAF 12 GeV electron beam,

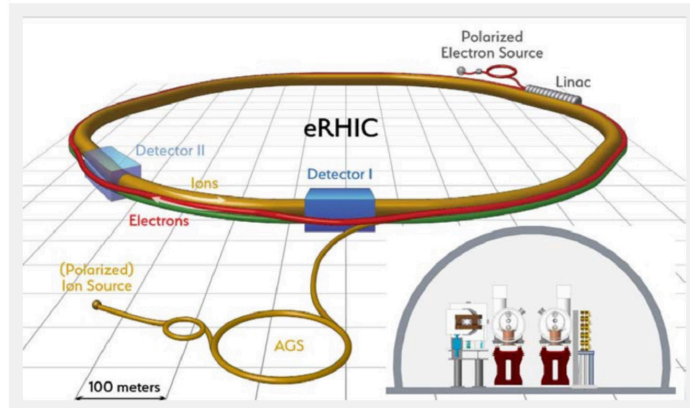


(from E. Sichterman, talk at POETIC 8, 2018)

- Some more details about two initiatives for future EIC in the US

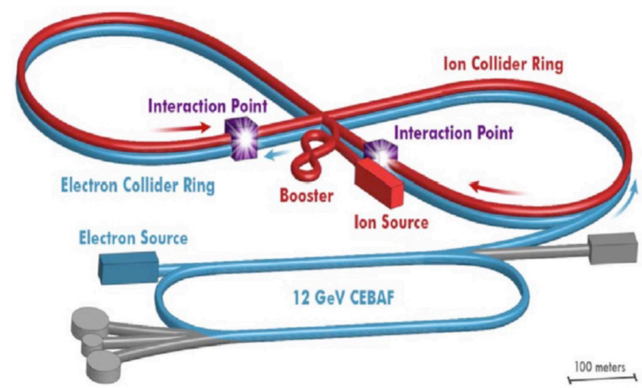
eRHIC:

- upgrade to RHIC hadron beam,
- new electron storage ring,
- 5 - 18 GeV e energy,
- Heavy Ions up to 100 GeV/u
- $\sqrt{s}$ up to 93 GeV
- L ~ 0.4×10^{34} $cm^{-2}s^{-1}$/A base design,
  1.0×10^{34} $cm^{-2}s^{-1}$/A w. strong cooling



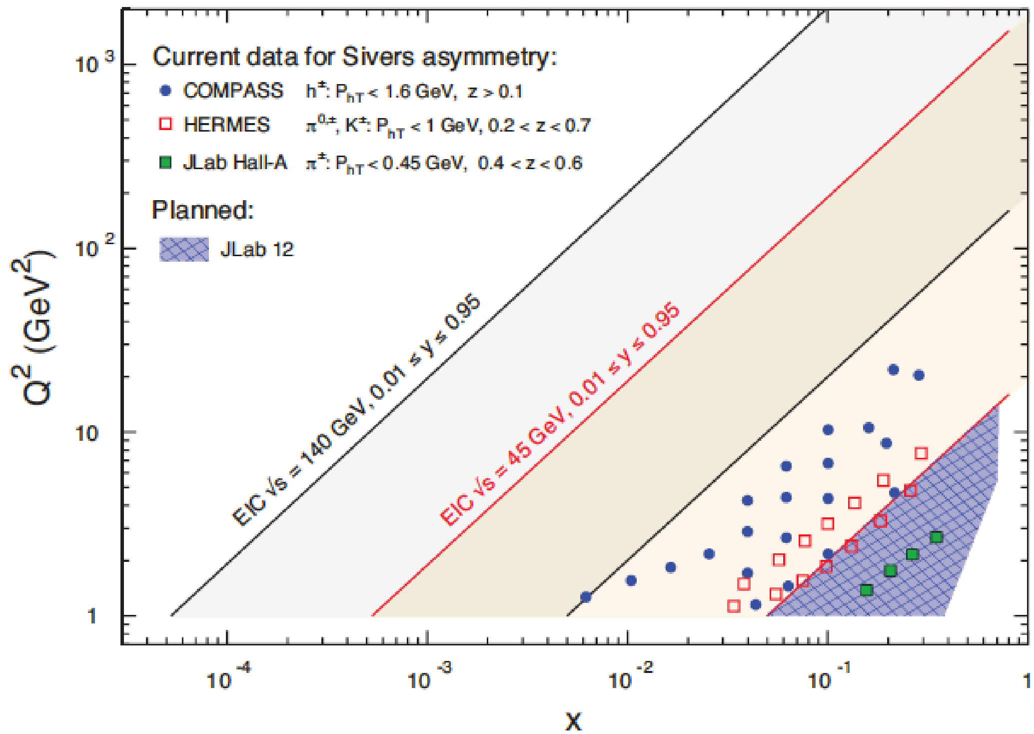
MEIC:

- upgrade to CEBAF 12 GeV electron
  beam facility,
- new hadron injector,
- new figure-8 collider configuration,
- 3-10 GeV electron energy,
- 12-40 GeV/u Heavy Ion energy,
  upgradable (ion arc dipole)
- L ~10^{34} $cm^{-2}s^{-1}$/A



(from E. Sichterman, talk at POETIC 8, 2018)

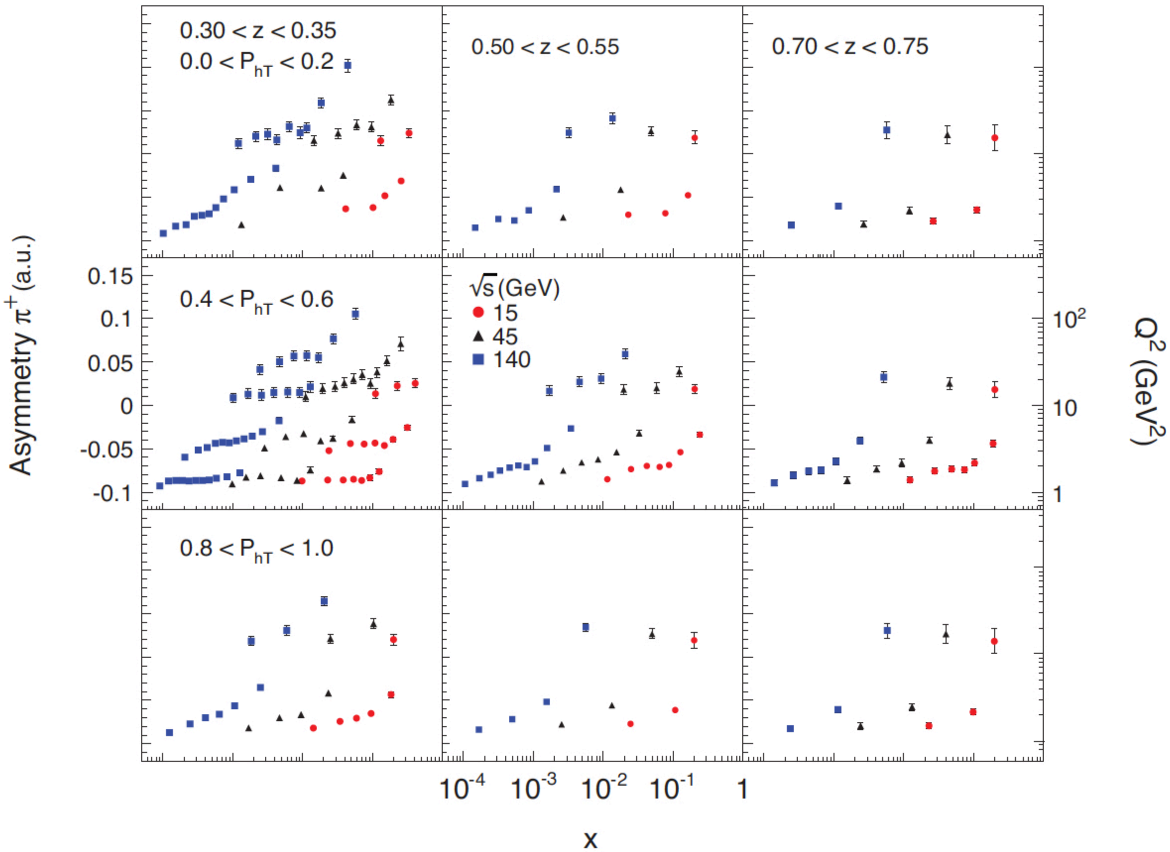
- Q^2 and x range at a future EIC in the US



(from arXiv:1212.1701)

 - some of existing data at (sufficiently) large Q^2, but often lack of precision
 - EIC can produce precision data
 - EIC can enter new kinematic domain (sea quarks and gluons)
 - EIC can move field of TMDs to next level

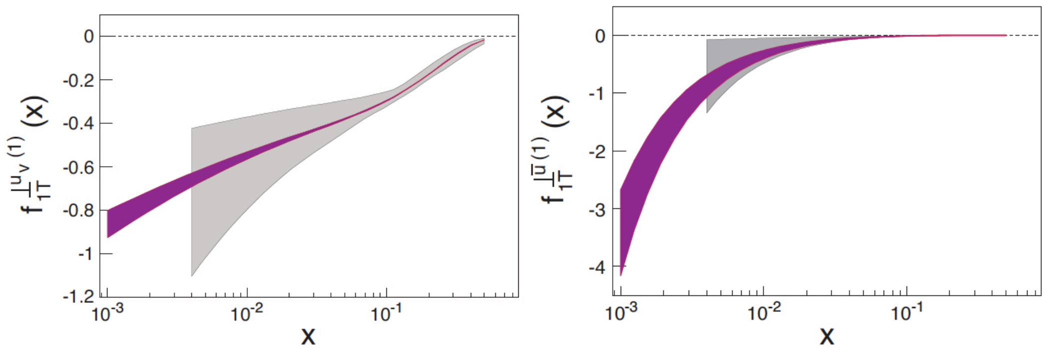
- Projected accuracy of Sivers asymmetry at a future EIC in the US



(from arXiv:1212.1701)

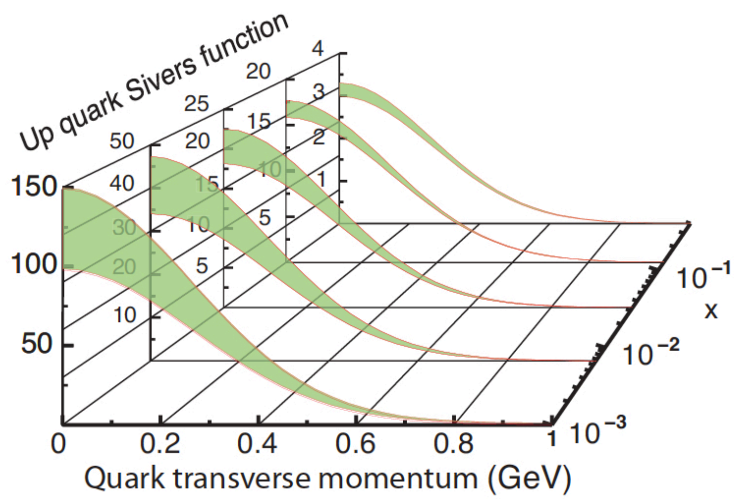
  - small errors
  - note the multi-dimensional binning

- Projected accuracy and kinematics of Sivers function at a future EIC in the US
  - x-dependence for $u_{\rm val}$ and $u_{\rm sea}$



(from arXiv:1212.1701)

  - 3-D representation

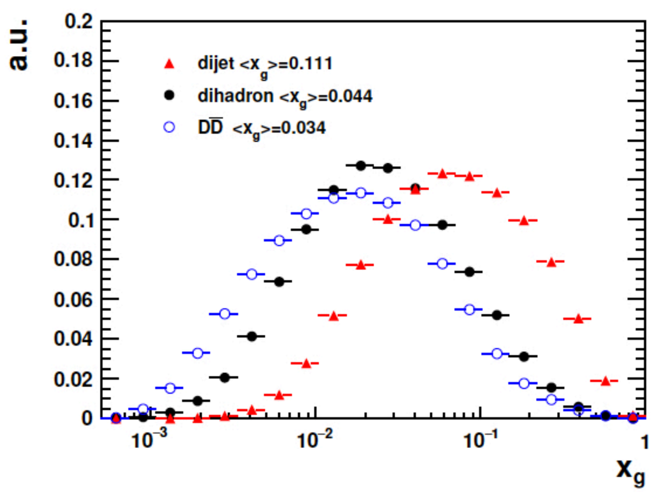


(from arXiv:1212.1701)

Addressing the Gluon Sivers Function

1. General strategy for addressing gluon content of nucleon in lepton-nucleon scattering
   - Typically, hunting for boson-gluon fusion process $\gamma^* g \to q\,\bar{q}$ via
     - di-hadron production (at high transverse momenta)
     - di-jet production
     - heavy-quark production (open charm)
     - quarkonium production
   - Each channel has advantages and drawbacks

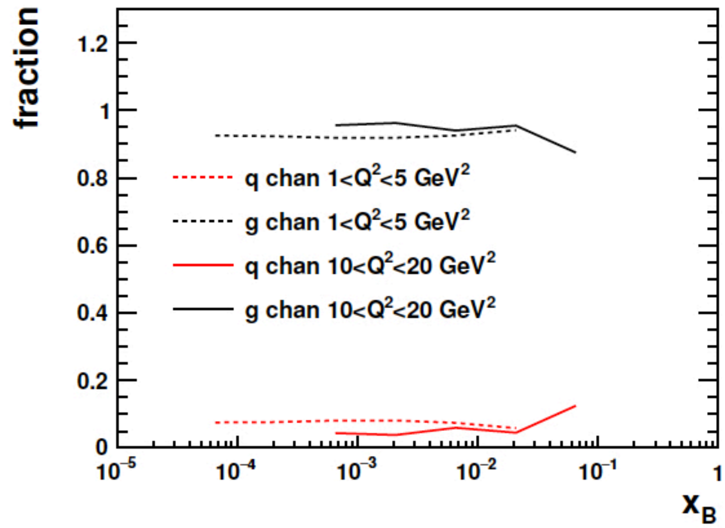
2. Feasibility study of gluon Sivers effect at EIC (Zheng et al, arXiv:1805.05290)
   - Study of open charm, di-hadron and di-jet production
   - Coverage of x-range for the three channels



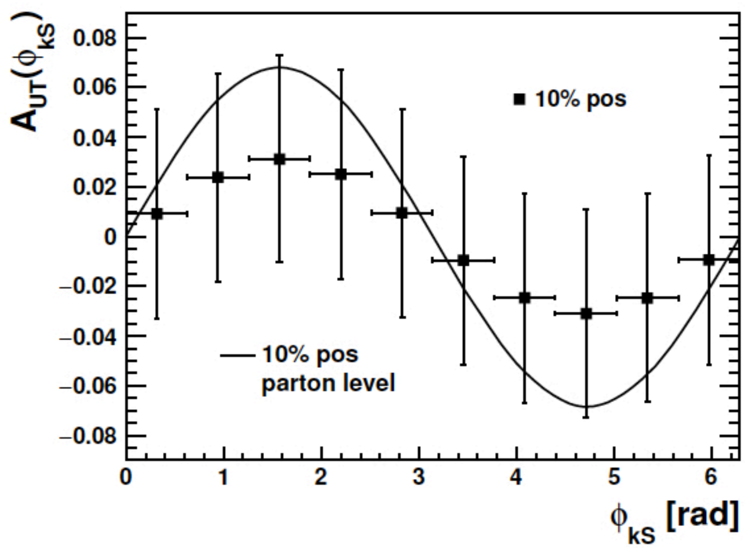
– channels have range of overlap
  $\to$ cross checks possible

– channels are also complementary

- Results for open charm production ($D\bar{D}$ pairs)

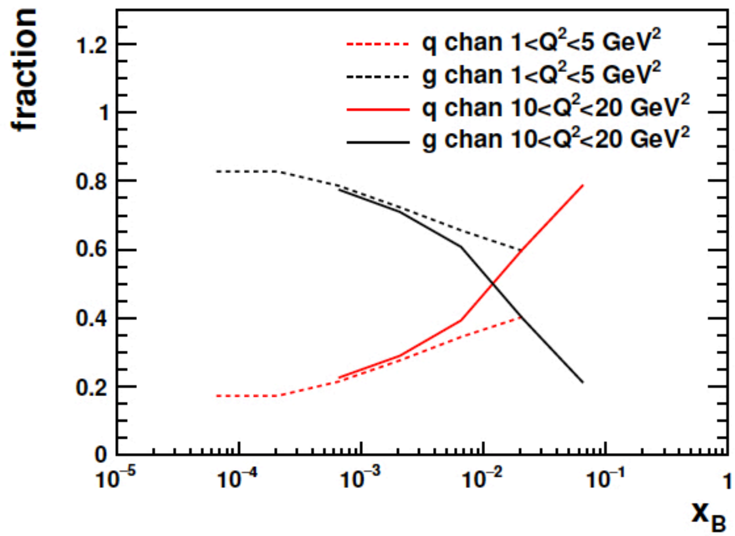


  - figure shows fraction of quark-initiated and gluon-initiated partonic process
  - boson-gluon fusion dominates

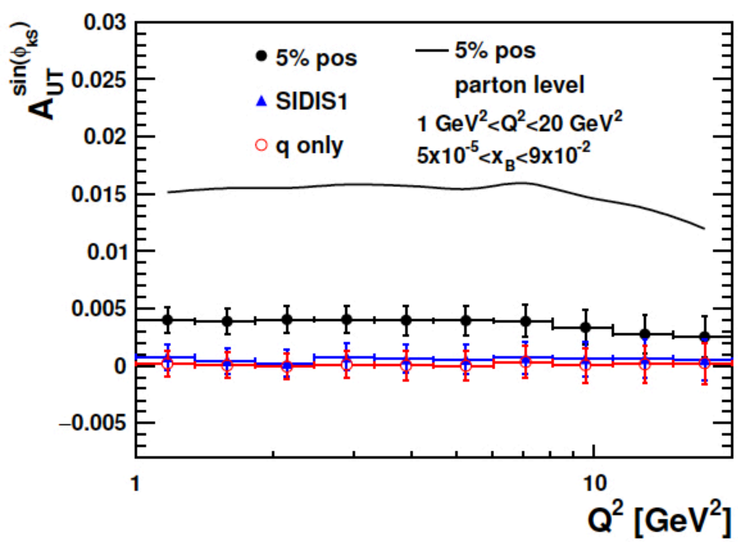


  - figure shows Sivers asymmetry
  - assumption: $f_{1T}^{\perp\,g} = 10\,\%$ of positivity bound
  - comparison to result at parton level
  - large errors since $D \to K\pi$ channel has $3.7\,\%$ branching ratio only

- Results for di-hadron production (light hadrons)



- figure shows fraction of quark-initiated and gluon-initiated partonic process
- boson-gluon fusion dominates at small x only



- figure shows Sivers asymmetry
- two models for $f_{1T}^{\perp g}$ (black and blue symbols)
- comparison to result at parton level and to quark contribution
- strong dilution of asymmetry due to fragmentation
- compared to open charm production, smaller asymmetry but smaller errors

Addressing the Density of Linearly Polarized Gluons

- Definition of $h_1^{\perp g}$ (Mulders, Rodrigues, hep-ph/0009343 / Meissner, Metz, Goeke, hep-ph/0703176)

$$\Phi^{g\,[ij]}(x,\vec{k}_\perp) = \frac{1}{xP^+}\int \frac{dz^-\,d^2\vec{z}_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle P|F_a^{+j}(-\tfrac{z}{2})\,\mathcal{W}_{ab}[-\tfrac{z}{2},\tfrac{z}{2}]\,F_b^{+i}(\tfrac{z}{2})|P\rangle\Big|_{z^+=0}$$

$$= \frac{1}{2}\,\delta_\perp^{ij}\,f_1^g(x,\vec{k}_T^2) + \frac{1}{2}\left(\frac{k_\perp^i\,k_\perp^j}{M^2} - \frac{1}{2}\,\delta_\perp^{ij}\,\frac{\vec{k}_\perp^{\,2}}{2M^2}\right) h_1^{\perp g}(x,\vec{k}_T^2)$$

  - no target polarization
  - no Wilson line required for nonzero $h_1^{\perp g}$ (unlike $h_1^{\perp q}$)
  - positivity bound

$$\frac{\vec{k}_\perp^{\,2}}{2M^2}\,h_1^{\perp g}(x,\vec{k}_\perp^{\,2}) \le f_1^g(x,\vec{k}_\perp^{\,2})$$

- At small x, positivity bound saturated in
  - quark target model (Meissner, Metz, Goeke, hep-ph/0703176)
  - McLerran-Venugopalan model (Metz, Zhou, arXiv:1105.1991 / Dominguez et al, arXiv:1109.6293)

- Observable in heavy-quark production: $\ell + N \to \ell + Q(k_1) + \bar{Q}(k_2) + X$
  - generic structure of unpolarized cross section (Boer et al, arXiv:1011.4225)

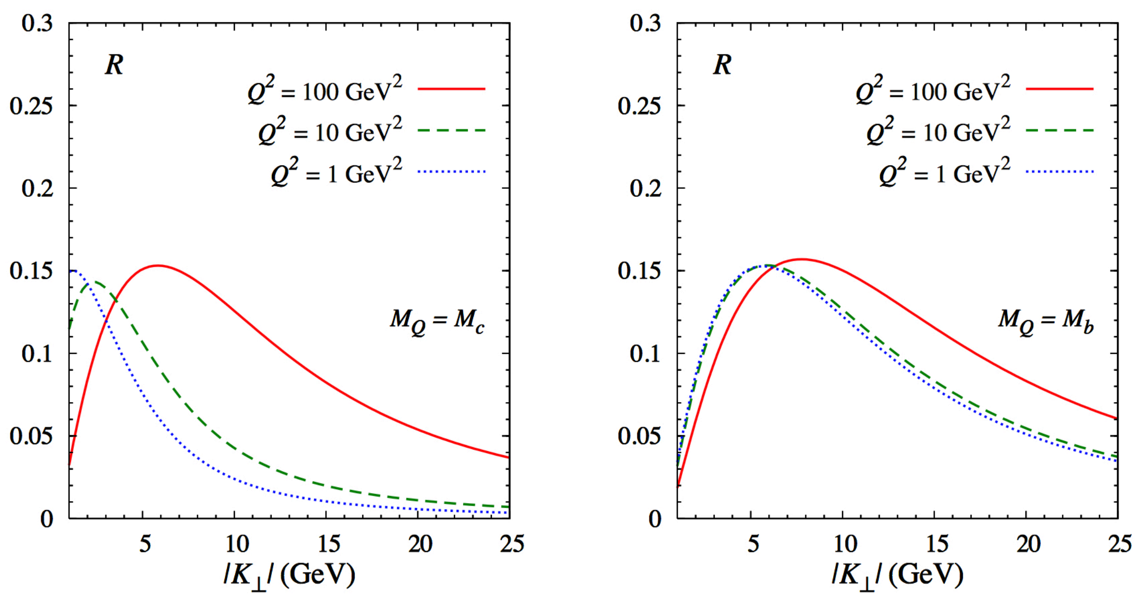
$$d\sigma \sim A + B \cos 2(\phi_K - \phi_\kappa) + \ldots \qquad \vec{K}_\perp = \frac{1}{2}(\vec{k}_{1\perp} - \vec{k}_{2\perp}) \quad \vec{\kappa}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$$

$$B \sim \sum_q e_q^2 \, h_1^{\perp g}(x, \vec{\kappa}_\perp^2) \times \text{kinematical factors}$$

B vanishes for $Q^2 = m_Q = 0$

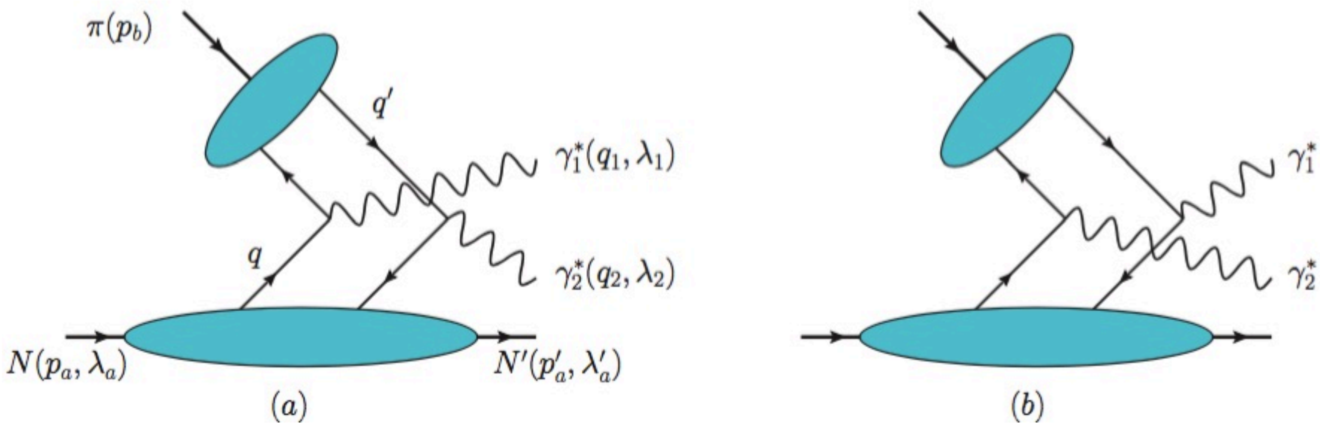
  - numerical estimate (for saturated positivity bound) (Pisano et al, arXiv:1307.3417)

(shown is essentially B/A)



Addressing GTMDs / Wigner Functions

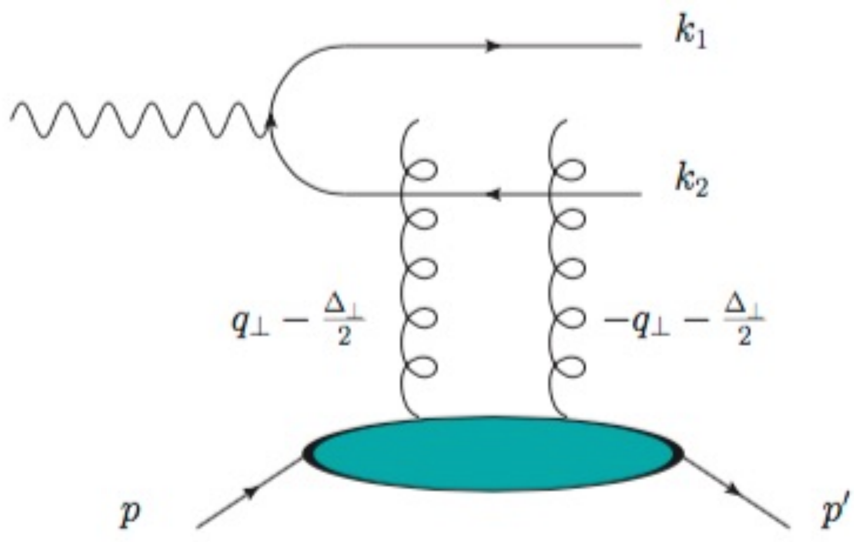
- For quite some time it was unclear how GTMDs can be measured
- Quark GTMDs in exclusive double Drell-Yan process
  (Bhattacharya, Metz, Zhou, arXiv:1702.04387)
  - process: $\pi\, N \to (\ell_1^+ \ell_1^-)(\ell_2^+ \ell_2^-) N'$
  - leading-order diagrams



  - access to (all) leading-twist chiral-even quark GTMDs
  - in particular, access to F_{14}^q (relation to OAM)
  - only ERBL region $(-\xi \le x \le \xi)$ enters in leading-order analysis

- Can quark GTMDs be measured in lepton-nucleon scattering ?

- Gluon GTMDs in lepton-nucleon and lepton-nucleus scattering
  - exclusive di-jet production in ℓN collisions and ℓA collisions at small x (Hatta, Xiao, Yuan, arXiv:1601.01585)



  - including polarization in same process may give access to gluon OAM (F_{14}^g) at small x (Hatta et al, arXiv:1612.02445)
  - same process may also give access to gluon OAM (F_{14}^g) at moderate x (Ji, Yuan, Zhao, arXiv:1612.02438)