Introduction to Quantum Chromodynamics (QCD)

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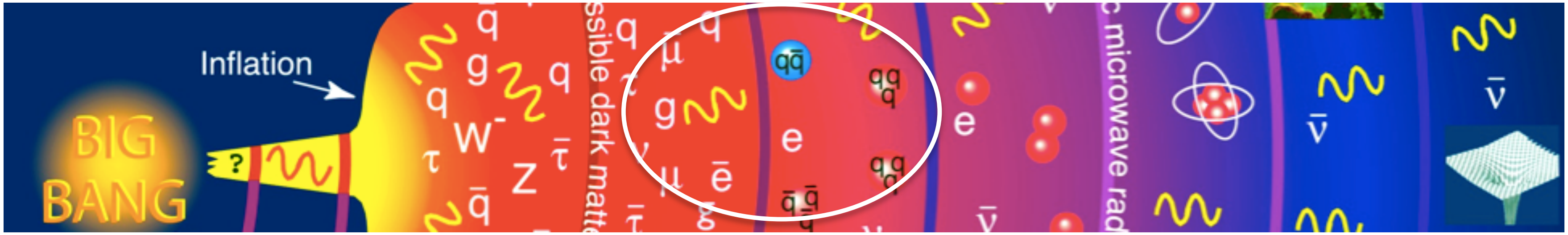
May 29 – June 15, 2018

Lecture Two

QCD is everywhere in our universe

- What is the role of QCD in the evolution of the universe?



- How hadrons are emerged from quarks and gluons?
- How does QCD make up the properties of hadrons?

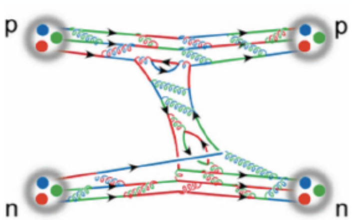
  Their mass, spin, magnetic moment, …

- What is the QCD landscape of nucleon and nuclei?

Color Confinement — Asymptotic freedom

Q (GeV)

200 MeV (1 fm) — 2 GeV (1/10 fm) — Probing momentum



- How do the nuclear force arise from QCD?
- …

Unprecedented Intellectual Challenge!

- **Facts:**

  No modern detector has been able to see quarks and gluons in isolation!

  Gluons are dark!

- **The challenge:**

  How to probe the quark-gluon dynamics, quantify the hadron structure, study the emergence of hadrons, …, if we cannot see quarks and gluons?

- **Answer to the challenge:**

  Theory advances:

  QCD factorization – matching the quarks/gluons to hadrons with controllable approximations!

  Experimental breakthroughs:

  Jets – ***Footprints of energetic quarks and gluons***

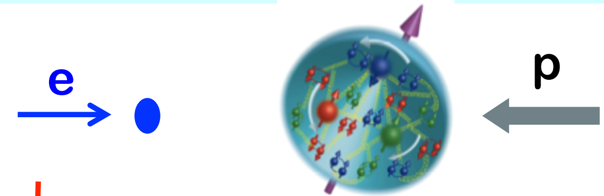
  Quarks – ***Need an EM probe to "see" their existence, …***

  Gluons – ***Varying the probe's resolution to "see" their effect, …***

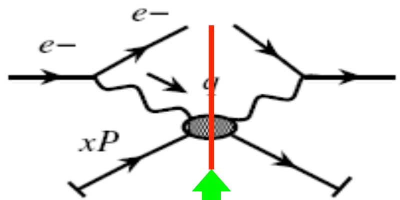
  Energy, luminosity and measurement – Unprecedented resolution, event rates, and precision probes, especially EM probes, like one at Jlab, …

Theoretical approaches – approximations

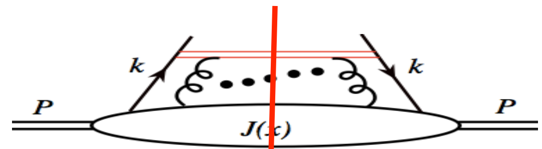
- **Perturbative QCD Factorization:**
  – Approximation at Feynman diagram level



$$\sigma_{\text{tot}}^{\text{DIS}}\ :$$



$$\otimes$$



$$+\ O\left(\frac{1}{QR}\right)$$

See Metz's lectures
Sokhan's lectures
Furletova's lectures

Probe Hard-part	Structure Parton-distribution	Approximation Power corrections

- **Effective field theory (EFT):**
  – Approximation at the Lagrangian level

See Stewart's lectures
Cirigliano's lectures

Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Other approaches:**

See Stevens' lectures
Pastore's lectures

Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

- **Lattice QCD:**
  – Approximation mainly due to computer power

See Stevens' lecture
Pastore's lectures

Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

Physical Observables

Cross sections with identified hadrons are non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not a perturbative scale

Purely infrared safe quantities

Observables without identified hadron(s)

Fully infrared safe observables – I

Fully inclusive, without any identified hadron!

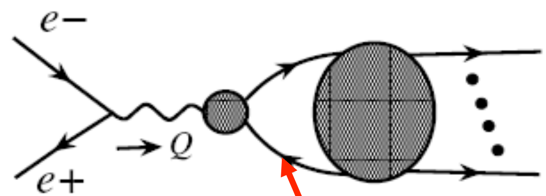
$$\sigma^{\rm total}_{e^+e^-\to{\rm hadrons}} = \sigma^{\rm total}_{e^+e^-\to{\rm partons}}$$

The simplest observable in QCD

e⁺e⁻ ➔hadrons inclsusive cross sections

❑ e⁺e⁻ ➔ hadron **total** cross section – not a specific hadron!

$$\sigma^{\rm tot}_{e^+e^-\to{\rm hadrons}} \propto \left| \quad \right|^2$$



Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to{\rm hadrons}} \propto \sum_n {\rm P}_{e^+e^-\to n} = \sum_n \sum_m {\rm P}_{e^+e^-\to m} {\rm P}_{m\to n} = \sum_m {\rm P}_{e^+e^-\to m} \sum_n {\rm P}_{m\to n}$$

=1 ← Unitarity

$$\sigma^{\rm tot}_{e^+e^-\to{\rm partons}} \propto \sum_m {\rm P}_{e^+e^-\to m}$$

$$\Rightarrow \quad \sigma^{\rm tot}_{e^+e^-\to{\rm hadrons}} = \sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$$

← Finite in perturbation theory – KLN theorem

❑ e⁺e⁻ ➔ parton total cross section:

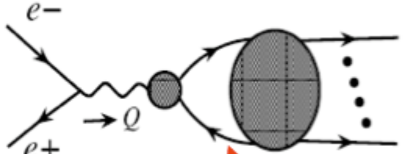
$$\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

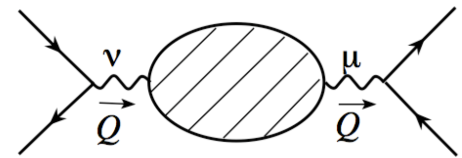
Calculable in pQCD

Infrared Safety of e^+e^- Total Cross Sections

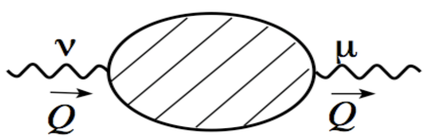
❑ Optical theorem:

$$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\left| \quad \right|^2 \propto {\rm Im}$$





❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

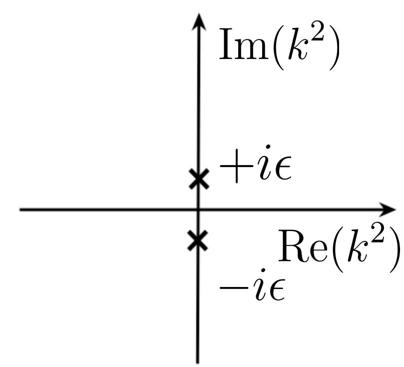
IR safety of $\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

❑ IR safety of $\Pi(Q^2)$

If there were pinched poles in Π (Q²),

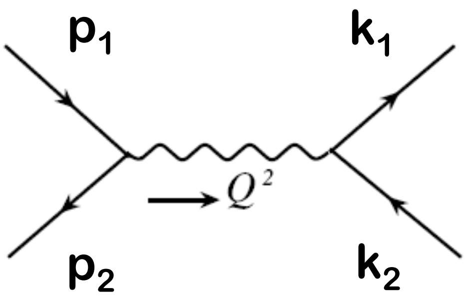
- ✧ **real partons moving away from each other**
- ✧ **cannot be back to form the virtual photon again!**

Rest frame of the virtual photon



Lowest order (LO) perturbative calculation

- **Lowest order Feynman diagram:**



- **Invariant amplitude square:**

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$

$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1 + m_Q\right)\gamma_\mu\left(\gamma\cdot k_2 - m_Q\right)\gamma_\nu\right]$$

$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right]$$

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - k_1)^2$$
$$u = (p_2 - k_1)^2$$

- **Lowest order cross section:**

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s = Q^2$$

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1 + \frac{2m_Q^2}{s}\right]\sqrt{1 - \frac{4m_Q^2}{s}}$$

Threshold constraint

One of the best tests for the number of colors

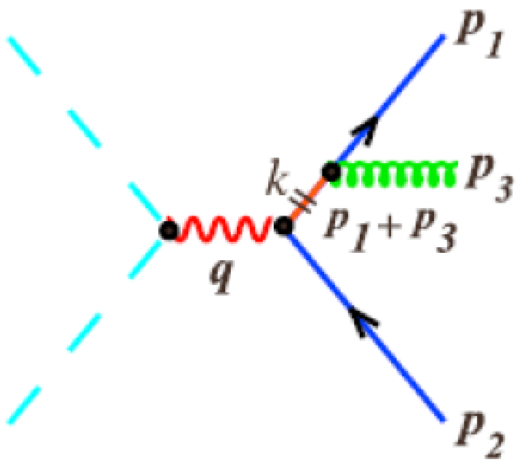
Next-to-leading order (NLO) contribution

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

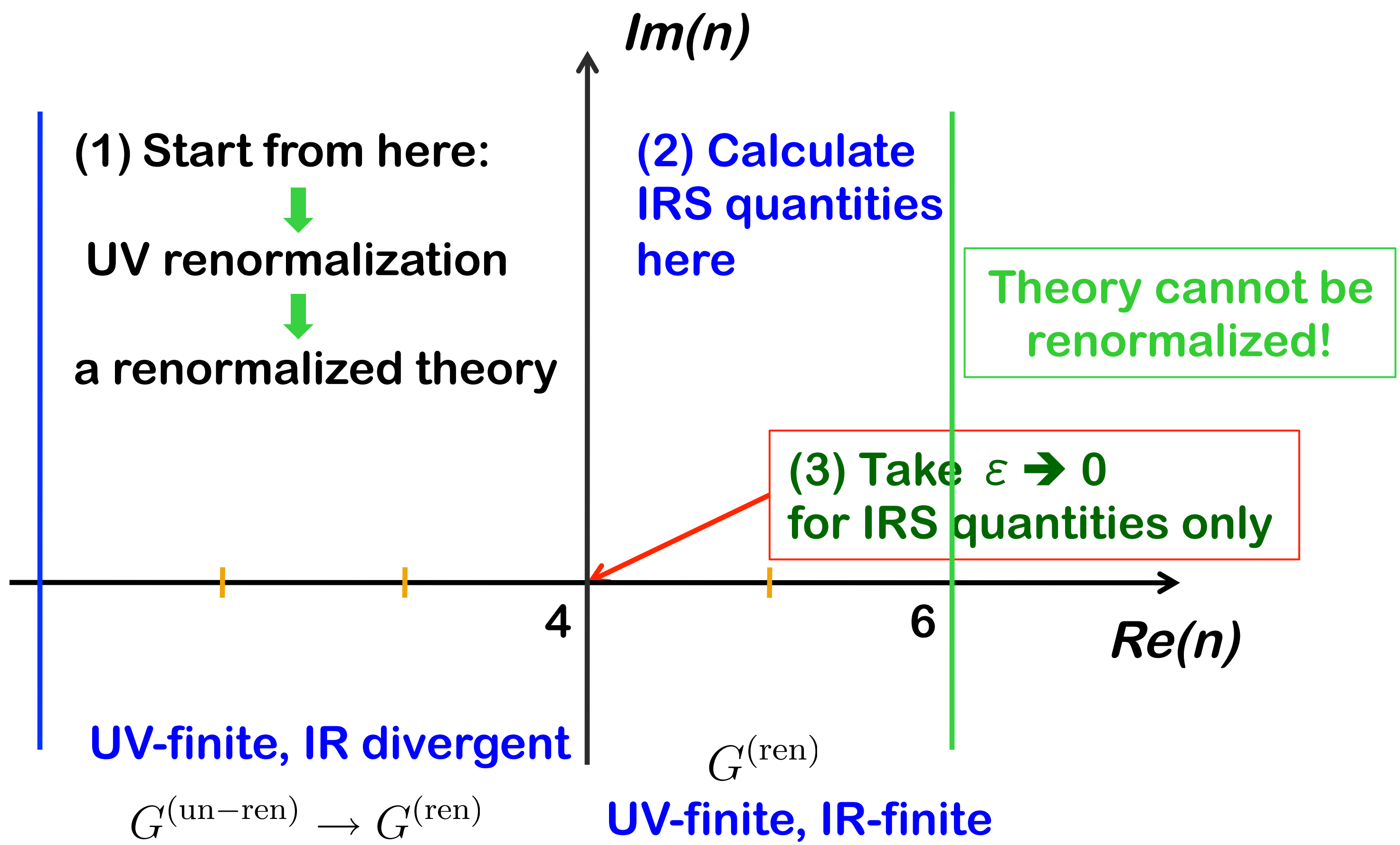
IR as $x3 \to 0$
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

- Complex *n*-dimensional space: $\int d^n k \, F(k, Q)$



Dimensional regularization for both IR and CO

❑ **NLO with a dimensional regulator:**

✧ **Real:**

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

✧ **Virtual:**

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

✧ **NLO:**

$$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi} + O(\varepsilon)\right]$$

No ε dependence!

✧ **Total:**

$$\sigma^{\mathrm{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O(\alpha_s^2)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Go beyond the inclusive total cross section?

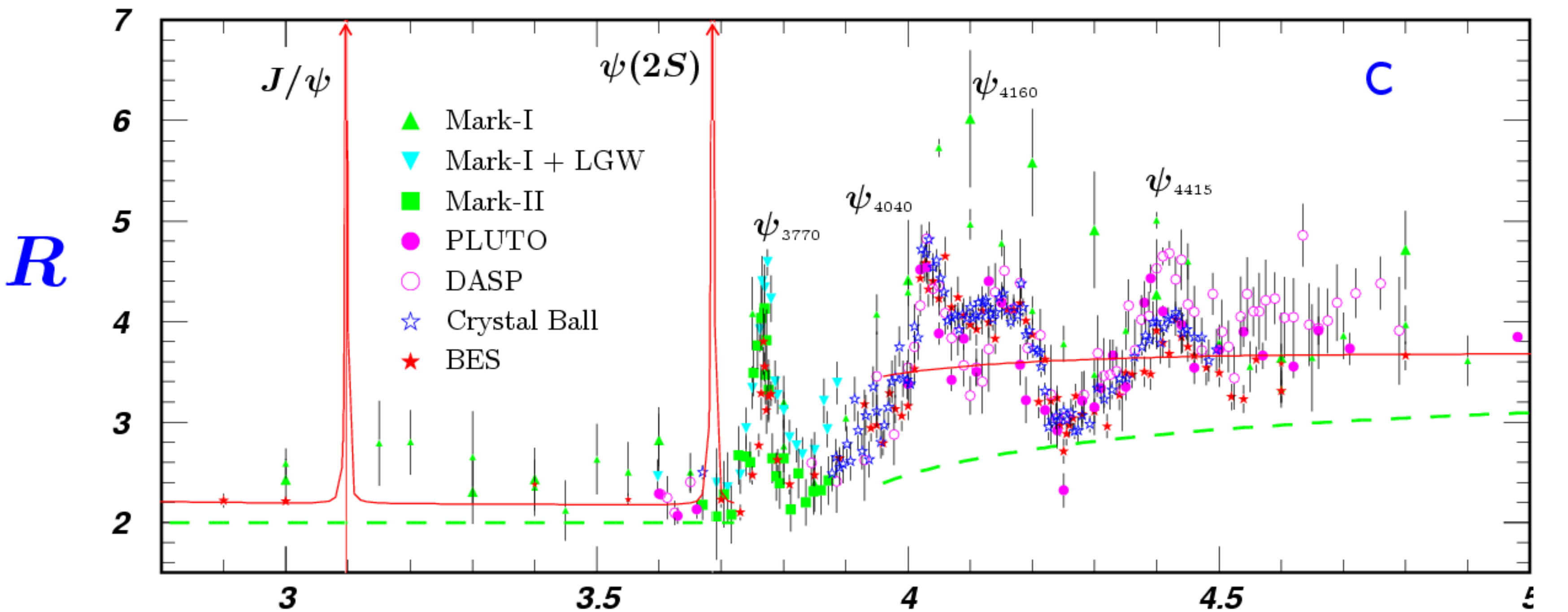
Hadronic cross section in e+e- collision

- **Normalized hadronic cross section:**

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+ N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Fully infrared safe observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\text{Jets}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{Jets}}_{e^+e^-\to\text{partons}}$$

Jets – "trace" or "footprint" of partons

Thrust distribution in e^+e^- collisions

etc.

Jets – trace of partons

- ❑ Jets – "total" cross-section with a limited phase-space

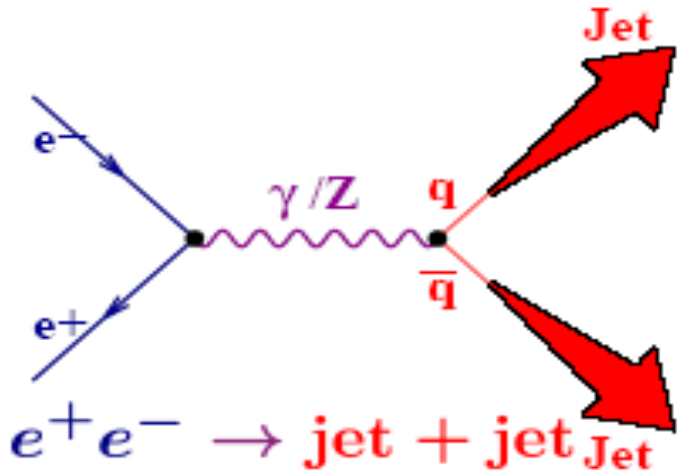
  Not any specific hadron!

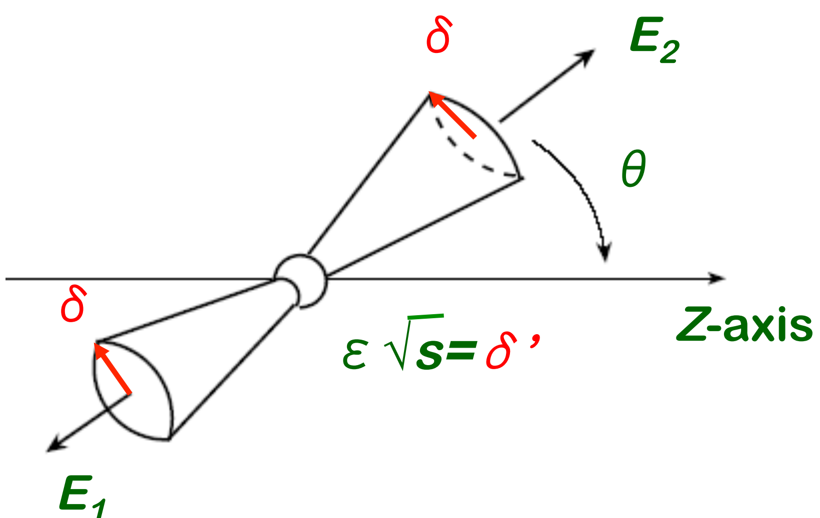
- ❑ Q: will IR cancellation be completed?

  - ✧ Leading partons are moving away from each other

  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton

- ❑ Many Jet algorithms





Sterman-Weinberg Jet

Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^{\mu},\lambda k_n^{\mu}\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^{\mu}\right) \quad \text{with} \quad 0 \le \lambda \le 1$$

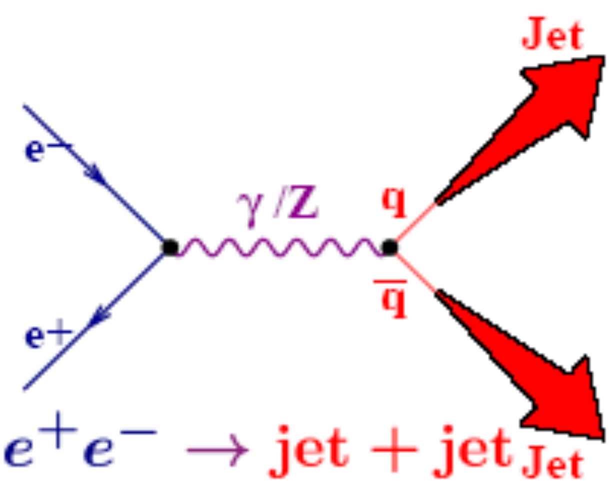
Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without the parton

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1 \;\text{ for all } n \quad \Rightarrow \quad \sigma^{(\text{tot})}$

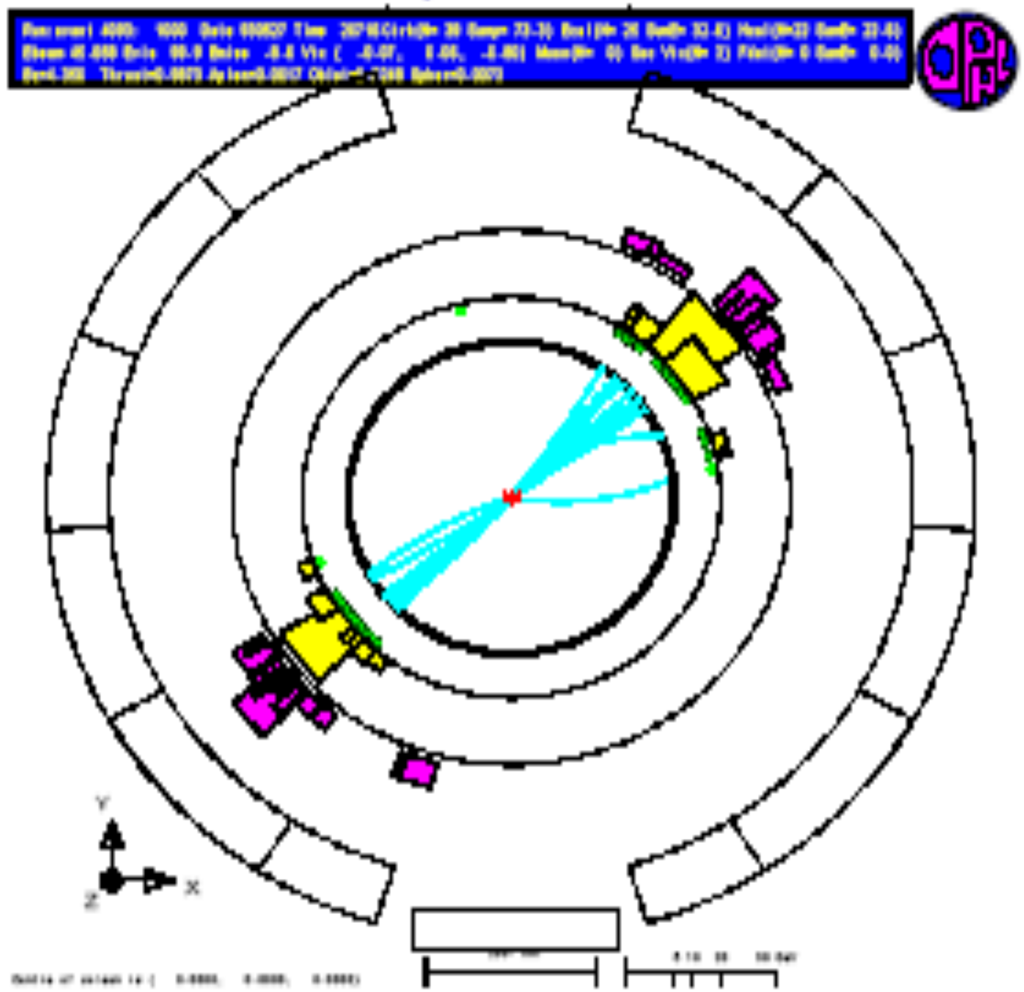
An early clean two-jet event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

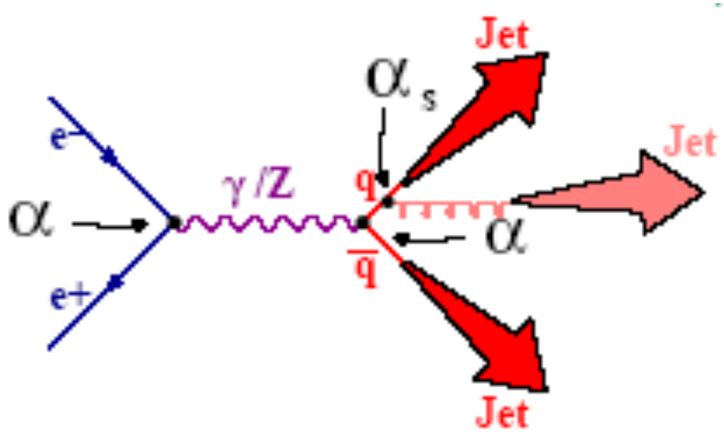
LEP ($\sqrt{s} = 90 - 205$ GeV)



A clean trace of two partons – a pair of quark and antiquark

Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

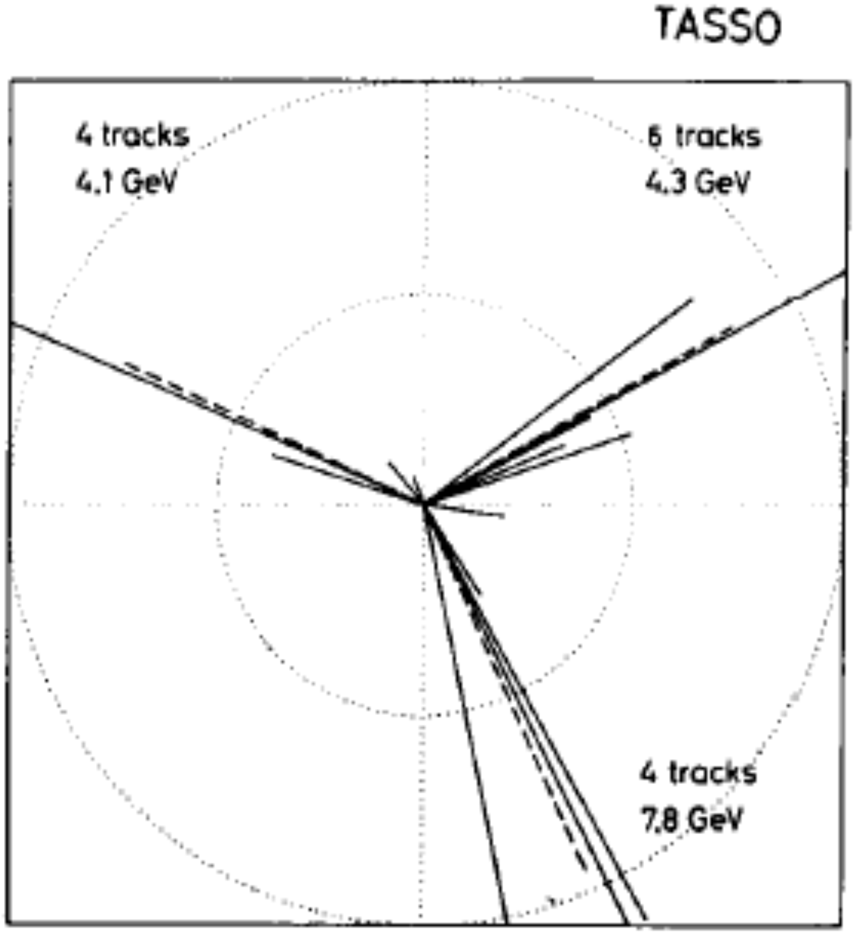
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

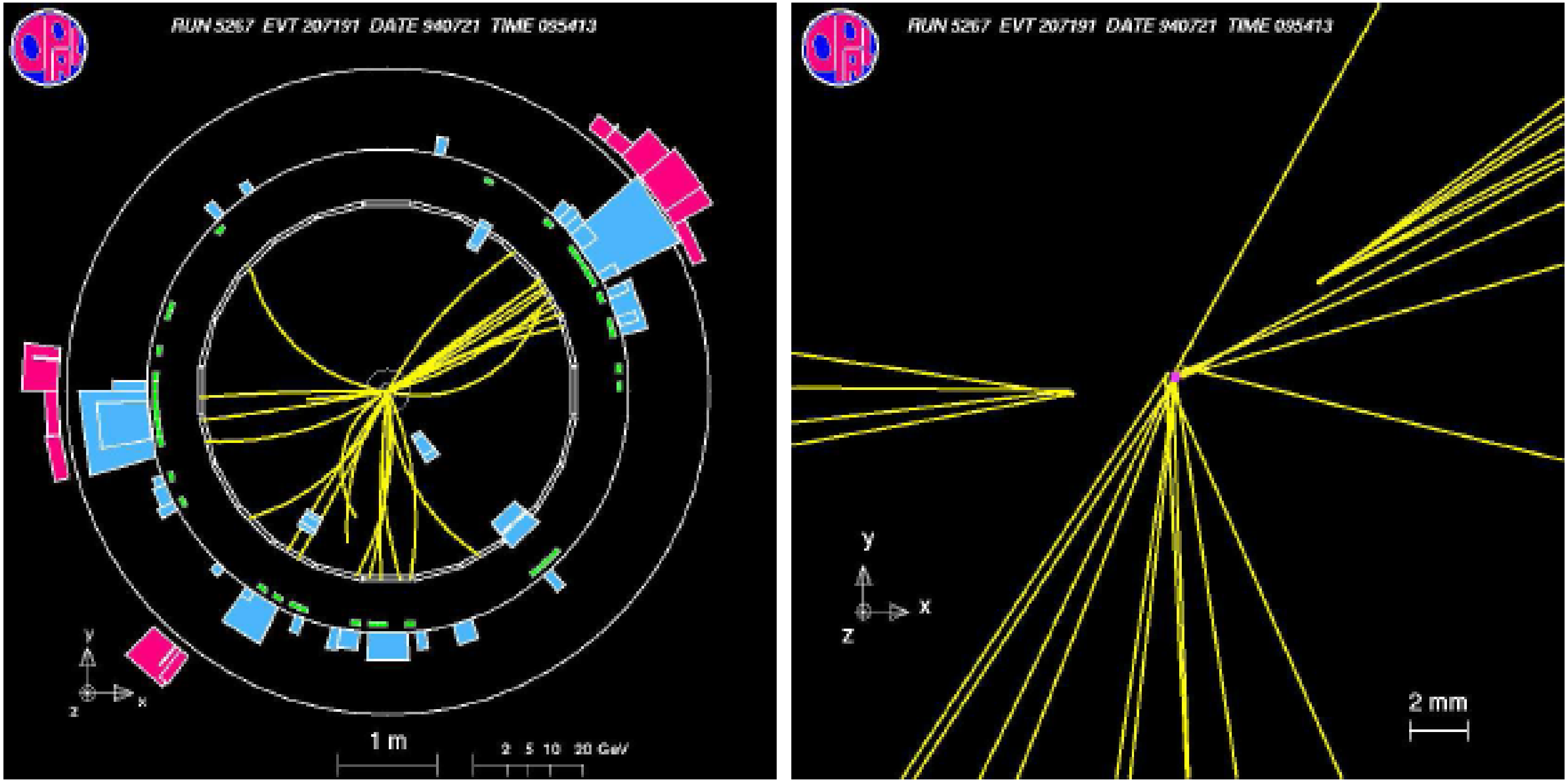
JADE Collab., Phys. Lett. B91 (1980) 142

PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ **GeV**



Tagged three-jet event from LEP

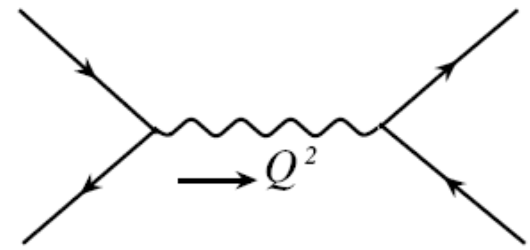


Gluon Jet

Two-jet cross section in e+e- collisions

❑ Parton-Model = Born term in QCD:

$$\sigma_{2\mathrm{Jet}}^{(\mathrm{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



❑ Two-jet in pQCD:

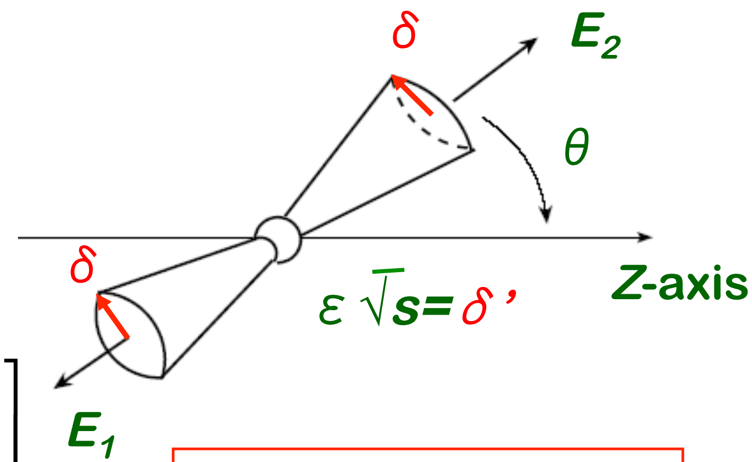
$$\sigma_{2\mathrm{Jet}}^{(\mathrm{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n(\delta)$

❑ Sterman-Weinberg jet:

$$\sigma_{2\mathrm{Jet}}^{(\mathrm{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$



Sterman-Weinberg Jet

$$\sigma_{\mathrm{total}} = \sigma_{2\mathrm{Jet}} \quad \text{as } Q\to\infty$$

Basics of jet finding algorithms

❑ Recombination jet algorithms (almost all e+e- colliders):

Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$ $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

for Durham k_T

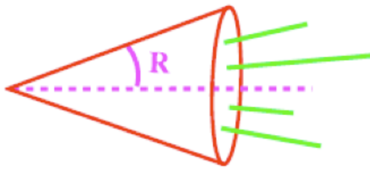
✧ different algorithm = different choice of M_{ij}^2 :

✧ Combine the particle pair (i,j) with the smallest y_{ij} : $(i,j) \rightarrow k$

$$e.g.\ \mathrm{E\ scheme}:\quad p_k = p_i + p_j$$

✧ iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$

❑ Cone jet algorithms (CDF, …, colliders):



✧ Cluster all particles into a cone of half angle R to form a jet:

✧ Require a minimum visible jet energy: $E_{jet} > \epsilon$

Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$

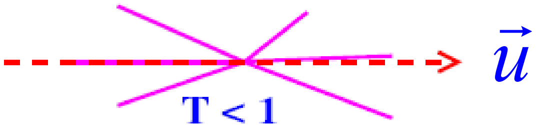
with $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

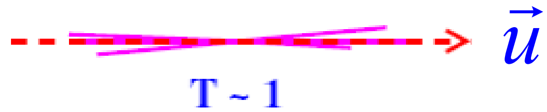
✧ Classical choices: ***p=1*** – "k_T algorithm", ***p= -1*** – "anti-k_T", …

Thrust distribution

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}}\left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|}\right)$$





- **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to \text{hadrons}}}{dT} \quad \text{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

- Contribution from p=0 particles drops out the sum

- Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

The harder question

❑ **Question:**

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

❑ **Facts:**

Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

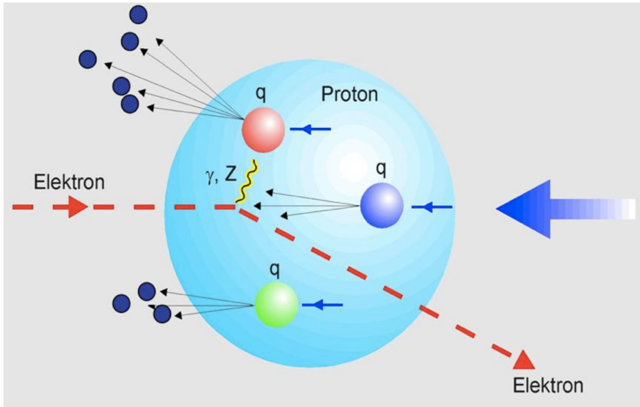
Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

❑ **Solution – Factorization:**

✧ Isolate the calculable dynamics of quarks and gluons

✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
– provide information on the partonic structure of the hadron

Observables with ONE identified hadron

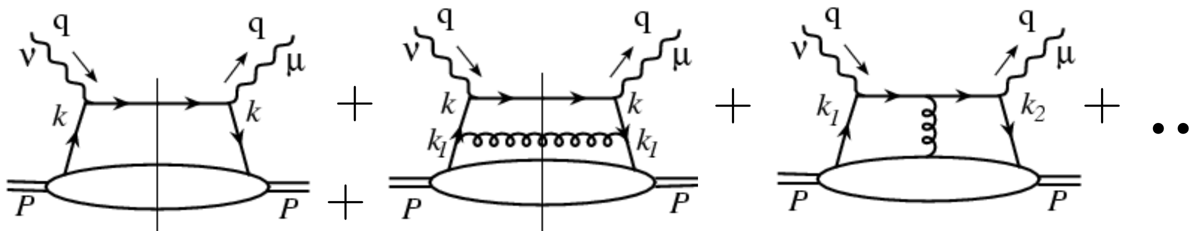


$$\sigma^{\text{DIS}}_{\ell p\to \ell' X(\text{everything})}$$

Identified initial-state hadron-proton!

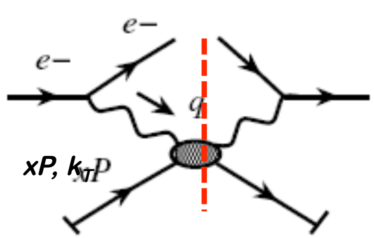
- **DIS cross section is infrared divergent, and nonperturbative!**

$$\sigma^{\text{DIS}}_{\ell p\to \ell' X(\text{everything})} \propto$$

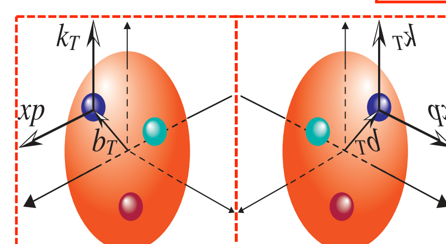


- **QCD factorization (approximation!)**

$$\sigma^{\text{DIS}}_{\ell p\to \ell' X(\text{everything})} = $$



$$\otimes$$



$$+ O\left(\frac{1}{QR}\right)$$

Color entanglement Approximation

Physical Observable

Controllable Probe

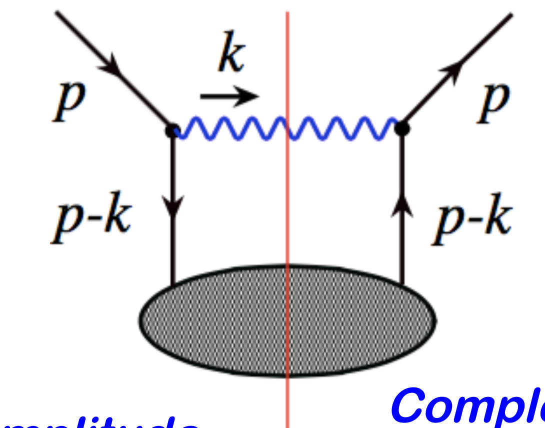
Quantum Probabilities Structure

Pinch singularity and pinch surface

- **"Square" of the diagram with a *"unobserved gluon"*:**

"Cut-line" – final-state

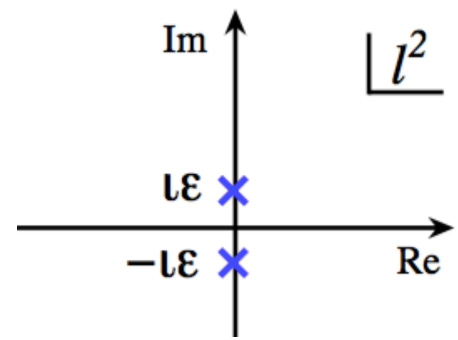
– in a "cut-diagram" notation



$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

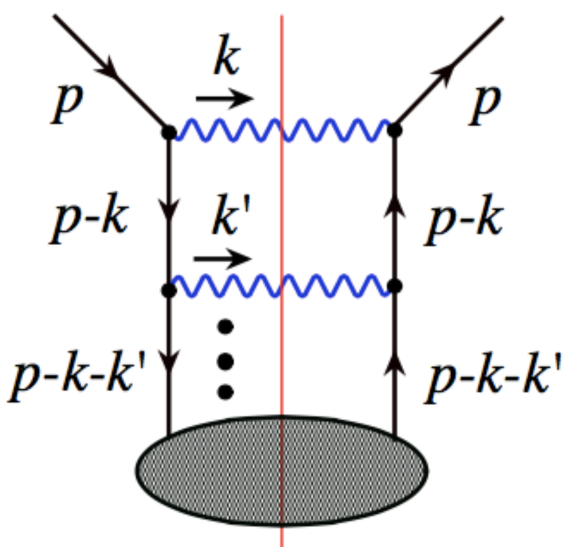
$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Amplitude

Complex conjugate of the Amplitude



Pinch surfaces

Pinch singularities "perturbatively"

= "surfaces" in k, k', …

determined by $(p\text{-}k)^2=0$, $(p\text{-}k\text{-}k')^2=0$, …

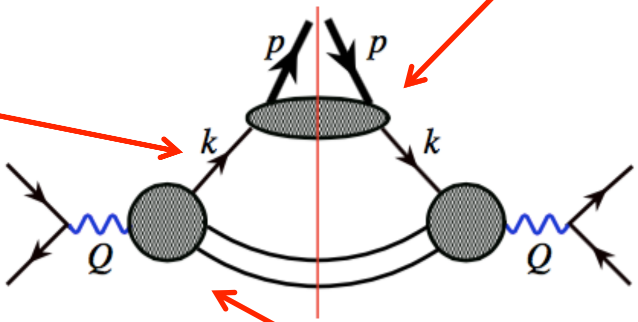
"perturbatively"

Hard collisions with identified hadron(s)

- ❑ **Creation of an identified hadron:**

Non-perturbative!

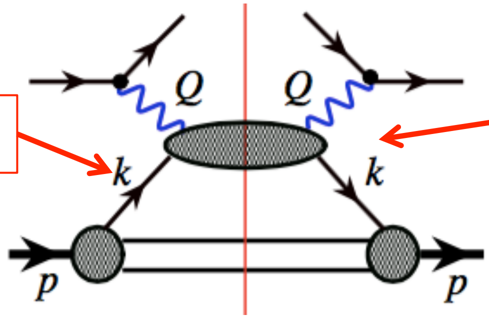
Pinch in k^2



Perturbative!

- ❑ **Identified initial hadron:**

Pinch in k^2

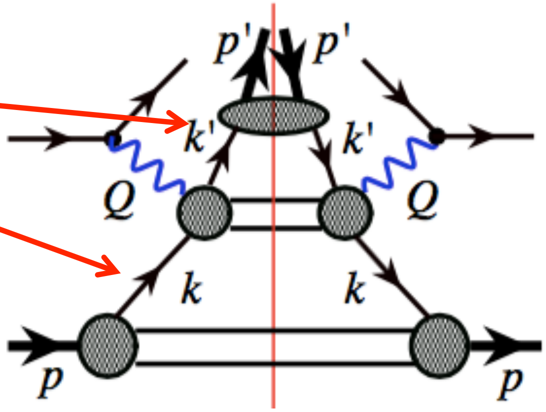


Perturbative!

Non-perturbative!

- ❑ **Initial + created identified hadron(s):**

Pinch in both k^2 and k'^2



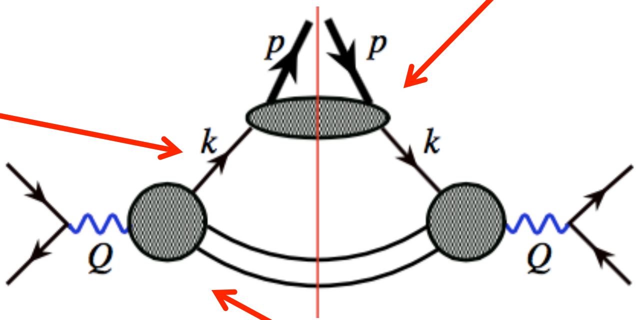
Cross section with identified hadron(s) is NOT perturbatively calculable

Hard collisions with identified hadron(s)

- Creation of an identified hadron:

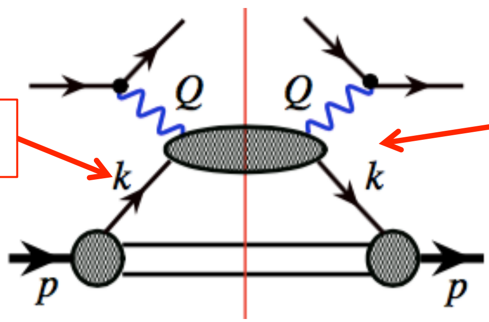
Non-perturbative!

Pinch in k^2



Perturbative!

- Identified initial hadron:



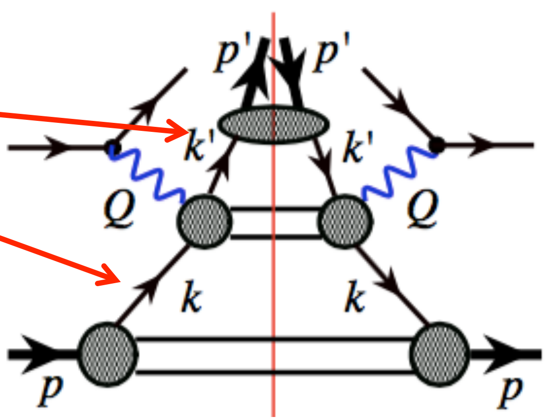
Pinch in k^2

Perturbative!

Non-perturbative!

- Initial + created identified hadron(s):

Pinch in both k^2 and k'^2



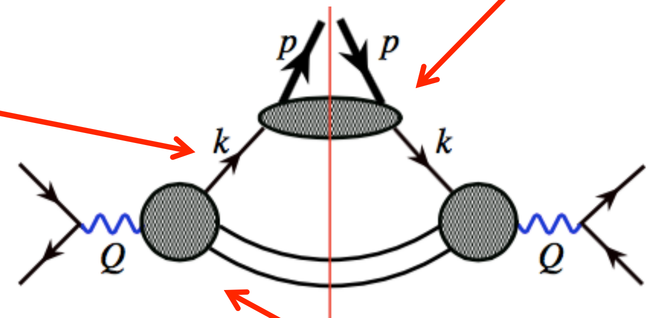
Dynamics at a HARD scale is linked by partons almost on Mass-Shell

Hard collisions with identified hadron(s)

- Creation of an identified hadron:

Non-perturbative!

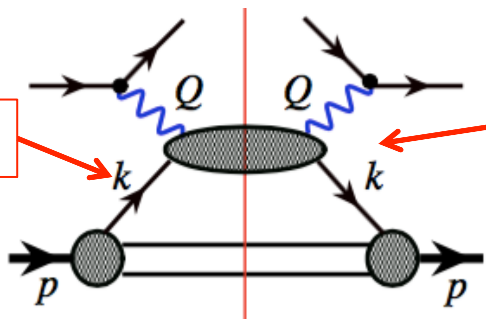
Pinch in k^2



- Identified initial hadron:

Perturbative!

Pinch in k^2

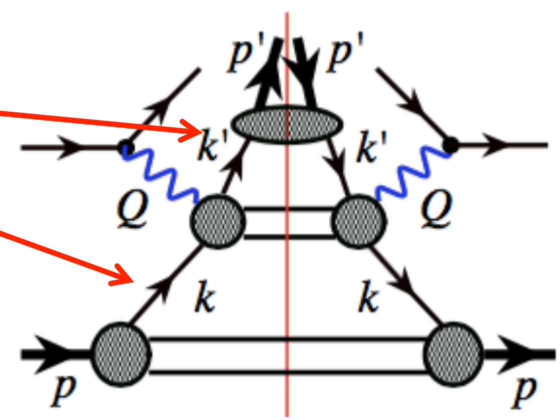


Perturbative!

Non-perturbative!

- Initial + created identified hadron(s):

Pinch in both k^2 and k'^2



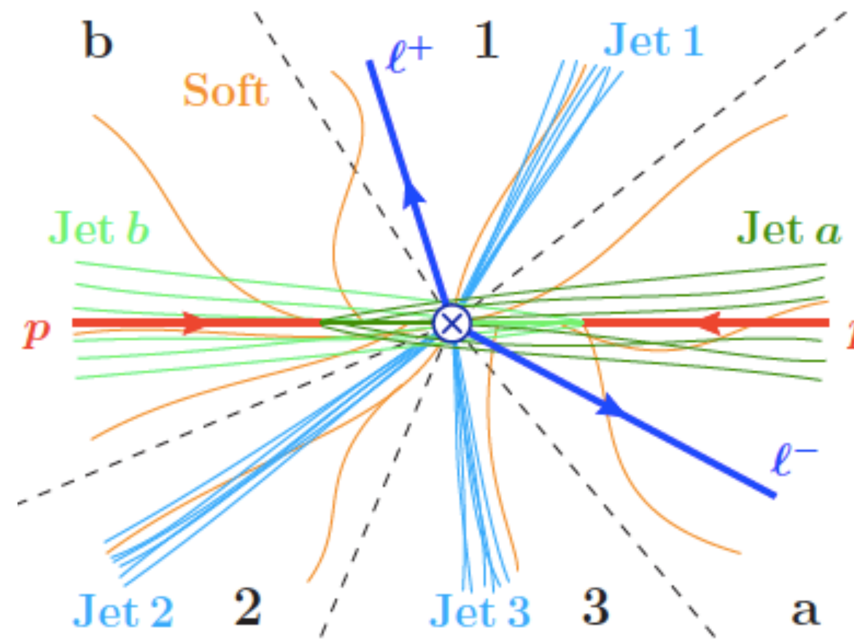
Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!

Backup slides

N-Jettiness

- **Event structure:**

  $pp \to$ leptons plus jets



- **N-Jettiness:**

  (Stewart, Tackmann, Waalewijin, 2010)

$$\tau_N = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

The sum include all final-state hadrons *excluding* more than N jets

Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust)

- **N-infinitely narrow jets (jet veto):**

  As a limit of N-Jettiness: $\tau_N \to 0$

Generalization of the thrust distribution in e^+e^- initial-state identified hadron!