HUGS

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Introduction to Quantum Chromodynamics (QCD)

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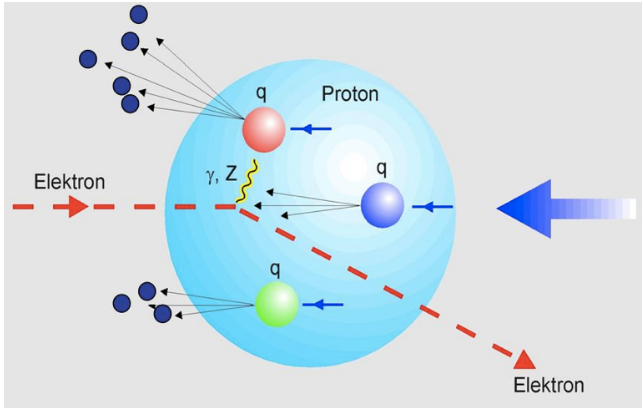
Lecture Three

Observables with ONE identified hadron

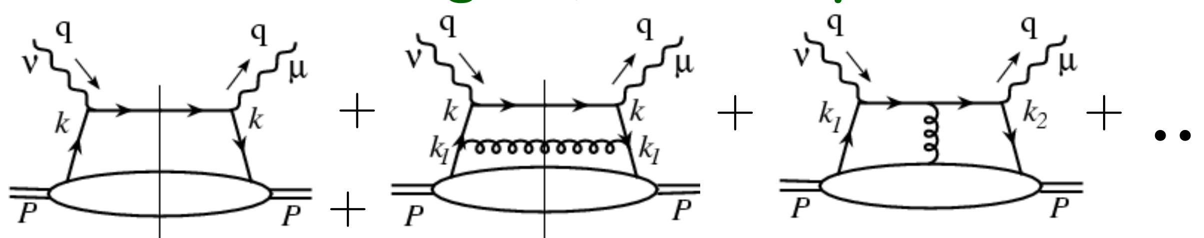


$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})}$$

Identified initial-state hadron-proton!

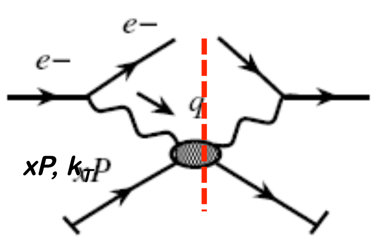
❑ DIS cross section is infrared divergent, and nonperturbative!

$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})} \propto$$

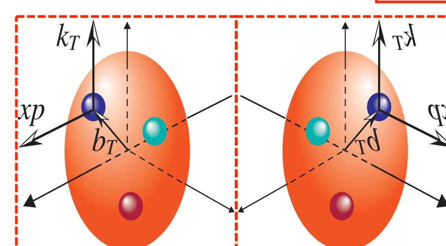


❑ QCD factorization (approximation!)

$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})} =$$



$$\otimes$$



$$+\ O\left(\frac{1}{QR}\right)$$

Color entanglement Approximation

Physical Observable

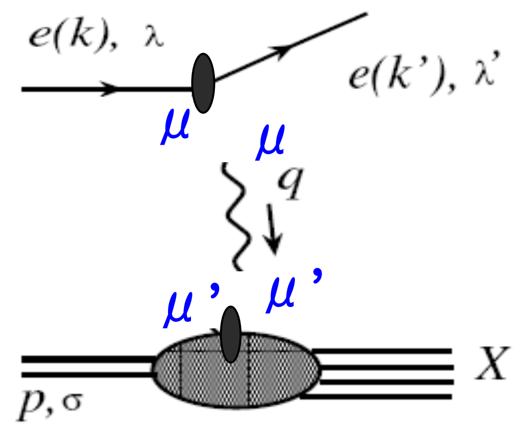
Controllable Probe

Quantum Probabilities Structure

Inclusive lepton-hadron DIS – one hadron

❑ Scattering amplitude:

$$
\begin{aligned}
\mathrm{M}(\lambda,\lambda';\sigma,q) &= \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k) \\
&* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right) \\
&* \left\langle X\left|eJ^{em}_{\mu'}(0)\right|p,\sigma\right\rangle
\end{aligned}
$$



❑ Cross section:

$$
d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_{X}\sum_{\lambda,\lambda',\sigma}\left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2\left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3\,2E_i}\right]\frac{d^3 k'}{(2\pi)^3\,2E'}(2\pi)^4\,\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)
$$

$$
\Rightarrow \quad \boxed{E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')\,W_{\mu\nu}(q,p)}
$$

❑ Leptonic tensor:

– **known from QED**

$$
L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)
$$

DIS structure functions

❑ **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\ \langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

❑ **Symmetries:**

- ✧ **Parity invariance (EM current)** ⟶ $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
- ✧ **Time-reversal invariance** ⟶ $W_{\mu\nu} = W^*_{\mu\nu}$ real
- ✧ **Current conservation** ⟶ $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2p\cdot q}$$

❑ **Structure functions – infrared sensitive:**

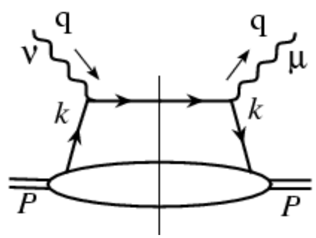
$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

Long-lived parton states

- **Feynman diagram representation of the hadronic tensor:**

$W^{\mu\nu} \propto$



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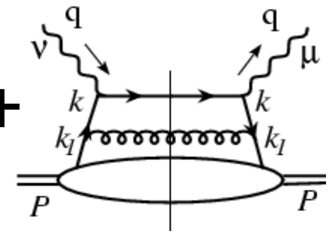

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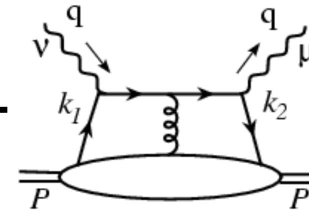
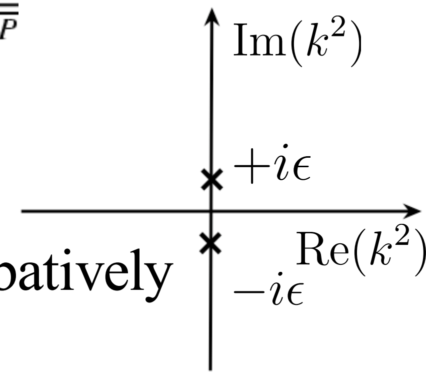


$+ \ldots$

- **Perturbative pinched poles:**

$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right) \Rightarrow \infty \text{ perturbatively}$$



- **Perturbative factorization:**

Light-cone coordinate:

$$v^\mu = (v^+, v^-, v^\perp),\ v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$$

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

$$\int\frac{dx}{x}d^2k_T\ \mathrm{H}(Q,k^2=0)\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{r_0}\right)+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

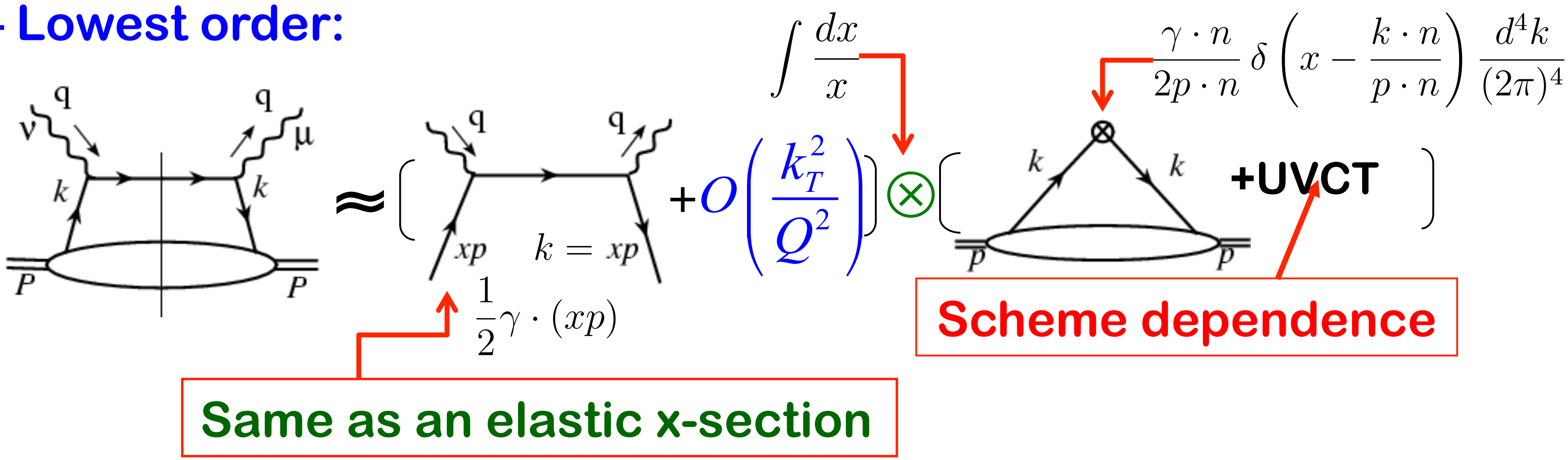
Short-distance

Nonperturbative matrix element

Collinear factorization – further approximation

- **Collinear approximation, if** $\boxed{Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}}$

  – **Lowest order:**



Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \longrightarrow \infty$, while x_B fixed

Feynman's parton model and Bjorken scaling

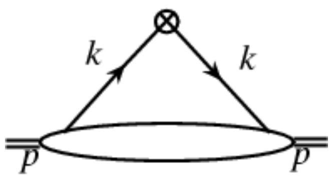
$$F_2(x_B,Q^2) = x_B \sum_f e_f^2\,\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

Spin-½ parton!

- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle/Q^2\right)$

Parton distribution functions (PDFs)

- PDFs as matrix elements of two parton fields:



  - combine the amplitude & its complex-conjugate

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

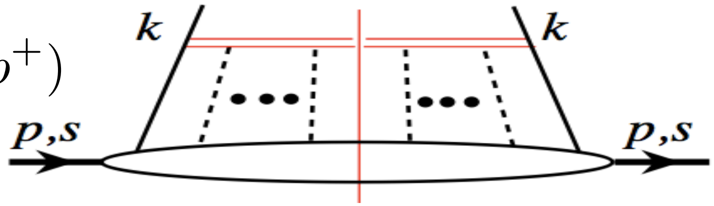
But, it is NOT gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

  - need a gauge link:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

  - corresponding diagram in momentum space:

$$\int \frac{d^4k}{(2\pi)^4}\, \delta(x - k^+/p^+)$$



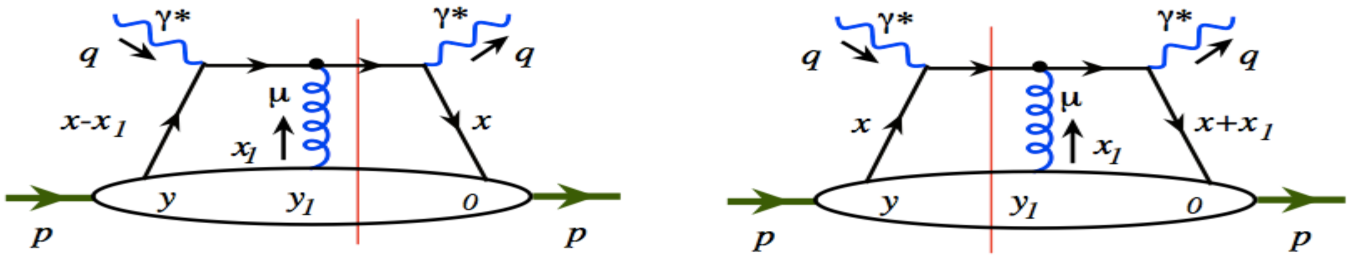
$$+\,\mathrm{UVCT}(\mu^2)$$

μ **-dependence**

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

- **Longitudinal gluon:**



- **Left diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, e^{ix_1p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

- **Right diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+ y_1^-}\, n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, e^{ix_1p^+y_1^-}\right]\mathcal{M} \;\; = ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

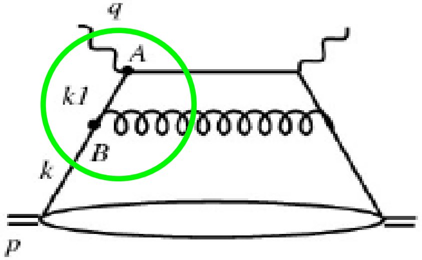
- **Total contribution:**

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\rm LO}$$

O(g)-term of the gauge link!

QCD high order corrections

❑ NLO partonic diagram to structure functions:



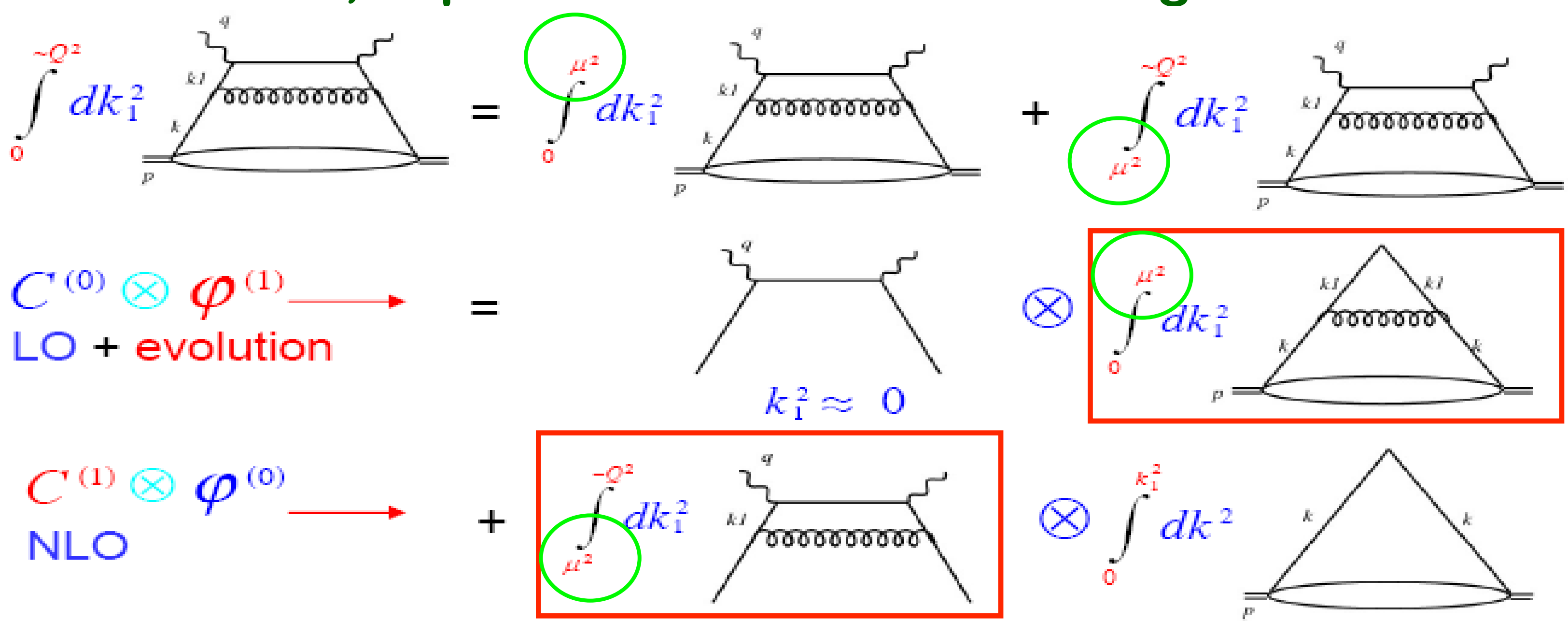
$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by

$$\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$$

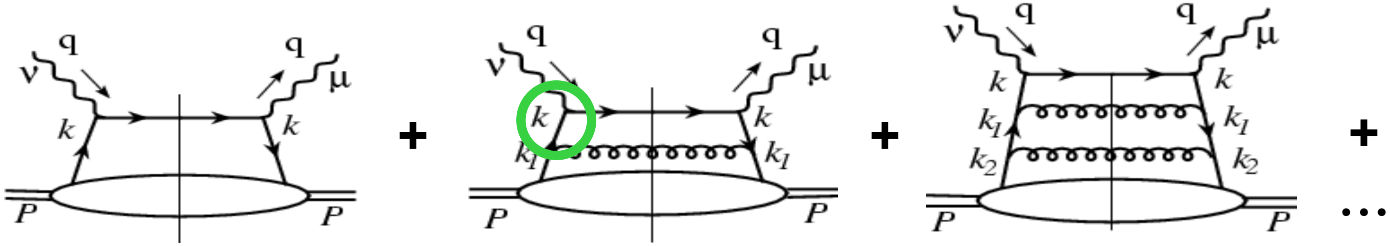
Diagram has both long- and short-distance physics

❑ Factorization, separation of short- from long-distance:

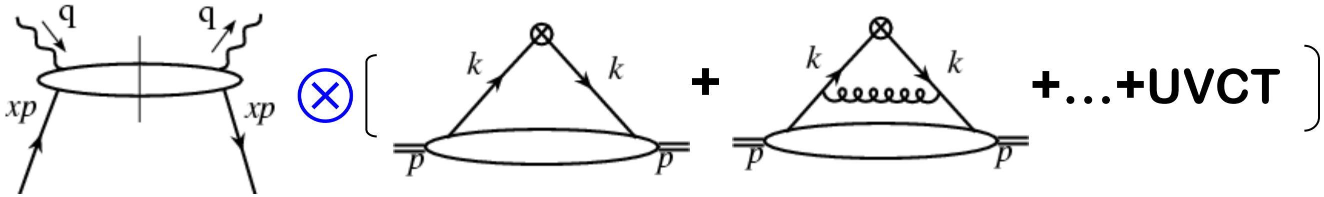


QCD leading power factorization

- QCD corrections: pinch singularities in $\int d^4 k_i$



- Logarithmic contributions into parton distributions:



$$F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- Factorization scale: μ_F^2

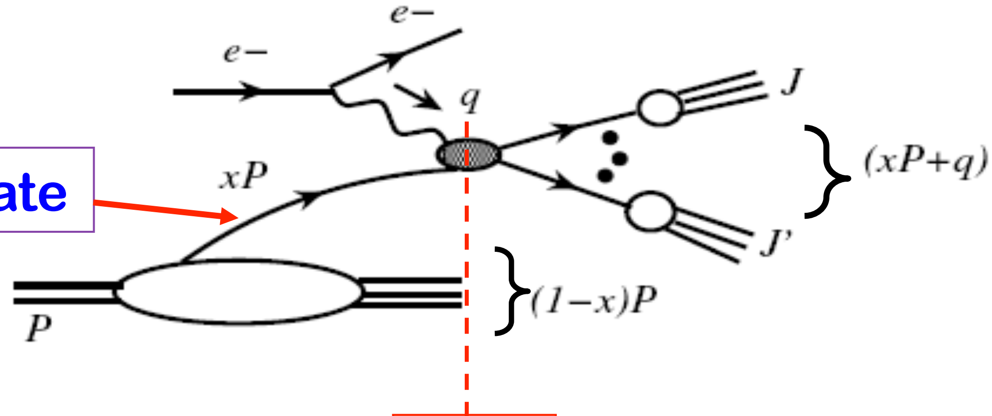
→ To separate the collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution

Picture of factorization for DIS

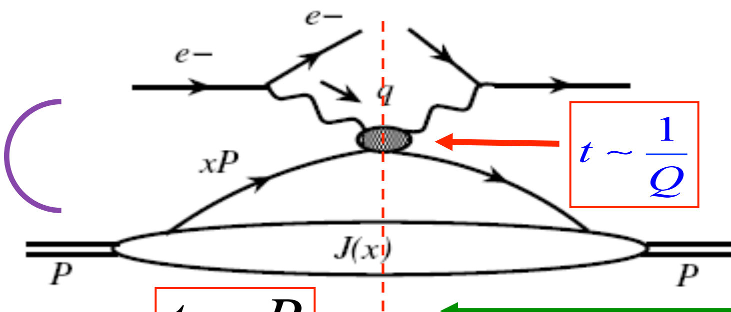
- **Time evolution:**

Long-lived parton state



Time: "Past" **Now** "Future"

- **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \mathrm{Im}\Big($$


$$\Big)$$

$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

How to calculate the perturbative parts?

❑ **Use DIS structure function F_2 as an example:**

$$\boxed{F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)}$$

✧ **Apply the factorized formula to parton states: $h \to q$**

Feynman diagrams $\longrightarrow$ $$F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ $\longleftarrow$ **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of α_s:**

0th order: $$F^{(0)}_{2q}(x_B,Q^2) = C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$

$$\Rightarrow \boxed{C^{(0)}_q(x) = F^{(0)}_{2q}(x)} \qquad \varphi^{(0)}_{q/q}\left(x\right) = \delta_{qq}\delta\left(1-x\right)$$

1th order: $$F^{(1)}_{2q}(x_B,Q^2) = C^{(1)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$
$$+ C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

$$\Rightarrow \boxed{C^{(1)}_q(x,Q^2/\mu^2) = F^{(1)}_{2q}(x,Q^2) - F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)}$$

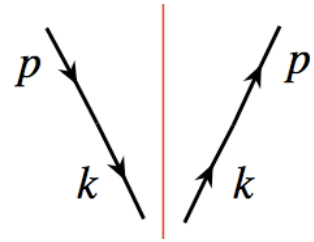
PDFs of a parton

❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\text{parton}(p)\rangle$ $\quad\phi_{f/q}(x,\mu^2)$ – **given by Feynman diagrams**

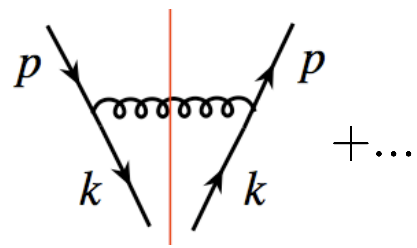
❑ Lowest order quark distribution:



✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:



✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

Partonic cross sections

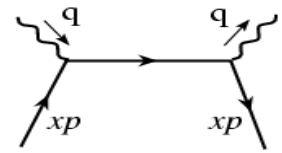
❑ Projection operators for SFs:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)F_2\left(x,Q^2\right)$$

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2) \\ F_2(x,Q^2) &= x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2) \end{aligned}}$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = xg^{\mu\nu}W_{\mu\nu,q}^{(0)} = xg^{\mu\nu}\left[\frac{1}{4\pi}\right.$$



$$\left.\vphantom{\frac{1}{4\pi}}\right]$$

$$= \left(xg^{\mu\nu}\right)\frac{e_q^2}{4\pi}\mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p\gamma_\mu\gamma\cdot\left(p+q\right)\gamma_\nu\right]2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x\delta(1-x)$$

$$\boxed{C_q^{(0)}(x) = e_q^2 x\delta(1-x)}$$

NLO coefficient function – complete example

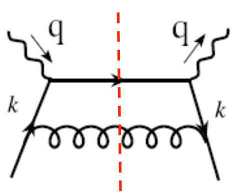
$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

- **Projection operators in n-dimension:** $\quad g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

$$\boxed{\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}}$$

- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$



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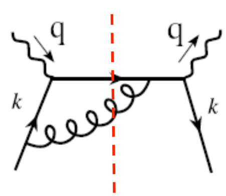

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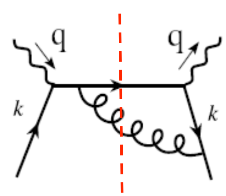

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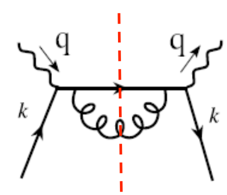
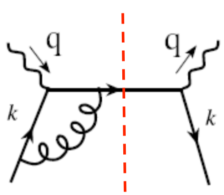
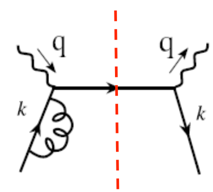


} **Real**

Virtual { +



+ c.c. (+



+ c.c. + UV CT)

- **Calculation:** $\quad -g^{\mu\nu}W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

- Lowest order in n-dimension:

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- NLO virtual contribution:

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- NLO real contribution:

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ln(1-z) f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu}W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon}P_{qq}(x) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+C_F\left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$
$$\left.\left. + 3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

❑ One loop contribution to F_2 of a quark:

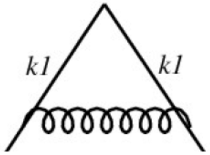
$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2}\right)\right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\mathrm{UV}} + \left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ **MS scheme:** $$\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$$

✧ **$\overline{\text{MS}}$ scheme:** $$\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$$

✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu^2)\Big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)}$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x) \ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) \right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Evolution

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\mathrm{QCD}}^2\right)$

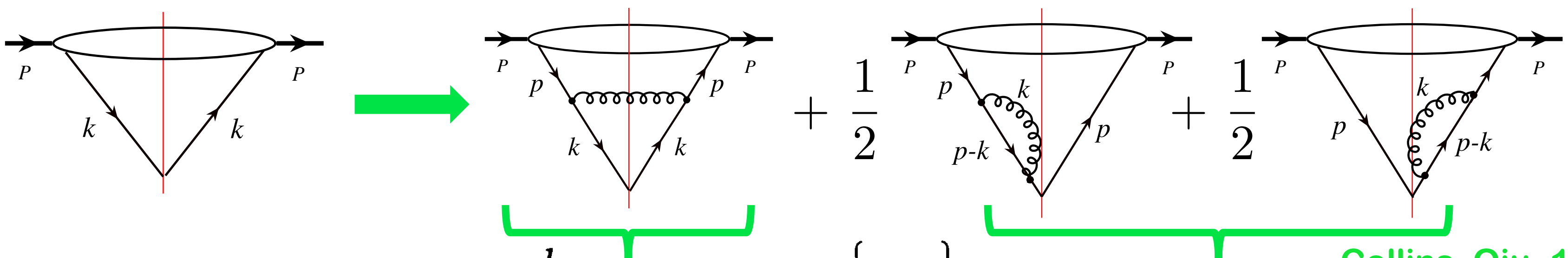
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ DGLAP evolution equation:

$$\boxed{\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)}$$

Calculation of evolution kernels

- **Evolution kernels are process independent**
  - ✧ Parton distribution functions are universal
  - ✧ Could be derived in many different ways
- **Extract from calculating parton PDFs' scale dependence**



Collins, Qiu, 1989

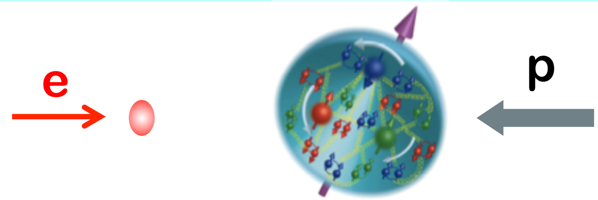
$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right)-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

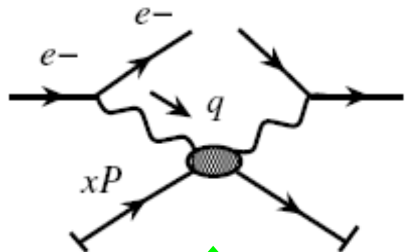
Change "Gain" "Loss"

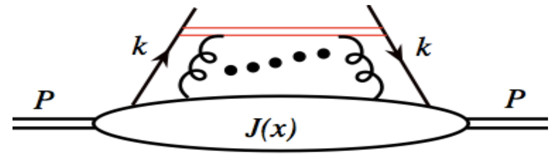
  - ✧ Same is true for gluon evolution, and mixing flavor terms
- **One can also extract the kernels from the CO divergence of partonic cross sections**

From one hadron to two hadrons

- One hadron:



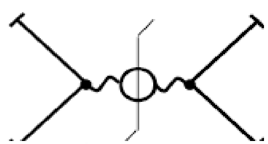
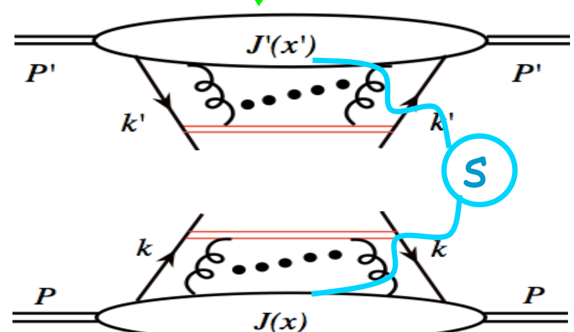
$\sigma_{\text{tot}}^{\text{DIS}} \sim$ 

$\otimes$ 

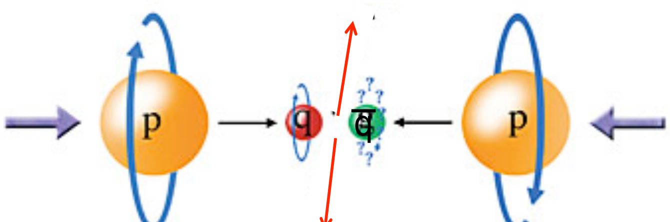
$+ \, O\left(\frac{1}{QR}\right)$

Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

- Two hadrons:

$\sigma_{\text{tot}}^{\text{DY}} \sim$  $\otimes$ 

$+ \, O\left(\frac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

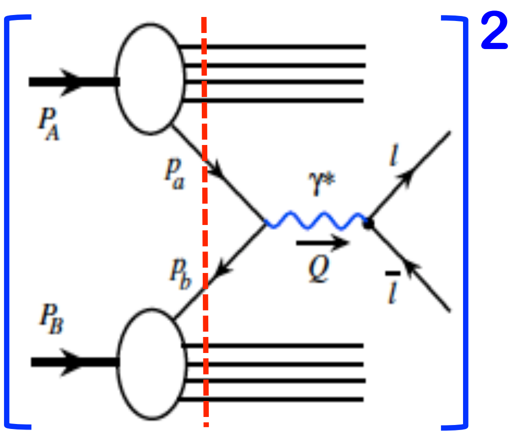
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

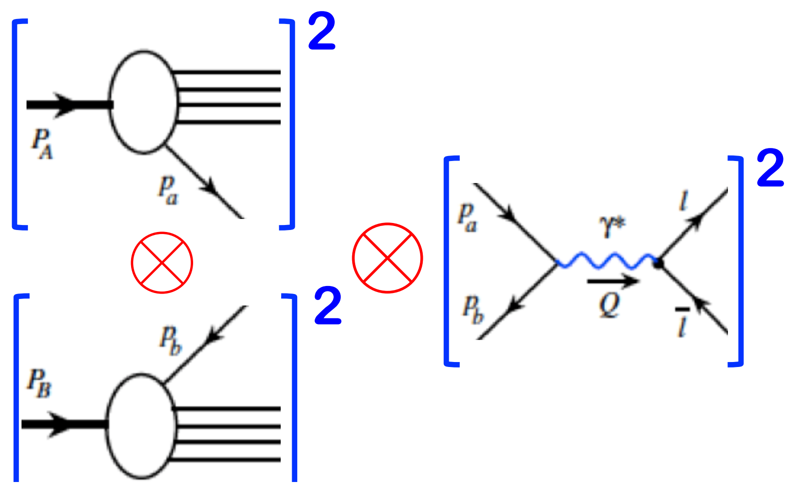
❑ Drell-Yan mechanism:

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \qquad \text{with} \quad q^2 \equiv Q^2 \gg \Lambda^2_{\mathrm{QCD}} \sim 1/\mathrm{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

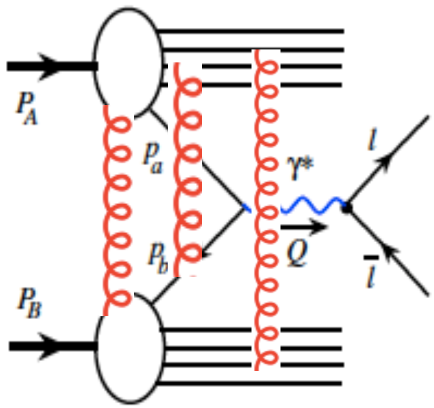
$$x_A = \frac{Q}{\sqrt{S}} e^{y} \qquad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

❑ Beyond the lowest order:

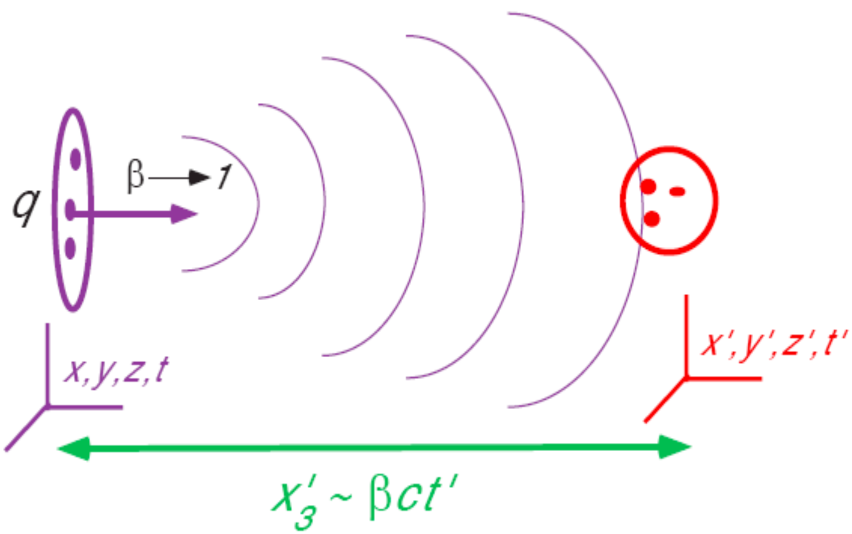
Collins, Soper and Sterman, Review in QCD, edited by AH Mueller 1989



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

→ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:

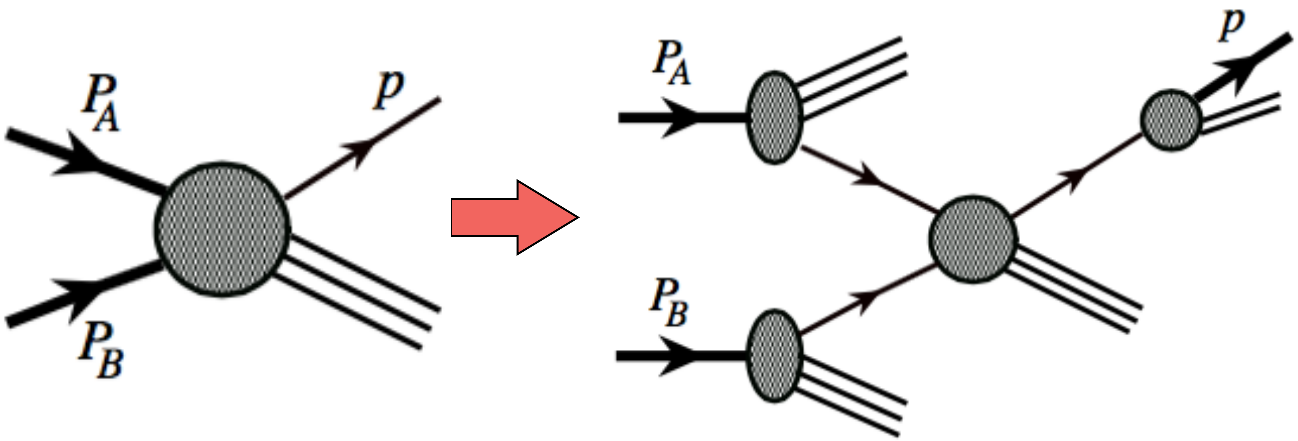


x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

Factorization for more than two hadrons

❑ Factorization for high p_T single hadron:

Nayak, Qiu, Sterman, 2006



$\gamma, W/Z, \ell(s), \text{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O $(1/P_T^2)$

$p_T \gg m \gtrsim \Lambda_{\rm QCD}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x,\mu_F^2) \otimes \phi_{B\to b}(x',\mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x,x',z,y,p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z,\mu_F^2)$$

✧ **Fragmentation function:** $D_{c\to C}(z,\mu_F^2)$

✧ **Choice of the scales:** $\mu_{\rm Fac}^2 \approx \mu_{\rm ren}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Global QCD analysis – Testing QCD

- **Factorization for observables with identified hadrons:**
  - **One-hadron (DIS):**

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - **Two-hadrons (DY, Jets, W/Z, …) :**

$$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

Testing QCD:
Universal PDFs for all cross sections?

  - **DGLAP Evolution:**

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

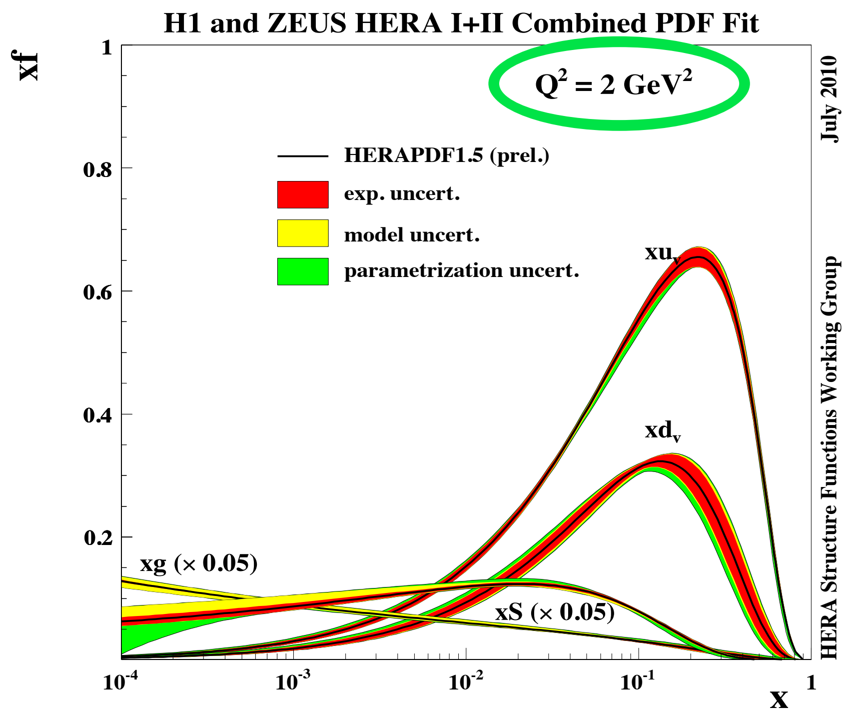
- **Input for QCD Global analysis/fitting:**
  - **World data with "Q" > 2 GeV**
  - **PDFs at an input scale:** $\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$

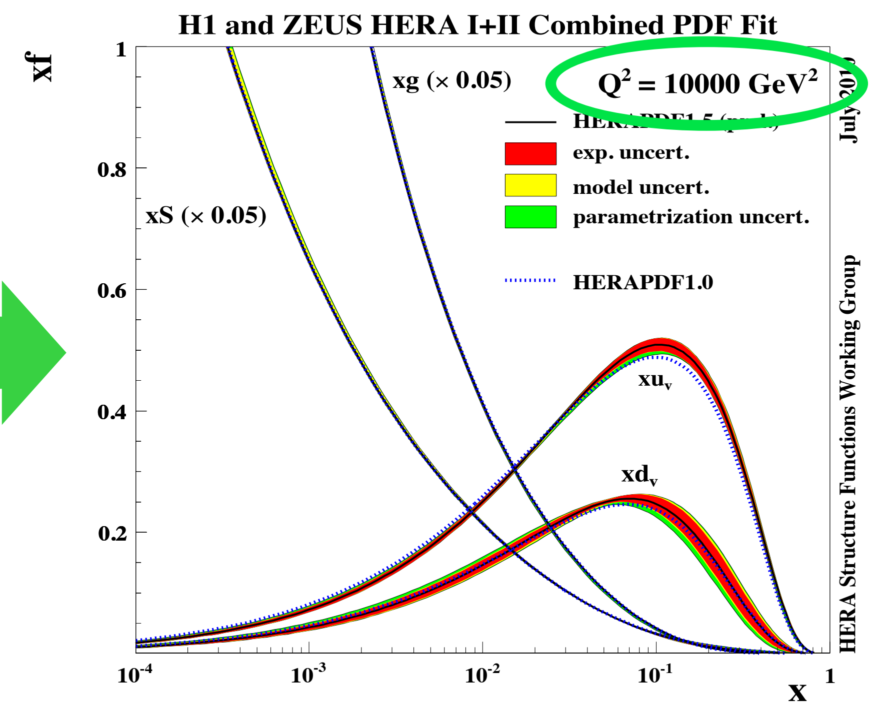
Input scale ~ GeV

Fitting paramters

PDFs from DIS

- Q2-dependence is a prediction of pQCD calculation:



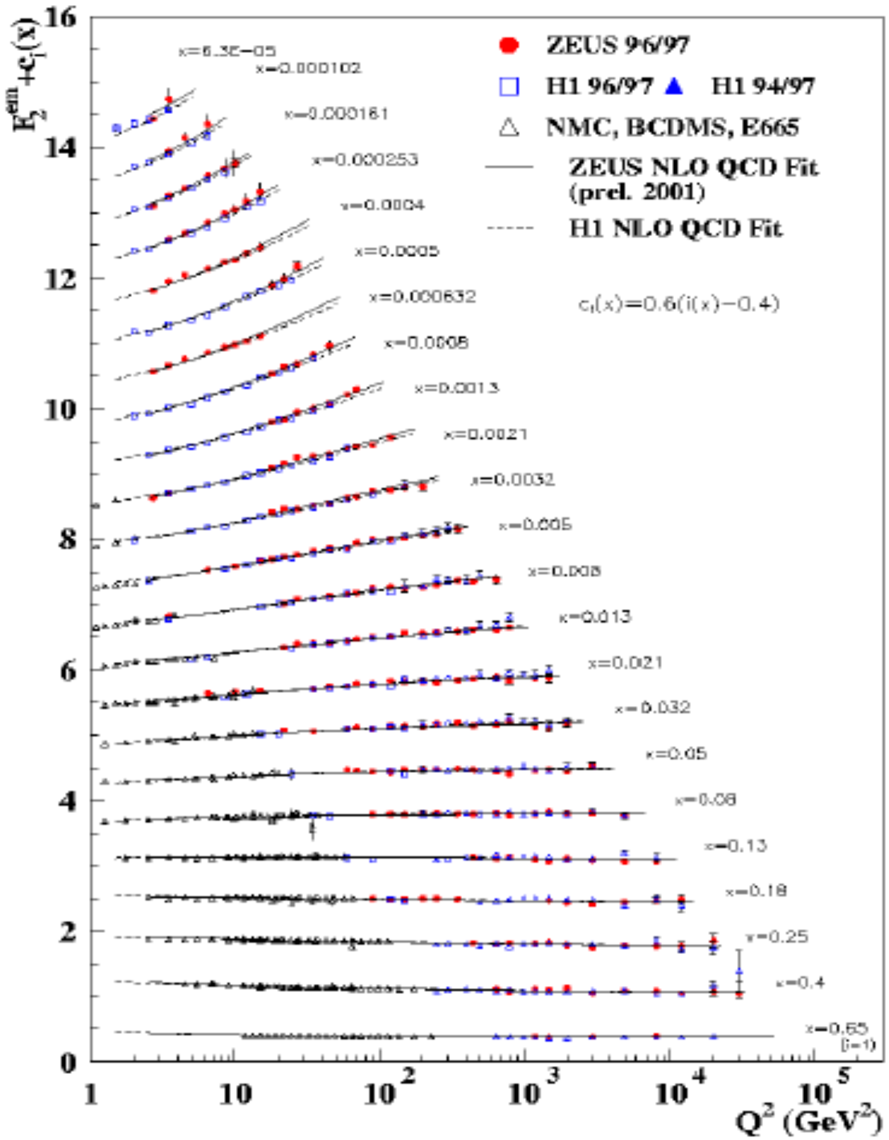


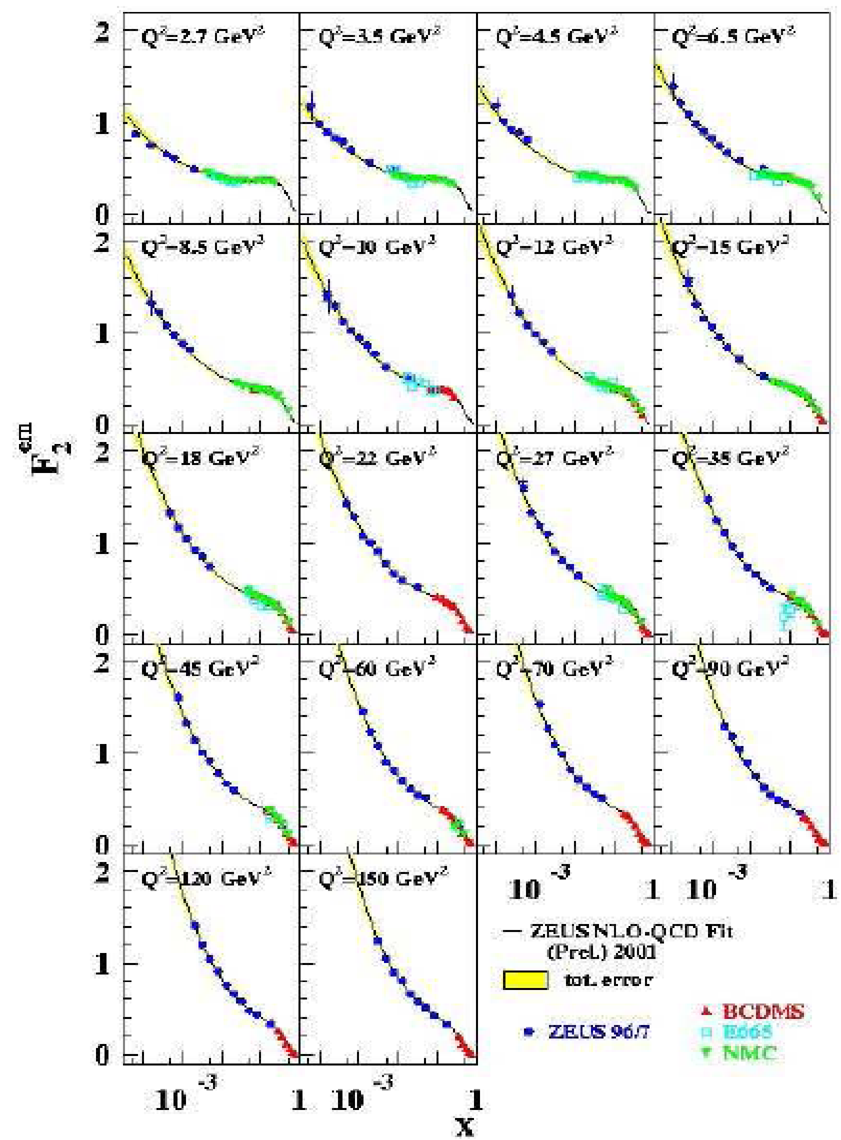
- Physics interpretation of PDFs:

$f(x,Q^2)$: Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q²"

- ✧ **Number of partons:** $\int_0^1 dx\, u_v(x,Q^2) = 2, \quad \int_0^1 dx\, d_v(x,Q^2) = 1$
- ✧ **Momentum fraction:** $\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x,Q^2) \longrightarrow \sum_f \langle x(Q^2)\rangle = 1$

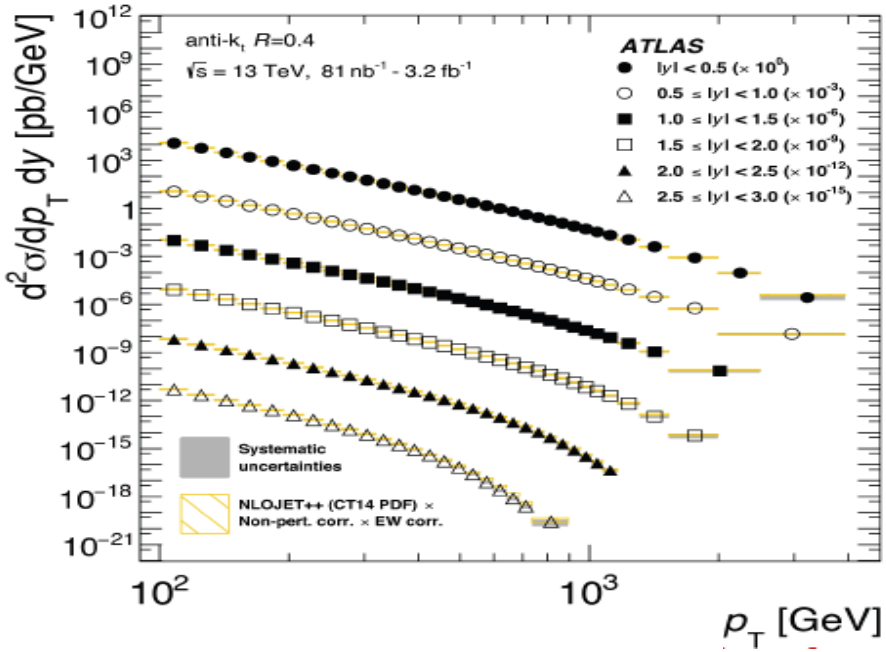
Scaling and scaling violation

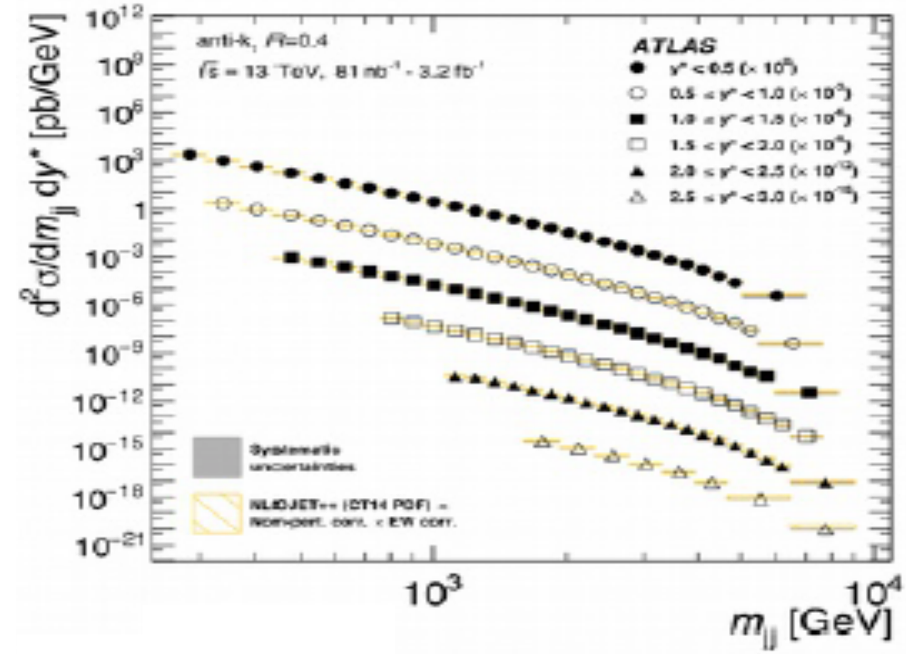


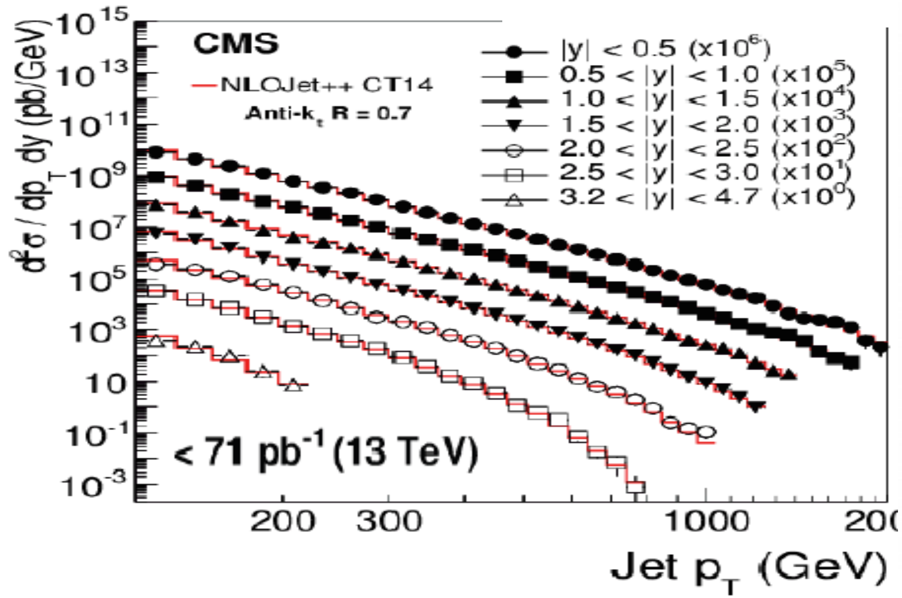


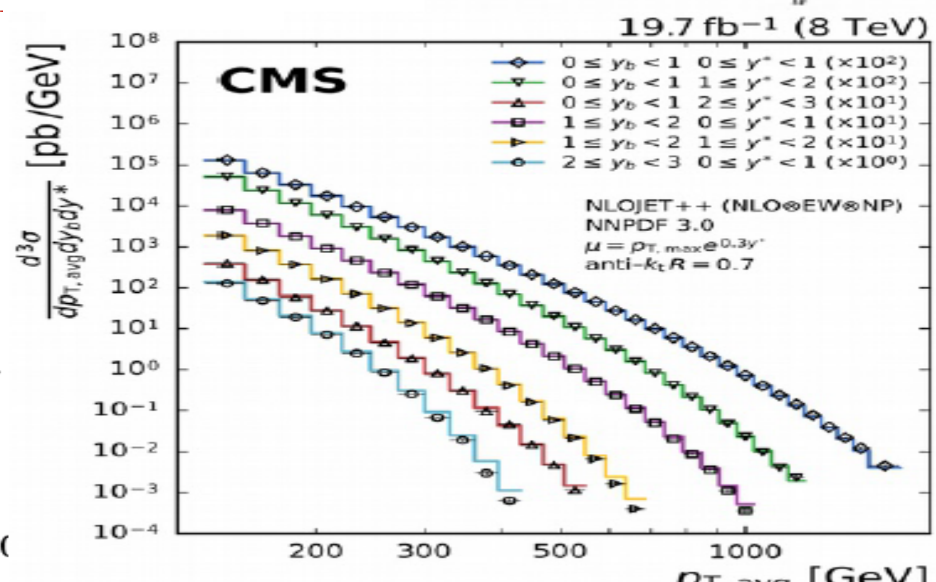
Q^2-dependence is a prediction of pQCD calculation

Jet production from the LHC



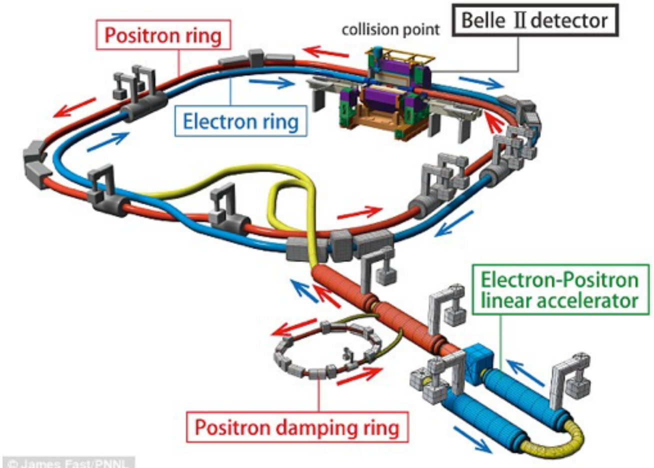


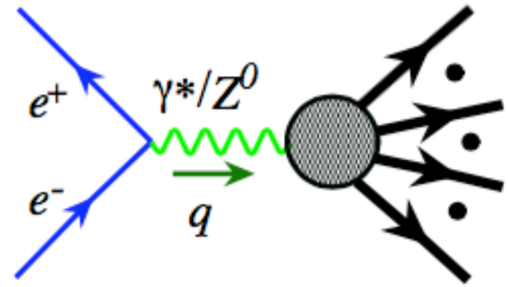




Hard probes from high energy collisions

- Lepton-lepton collisions:

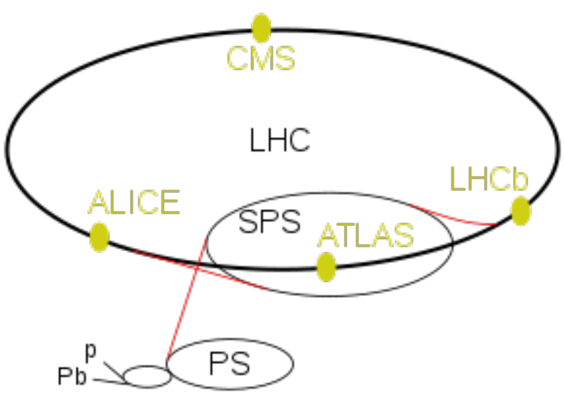


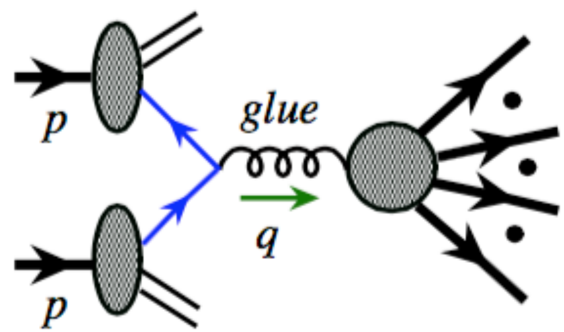


Hadrons

- ✧ No hadron in the initial-state
- ✧ Hadrons are emerged from energy
- ✧ Not ideal for studying hadron structure

- Hadron-hadron collisions:





Hadrons

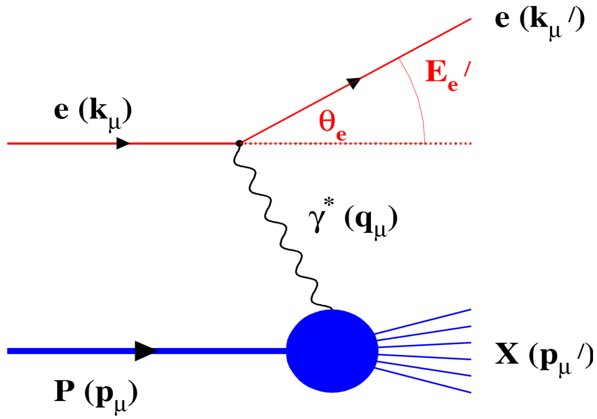
- ✧ Hadron structure – motion of quarks, …
- ✧ Emergence of hadrons, …
- ✧ Initial hadrons broken – collision effect, …

- Lepton-hadron collisions:

Hard collision without breaking the initial-state hadron – spatial imaging, …

Why a lepton-hadron facility is special?

- Many complementary probes at one facility:



Q^2 → Measure of resolution

y → Measure of inelasticity

x → Measure of momentum fraction of the struck quark in a proton

$$Q^2 = S\ x\ y$$

Inclusive events: e+p/A → e'+X

Detect only the scattered lepton in the detector

(Modern Rutherford experiment!)

Semi-Inclusive events: e+p/A → e'+h(π,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

(Initial hadron is broken – confined motion! – cleaner than h-h collisions)

Exclusive events: e+p/A → e'+ p'/A'+ h(π,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)

The Electron-Ion Collider (EIC) – the Future!

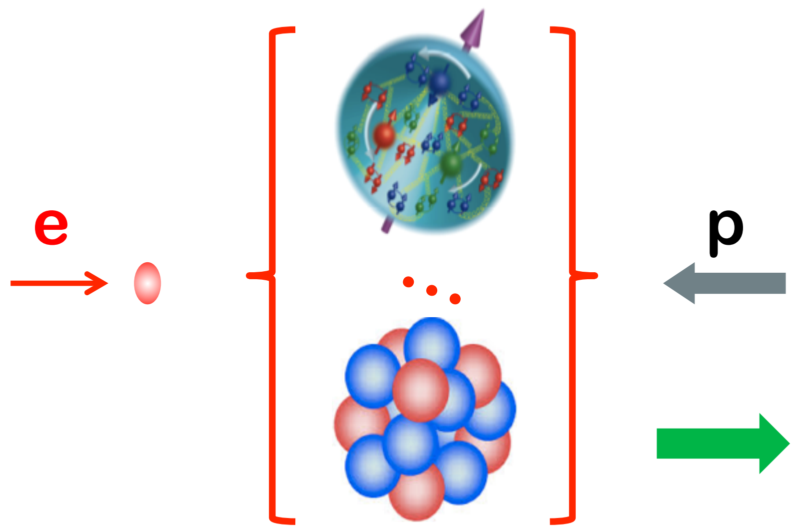
- **A sharpest "CT"** – **"imagine" quark/gluon structure without breaking the hadron**

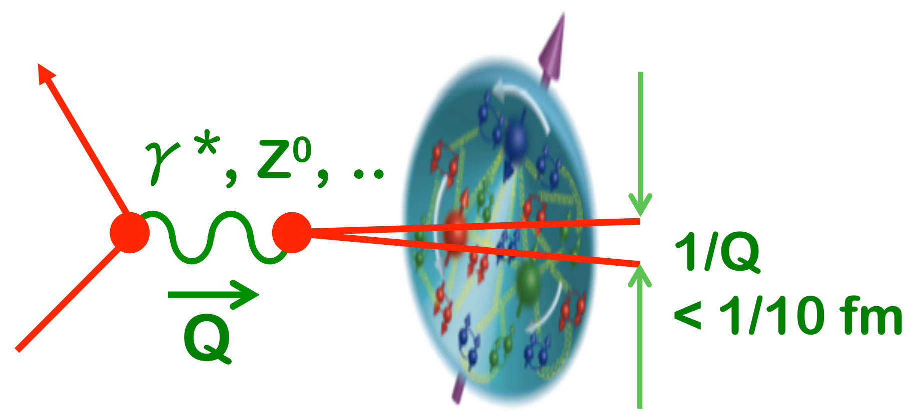
  – **"cat-scan" the nucleon and nuclei with a better than 1/10 fm resolution**

  – **"see" proton "radius" of quark/gluon density comparing with the radius of EM charge density**

➡ ***To discover color confining radius, hints on confining mechanism!***

- **A giant "Microscope"** – **"see" quarks and gluons by breaking the hadron**





➡ ***To discover/study color entanglement of the non-linear dynamics of the glue!***

Backup slides

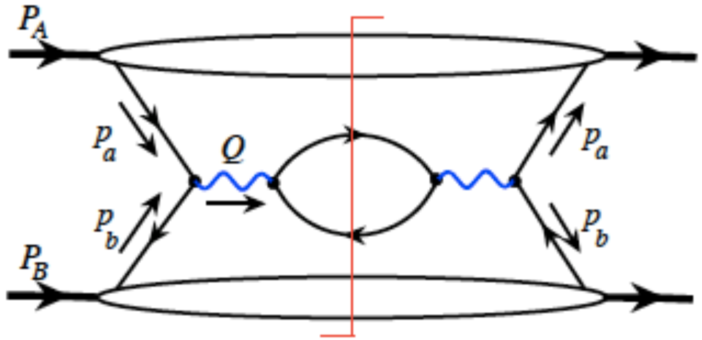
Drell-Yan process in QCD – factorization

Collins, Soper, Sterman, 1988

❑ Factorization – approximation:

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

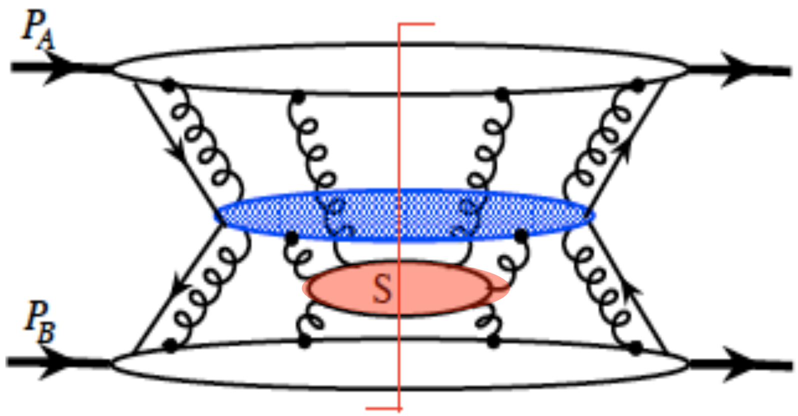
$$\begin{aligned} &\text{on-shell:} && p_a^2,\ p_b^2 \ll Q^2; \\ &\text{collinear:} && p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ &\text{higher-power:} && p_a^- \ll q^-; \quad \text{and} \\ & && p_b^+ \ll q^+ \end{aligned}$$

✧ Infrared safe of partonic parts:

Cancelation of IR behavior
Absorb all CO divergences into PDFs

Drell-Yan process in QCD – factorization

❑ **Leading singular integration regions (pinch surface):**



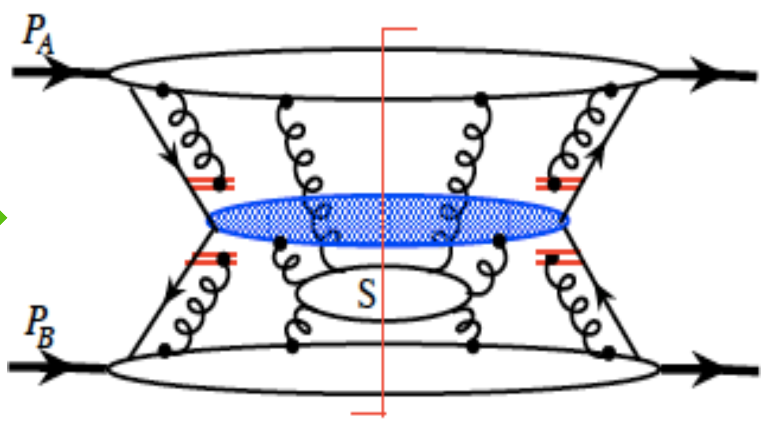
Hard: all lines off-shell by Q

Collinear:
- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

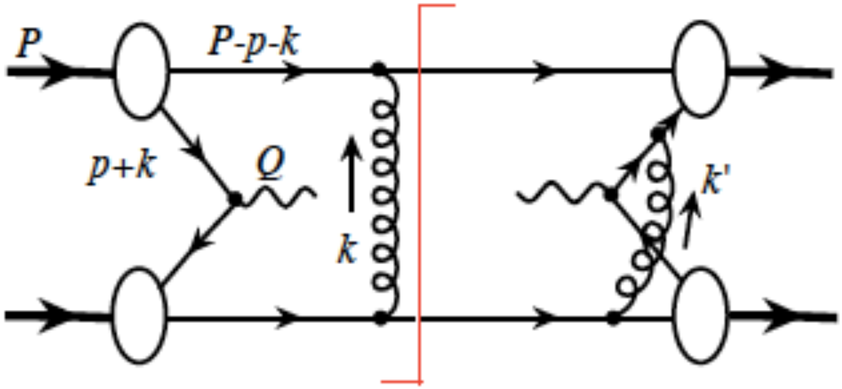
- ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
- ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ **Trouble with soft gluons:**



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

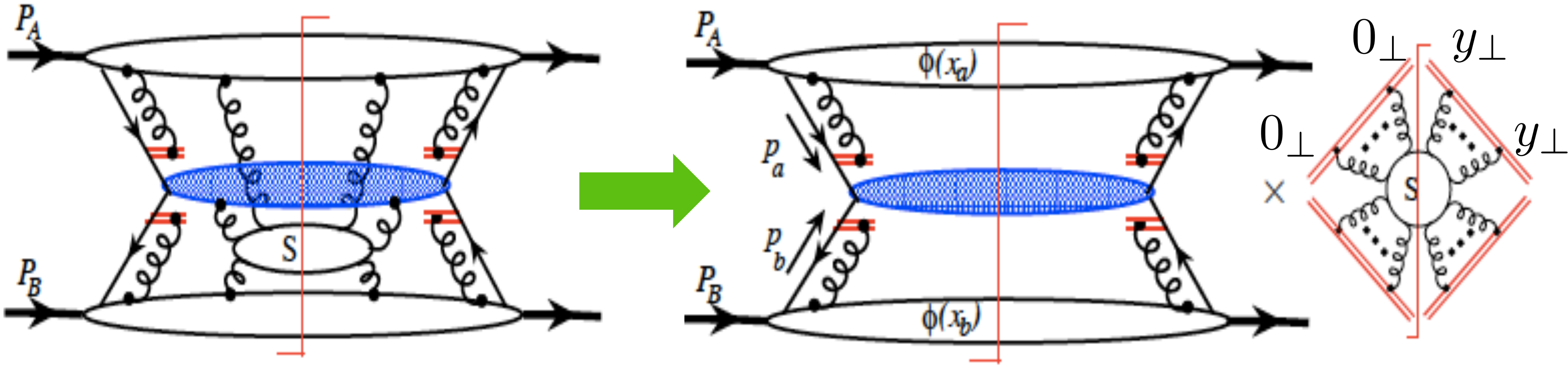
✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

✧ **The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^{\pm}$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- TMD factorization ($q_\perp \ll Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- Collinear factorization ($q_\perp \sim Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- Spin dependence:

The factorization arguments are independent of the spin states of the colliding hadrons

→ same formula with polarized PDFs for γ^*,W/Z, H^0...

Global QCD analysis – Testing QCD

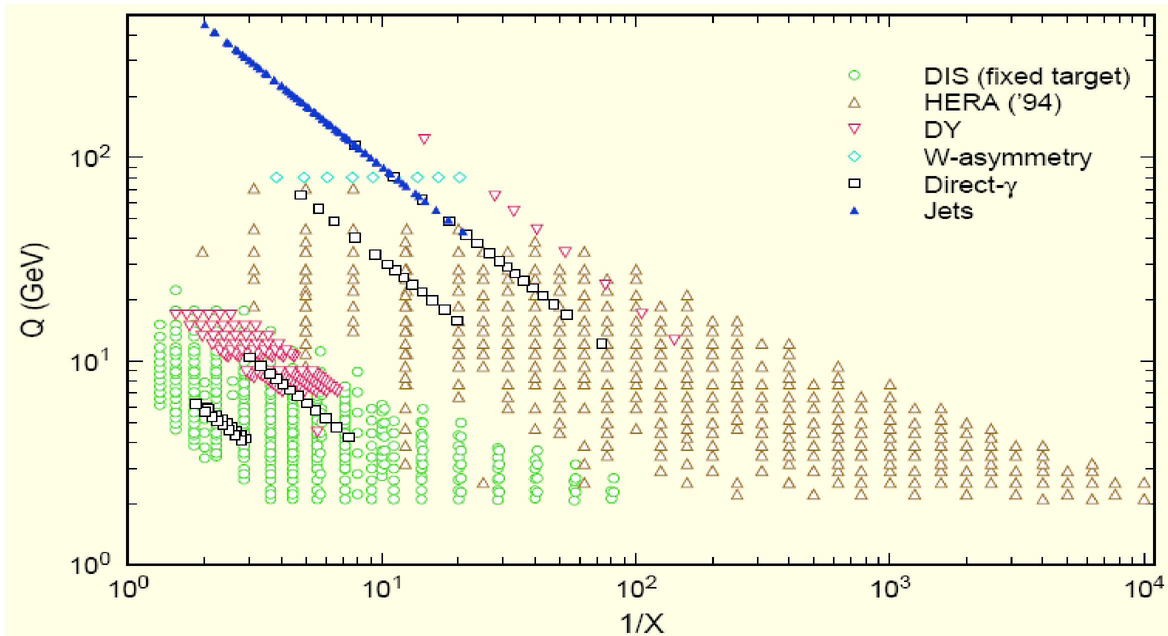
Input DPFs at Q_0

$\varphi_{f/h}\left(x,\{a_j\}\right)$

Vary $\{a_j\}$

Minimize Chi^2

DGLAP



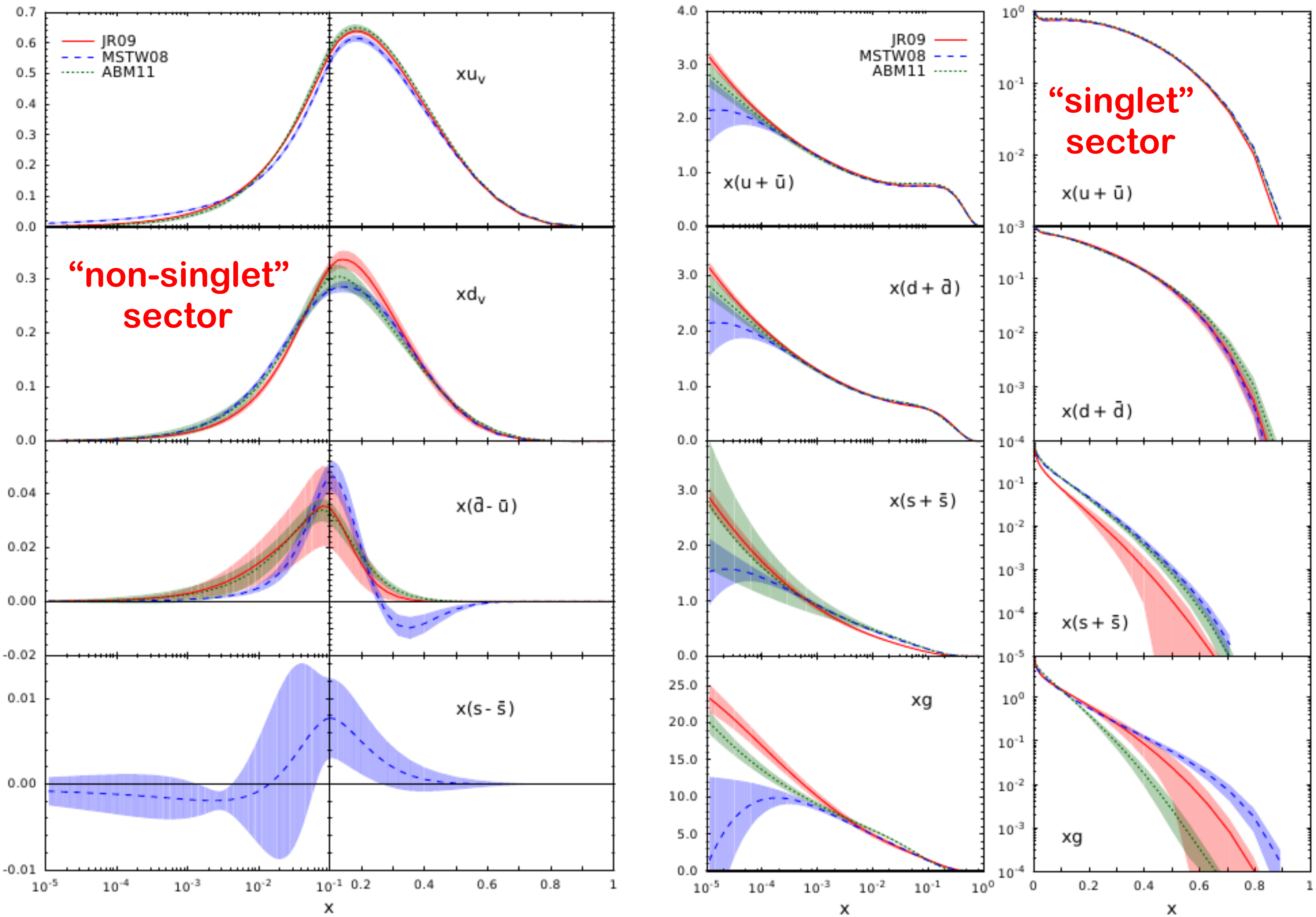
$\varphi_{f/h}(x)$ **at $Q>Q_0$**

QCD calculation

Comparison with Data at various x and Q

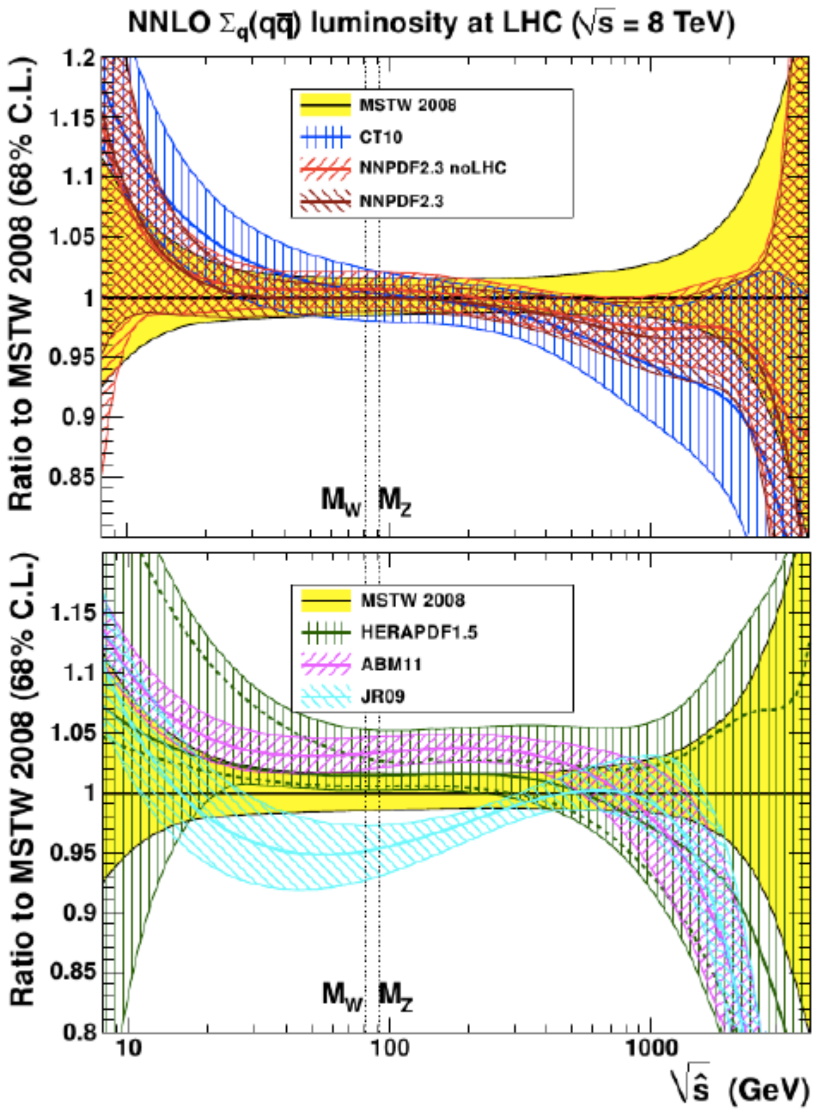
Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

Uncertainties of PDFs



Partonic luminosities

q - qbar



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