HUGS

JEFFERSON SCIENCE ASSOCIATES & HUGS FELLOWSHIP FOR GRADUATE STUDENTS IN DEVELOPING COUNTRIES

Introduction to Quantum Chromodynamics (QCD)

Jianwei Qiu
Theory Center, Jefferson Lab

May 29 – June 15, 2018

Lecture Four

Theory Center

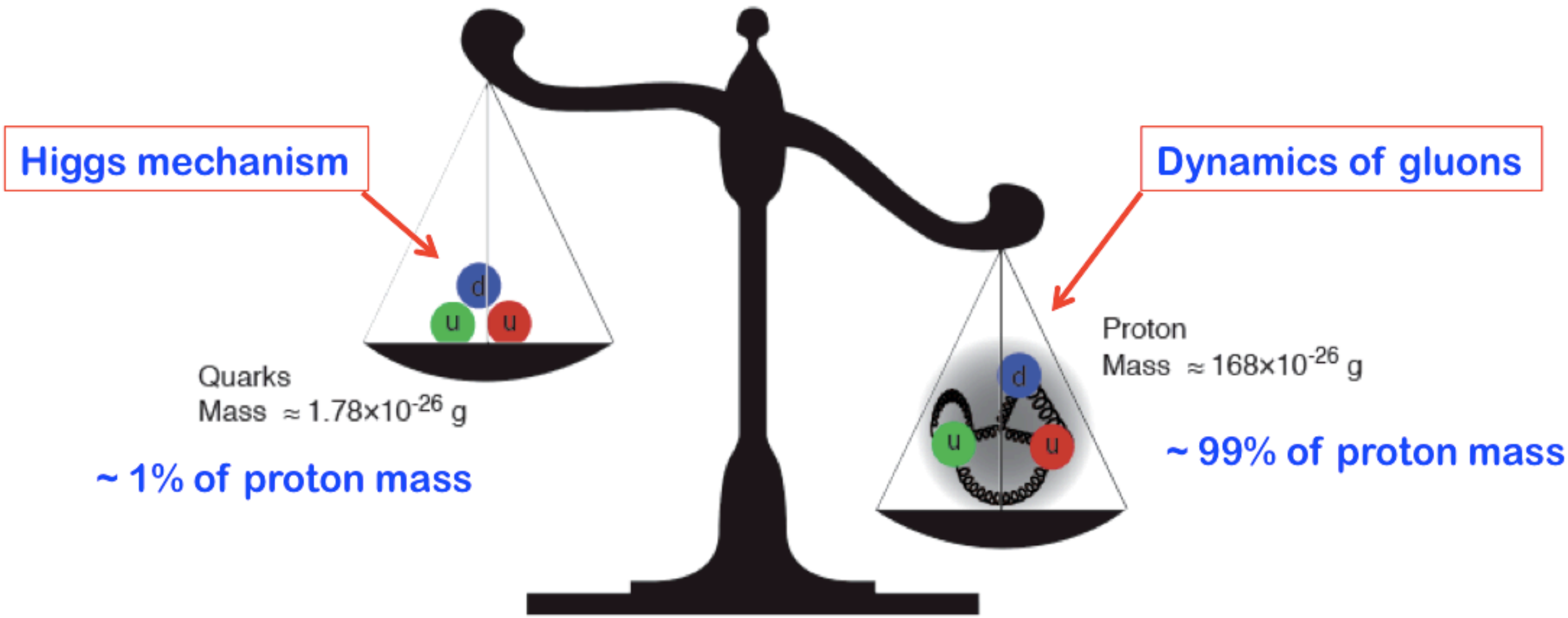
Jefferson Lab
EXPLORING THE NATURE OF MATTER

Hadron properties – the mass?

- How does QCD generate the nucleon mass?

  "… The vast majority of the nucleon's mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. …" *The 2015 Long Range Plan for Nuclear Science*

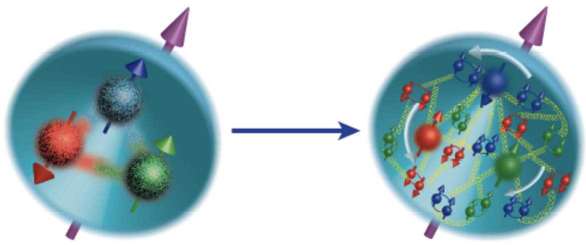
- Higgs mechanism is not relevant to hadron mass!



"Mass without mass!"

Hadron Mass

❑ Proton's mass:



- ✧ QCD Lagrangian does not have mass dimension parameters, other than current quark masses
- ✧ Asymptotic freedom ⟷ confinement:

➡ A dynamical scale, $\Lambda_{\rm QCD}$, consistent with $\dfrac{1}{R} \sim 200\ {\rm MeV}$

❑ Bag model:

- ✧ Kinetic energy of three quarks: $K_q \sim 3/R$
- ✧ Bag energy (bag constant B): $T_b = \dfrac{4}{3}\pi R^3\, B$
- ✧ Minimize $K_q + T_b$; $M_p \sim \dfrac{4}{R} \sim \dfrac{4}{0.88 fm} \sim 912{\rm MeV}$

❑ Constituent quark model:

- ✧ Spontaneous chiral symmetry breaking:

Massless quarks gain ~300 MeV mass when traveling in vacuum

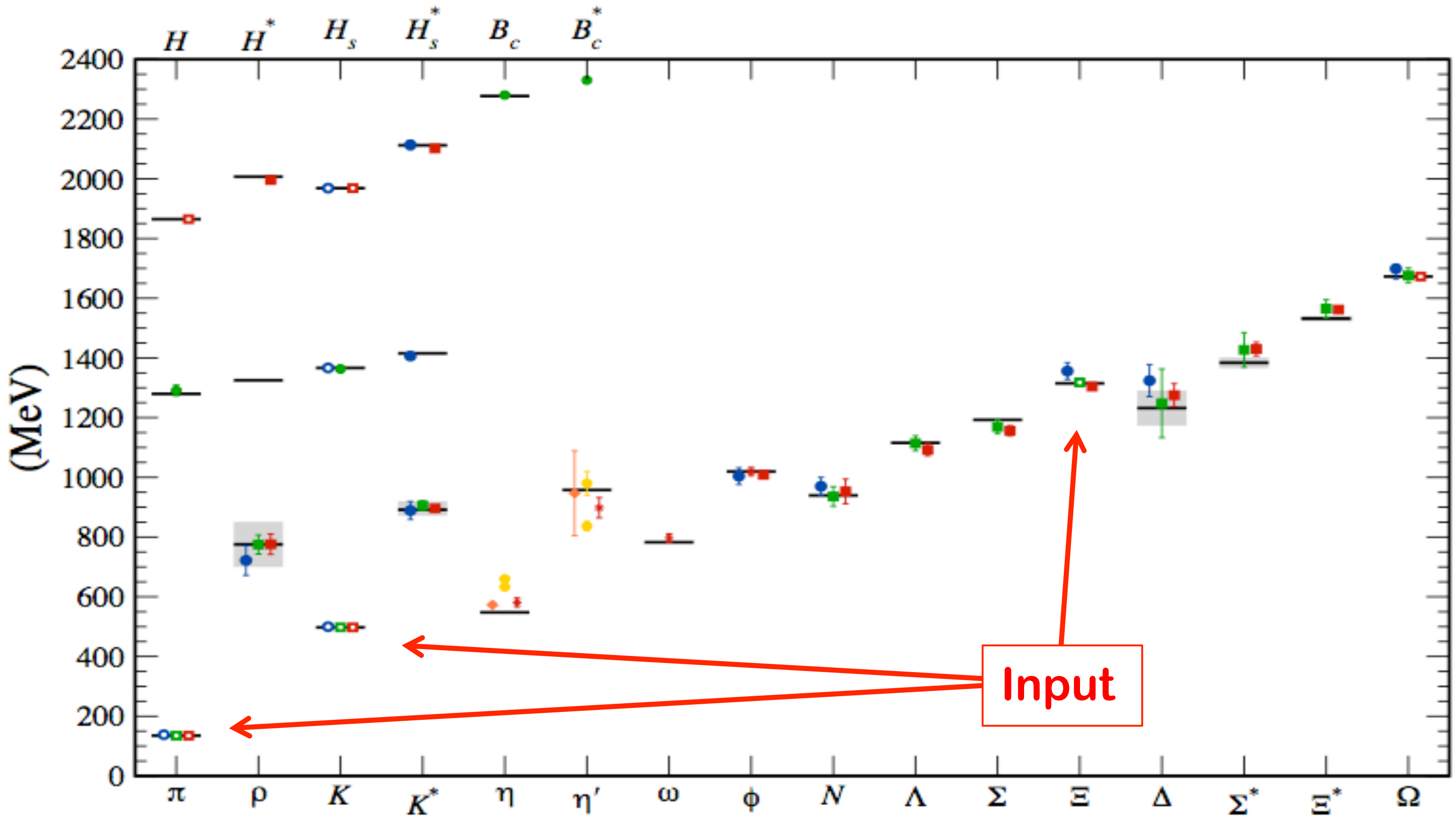
➡ $M_p \sim 3\, m_q^{\rm eff} \sim 900\ {\rm MeV}$

❑ Lattice QCD:

Ratios of hadron masses

Hadron Mass in QCD

- From Lattice QCD calculation:



A major success of QCD – is the right theory for the Strong Interaction!

How does QCD generate this? The role of quarks vs that of gluons?

If we do not understand proton mass, we do not understand QCD

New community effort

- ❑ **Three-pronged approach to explore the origin of hadron mass**
  - ✧ **Lattice QCD**
  - ✧ **Mass decomposition – roles of the constituents**
  - ✧ **Model calculation – approximated analytical approach**

https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

http://www.ectstar.eu/node/2218

Hadron properties – the spin?

❑ Spin:

- ✧ Pauli (1924): two-valued quantum degree of freedom of electron
- ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant ħ)
- ✧ Composite particle = Total angular momentum when it is at rest

❑ Spin of a nucleus:

- ✧ Nuclear binding: 8 MeV/nucleon << mass of nucleon
- ✧ Nucleon number is fixed inside a given nucleus
- ✧ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

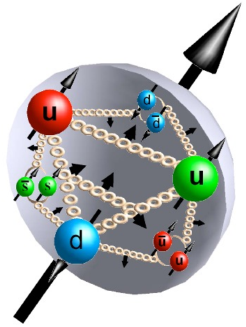
- ✧ If the probing energy << mass of constituent quark
- ✧ Nucleon is made of three constituent (valence) quark
- ✧ Spin of a nucleon = sum of the constituent quark spin

State: $$|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$$

Spin: $$S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \qquad S = \sum_i S_i$$

Carried by valence quarks

Hadron spin in QCD



❑ **Spin of a nucleon – QCD:**

- ✧ **Current quark mass << energy exchange of the collision**
- ✧ **Number of quarks and gluons depends on the probing energy**

❑ **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

❑ **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x\; M^{0jk}_{\rm QCD} \Longleftarrow M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

- ✧ **Quark angular momentum operator:**

Angular momentum density

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q\right] \qquad \longrightarrow \Delta q + L_q ?$$

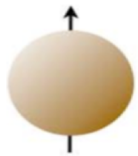
- ✧ **Gluon angular momentum operator:**

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B})\right] \qquad \longrightarrow \Delta g + L_g ?$$

Need to have the matrix elements of these partonic operators measured independently

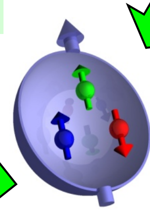
Proton spin – current status

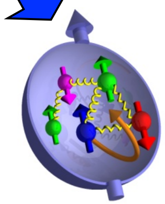
- How does QCD make up the nucleon's spin?



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g) = \sum \langle P,S|\hat{J}^z_f(\mu)|P,S\rangle$$

Proton Spin





Quark helicity
Best known

$$\frac{1}{2}\int dx\left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Spin "puzzle"

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

$$\sim 20\%(\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

If we do not understand proton spin, we do not understand QCD

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

- **Cross section:**

  Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ – Positive definite

- **Asymmetries or difference of cross sections:**

  – Not necessary positive!

  - both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - one beam polarized A_L, A_N

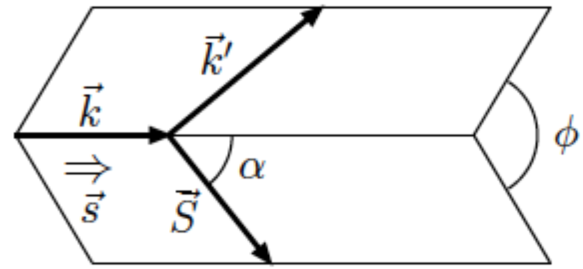
$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Polarized deep inelastic scattering

❑ **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) \; - \; \mathcal{W}^{\mu\nu}(P,q,-S)$$



✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$ **,**
and lepton helicity λ

✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \left\{ \cos\alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2 x^2 y^2}{Q^2} \right] g_1(x,Q^2) \; - \; \frac{2m^2x^2y}{Q^2}\, g_2(x,Q^2) \right\} \right.$$

$$\left. - \sin\alpha\cos\phi\, \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2x^2y^2}{Q^2}\right)} \left(\frac{y}{2}\, g_1(x,Q^2) \; + \; g_2(x,Q^2) \right) \right\}$$

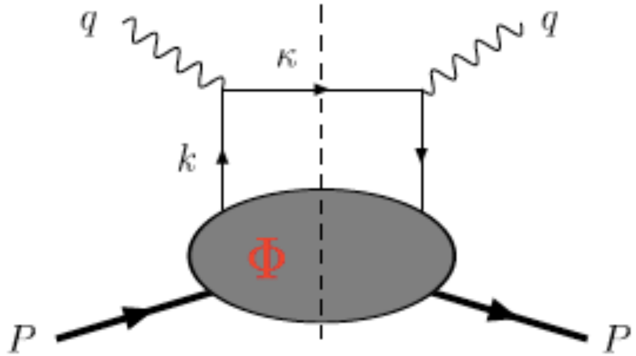
✧ **Spin orientation:**

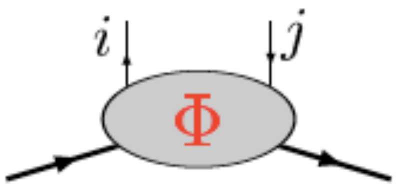
$$\alpha \;=\; 0 \;:\; \Rightarrow \quad g_1$$

$$\alpha \;=\; \pi/2 :\; \Rightarrow \quad y\, g_1 \; + \; 2\, g_2$$, suppressed m/Q

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:





✧ **Two-quark correlator:**

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4 \delta^4(P-k-P_X) \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\, e^{ik\cdot z} \langle PS|\, \bar{\psi}_j(0)\, \psi_i(z)\, |PS\rangle$$

✧ **Hadronic tensor (one –flavor):**

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4} \delta\left((k+q)^2\right) \text{Tr}\left[\Phi\, \gamma^\mu (\not{k}+\not{q})\gamma^\nu\right]$$

Polarized deep inelastic scattering

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of P, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\|}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$

✧ **3-leading power quark parton distribution:**

$$q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Basics for spin observables

- **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p, \vec{s}|\mathcal{O}(\psi, A^\mu)|p, \vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

- **Parity and Time-reversal invariance:**

$$\langle p, \vec{s}|\mathcal{O}(\psi, A^\mu)|p, \vec{s}\rangle = \langle p, -\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p, -\vec{s}\rangle$$

- **IF:** $\langle p, -\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p, -\vec{s}\rangle = \pm\langle p, -\vec{s}|\mathcal{O}(\psi, A^\mu)|p, -\vec{s}\rangle$

  or $\langle p, \vec{s}|\mathcal{O}(\psi, A^\mu)|p, \vec{s}\rangle = \pm\langle p, -\vec{s}|\mathcal{O}(\psi, A^\mu)|p, -\vec{s}\rangle$

Operators lead to the "+" sign $\Longrightarrow$ **spin-averaged cross sections**

Operators lead to the "-" sign $\Longrightarrow$ **spin asymmetries**

- **Example:**

$$\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

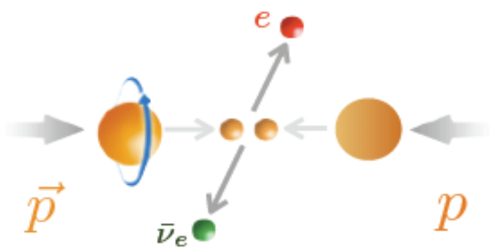
Quark helicity: $\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$

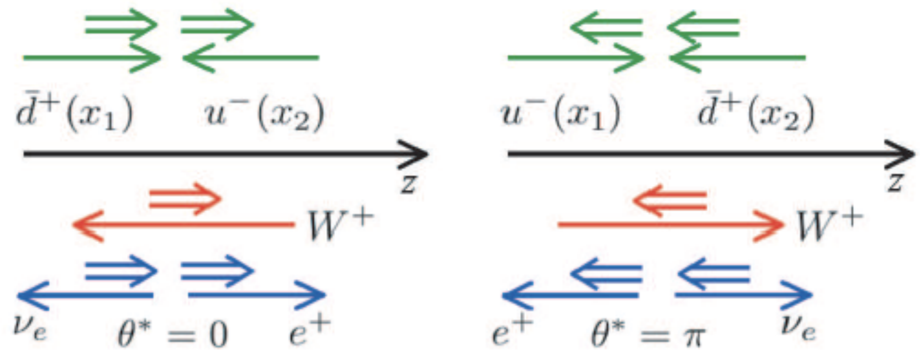
Transversity: $\mathcal{O}(\psi, A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \longrightarrow h(x)$

Gluon helicity: $\mathcal{O}(\psi, A^\mu) = \dfrac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$

Determination of Δq and $\Delta\bar{q}$

❑ W's are left-handed:





❑ Flavor separation:

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}$$

Forward W^+ (backward e^+):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W^+ (forward e^+):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

❑ Complications:

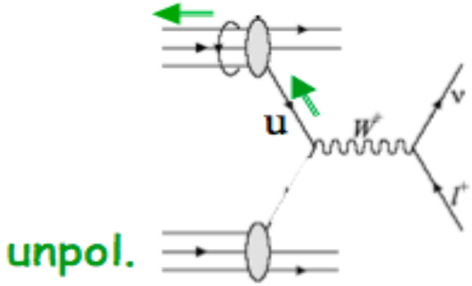
High order, W's p_T-distribution at low p_T

Sea quark polarization – RHIC W program

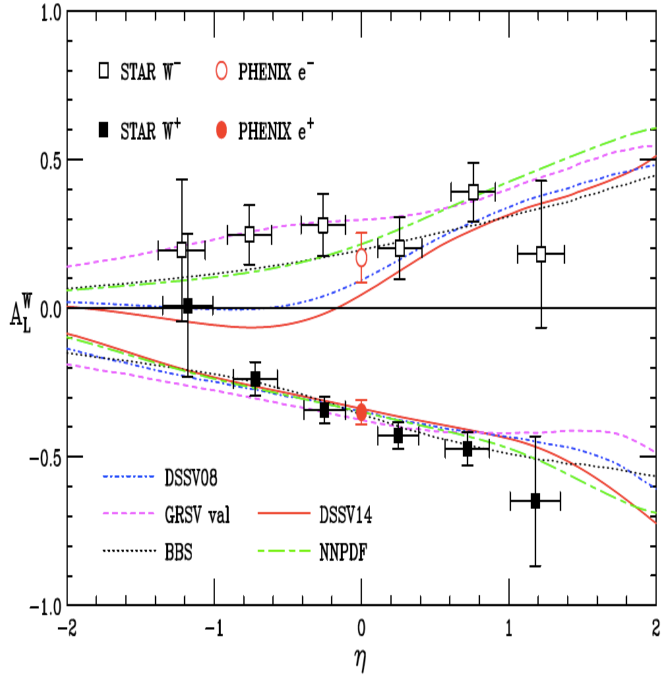
❑ Single longitudinal spin asymmetries:

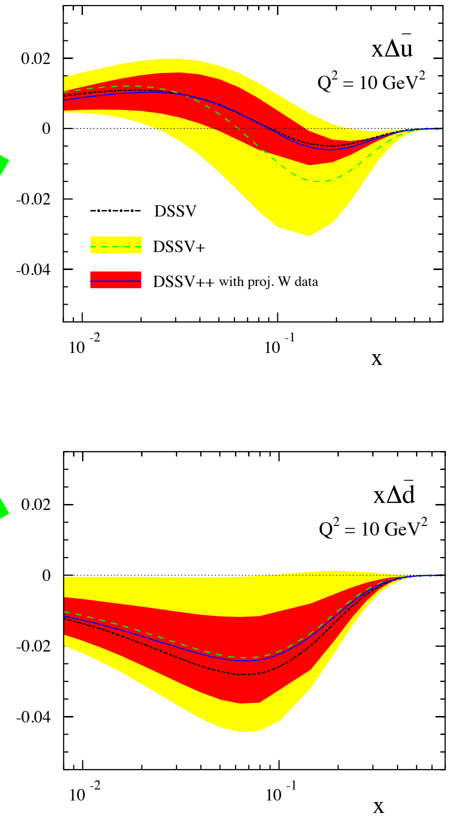
$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s)$$

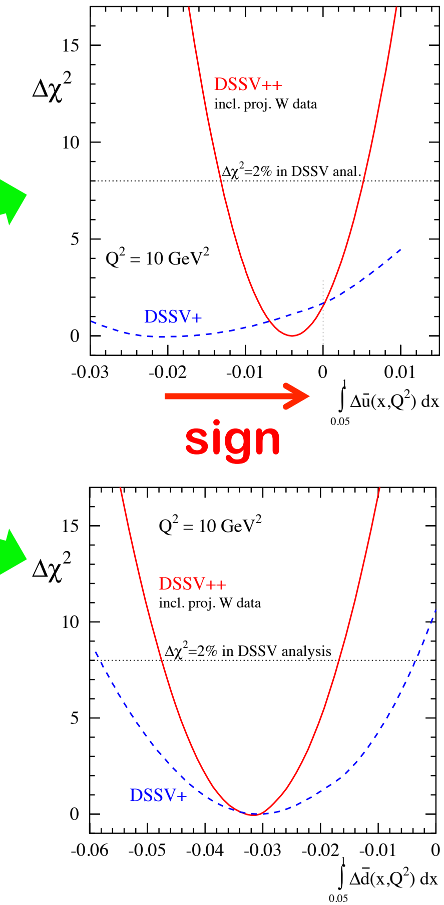
Parity violating weak interaction



❑ From 2013 RHIC data:

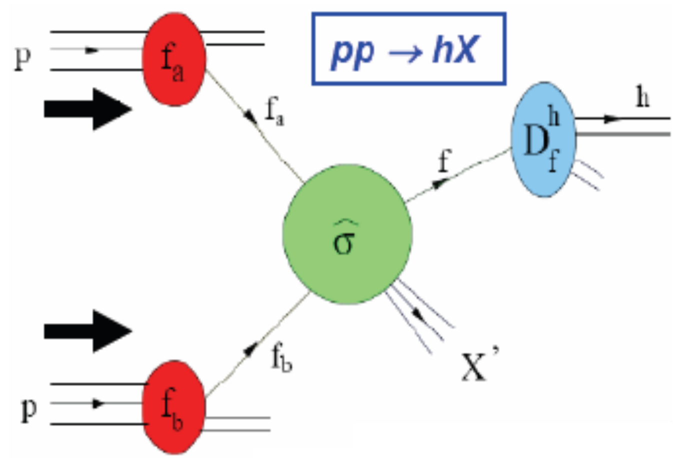


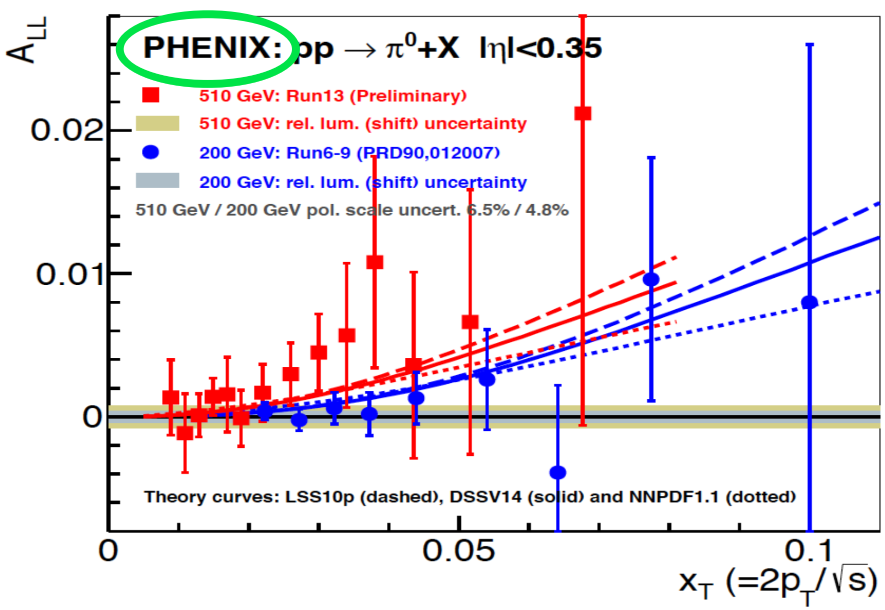




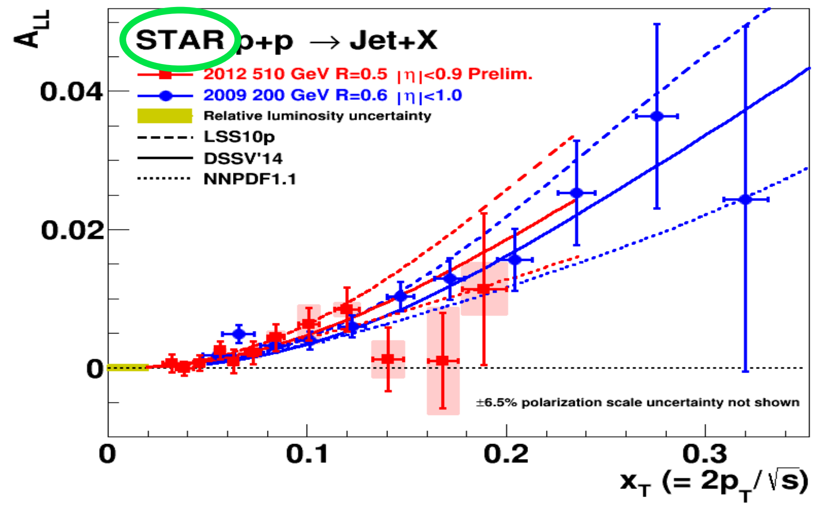
RHIC Measurements on ΔG

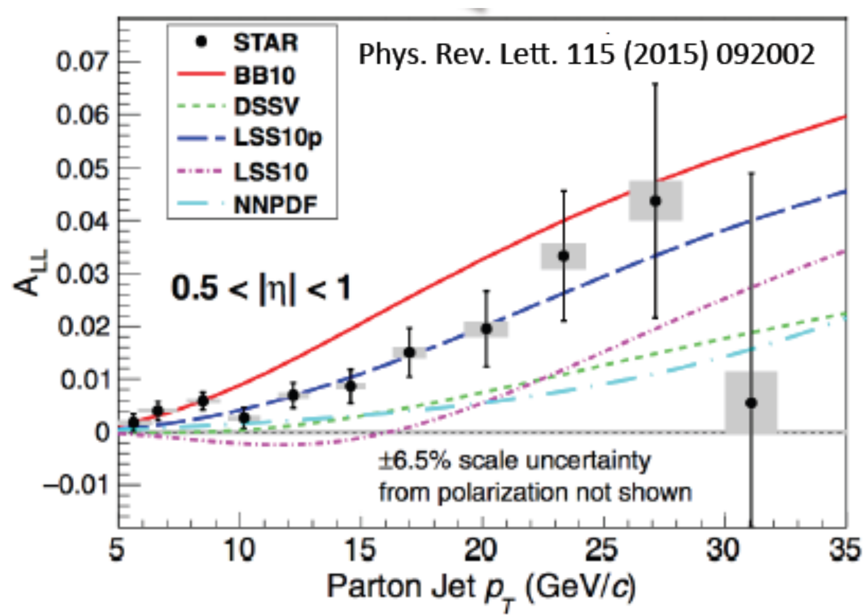
❑ PHENIX – π^0:





❑ STAR – jet:



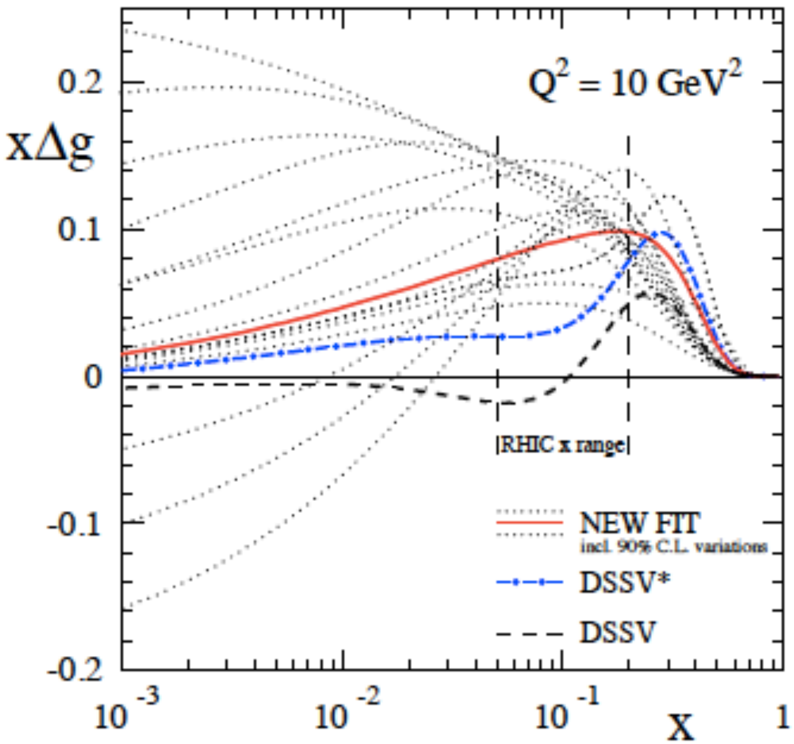


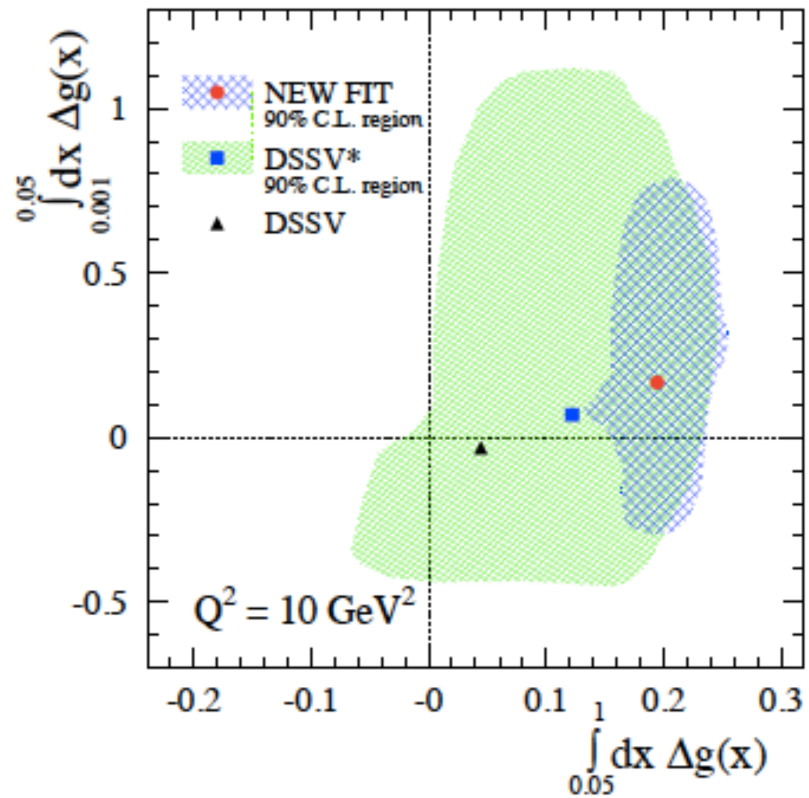
Global QCD analysis of helicity PDFs

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014) 012001

results featured in Sci. Am., Phys. World, ...

❑ Impact on gluon helicity:





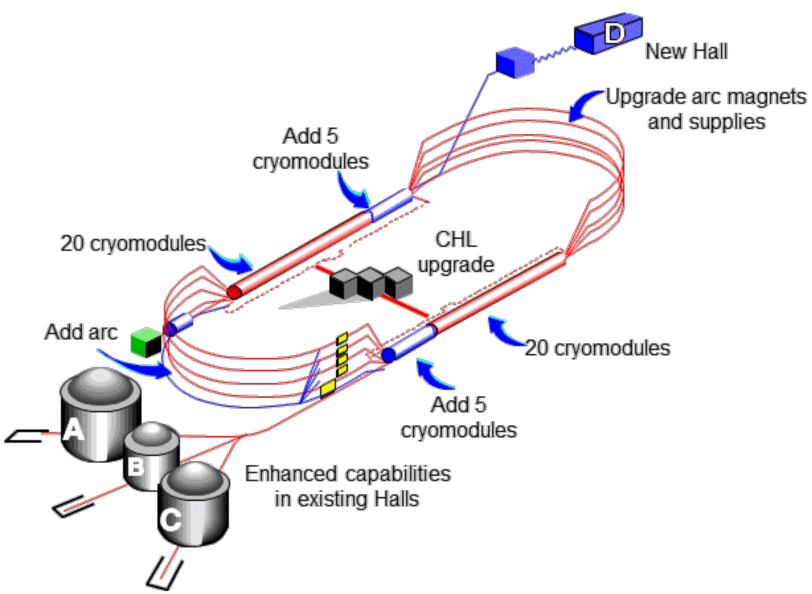
- ✧ Red line is the new fit
- ✧ Dotted lines = other fits with 90% C.L.

- ✧ 90% C.L. areas
- ✧ Leads ΔG to a positive #

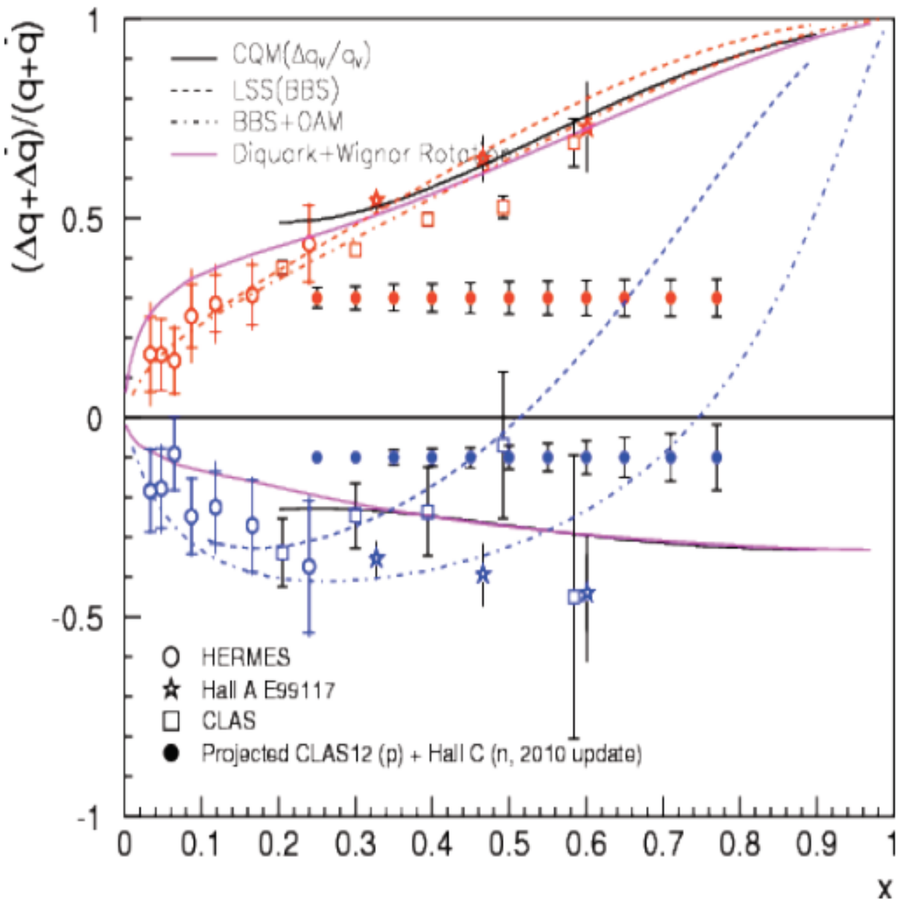
What is next?

- JLab 12GeV – upgrade project just completed:

12 GeV CEBAF Upgrade Project
is just complete, and
all 4-Halls are taking data



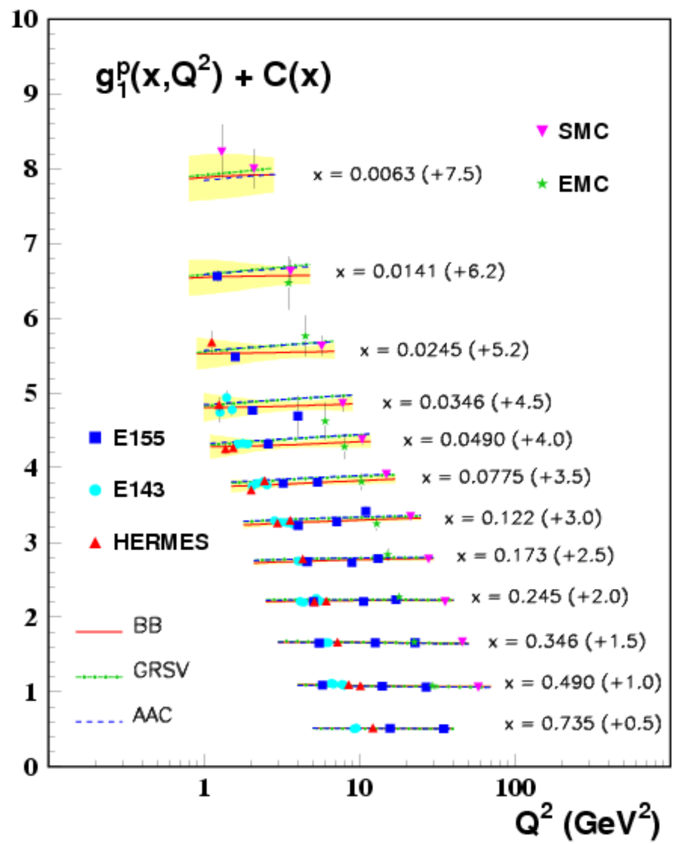
CLAS12



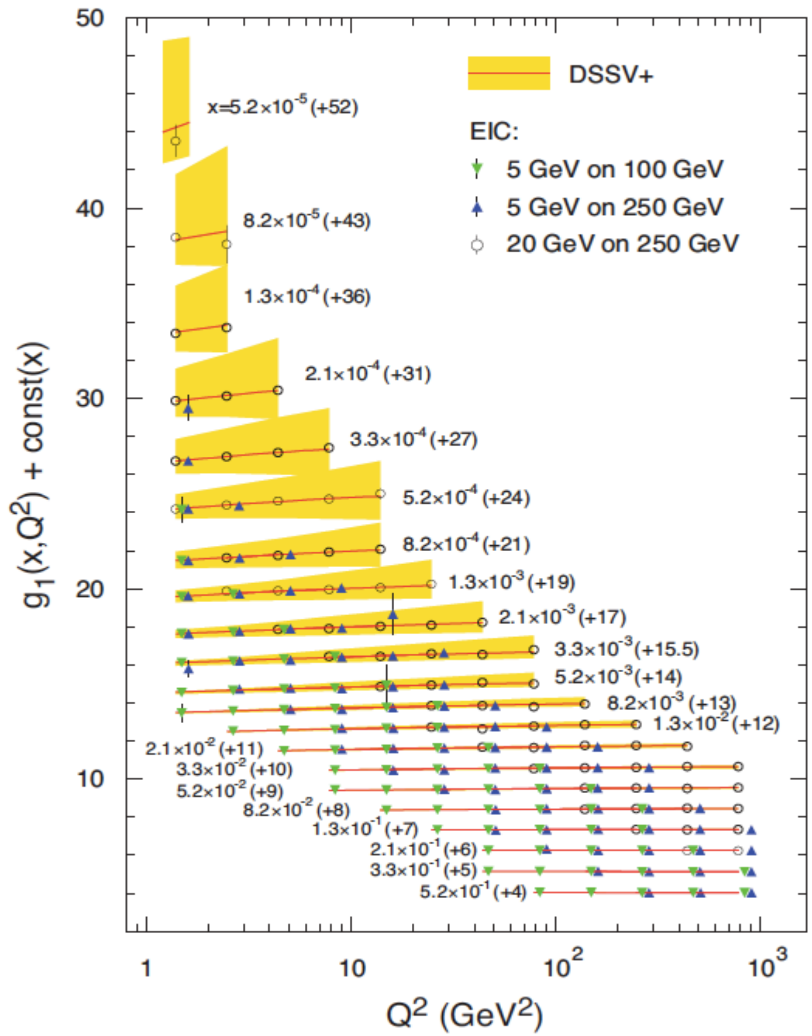
Plus many more JLab experiments,
COMPASS, Fermilab-fixed target expts
…

The Future: Challenges & opportunities

- The power & precision of EIC:



at EIC



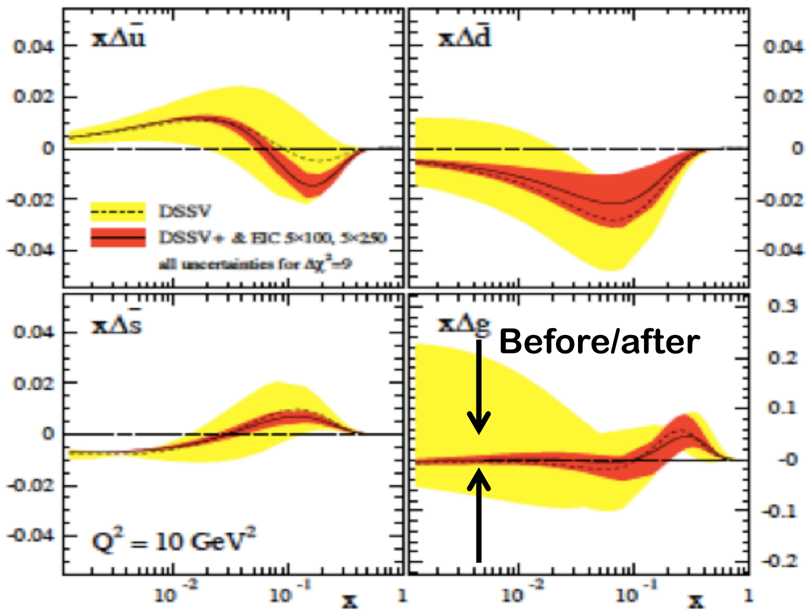
- Reach out the glue:

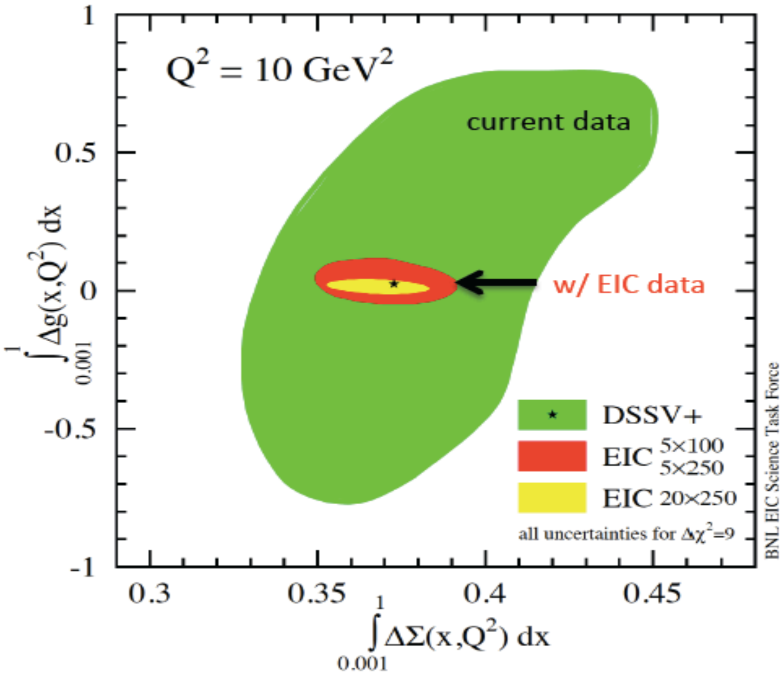
$$\frac{dg_1(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g(x,Q^2) + \cdots$$

The future – what the EIC can do?

❑ **One-year of running at EIC:**

Wider Q^2 and x range including low x at EIC!





No other machine in the world can achieve this!

❑ **Ultimate solution to the proton spin puzzle:**

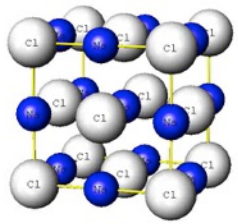
✧ ***Precision measurement of $\Delta g(x)$ – extend to smaller x regime***

✧ ***Orbital angular momentum contribution – measurement of TMDs & GPDs!***

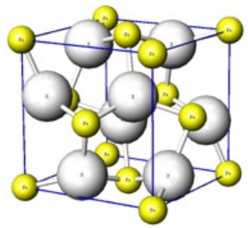
Hadron's partonic structure in QCD

- **Structure – "a still picture"**

Crystal Structure:

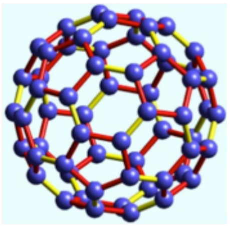


NaCl,
B1 type structure



FeS2,
C2, pyrite type structure

Nano-material:



Fullerene, C60

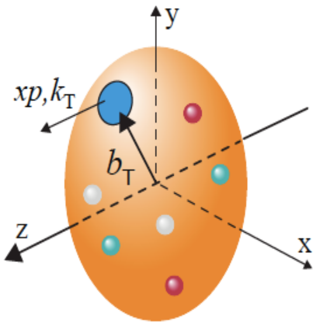
Motion of nuclei is much slower than the speed of light!

- **No "still picture" for hadron's partonic structure!**

Motion of quarks/gluons is relativistic!

Partonic Structure:

Quantum "probabilities" $\langle P, S | \mathcal{O}(\overline{\psi}, \psi, A^{\mu}) | P, S \rangle$



None of these matrix elements is a direct physical observable in QCD – color confinement!

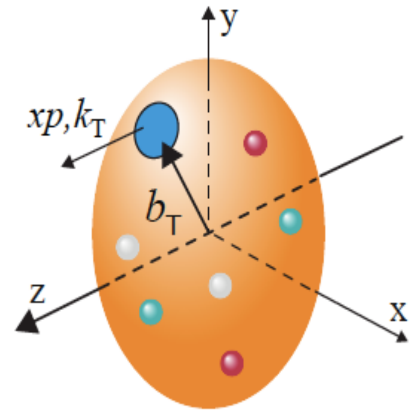
- **Accessible hadron's partonic structure?**

= Universal matrix elements of quarks and/or gluons
1) can be related to good physical cross sections of hadron(s) *with controllable approximation*,
2) can be calculated in lattice QCD, …

Paradigm shift: 3D confined motion

- **Cross sections with two-momentum scales observed:**

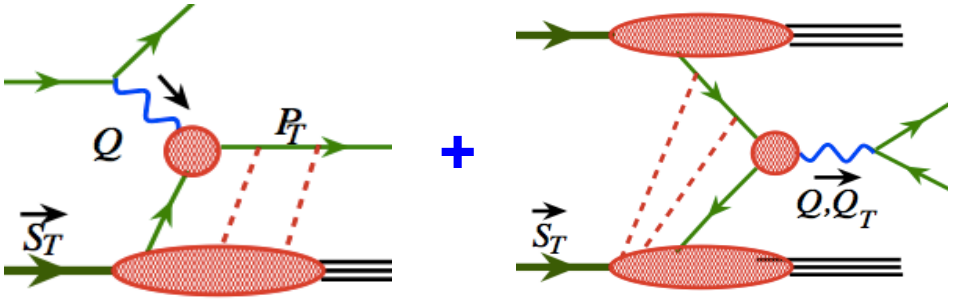
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$



✧ Hard scale: Q_1 localizes the probe to see the quark or gluon d.o.f.

✧ "Soft" scale: Q_2 could be more sensitive to hadron structure, e.g., confined motion

- **Two-scale observables with the hadron broken:**



Two-jet momentum imbalance in SIDIS, …

SIDIS: $Q \gg P_T$

DY: $Q \gg P_T$

➡

✧ Natural observables with TWO very different scales

✧ TMD factorization: partons' confined motion is encoded into TMDs

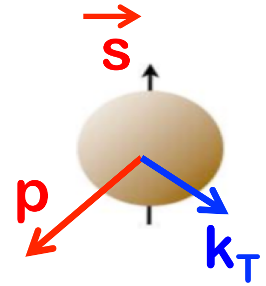
TMDs: confined motion, its spin correlation

- Power of spin – many more correlations:

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^{\perp} =$ − Boer-Mulders
	L		$g_{1L} =$ − Helicity	$h_{1L}^{\perp} =$ −
	T	$f_{1T}^{\perp} =$ − Sivers	$g_{1T}^{\perp} =$ −	$h_1 =$ − Transversity $h_{1T}^{\perp} =$ −

Nucleon Spin   Quark Spin

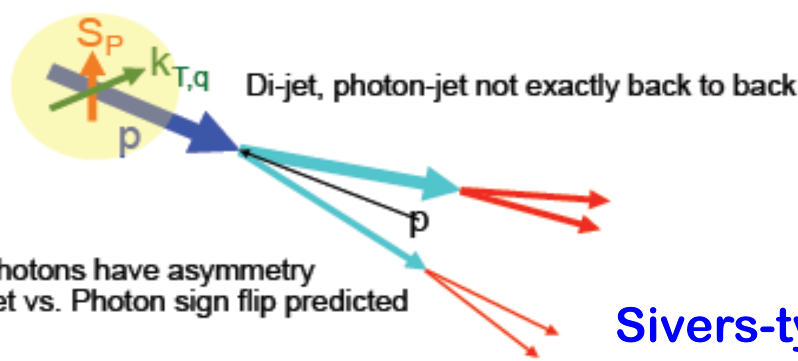
Similar for gluons



Require two Physical scales

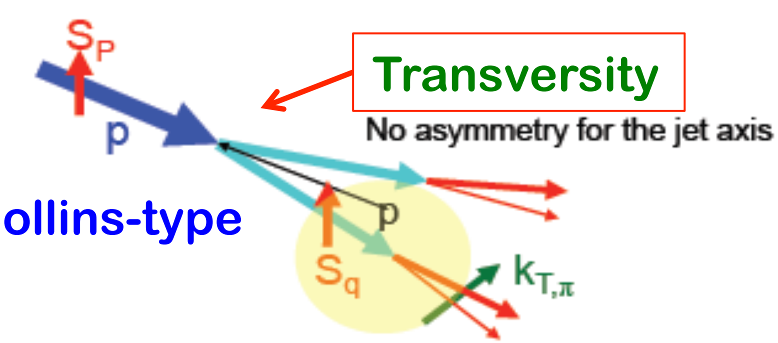
More than one TMD contribute to the same observable!

- A_N – single hadron production:



Sivers-type

Collins-type



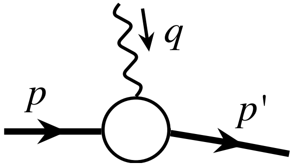
Proton's radius in color distribution?

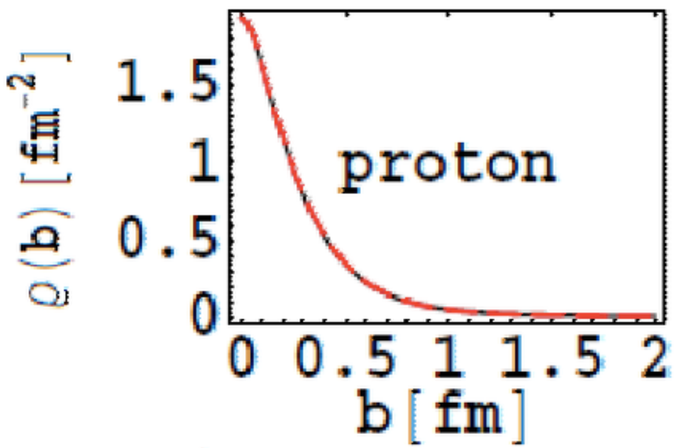
- The "big" question:

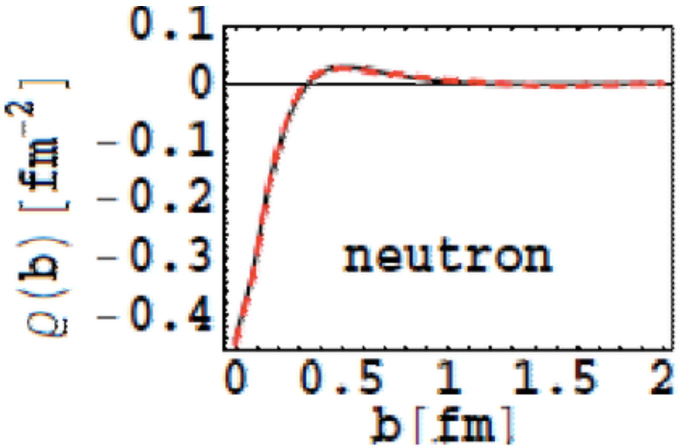
  How color is distributed inside a hadron? (clue for color confinement?)

- Electric charge distribution:

  Elastic electric form factor ➡ Charge distributions

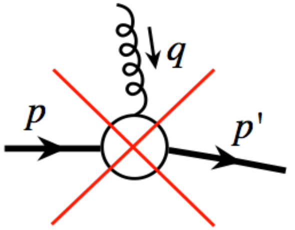






- But, NO color elastic nucleon form factor!

  Hadron is colorless and gluon carries color



➡ *Parton density's spatial distributions – a function of x as well (more "proton"-like than "neutron"-like?) – GPDs*

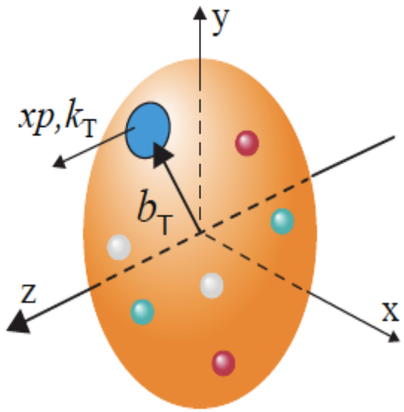
Paradigm shift: 2D spatial distributions

- **Cross sections with two-momentum scales observed:**

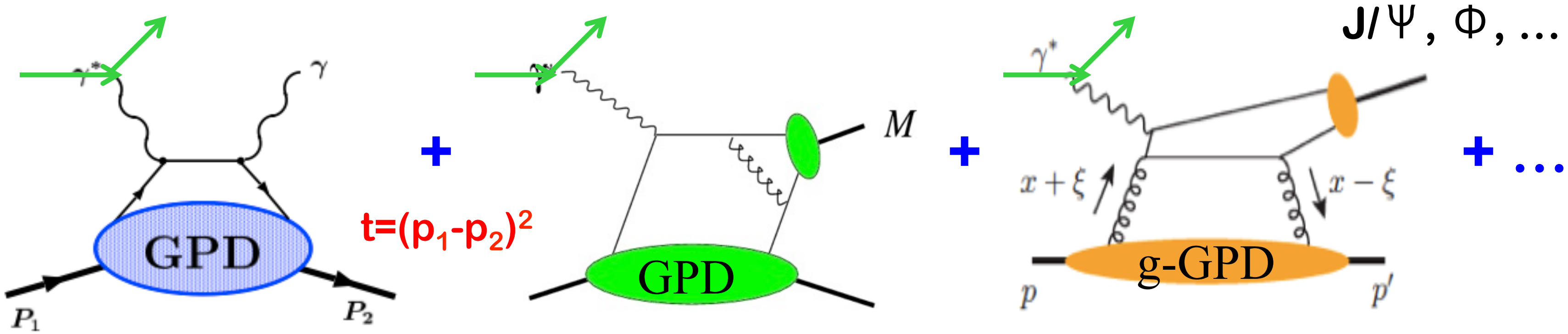
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

✧ **Hard scale:** Q_1 **localizes the probe to see the quark or gluon d.o.f.**

✧ **"Soft" scale:** Q_2 **could be more sensitive to hadron structure, e.g., confined motion**



- **Two-scale observables with the hadron unbroken:**



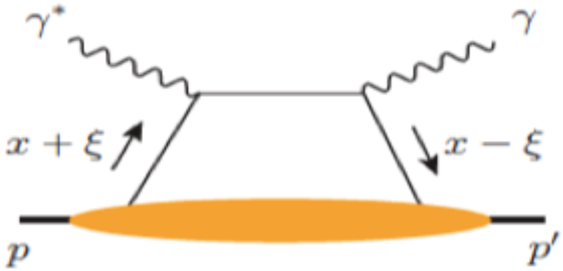
DVCS: $Q^2 >> |t|$ DVEM: $Q^2 >> |t|$ EHMP: $Q^2 >> |t|$

✧ **Natural observables with TWO very different scales**

✧ **GPDs: Fourier Transform of t-dependence gives spatial b_T-dependence**

Deep virtual Compton scattering

❑ **The LO diagram:**



$$\xi \;=\; Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

❑ **Scattering amplitude:**

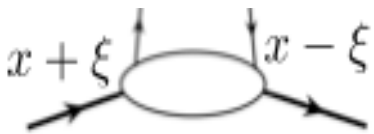
$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int dx\left(\frac{1}{x-\xi/2+i\epsilon}+\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[H(x,\Delta^2,\Delta\cdot n)\underline{\bar{U}(P')\not{n}U(P)} + E(x,\Delta^2,\Delta\cdot n)\underline{\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)}\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\int dx\left(\frac{1}{x-\xi/2+i\epsilon}-\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\underline{\bar{U}(P')\not{n}\gamma_5 U(P)} + \tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\underline{\bar{U}(P')\gamma_5 U(P)}\right]$$

❑ **GPDs:**

$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P)+\ldots$$
$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5 U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P)+\ldots$$

What can GPDs tell us?

- ❑ **GPDs of quarks and gluons:**



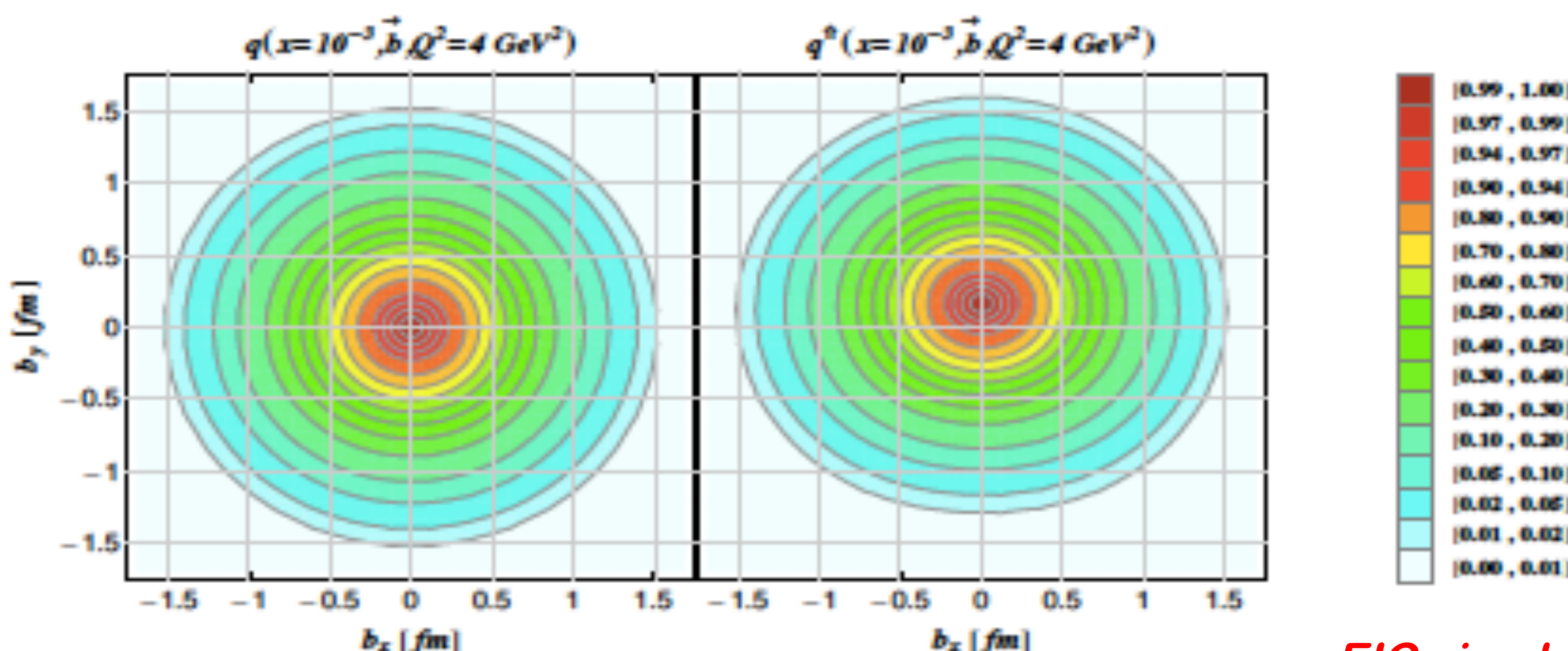
$$H_q(x,\xi,t,Q), \quad E_q(x,\xi,t,Q),$$
$$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$$

Evolution in Q

– gluon GPDs

- ❑ **Imaging ($\xi \to 0$):** $$q(x,b_\perp,Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x,\xi=0,t=-\Delta_\perp^2,Q)$$

- ❑ **Influence of transverse polarization – shift in density:**



EIC simulation

Advantages of the lepton-hadron facilities

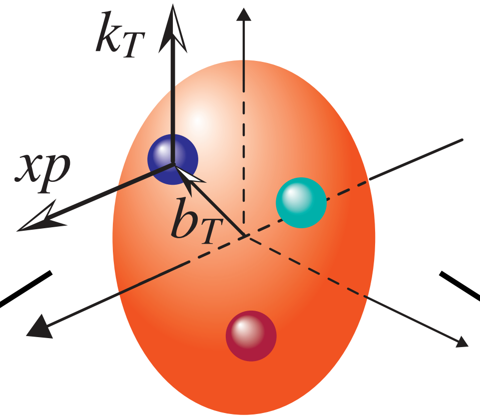
- **3D boosted partonic structure:**

Momentum Space

TMDs

Confined motion

$\int d^2 b_T$



$\int d^2 k_T$

Coordinate Space

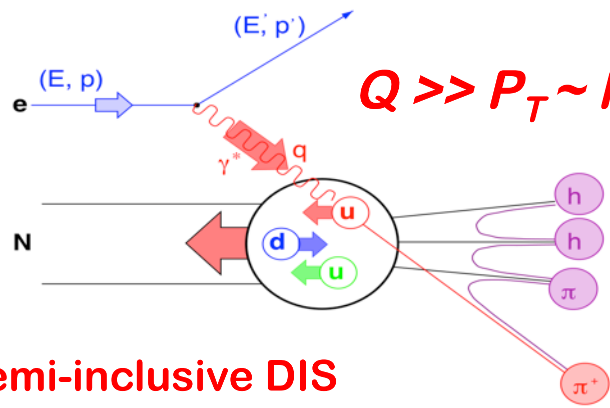
GPDs

Spatial distribution

$f(x,k_T)$ | Two-scales observables | $f(x,b_T)$

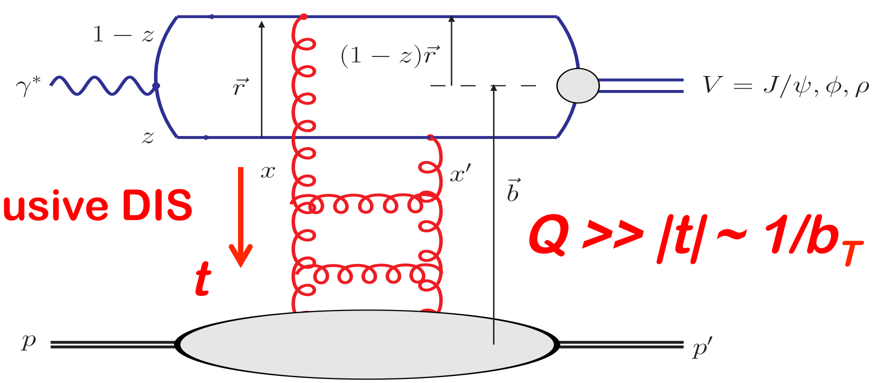
3D momentum-space images

2+1D coordinate-space images



$Q >> P_T \sim k_T$

Semi-inclusive DIS



Exclusive DIS

t

$Q >> |t| \sim 1/b_T$

JLab12 – valence quarks, EIC – sea quarks and gluons

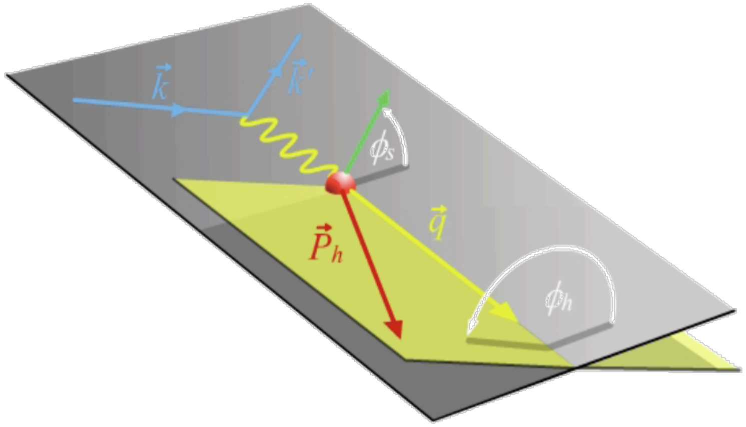
SIDIS is the best for probing TMDs

- Naturally, two scales & two planes:

$$A_{UT}(\varphi_h^l,\varphi_S^l)=\frac{1}{P}\frac{N^{\uparrow}-N^{\downarrow}}{N^{\uparrow}+N^{\downarrow}}$$

$$=A_{UT}^{Collins}\sin(\phi_h+\phi_S)+A_{UT}^{Sivers}\sin(\phi_h-\phi_S)$$

$$+A_{UT}^{Pretzelosity}\sin(3\phi_h-\phi_S)$$



- Separation of TMDs:

$$A_{UT}^{Collins}\propto\langle\sin(\phi_h+\phi_S)\rangle_{UT}\propto h_1\otimes H_1^{\perp}$$

$$A_{UT}^{Sivers}\propto\langle\sin(\phi_h-\phi_S)\rangle_{UT}\propto f_{1T}^{\perp}\otimes D_1$$

$$A_{UT}^{Pretzelosity}\propto\langle\sin(3\phi_h-\phi_S)\rangle_{UT}\propto h_{1T}^{\perp}\otimes H_1^{\perp}$$

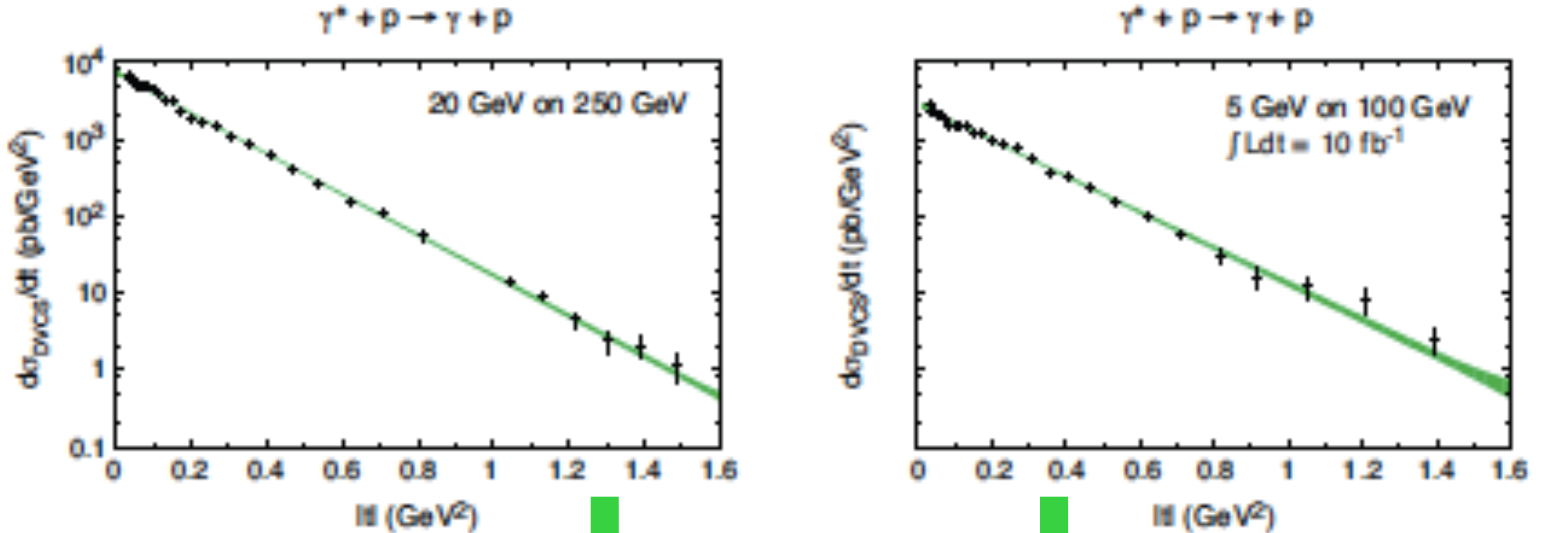
Collins frag. Func. from e^+e^- collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

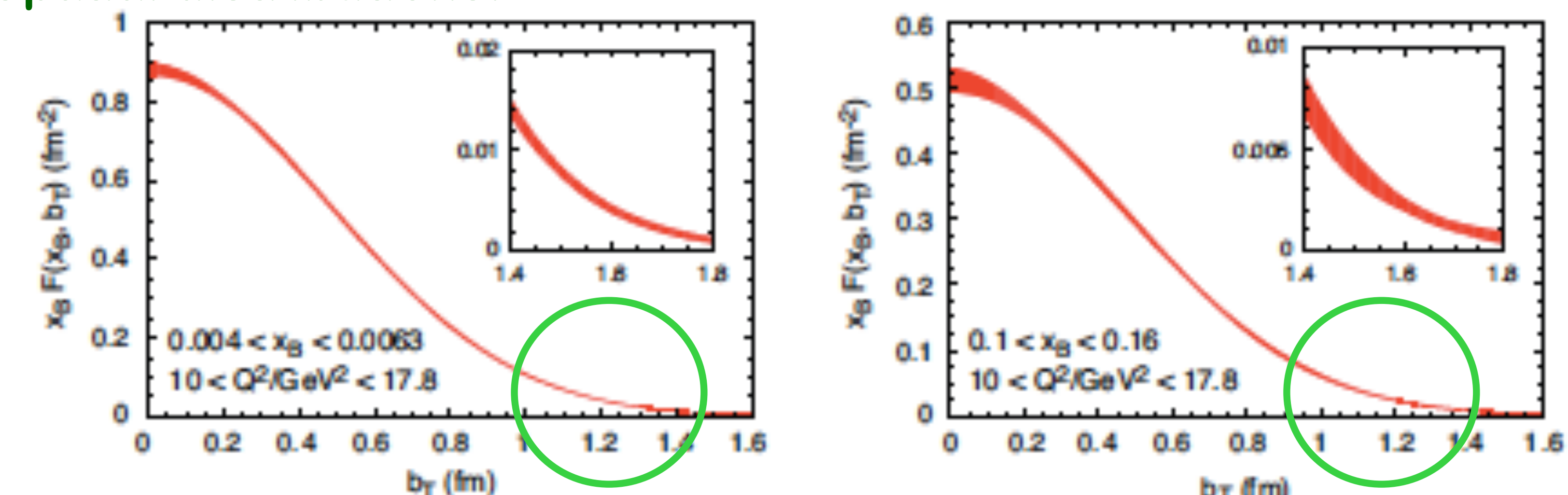
Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

DVCS @ EIC

- Cross Sections:



- Spatial distributions:

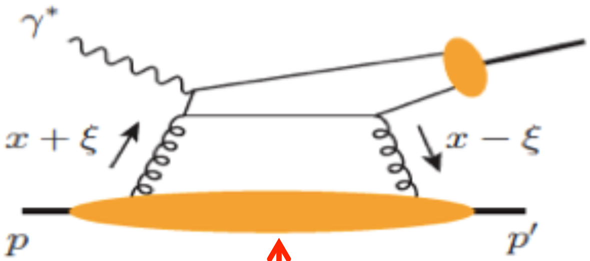


Quark radius (x)!

Spatial distribution of gluons

❑ **Exclusive vector meson production:**

$$\frac{d\sigma}{dx_B dQ^2 dt}$$ EIC-WhitePaper



J/Ψ, Φ, …

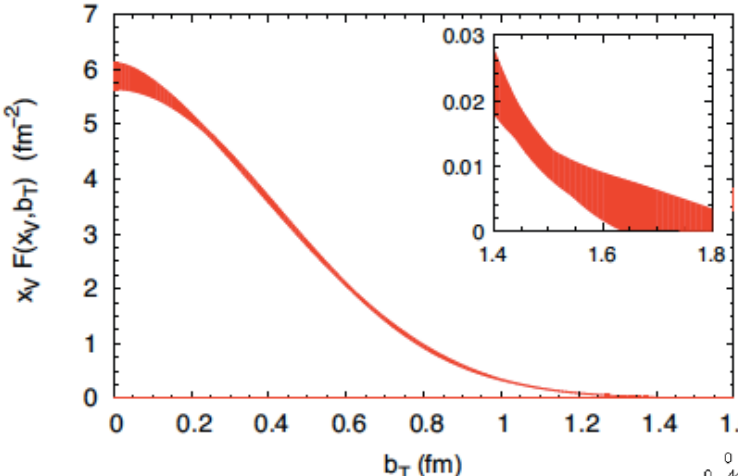
t-dep

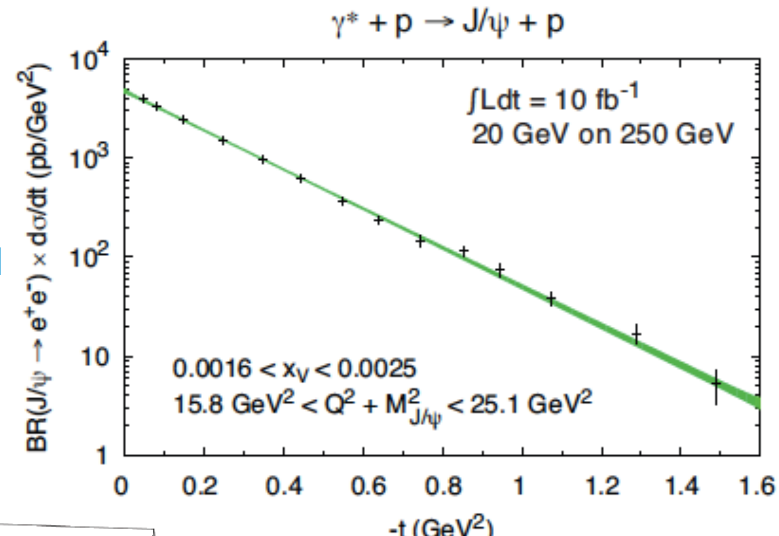
✧ Fourier transform of the t-dep

Spatial imaging of glue density

✧ Resolution ~ 1/Q or $1/M_Q$

❑ **Gluon imaging from simulation:**

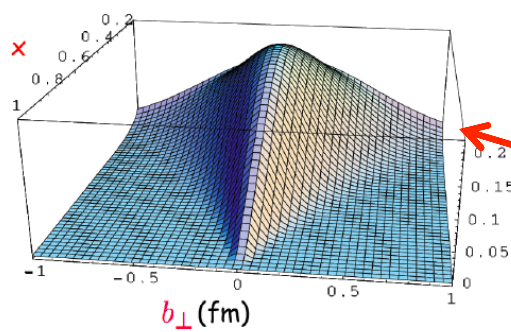




Only possible at the EIC

Gluon radius?

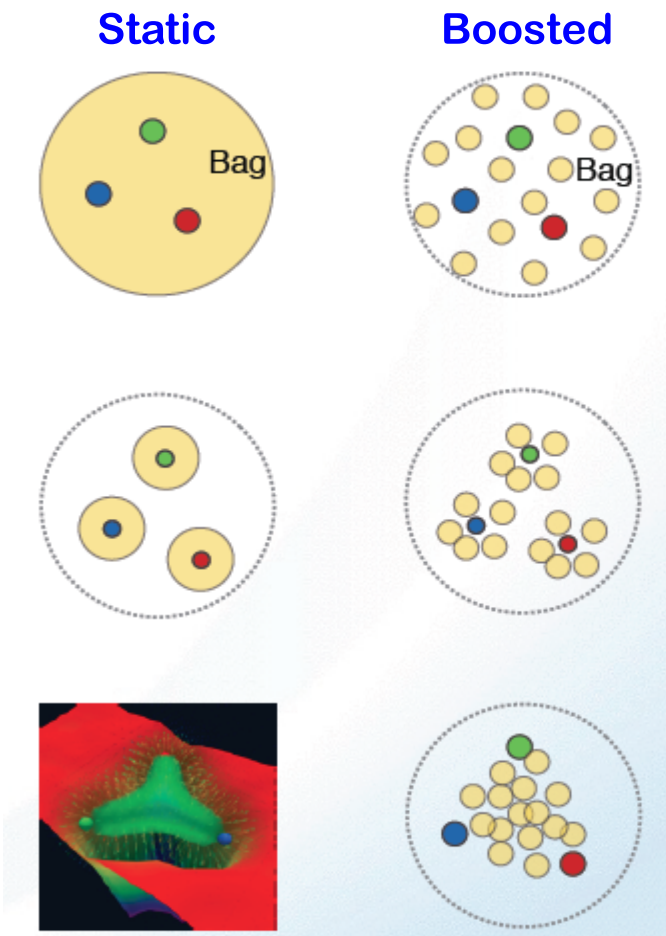
Gluon radius (x)!



How spread
at small-x?
Color confinement

Why 3D nucleon structure?

❑ **Spatial distributions of quarks and gluons:**



Bag Model:
Gluon field distribution is wider than the fast moving quarks.
Gluon radius > Charge Radius

Constituent Quark Model:
Gluons and sea quarks hide inside massive quarks.
Gluon radius ~ Charge Radius

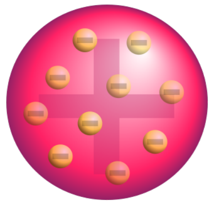
Lattice Gauge theory (with slow moving quarks):
Gluons more concentrated inside the quarks
Gluon radius < Charge Radius

3D confined motion (TMDs) + spatial distribution (GPDs)
Hints on the color confining mechanism
Relation between charge radius, quark radius (x), and gluon radius (x)?

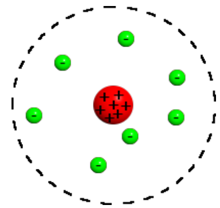
Why 3D hadron structure?

❑ Rutherford's experiment – atomic structure (100 years ago):

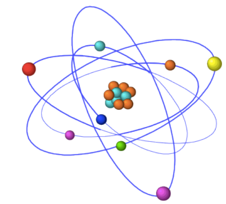
Atom:



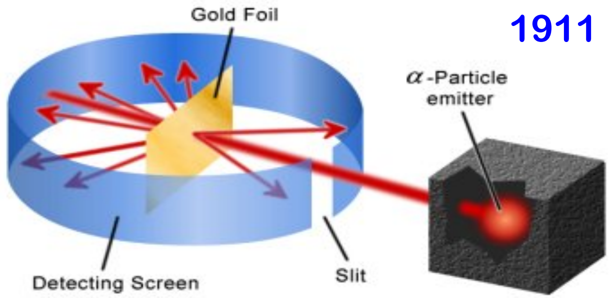
J.J. Thomson's
plum-pudding model



Rutherford's
planetary model



Modern model
Quantum orbitals



1911

Discovery of nucleus
A localized
charge/force center
A vast
"open" space

Discovery of
Quantum Mechanics,
and
the Quantum World!

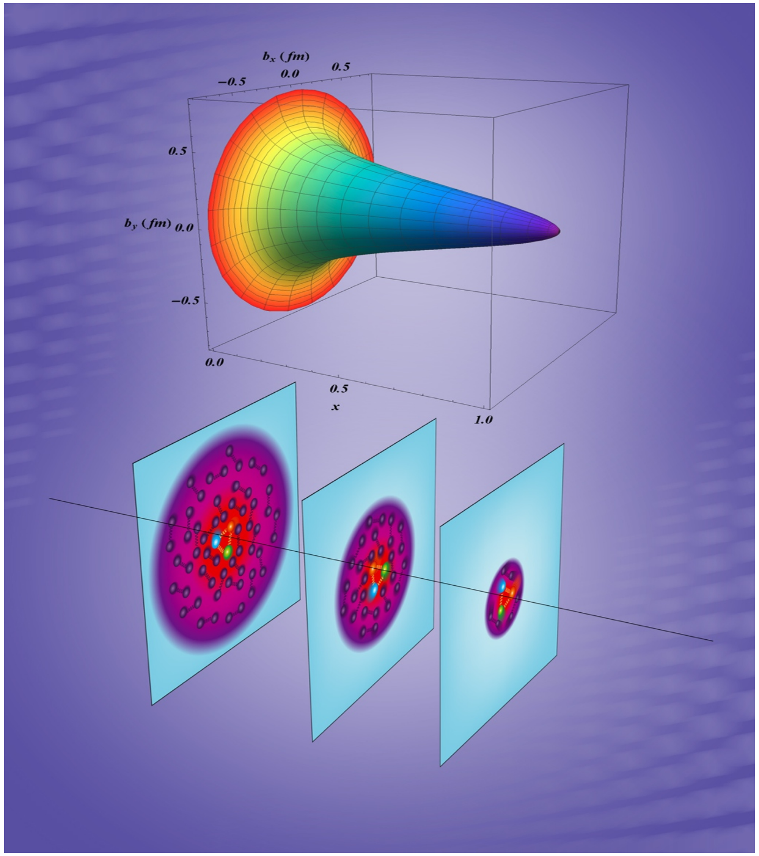
❑ Completely changed our "view" of the visible world:

- ✧ Mass by "tiny" nuclei – *less than 1 trillionth in volume of an atom*
- ✧ Motion by quantum probability – *the quantum world!*
- ✧ Provided infinite opportunities to improve things around us, …

What could we learn from the hadron structure in QCD, …?

Paradigm shift: 3D imaging of the "Proton"

❑ **This is transformational!**

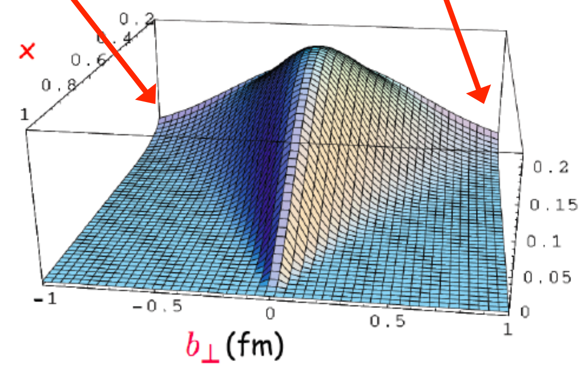


JLab12 – valence quarks,
EIC – sea quarks and gluons

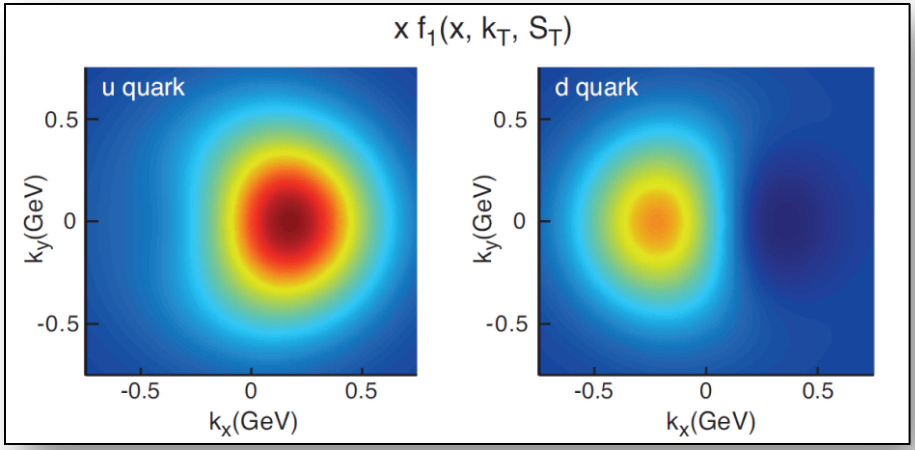
✧ **How color is confined?**

How far does glue density spread?

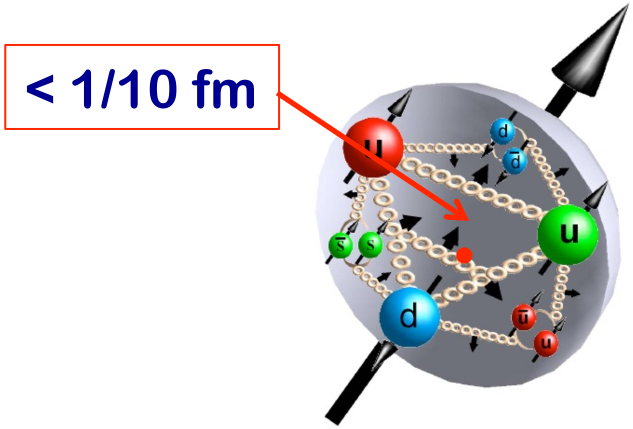
How fast does glue density fall?



✧ **Why there is preference in motion?**



Summary



- QCD has been extremely successful in interpreting and predicting high energy experimental data!

- But, we still do not know much about hadron structure – work just started!

- Cross sections with large momentum transfer(s) and identified hadron(s) are the source of structure information

- QCD factorization is necessary for any controllable "probe" for hadron's quark-gluon structure!

- TMDs and GPDs, accessible by high energy scattering with polarized beams at EIC, carry important information on hadron's 3D structure, and its correlation with hadron's spin!

No "still pictures", but quantum distributions, for hadron structure in QCD!

Thank you!

Backup slides

Mass vs. Spin

❑ Mass – intrinsic to a particle:

= Energy of the particle when it is at the rest

✧ QCD energy-momentum tensor in terms of quarks and gluons

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\,g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

✧ Proton mass:

X. Ji, PRL (1995)

$$m = \frac{\langle p|\int d^3x\, T^{00}\,|p\rangle}{\langle p|p\rangle} \sim \mathrm{GeV}$$

when proton is at rest!

❑ Spin – intrinsic to a particle:

= Angular momentum of the particle when it is at the rest

✧ QCD angular momentum density in terms of energy-momentum tensor

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad\qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

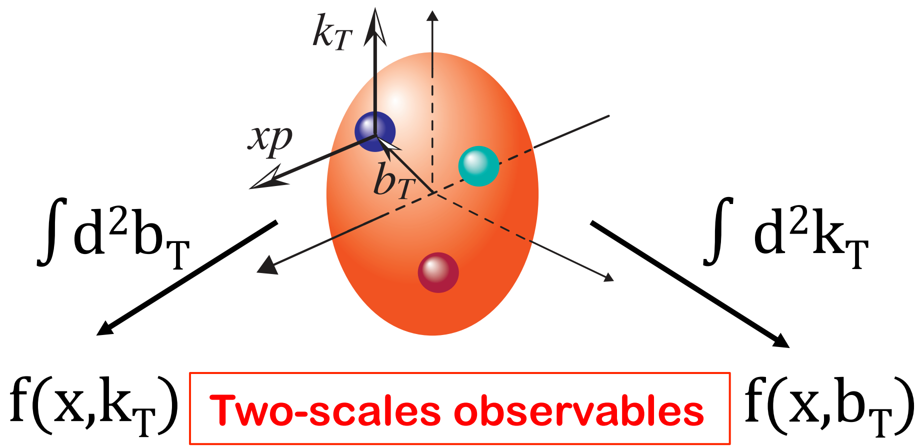
✧ Proton spin:

$$S(\mu) = \sum_f \langle P,S|\hat{J}^z_f(\mu)|P,S\rangle = \frac{1}{2}$$

Unified description of hadron structure

❑ Wigner distributions in 5D (or GTMDs):

Momentum Space	Coordinate Space
TMDs	*GPDs*
Confined motion	Spatial distribution



❑ Theory is solid – *TMDs & SIDIS as an example*:

✧ **Low P_{hT} ($P_{hT} << Q$) – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

✧ **High P_{hT} ($P_{hT} \sim Q$) – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

✧ **P_{hT} Integrated - Collinear factorization:**

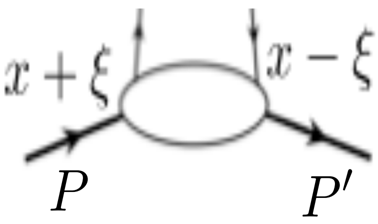
$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

✧ **Very high $P_{hT} >> Q$ – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \sum_{abc} \hat{H}_{ab\to c} \otimes \phi_{\gamma\to a} \otimes \phi_b \otimes D_{c\to h} + \mathcal{O}\left(\frac{1}{Q}, \frac{Q}{P_{h\perp}}\right)$$

Definition of GPDs

- Quark "form factor":



$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' | \bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2) | P \rangle$$

$$\equiv \; H_q(x,\xi,t,\mu^2) \left[\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P) \right] \frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2) \left[\bar{\mathcal{U}}(P') \frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M} \mathcal{U}(P) \right] \frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2$ and $t = (P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2$ if $\xi \to 0$

$\tilde{H}_q(x,\xi,t,Q)$, $\tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

- Total quark's orbital contribution to proton's spin: Ji, PRL78, 1997

$$J_q = \frac{1}{2} \lim_{t\to 0} \int dx\, x \left[H_q(x,\xi,t) + E_q(x,\xi,t) \right]$$

$$= \frac{1}{2}\Delta q + L_q$$

- Connection to normal quark distribution:

$$H_q(x,0,0,\mu^2) = q(x,\mu^2)$$ **The limit when** $\xi \to 0$

Orbital angular momentum

OAM: Correlation between parton's position and its motion
– in an averaged (or probability) sense

❑ **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

❑ **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

❑ **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ compensated by difference between gluon OAM density

✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\,\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\,\underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\,\psi(y)\,|P\rangle_{y^+=0}$$

JM: "staple" gauge link
Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a "torque" of color Lorentz force

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P' | \overline{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-)$$

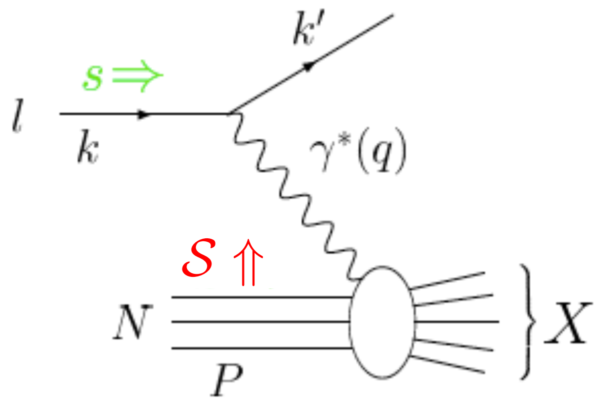
$$\times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{"Chromodynamic torque"}} \Phi(z^-, y)\psi(y) | P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Transverse single-spin asymmetry (TSSA)

- Over 50 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- DIS cross section: $\sigma(\vec{s}_\perp) \propto L^{\mu\nu} W_{\mu\nu}(\vec{s}_\perp)$
- Leptonic tensor is symmetric:

$$L^{\mu\nu} = L^{\nu\mu}$$

- Hadronic tensor:

$$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle$$

- Polarized cross section:

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right]$$

- Vanishing single spin asymmetry:

$$\boxed{A_N = 0 \quad \Leftrightarrow \quad \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle \overset{?}{=} \langle P, -\vec{s}_\perp | j^\dagger_\nu(0)\, j_\mu(y) | P, -\vec{s}_\perp \rangle}$$

A_N for inclusive DIS

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T\,|P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned}|\beta_T\rangle &= V_T\,\left[j^\dagger_\mu(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j^\dagger_\nu(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle\end{aligned}$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned}\longrightarrow\ &\langle -P, -\vec{s}_\perp|\,\left(V_T j^\dagger_\nu(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp|\, j^\dagger_\mu(0)\, j_\nu(y)\,|P, \vec{s}_\perp\rangle\end{aligned}}$$

A_N for inclusive DIS

- **Parity invariance:**

$$\boxed{1 = U_P^{-1} U_P = U_P^\dagger U_P}$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(-y)\, j_\mu(0) | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\boxed{\begin{aligned} &\langle P, -\vec{s}_\perp | j_\nu^\dagger(0)\, j_\mu(y) | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | j_\mu^\dagger(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle \end{aligned}}$$

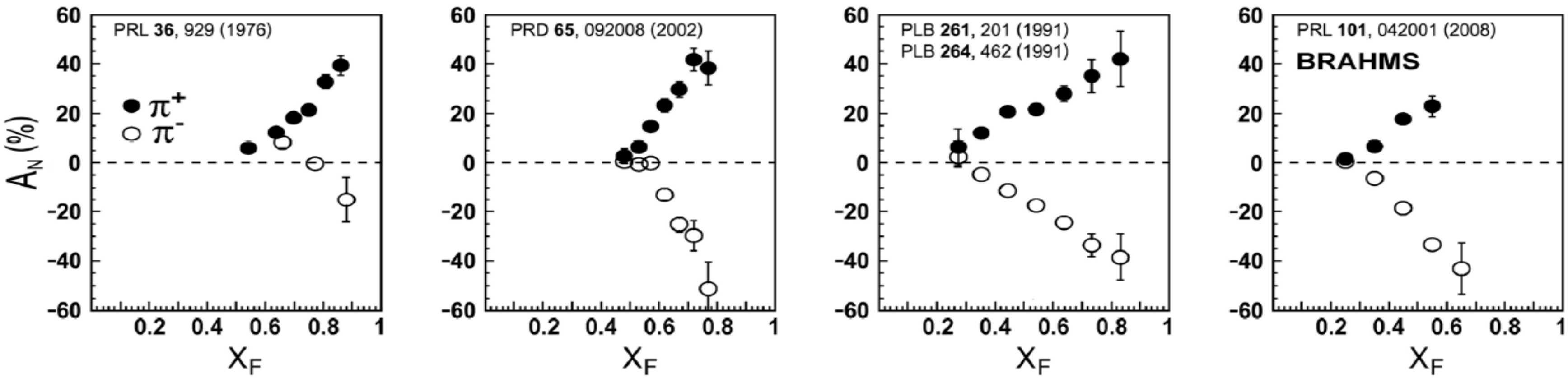
- **Polarized cross section:**

$$\boxed{\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp)\right] = 0 \end{aligned}}$$

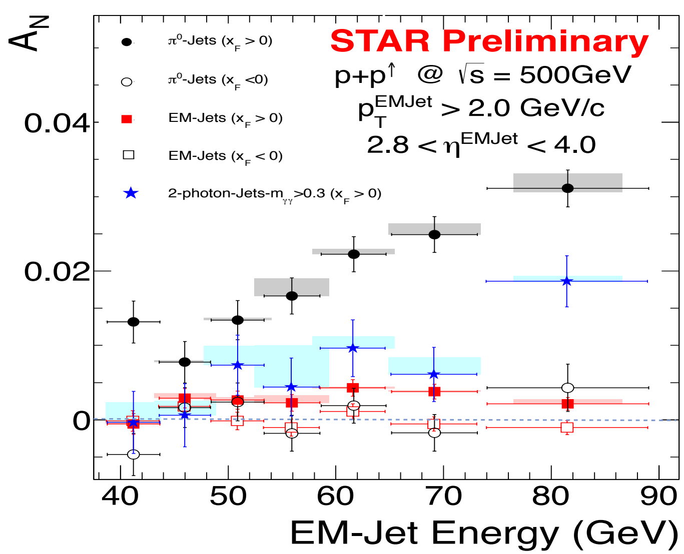
A_N in hadronic collisions

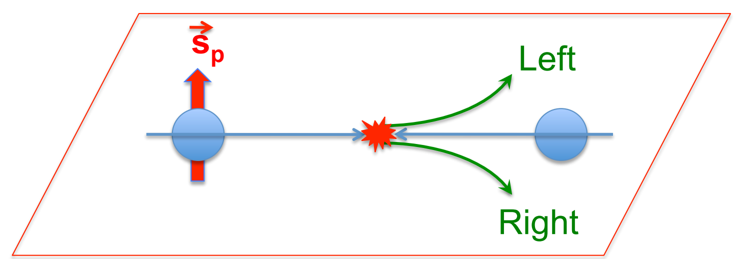
- A_N - consistently observed for over 35 years!

ANL – 4.9 GeV | BNL – 6.6 GeV | FNAL – 20 GeV | BNL – 62.4 GeV



- Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$$

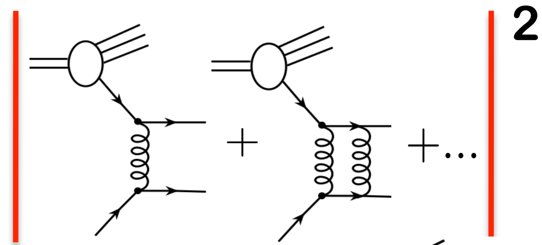
Do we understand this?

Do we understand it?

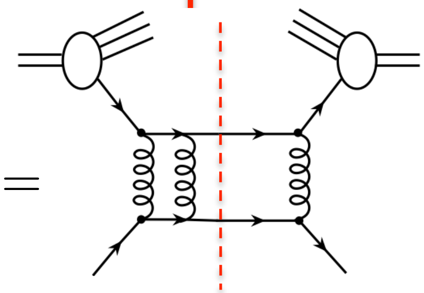
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) \; =$



$\propto \; \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \;\propto\; i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \;\Rightarrow\; i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

❑ Vanish without parton's transverse motion:

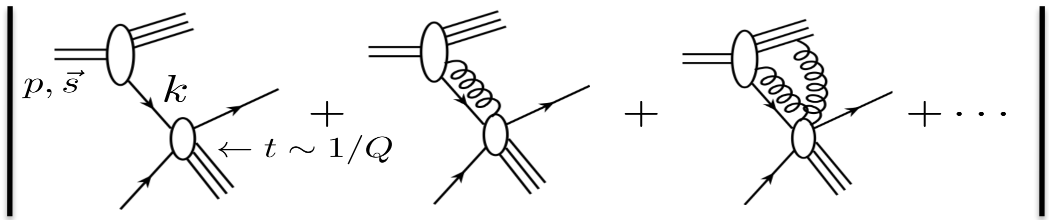
➡ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

How collinear factorization generates TSSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$



$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q,s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

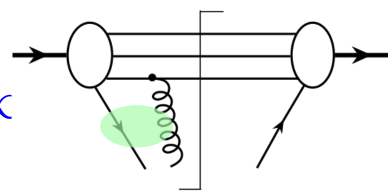
Too large to compete!

Three-parton correlation

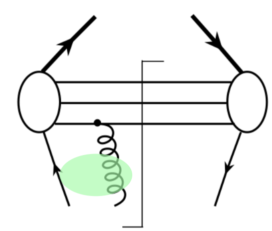
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$ 

Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

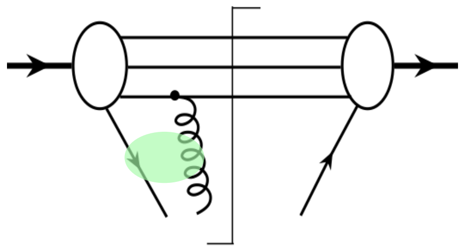
Needed Phase: Integration of "dx" using unpinched poles

“Interpretation” of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$$T^{(3)}(x, x, S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

❑ “Expectation value” of QCD operators:

$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+ \left[\epsilon_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- \, F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

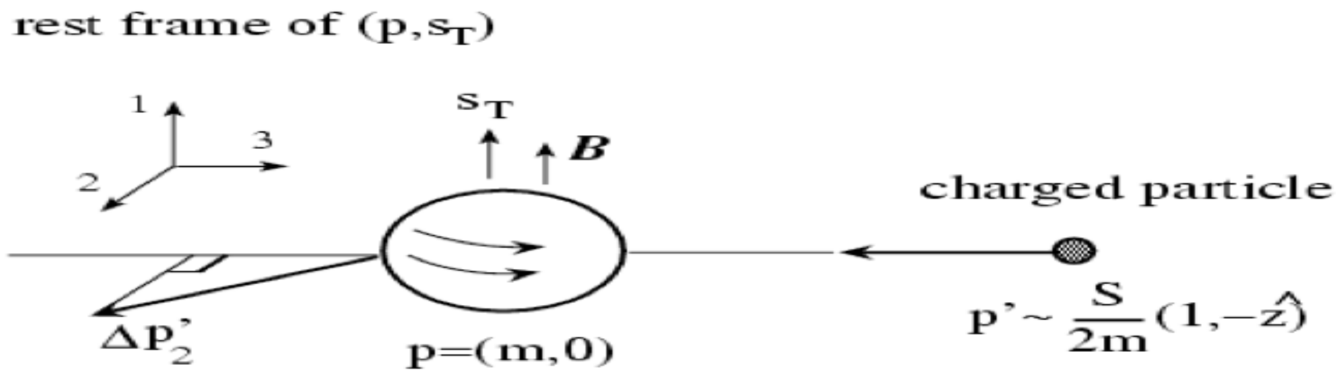
$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \;\longrightarrow\; \langle P, s|\overline{\psi}(0)\gamma^+ \left[i\, g_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- \, F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

How to interpret the “expectation value” of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change: $\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Transversity distributions

- **Transversity:** $\delta q(x)$ or $h_1(x)$

Jaffe and Ji, 1991

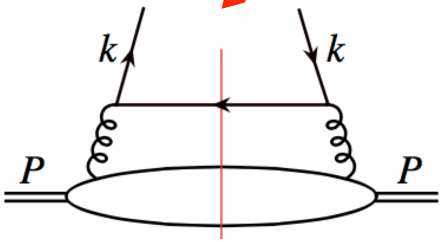
$$h_1(x) = \frac{1}{\sqrt{2}p^+}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\perp|\psi_+^\dagger(0)\gamma_\perp\gamma_5\psi_+(\lambda n)|PS_\perp\rangle \text{ + UVCT}$$

with $\psi_\pm = P_\pm\psi$ and $P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$

- **Unique for the quarks:**

No mixing with gluons!

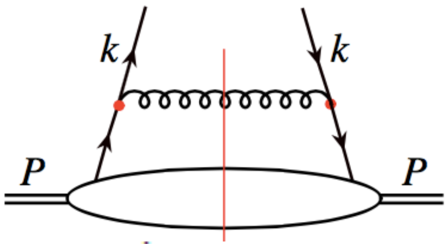
$\boxed{\gamma\cdot n\gamma_\perp\gamma_5}$ Even # of γ's



= 0

No mixing with PDFs, helicity distributions

- **Perturbatively UV and CO divergent:**



+ wave function renormalization

$$\Delta_T P_{qq}^{(0)}(x) = C_F\left[\frac{2x}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

→ "DGLAP" evolution kernels

NLO - Vogelsang, 1998

Soffer's inequality

- Relation between quark distributions:

$$h_1(x) \leq \frac{1}{2}\left[q(x) + \Delta q(x)\right] = q^+(x)$$

Derived by using the positivity constraint of
quark + nucleon -> quark + nucleon
forward scattering helicity amplitudes

Cautions:

- ✧ Quark field of the Transversity distribution is NOT on-shell
- ✧ Quark + nucleon -> quark + nucleon
  forward scattering amplitude is perturbatively divergent

- Testing vs using as a constraint:

It is important to test this inequality, rather than using it as a constraint for fitting the transversity

Perturbatively calculated evolution kernels seem to be consistent with the inequality – the scale dependence