Few-Body Physics with Relation to Neutrinos

Saori Pastore
HUGS Summer School
Jefferson Lab - Newport News VA, June 2018

Thanks to the Organizers

Neutrinos (Fundamental Symmetries) and Nuclei

Topics (5 hours)

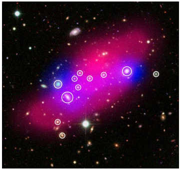
* Nuclear Theory for the Neutrino Experimental Program
* Microscopic (or *ab initio*) Description of Nuclei
* "Realistic" Models of Two- and Three-Nucleon Interactions
* "Realistic" Models of Many-Body Nuclear Electroweak Currents
* Short-range Structure of Nuclei and Nuclear Correlations
* Quasi-Elastic Electron and Neutrino Scattering off Nuclei
* Validation of the theory against available data

Jefferson Lab

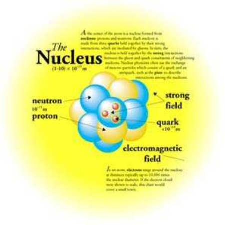
Nuclear Physics for the Experimental Neutrino Program

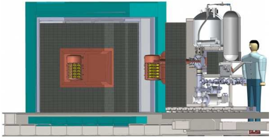
Understand Nuclei to Understand the Cosmos

Jefferson Lab

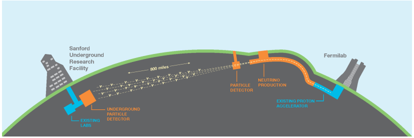


ESA, XMM-Newton, Gastaldello, CFHTL





Majorana Demonstrator



LBNF

(Some) Neutrino's Facts

1930 Pauli postulates the existence of an undetected particle to preserve energy/momentum conservation in β-decay

Wolfgang Pauli

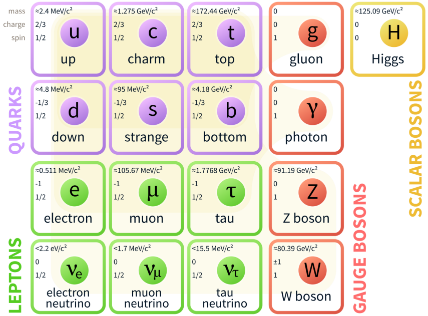
Enrico Fermi

1934 Fermi develops the theory for beta-decay and names the new particle "neutrino"

1956 Neutrinos are detected by Reines and Cowan at Savannah River!

Fred Reines Clyde Cowan

The Standard Model



Wikipedia

Neutrinos *i*) are chargeless elementary particles; *ii*) come in 3 flavors ν_e, ν_μ, and ν_τ; *iii*) only interact via the weak interaction (10^{-4} EM and 10^{-9} Strong)

the Sun is a huge source of ν's on Earth, every sec $\sim 10^{11}$ solar ν's cross 1 cm^2

The Standard Model says neutrinos are massless... to be continued

A Happy Ending Neutrino Tale

1968 Solar Neutrino Problem:
only 1/3 of the solar ν_e neutrinos predicted by the Standard Solar Model of Bahcall is observed by Davis

Ray Davis and John Bahcall, 1964

Bruno Pontecorvo

1968 Pontecorvo's idea:
neutrinos oscillate between flavors, *e.g.*, electron neutrinos change into muon neutrinos

since the '80 Underground atmospheric neutrino experiments demonstrated that neutrinos oscillate. Measurements of solar neutrinos of all flavors are in excellent agreement with the Standard Solar Model prediction! Go Bahcall!

Takaaki Kajita and Art McDonald

* 2016 APS April meeting talks by Kajita and McDonald https://meetings.aps.org/Meeting/APR16/Session/Q1
plus a book on neutrino's history "Neutrino" by Frank Close 2010 Oxford University Press

Fundamental Physics Quests I: Neutrino Oscillation

neutrinos oscillate

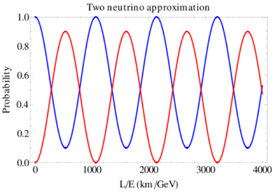
$\rightarrow$

they have tiny masses

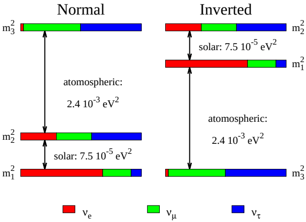
=

BSM physics

Beyond the Standard Model



Wikipedia



JUNO coll. - J.Phys.G**43**(2016)030401

$$P(\nu_\mu \rightarrow \nu_e) = sin^2 2\theta \, sin^2 \left(\frac{(m_2^2 - m_1^2)\, L}{2\, E_\nu} \right)$$

Simplified 2 flavors picture:

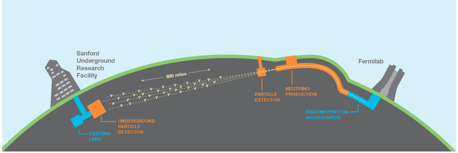
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} cos\,\theta & sin\,\theta \\ -sin\,\theta & cos\,\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

with $|\nu_1\rangle$ and $|\nu_2\rangle$ mass-eigenstates

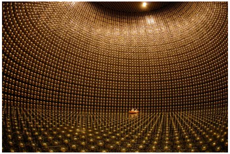
* Unknown *

ν-mass hierarchy, CP-violation, accurate mixing angles, Majorana vs Dirac ν

Nuclei for Accelerator Neutrinos' Experiments

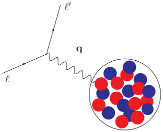


LBNF



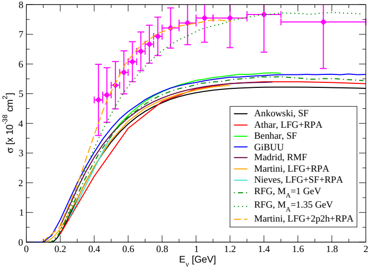
T2K

Neutrino-Nucleus scattering



$$P(\nu_\mu \to \nu_e) = sin^2 2\theta sin^2\left(\frac{\Delta m^2_{21} L}{2E_\nu}\right)$$

CCQE on ^{12}C

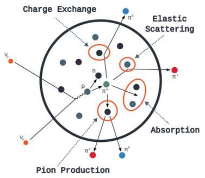


Alvarez-Ruso arXiv:1012.3871

* Nuclei of ^{12}C, ^{40}Ar, ^{16}O, ^{56}Fe, ... *

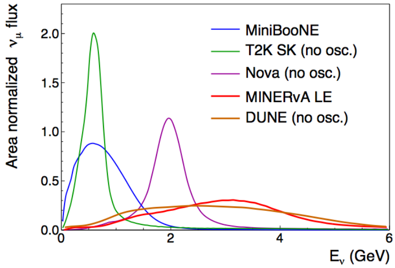
are the DUNE, MiniBoone, T2K, Minerνa ... detectors' active material

Nuclei for Accelerator Neutrinos' Experiments: More in Detail



Tomasz Golan

Neutrino Flux



Phil Rodrigues

* Oscillation Probabilities depend on the initial neutrino energy E_ν

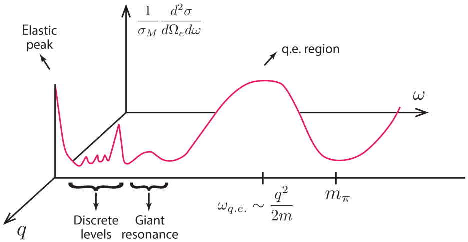
* Neutrinos are produced via decay-processes, E_ν is unknown!

$$P(\nu_\mu \to \nu_e) = sin^2 2\theta sin^2 \left(\frac{\Delta m^2_{21} L}{2E_\nu} \right)$$

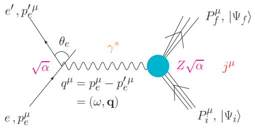
* E_ν is reconstructed from the final state observed in the detector

* !! Accurate theoretical neutrino-nucleus cross sections are vital !!
to E_ν reconstruction

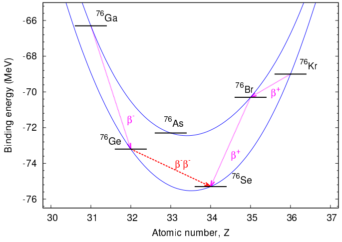
Nuclei for Accelerator Neutrinos' Experiments: Kinematics



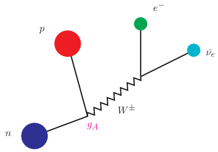
* probe's spatial resolution $\propto 1/|\mathbf{q}|$
* $\omega \sim$ few MeV, $q \sim 0$: EM decay, β-decay, $\beta\beta$-decays
* $\omega \lesssim$ tens MeV: Nuclear Rates for Astrophysics
$\Rightarrow$ $\omega \sim 10^2$ MeV: Accelerator neutrinos, ν-nucleus scattering $\Leftarrow$



Standard Single and Double Beta Decays



J. Menéndez - arXiv:1703.08921v1

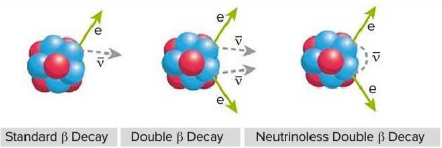


Maria Geoppert-Mayer

$$\text{single beta decay: } (Z,N) \to (Z+1,N-1) + e + \bar{\nu}_e$$

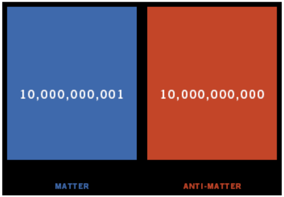
$$\text{double beta decay: } (Z,N) \to (Z+2,N-2) + 2e + 2\bar{\nu}_e$$

$$\text{lepton \# } L = l - \bar{l} \text{ is conserved}$$

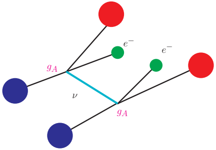


2015 Long Range Plane for Nuclear Physics

Fundamental Physics Quests II: Neutrinoless Double Beta Decay



H. Murayama

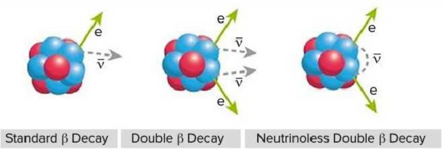


Ettore Majorana

$0\nu\beta\beta$ neutrinoless double beta decay

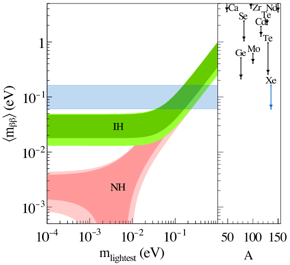
$$(Z,N) \to (Z+2,N-2)+2e$$

lepton # $L = l - \bar{l}$ is not conserved

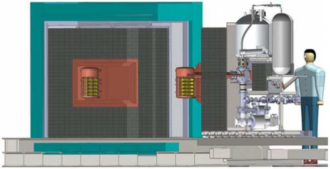


2015 Long Range Plane for Nuclear Physics

Nuclear Physics for Neutrinoless Double Beta Decay Searches



J. Engel and J. Menéndez - arXiv:1610.06548

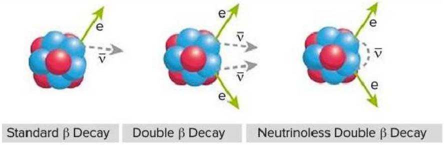


Majorana Demonstrator

$0\nu\beta\beta$-decay $\tau_{1/2} \gtrsim 10^{25}$ years (age of the universe 1.4×10^{10} years)

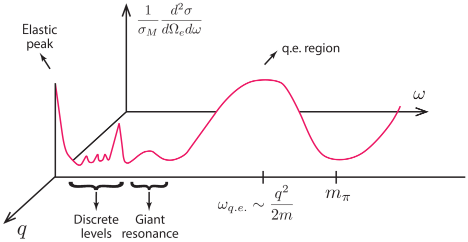
need 1 ton of material to see (if any) ~ 5 decays per year

* Decay Rate $\propto$ (nuclear matrix elements)$^2 \times \langle m_{\beta\beta} \rangle^2$ *



2015 Long Range Plane for Nuclear Physics

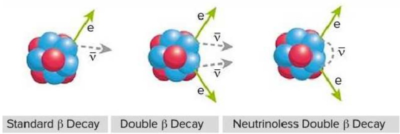
Nuclear Physics for Neutrinoless Double Beta Decay: Kinematics



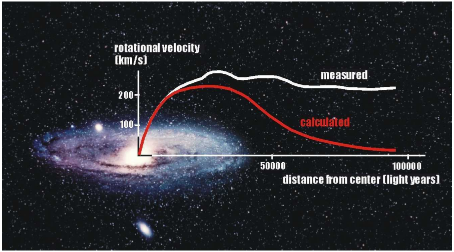
⇒ $\omega \sim$ few MeV, $q \sim 0$: EM decay, β-decay, $\beta\beta$-decays ⇐

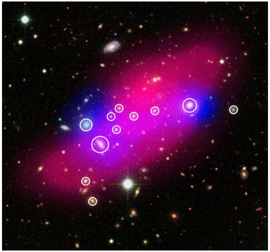
⇒ $\omega \sim$ few MeV, $q \sim$ hundreds of MeVs: $0\nu\beta\beta$-decays ⇐

* $\omega \sim 10^2$ MeV: Accelerator neutrinos, ν-nucleus scattering



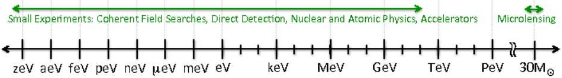
Fundamental Physics Quests III: Dark Matter





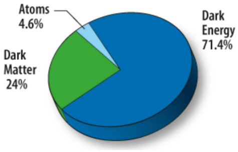
ESA, XMM-Newton, Gastaldello, CFHTL

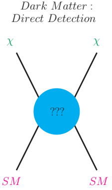
Dark Matter Candidates

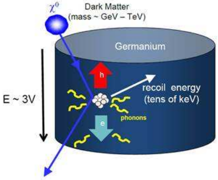


US Cosmic Vision 2017 arXiv:1707.04591

Dark Matter Direct Detection with Nuclei





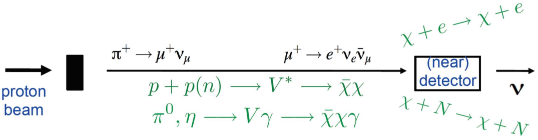


CDMS

Dark Matter Beam Production and Direct detection:

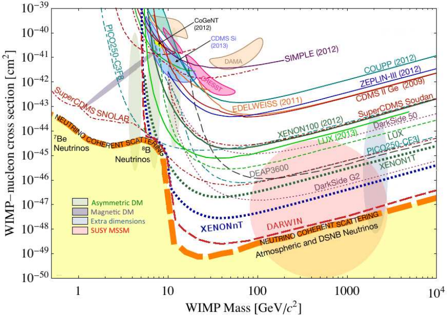
$$\chi + A \to \chi + A$$

Dark Matter is detected via scattering on nuclei in the detector
Couplings of Sub-GeV Dark Matter requires knowledge of nuclear responses

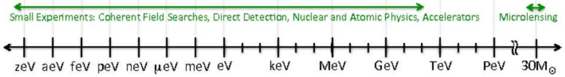


A. A. Aguilar-Arevalo *et al.* arXiv:1211.2258

Dark Matter Direct Detection with Nuclei

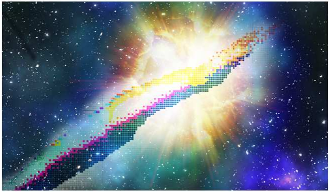


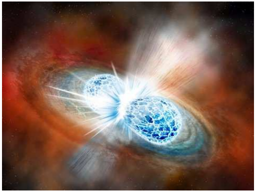
L. Baudis Phys.Dark Univ. 4 (2014) 50 adapted from P. Cushman *et al.* FERMILAB-CONF13688AE (2013)



US Cosmic Vision 2017 arXiv:1707.04591

Impact on Astrophysics



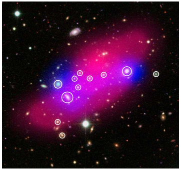


NASA

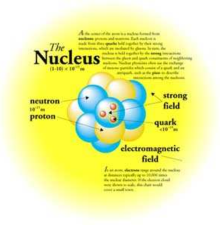
* Neutrinos and nuclei in dense environments *
* Weak reactions and astrophysical modeling *

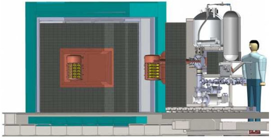
FRIB

Jefferson Lab

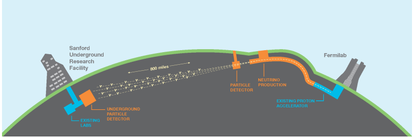


ESA, XMM-Newton, Gastaldello, CFHTL





Majorana Demonstrator

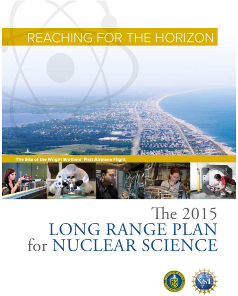


LBNF

The Science Questions

... overarching questions "that are central to the field as a whole, that reach out to other areas of science, and that together animate nuclear physics today:

1. How did visible matter come into being and how does it evolve?
2. How does subatomic matter organize itself and what phenomena emerge?
3. Are the fundamental interactions that are basic to the structure of matter fully understood?
4. How can the knowledge and technical progress provided by nuclear physics best be used to benefit society? "

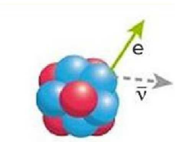


Fundamental Physics Quests rely on Nuclear Physics

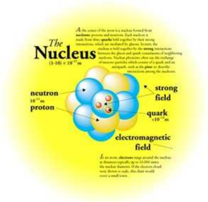
* An accurate understanding of nuclear structure and dynamics is required to extract new physics from nuclear effects *

Outline

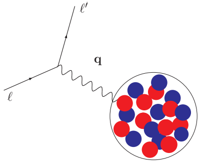
Decays



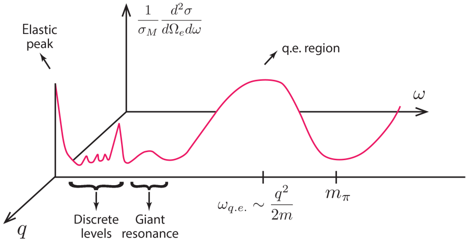
Energies and Structure



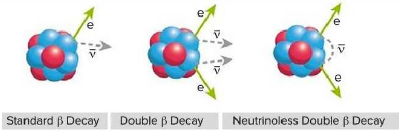
Scattering



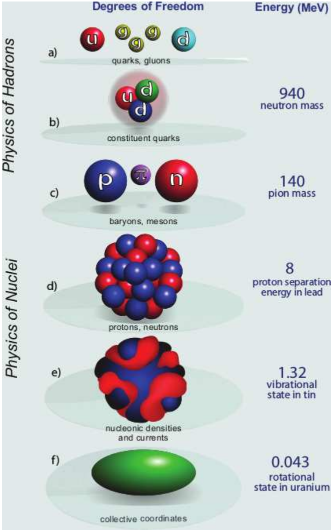
Nuclear Structure and Dynamics



- $\omega \sim$ few MeV, $q \sim 0$: EM decay, β-decay, $\beta\beta$-decays
- $\omega \lesssim$ tens MeV: Nuclear Rates for Astrophysics
- $\omega \sim 10^2$ MeV: Accelerator neutrinos, ν-nucleus scattering



Scales and Models



2007 Long Range Plane for Nuclear Physics

Reading Material

* On line material *

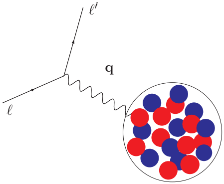
- Notes from Prof Rocco Schiavilla (for personal use only)
  https://indico.fnal.gov/event/8047/material/0/0
- Notes from Prof Luca Girlanda (for personal use only)
  http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez1.pdf
  http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez2.pdf
  http://chimera.roma1.infn.it/OMAR/ECTSTAR_DTP/girlanda/lez3.pdf
- Review Articles on *Ab initio* calculations of electromagnetic properties of light nuclei
  * Carlson & Schiavilla - Rev.Mod.Phys. 70 (1998) 743-842: http://inspirehep.net/record/40882
  * Bacca & Pastore - J.Phys. G41 (2014) no.12, 123002: http://inspirehep.net/record/1306337
  * Marcucci & F. Gross & M.T. Pena & M. Piarulli & R. Schiavilla & I. Sick & A. Stadler & J.W. Van Orden & M. Viviani - J.Phys. G43 (2016) 023002: https://inspirehep.net/record/1362209

* Textbooks *

- *Pions and Nuclei* by Torleif Ericson and Wolfram Weise, Oxford University Press (October 6, 1988)
- *Theoretical Nuclear and Subnuclear Physics* by John Dirk Walecka, Oxford University Press (March 23, 1995)
- *Foundations of Nuclear and Particle Physics* by T. William Donnelly, Joseph A. Formaggio, Barry R. Holstein, Richard G. Milner, Bernd Surrow, Cambridge University Press; 1st edition (February 1, 2017) new item!
- *A Primer for Chiral Perturbation Theory* by Stefan Scherer and Matthias R. Schindler, Springer; 2012 edition (September 30, 2011) (somewhat) new item!

saori.pastore@gmail.com

The Microscopic (or *ab initio*) Description of Nuclei



Develop a comprehensive theory that describes quantitatively and predictably all nuclear structure and reactions

* Accurate understanding of interactions between nucleons, p's and n's

* and between e's, ν's, **DM**, . . . , with nucleons, nucleons-pairs, . . .

$$H\Psi = E\Psi$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

Erwin Schrödinger

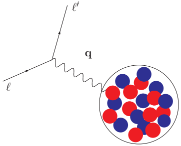
The *ab initio* Approach

The nucleus is made of A interacting nucleons and its energy is

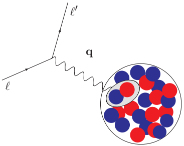
$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

where υ_{ij} and V_{ijk} are two- and three-nucleon operators based on EXPT data fitting and fitted parameters subsume underlying QCD

1b



2b



$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots ,$$

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

Two-body 2b currents essential to satisfy current conservation

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = \left[t_i + \upsilon_{ij} + V_{ijk}, \rho\right]$$

* “Longitudinal” component fixed by current conservation

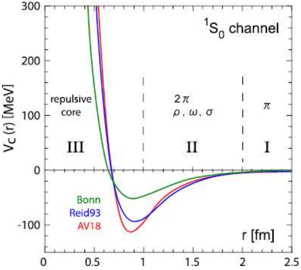
* “Transverse” component “model dependent”

The Basic Model
Requirement 1: Nuclear Interactions

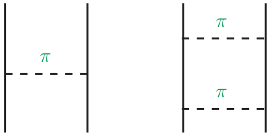
$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

Step 1. Construct two- and three-body interactions

* Chiral Effective Field Theory Interactions
* "Conventional" or "Phenomenological" Interactions



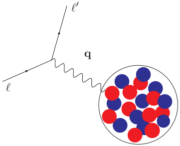
Aoki *et al.* Comput.Sci.Disc.1(2008)015009



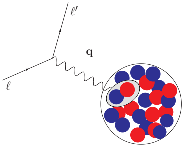
* One-pion-exchange: range$\sim \frac{1}{m_\pi} \sim 1.4$ fm
* Two-pion-exchange: range$\sim \frac{1}{2m_\pi} \sim 0.7$ fm

The Basic Model
Requirement 2: Nuclear Many-Body Currents

1b



2b



$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots \,,$$

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

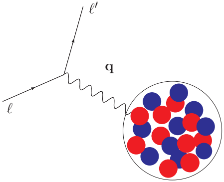
Step 2. Understand how external probes (e, ν, **DM** ...) interact with nucleons, nucleon pairs, nucleon triplets...

* Chiral Effective Field Theory Electroweak Many-Body Currents
* "Conventional" or "Phenomenological" Electroweak Many-Body Currents

Step 2.a First validate and then use the model

* Validate the theory against EM data in a wide range of energies
* Neutrino-Nucleus Observables from low to high energies and momenta

The Basic Model
Requirement 3: Solve the Many-Body Nuclear Problem



Step 3. Develop Computational Methods to solve (numerically) exactly or within approximations that are under control

$$H\Psi = E\Psi$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A, s_1, s_2, ..., s_A, t_1, t_2, ..., t_A)$$

Ψ are spin-isospin vectors in $3A$ dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

^{4}He : 96
^{6}Li : 1280
^{8}Li : 14336
^{12}C : 540572

Requirement 1: Nuclear Interactions

(Some) Nuclear Force Facts

* Binding Energy per Nucleon $\sim$ 8.5 MeV in all nuclei
* Nucleon-nucleon interaction is short-ranged w.r.t. nuclear radius

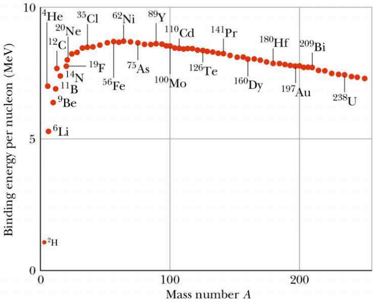
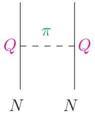
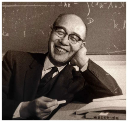


figure from ohio.edu

* 1930s Yukawa Potential
NN force is mediated by massive particle
* 1947 The pion is observed
$m \sim$ 140 MeV implying a range $\propto$ 1.4 fm

$$v_Y \sim -\frac{e^{-mr}}{r} \qquad \text{range} \propto \frac{1}{m}$$





Hideki Yukawa

(Some) Nuclear Force Facts

* Charge-density inside nuclei is constant and independent of A

* Nucleon-nucleon force is strongly repulsive at small interparticle distances

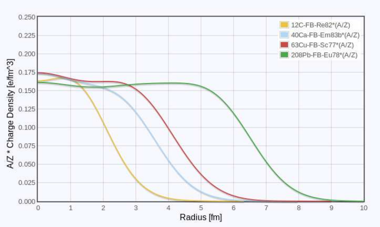


Fig. from virginia.edu

^{7}Li and ^{7}Be spectra $(3p, 4n) \to (4p, 3n)$

* NN forces exhibit charge-independence, *i.e.*, do not recognize p's from n's

* Nuclear interactions depend on the total isospin T of the NN pair, but not on T_z

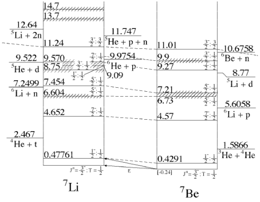
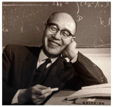


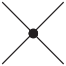
figure from TUNL database

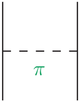
Nuclear Force These Days

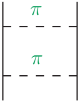
* 1930s Yukawa Potential
* 1960–1990 Highly sophisticated meson exchange potentials
* 1990s– Highly sophisticated Chiral Effective Field Theory based potentials



Hideki Yukawa







Steven Weinberg

* Contact terms: short-range
* One-pion-exchange: range$\sim \frac{1}{m_\pi}$
* Two-pion-exchange: range$\sim \frac{1}{2m_\pi}$

Constructing the Nuclear Many-Body Hamiltonian
(The Chiral Effective Filed Theory Perspective)

The nucleus is made of A interacting nucleons and its energy is

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

* υ_{ij} correlates nucleons in pairs; and * V_{ijk} correlates nucleons in triples
* … indicate that the expansion in many-body operators "is" convergent
* υ_{ij} and V_{ijk} involve parameters that subsume underlying QCD, fitted to large number (order of thousands) of NN-scattering data

Three-body force: an example

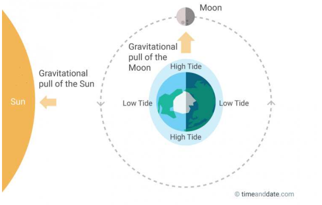


figure from www.timeanddate.com

Time-Ordered-Perturbation Theory

The relevant degrees of freedom of nuclear physics are bound states of QCD

* non relativistic nucleons N*
* pions π as mediators of the nucleon-nucleon interaction
* non relativistic Delta's Δ with $m_\Delta \sim m_N + 2m_\pi$

Transition amplitude in time-ordered perturbation theory

$$T_{fi} = \langle N'N' \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid NN \rangle^*$$

$H_0 =$ free π, N, Δ Hamiltonians
$H_1 =$ interacting π, N, Δ, and external electroweak fields Hamiltonians

$$T_{fi} = \langle N'N' \mid T \mid NN \rangle \propto \upsilon_{ij} , \qquad T_{fi} = \langle N'N' \mid T \mid NN;\gamma \rangle \propto (A^0 \rho_{ij}, \mathbf{A} \cdot \mathbf{j}_{ij})$$

* Based on the fact that $v_{nucleon} \sim 0.2c$; relativity included perturbatively

* Note no pions in the initial or final states, *i.e.*, pion-production not accounted in the theory

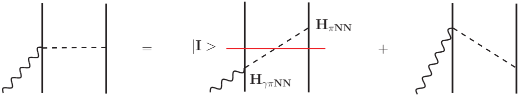
Transition amplitude in time-ordered perturbation theory

Insert complete sets of eigenstates of H_0 between successive terms of H_1

$$T_{fi} = \langle N'N' \mid H_1 \mid NN;\gamma\rangle + \sum_{|I\rangle} \langle N'N' \mid H_1 | I\rangle \frac{1}{E_i - E_I} \langle I | H_1 \mid NN;\gamma\rangle + \ldots$$

The contributions to the T_{fi} are represented by time ordered diagrams

Example: seagull pion exchange current



N number of H_1's (vertices) → N! time-ordered diagrams

* H_1 by construction satisfies the symmetries exhibited by QCD (in the low-energy regime), *i.e.*, Parity, Charge Conjugation, Isospin, ..., and Chiral

Conceptual Perturbation Theory

$$\frac{1}{1-x} = \sum_{n=0} x^n = 1 + x + x^2 + x^3 \ldots$$

* x is small expansion parameter
* one only needs to evaluate few terms in the expansion (if lucky)
* the error is given by the truncation in the expansion

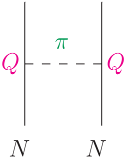
* Examples *

* Chiral Effective Field Theory: $x = Q$
* Large N_c: $x = \frac{1}{N_c}$
* $\ldots$

Nuclear Chiral Effective Field Theory (χEFT) approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

* χEFT is a low-energy ($Q \ll \Lambda_\chi \sim 1$ GeV) approximation of QCD
* It provides effective Lagrangians describing π's, N's, Δ's, ... interactions that are expanded in powers n of a perturbative (small) parameter Q/Λ_χ



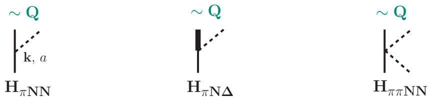
$$\mathscr{L}_{eff} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots + \mathscr{L}^{(n)} + \ldots$$

* The coefficients of the expansion, Low Energy Constants (LECs), are unknown and need to be fixed by comparison with exp data, or take them from LQCD
* The systematic expansion in Q naturally has the feature

$$\langle \mathscr{O} \rangle_{1-\text{body}} > \langle \mathscr{O} \rangle_{2-\text{body}} > \langle \mathscr{O} \rangle_{3-\text{body}}$$

* A theoretical error due to the truncation of the expansion can be assigned

π, N and Δ Strong Vertices



$$H_{\pi NN} = \frac{g_A}{F_\pi} \int \mathrm{d}\mathbf{x} N^\dagger(\mathbf{x}) \left[\boldsymbol{\sigma} \cdot \nabla \pi_a(\mathbf{x})\right] \tau_a N(\mathbf{x}) \longrightarrow V_{\pi NN} = -i \frac{g_A}{F_\pi} \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\sqrt{2\,\omega_k}} \tau_a \sim Q^1 \times Q^{-1/2}$$

$$H_{\pi N\Delta} = \frac{h_A}{F_\pi} \int \mathrm{d}\mathbf{x} \Delta^\dagger(\mathbf{x}) \left[\mathbf{S} \cdot \nabla \pi_a(\mathbf{x})\right] T_a N(\mathbf{x}) \longrightarrow V_{\pi N\Delta} = -i \frac{h_A}{F_\pi} \frac{\mathbf{S} \cdot \mathbf{k}}{\sqrt{2\,\omega_k}} T_a \sim Q^1 \times Q^{-1/2}$$

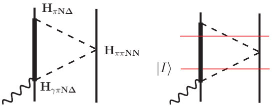
$g_A \simeq 1.27$; $F_\pi \simeq 186$ MeV; $h_A \sim 2.77$ (fixed to the width of the Δ) are 'known' LECs

$$\begin{aligned} \pi_a(\mathbf{x}) &= \sum_{\mathbf{k}} \frac{1}{\sqrt{2\,\omega_k}} \left[c_{\mathbf{k},a}\, \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} + \mathrm{h.c.}\right] , \\ N(\mathbf{x}) &= \sum_{\mathbf{p},\sigma\tau} b_{\mathbf{p},\sigma\tau}\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{x}} \chi_{\sigma\tau} , \end{aligned}$$

(Naïve) Power Counting

Each contribution to the T_{fi} scales as

$$\underbrace{\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i}\right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



$\alpha_i =$ # of derivatives (momenta) in H_1;
$\beta_i =$ # of π's;
$N =$ # of vertices; $N-1 =$ # of intermediate states;
$L =$ # of loops

$$H_1 \text{ scaling} \sim \underbrace{Q^1}_{H_{\pi N\Delta}} \times \underbrace{Q^1}_{H_{\pi\pi NN}} \times \underbrace{Q^0}_{H_{\pi\gamma N\Delta}} \times Q^{-2} \sim Q^0$$

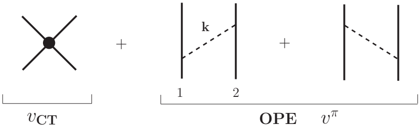
$$\text{denominators} \sim \frac{1}{E_i - H_0}|I\rangle \sim \frac{1}{2m_N - (m_\Delta + m_N + \omega_\pi)}|I\rangle = -\frac{1}{m_\Delta - m_N + \omega_\pi}|I\rangle \sim \frac{1}{Q}|I\rangle$$

$$Q^1 = Q^0 \times Q^{-2} \times Q^3$$

* This power counting also follows from considering Feynman diagrams, where loop integrations are in 4D

χEFT nucleon-nucleon potential at LO

$v_{\rm NN}^{\rm LO} =$



$\sim \mathbf{Q}^0$

$$T_{fi}^{\rm LO} = \langle N'N' \mid H_{{\rm CT},1} \mid NN \rangle + \sum_{|I\rangle} \langle N'N' \mid H_{\pi NN} \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_{\pi NN} \mid NN \rangle$$

Leading order nucleon-nucleon potential in χEFT

$$v_{\rm NN}^{\rm LO} = v_{\rm CT} + v_\pi = C_S + C_T\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

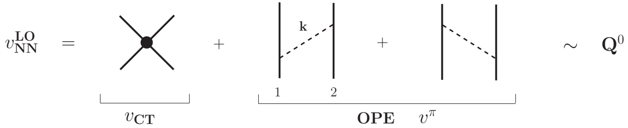
* Configuration space *

$$v_{12} = \sum_p v_{12}^p(r)\, O_{12}^p; \qquad O_{12} = 1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$$S_{12} = 3\, \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}\, \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$\Leftarrow$ Tensor Operator

One Pion Exchange in Configuration Space



One-Pion-Exchange Potential (OPEP)

$$\upsilon_\pi(\mathbf{k}) = -\frac{g_A^2}{F_\pi^2}\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k}}{\omega_k^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$$\upsilon_\pi(\mathbf{r}) = \frac{f_{\pi NN}^2}{4\pi}\frac{m_\pi}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[T_\pi(r)S_{12}+\left[Y_\pi(r)-\frac{4\pi}{m_\pi^3}\delta(\mathbf{r})\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]$$

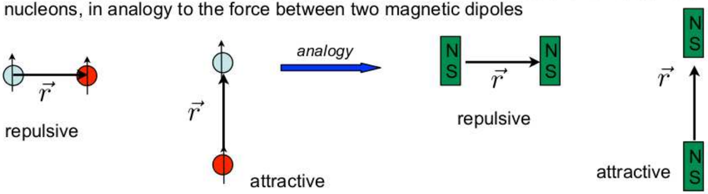
$$Y_\pi(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$ $\Leftarrow$ Yukawa Function

$$T_\pi(r) = \left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)Y_\pi(r)$$

$$S_{12} = 3\,\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$ $\Leftarrow$ Tensor Operator

Tensor Operator: An Analogy

- Tensor force: depends on the angle between relative distance vector and the spins of the nucleons, in analogy to the force between two magnetic dipoles



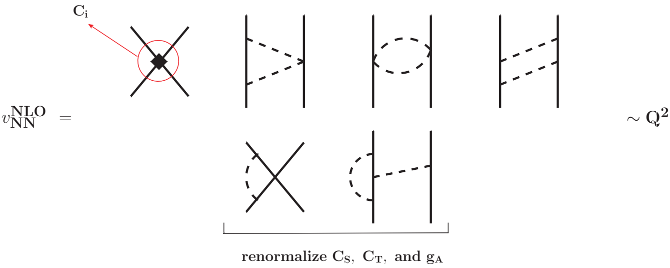
$$S_{12} \sim [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma_2}] \qquad U \sim [3(\vec{\mu_1} \cdot \hat{r})(\vec{\mu_2} \cdot \hat{r}) - \vec{\mu}_1 \cdot \vec{\mu_2}]$$

figure from Sonia Bacca

* Tensor Force is non-spherical and spin dependent

* Tensor Force correlates spatial and spin orientations

χEFT nucleon-nucleon potential at NLO (without Δ's)

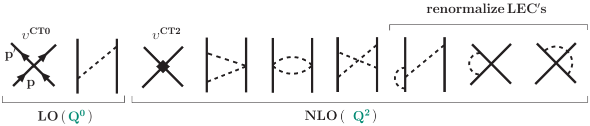


* At NLO there are 7 LEC's, C_i, fixed so as to reproduce nucleon-nucleon scattering data (order of k data)
* C_i's multiply contact terms with 2 derivatives acting on the nucleon fields (∇N)
* Loop-integrals contain ultraviolet divergences reabsorbed into g_A, C_S, C_T, and C_i's (for example, use dimensional regularization)

* Configuration space *

$$v_{12} = \sum_p v^p_{12}(r)\, O^p_{12}; \qquad O_{12} = [1,\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,\, S_{12},\mathbf{L}\cdot\mathbf{S}] \otimes [1,\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

Fitting the NN interaction

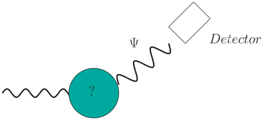


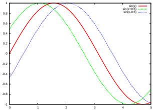
* At NLO there are 9 free parameters to be determined

C_S, C_T, and 7 C_i

* Solve for the scattering waves of the Schrödinger Equation

* Fit the LECs to the phase shifts





plane wave
shift by attractive potential
shift by repulsive potential

$$\Psi \quad = \quad A\, sin(kr + \delta) \sim (e^{i2\delta} e^{ikr} - e^{-ikr})$$

* Curiosity: Indirect evidence of one-pion-exchange potential comes from the 1993 Nijmegen phase-shift analysis with m_π left as free-parameter; best fit obtained with actual pion mass

Technicalities: The Cutoff

* χEFT operators have a power law behavior in Q

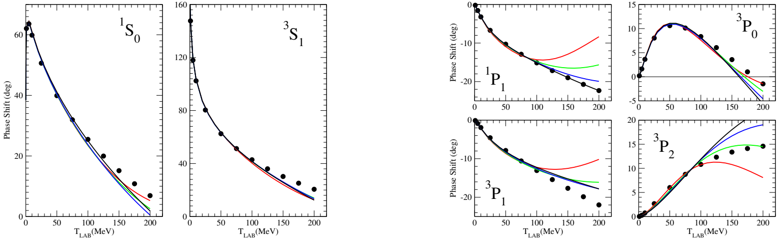
1. introduce a regulator to kill divergencies at large Q, *e.g.*, $C_\Lambda = e^{-(Q/\Lambda)^n}$
2. pick n large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \ldots$$

3. for each cutoff Λ re-fit the LECs
4. ideally, your results should be cutoff-independent

* In r_{ij}-space this corresponds to cutting off the short-range part of the operators that make the matrix elements diverge at $r_{ij} = 0$

Determining LEC's: fits to *np* phases $*$ up to $T_{\rm LAB} = 100$MeV
NLO Chiral Potential

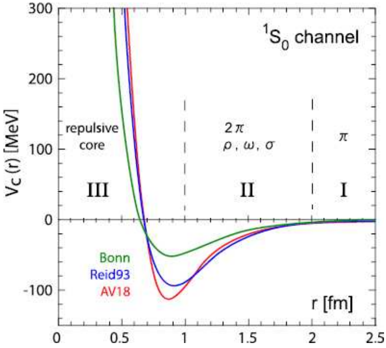


LS-equation regulator $\sim \exp(-2\,Q^4/\Lambda^4)$, (cutting off momenta $Q \gtrsim$ 3–4 m_π),
Λ=500, 600, and 700 MeV

$*$ F.Gross and A.Stadler PRC78(2008)104405

Pastore *et al.* PRC80(2009)034004

Nucleon-nucleon potential



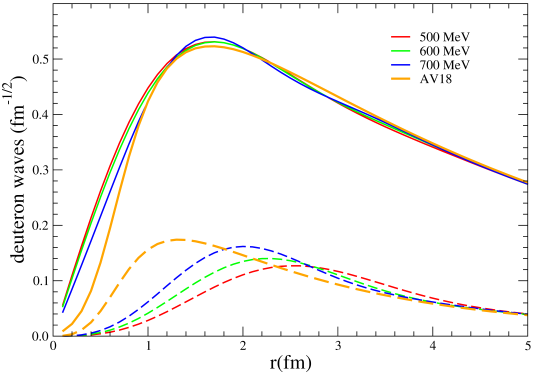
Aoki *et al.* Comput.Sci.Disc.1(2008)015009

CT = Contact Term* - short-range;
OPE = One Pion Exchange - range $\sim \frac{1}{m_\pi}$;
TPE = Two Pion Exchange - range $\sim \frac{1}{2m_\pi}$

$*$ in practice CT's in r-space are coded with representations of a δ-function (*e.g.*, a Gaussian function), or special functions such as Wood-Saxon functions

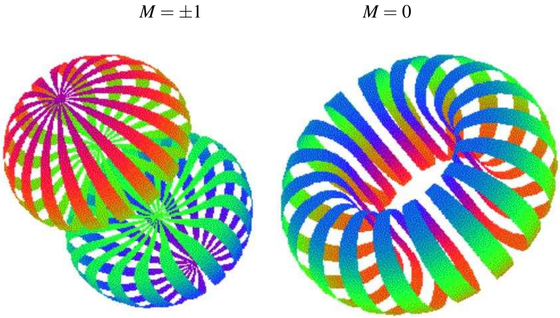
Nucleon-Nucleon Potential and the Deuteron

Deuteron Waves



Pastore *et al.* PRC80(2009)034004

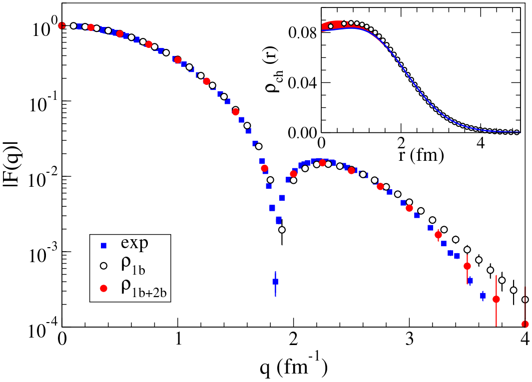
Nucleon-Nucleon Potential and the Deuteron



Constant density surfaces for a polarized deuteron in the $M = \pm 1$ (left) and $M = 0$ (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743

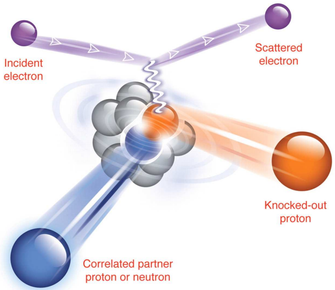
Shape of Nuclei

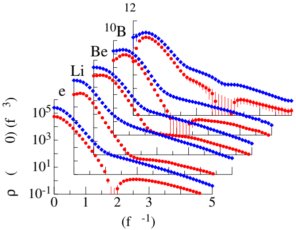


Lovato *et al.*

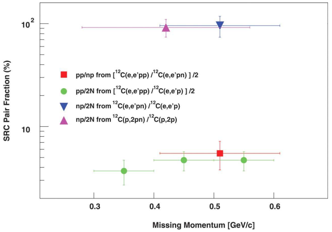
PRL111(2013)092501

Back-to-back *np* and *pp* Momentum Distributions





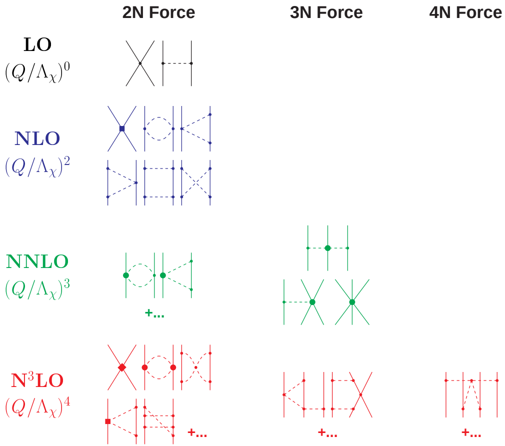
Wiringa *et al.* PRC89(2014)024305



JLab, Subedi *et al.* Science320(2008)1475

Nuclear properties are strongly affected by two-nucleon interactions!

χEFT many-body potential: Hierarchy

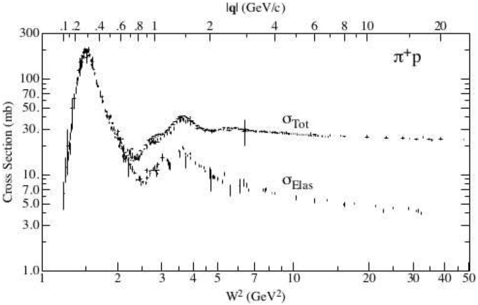


Machleidt & Sammarruca - PhysicaScripta91(2016)083007

* NN potential at N3LO: 15 additional LECs allow to get fits with χ^2/datum ~ 1
* Additional operatorial structures emerges (same as Argonne υ_{14})

$$O_{12} = [1,\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,\, S_{12},\, \mathbf{L}\cdot\mathbf{S},\, \mathbf{L}^2,\, \mathbf{L}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,\, (\mathbf{L}\cdot\mathbf{S})^2] \otimes [1,\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]$$

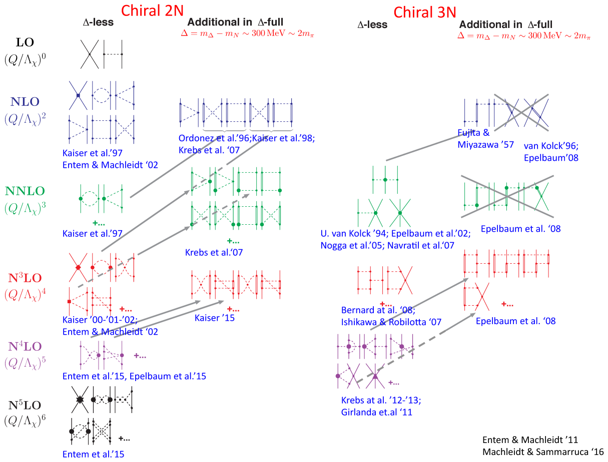
Nucleon's excitations



$\pi^+ p$ scattering

* Δ resonance has large strength and low-energy ($m_\Delta - m_N \sim 2m_\pi$)
* Δ's play important role in π-exchange interactions between nucleons
* LECs in chiral potentials are making up for d.o.f. not included in the theory
* Explicit inclusion of Δ's improves on chiral's formulation and convergence

Nuclear Interactions and the role of the Δ

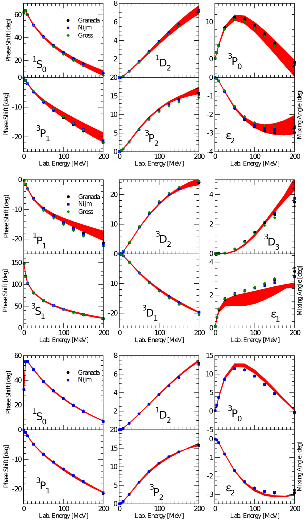


Courtesy of Maria Piarulli

* N3LO with Δ nucleon-nucleon interaction constructed by Piarulli *et al.* in PRC91(2015)024003-PRC94(2016)054007-arXiv:1707.02883 with $\Delta's$ fits ∼ 2000 (∼ 3000) data up 125 (200) MeV with χ^2/datum ∼ 1;

* N2LO with Δ 3-nucleon force fits ^{3}H binding energy and the *nd* scattering length

Phase Shifts from Chiral NN with Δ's



Piarulli *et al.* PRC 94(2016)054007

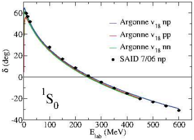
Phenomenological aka Conventional aka Traditional aka Realistic Two- and Three- Nucleon Potentials

NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

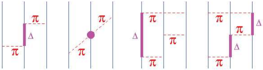
Argonne v_{18}: $v_{ij} = v^{\gamma}_{ij} + v^{\pi}_{ij} + v^{I}_{ij} + v^{S}_{ij} = \sum v_p(r_{ij})O^p_{ij}$



- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1

Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V^{2\pi}_{ijk} + V^{3\pi}_{ijk} + V^{R}_{ijk}$



- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

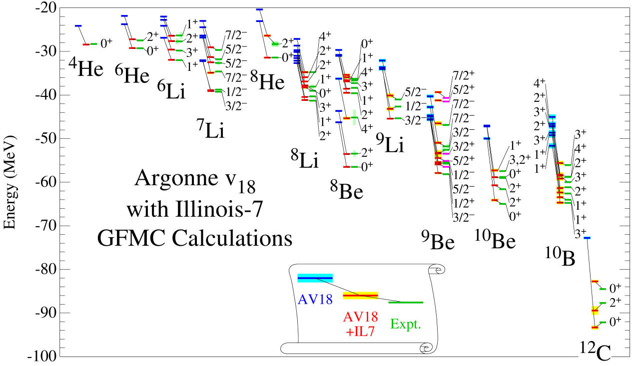
Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

Courtesy of Bob Wiringa

* AV18 fitted up to 350 MeV, reproduces phase shifts up to $\sim$ 1 GeV *

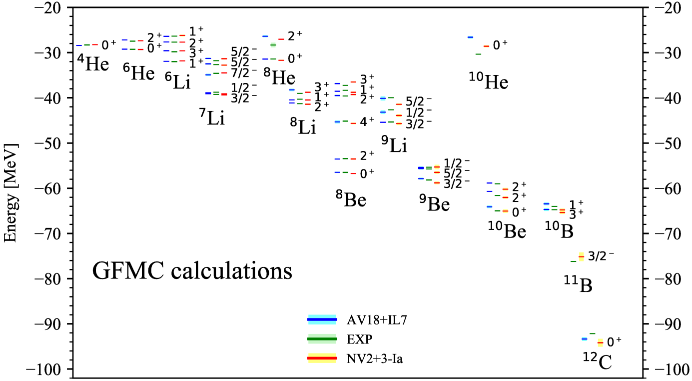
* IL7 fitted to 23 energy levels, predicts hundreds of levels *

Spectra of Light Nuclei



Carlson *et al.* Rev.Mod.Phys.87(2015)1067

Spectra of Light Nuclei



M. Piarulli *et al.* - arXiv:1707.02883

* one-pion-exchange physics dominates *
* it is included in both chiral and "conventional" potentials *

Three-body forces

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$$V_{ijk} \sim (0.2 - 0.9)\, v_{ij} \sim (0.15 - 0.6)\, H$$

$$v_\pi \sim 0.83\, v_{ij}$$

^{10}B VMC code output

```
Ti + Vij =  -38.2131 (0.1433)          + Vijk =  -46.7975 (0.1150)

Ti   = 290.3220 (1.2932)  Vij  =-328.5351 (1.1983)  Vijk =  -8.5844 (0.0892)
```

(Very) Incomplete List of Credits and Reading Material

* Pieper and Wiringa; Ann.Rev.Nucl.Part.Sci.51(2001)53
* Carlson *et al.*; Rev.Mod.Phys.87(2015)1067
* van Kolck *et al.*; PRL72(1994)1982-PRC53(1996)2086
* Kaiser, Weise *et al.*; NPA625(1997)758-NPA637(1998)395
* Epelbaum, Glöckle, Meissner*; RevModPhys81(2009)1773 and references therein
* Entem and Machleidt*; PhysRept503(2011)1 and references therin

* NN Potentials suited for Quantum Monte Carlo calculations *

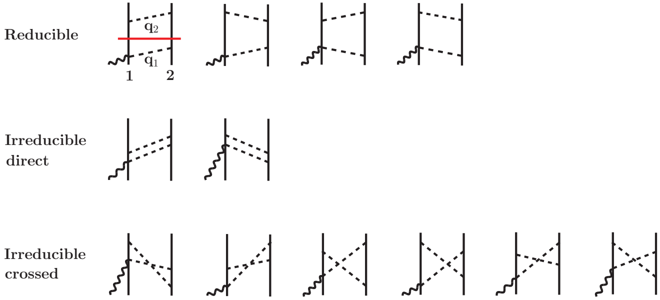
* Pieper and Wiringa; Ann.Rev.Nucl.Part.Sci.51(2001)53
* Gezerlis *et al.* and Lynn *et al.*; PRL111(2013)032501,PRC90(2014)054323,PRL113(2014)192501;
* Piarulli *et al.*; PRC91(2015)024003-PRC94(2016)054007-arXiv:1707.02883

Summary: Nuclear Interactions

* The Microscopic description of Nuclei is very successful
* Nuclear two-body forces contain a number of parameters (up to ~ 40) fitted to a large $\sim 4k$ ($\sim 3k$) data base up to 350 (~ 200) MeV in the case of AV18 (Chiral) model
* Intermediate and long components are described in terms of one- and two-pion exchange potentials
* Short-range parts are described by contact terms or special functions
* Due to a cancellation between kinetic and two-body contribution, three-body potentials are (small but) necessary to reach agreement with the data
* Calculated spectra of light nuclei are reproduced within $1-2\%$ of expt data
* Two-body one-pion-exchange contributions dominate and are crucial to explain the data
* AV18 potential is hard to be systematically improved but has a range of applicability up to ~ 1 GeV

Technicalities I: Reducible Contributions

4 interaction Hamiltonians $\longrightarrow$ 4! time ordered diagrams



$$|\Psi\rangle \simeq |\phi\rangle + \frac{1}{E_i - H_0}\upsilon^\pi|\phi\rangle + \ldots$$

$$\langle\Psi_f|\mathbf{j}|\Psi_i\rangle \simeq \langle\phi_f|\mathbf{j}|\phi_i\rangle + \langle\phi_f|\upsilon^\pi\frac{1}{E_i - H_0}\mathbf{j} + \text{h.c.}|\phi_i\rangle + \ldots$$

* Need to carefully subtract contributions generated by the iterated solution of the Schrödinger equation