

Few-Body Physics with Relation to Neutrinos

Saori Pastore
HUGS Summer School
Jefferson Lab - Newport News VA, June 2018



Thanks to the Organizers

Neutrinos (Fundamental Symmetries) and Nuclei

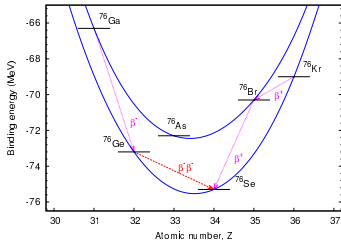
Topics (5 hours)

- * Nuclear Theory for the Neutrino Experimental Program ✓
- * Microscopic (or *ab initio*) Description of Nuclei ✓
- * “Realistic” Models of Two- and Three-Nucleon Interactions ✓
- * “Realistic” Models of Many-Body Nuclear Electroweak Currents ✓
- * Short-range Structure of Nuclei and Nuclear Correlations ✓
- * Quasi-Elastic Electron and Neutrino Scattering off Nuclei ✓
- * Validation of the theory against available data ✓

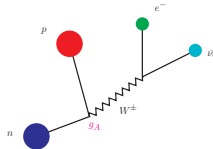
Correlations and Currents in Neutrinoless Double Beta Decay



Standard Single and Double Beta Decays



J. Menéndez - arXiv:1703.08921v1

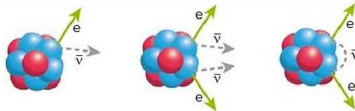


Maria Geopert-Mayer

single beta decay: $(Z, N) \rightarrow (Z + 1, N - 1) + e + \bar{\nu}_e$

double beta decay: $(Z, N) \rightarrow (Z + 2, N - 2) + 2e + 2\bar{\nu}_e$

lepton # $L = l - \bar{l}$ is conserved

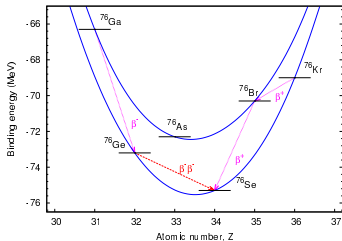


Standard β Decay

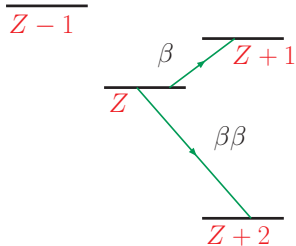
Double β Decay

Neutrinoless Double β Decay

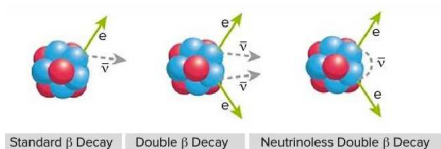
Double Beta Decay



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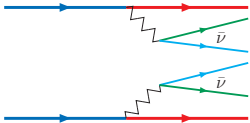
Standard β Decay Double β Decay Neutrinoless Double β Decay

Majorana Neutrino



Maria Geoppert-Mayer

$$\nu \neq \bar{\nu}$$



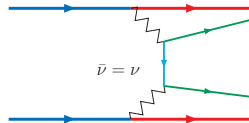
$$(Z, N) \rightarrow (Z+2, N-2) + 2e + 2\bar{\nu}_e$$

lepton # $L = l - \bar{l}$ is conserved



Ettore Majorana

$$\nu = \bar{\nu}$$



$$(Z, N) \rightarrow (Z+2, N-2) + 2e$$

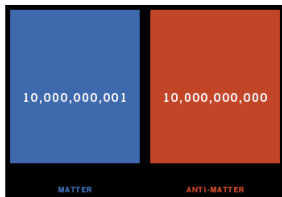
lepton # $L = l - \bar{l}$ is not conserved

$\beta\beta$ and $0\nu\beta\beta$ decays kinematics are different

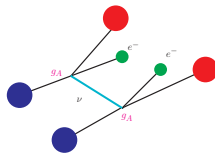
$\omega \sim$ few MeV in both processes

$\mathbf{q} \sim 0$ in $\beta\beta$ -decay and $\mathbf{q} \sim$ hundreds of MeVs in $0\nu\beta\beta$ -decay

Neutrinoless Double Beta Decay



H. Murayama

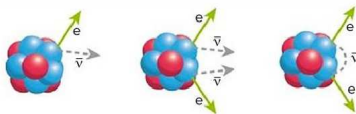


Ettore Majorana

$0\nu\beta\beta$ neutrinoless double beta decay

$$(Z, N) \rightarrow (Z+2, N-2) + 2e$$

lepton # $L = l - \bar{l}$ is not conserved

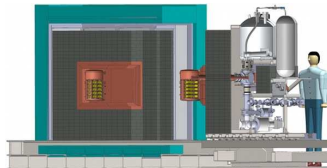
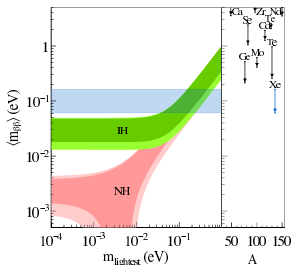


Standard β Decay

Double β Decay

Neutrinoless Double β Decay

Nuclear Physics for Neutrinoless Double Beta Decay Searches



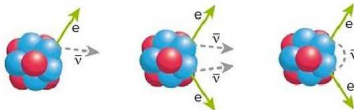
Majorana Demonstrator

J. Engel and J. Menéndez - arXiv:1610.06548

$0\nu\beta\beta$ -decay $\tau_{1/2} \gtrsim 10^{25}$ years (age of the universe 1.4×10^{10} years)

need 1 ton of material to see (if any) ~ 5 decays per year

* Decay Rate \propto (nuclear matrix elements) $^2 \times \langle m_{\beta\beta} \rangle^2$ *

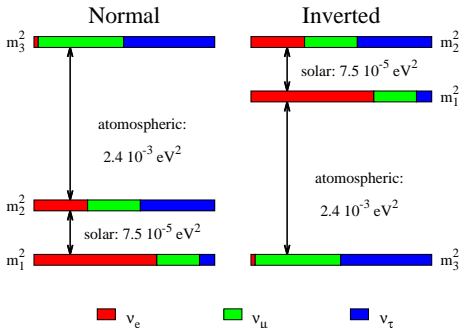


Standard β Decay

Double β Decay

Neutrinoless Double β Decay

Neutrinos' Mass Hierarchy



JUNO coll. - J.Phys.G43(2016)030401

$$m_{\beta\beta} = f(m_1, m_2, m_3)$$

We will number (just for convenience) the massive neutrinos in such a way that $m_1 < m_2$, so that $\Delta m_{21}^2 > 0$. cit. PDG2017

Neutrinoless Double Beta Decay Candidates

Half Life and Nuclear Matrix Element

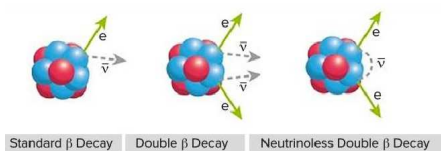
$$\frac{1}{\tau_{1/2}} \propto |M^{0\nu}|^2 \times m_{\beta\beta}^2$$

$M^{0\nu} = 0\nu\beta\beta$ nuclear matrix element

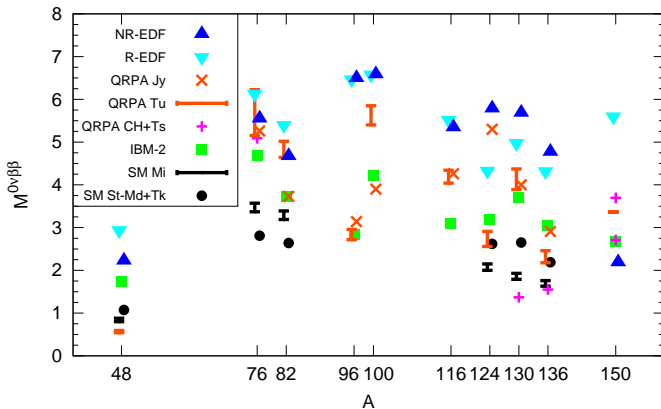
$$m_{\beta\beta} = f(m_1, m_2, m_3)$$

Candidates (smallest nucleus $A = 48$)

^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{116}Cd ^{128}Te ^{130}Te ^{136}Xe ^{150}Nd



Neutrinoless Double Beta Decay Matrix Elements: Status

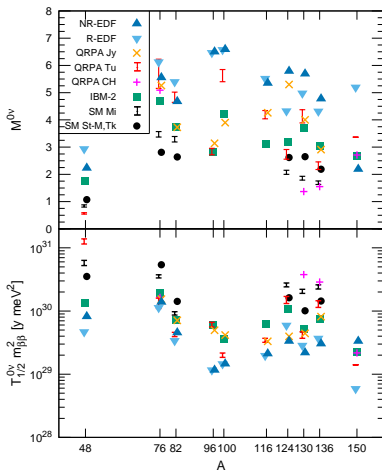


J. Engel and J. Menéndez - arXiv:1610.06548

^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{116}Cd ^{128}Te ^{130}Te ^{136}Xe ^{150}Nd

calculated M^{0v} can differ by a factor of 3

Neutrinoless Double Beta Decay Half Life: Status

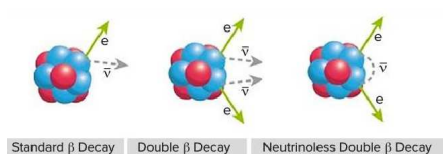


J. Engel and J. Menéndez - arXiv:1610.06548

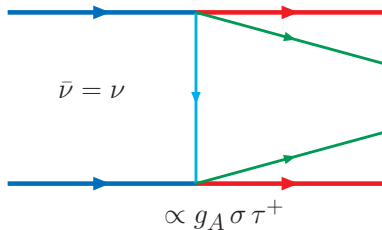
$$\tau_{1/2} \times m_{\beta\beta}^2 = |M^{0\nu}|^{-2}$$

Quantum Monte Carlo calculations of $0\nu\beta\beta$ -decay matrix elements

- * Quantum Monte Carlo (QMC) Methods solve the many-nuclear problem exactly
 - * QMC (currently) computationally limited to $A \leq 12$ nuclei
 - * QMC method, many-body Hamiltonians and electroweak currents largely and successfully tested against available experimental data
- * Calculate $0\nu\beta\beta$ -decay matrix elements in light nuclei within the above framework
 - * Not directly relevant to the experimental program ($0\nu\beta\beta$ -decay not occurring in $A \leq 12$ nuclei)
 - * Very important from the theoretical stand point of view to
 - benchmark different computational methods
 - identify most relevant contributions to the matrix elements
 - have an insight into the dynamics of the process



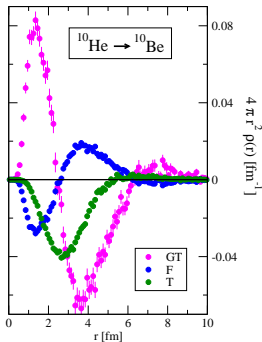
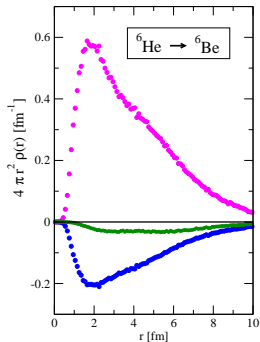
$0\nu\beta\beta$ -decay mediated by a neutrino



$$v_{0\nu} = \sum_i h_i(r) O_{12} \quad O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes \tau_1^+ \tau_2^+$$

F, GT, T = Fermi, Gamow-Teller, Tensor

F, GT, and T Transition Densities



* $\Delta T = 0$

${}^6\text{He}(1) \rightarrow {}^6\text{Be}(1)$

${}^8\text{He}(2) \rightarrow {}^8\text{Be}^*(2)$

${}^{10}\text{Be}(1) \rightarrow {}^{10}\text{C}(1)$

* $\Delta T = 2$

${}^8\text{He}(1) \rightarrow {}^8\text{Be}(0)$

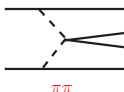
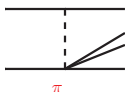
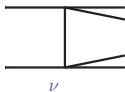
${}^{10}\text{He}(3) \rightarrow {}^{10}\text{Be}(1)$

${}^{12}\text{Be}(2) \rightarrow {}^{12}\text{C}(0)$

$$F = \tau_{1,+} \tau_{2,+} ; GT = \tau_{1,+} \tau_{2,+} \sigma_1 \cdot \sigma_2 ; T = \tau_{1,+} \tau_{2,+} S_{12}$$

arXiv:1710.05026

Lepton-number violating transition operators



$$v_{\nu} \sim L_{\nu} \tau_{1,+} \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{m_{\pi} \mathbf{q}^2} + \dots + v_{\nu}^{\text{N2LO-loop}^*}$$

$$v_{\pi\pi} \sim L_{\pi\pi} \tau_{1,+} \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{m_{\pi} (\mathbf{q}^2 + m_{\pi}^2)^2}$$

$$v_{\pi} \sim L_{\pi} \tau_{1,+} \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{m_{\pi}^3 (\mathbf{q}^2 + m_{\pi}^2)}$$

$$v_{NN} \sim L_{NN} \tau_{1,+} \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{m_{\pi}^3}$$

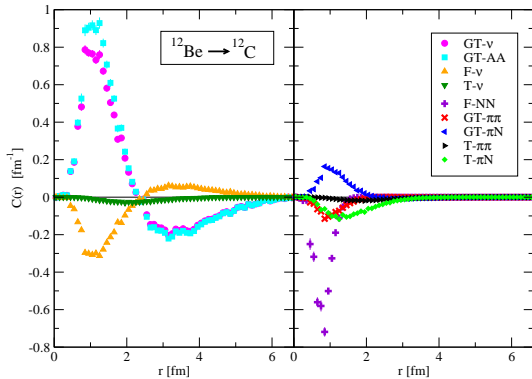
$L_{\pi\pi}$, L_{π} , L_{NN} encode hadronic and **model dependent** particle physics

* Cirigliano & Dekens & Mereghetti & Walker-Loud in arXiv:1710.01729

$$v_{0\nu} = \sum_i h_i(r) O_{12} \quad O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes \tau_1^+ \tau_2^+$$

with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa

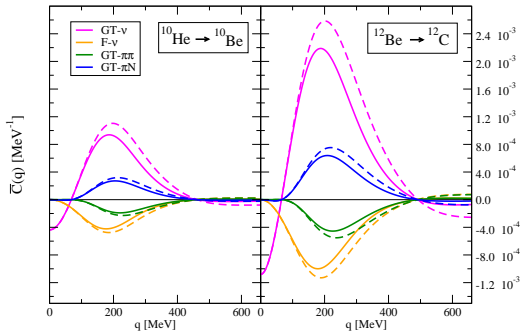
Double beta-decay Matrix Elements in $A = 12$



with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa

PRC97(2018)014606

Momentum Dependence



- * Peaks at ~ 200 MeV
- * Form factors on/off $\rightarrow \sim 10\%$ variation same size as $v_v^{\text{N2LO-loop}}$ from Cirigliano *et al.* arXiv:1710.01729
- * $A = 10$ highly suppressed w.r.t. $A = 12$
- * $A = 10$ **small overlap** between initial diffuse w.f. ($r_n \sim 3.66$ fm) and final compact w.f. ($r_p \sim 2.32$ fm)
- * $A = 12$ **large overlap** between initial compact w.f. ($r_n \sim 2.99$ fm) and final compact w.f. ($r_p \sim 2.48$ fm)
- * $A = 12$ 'most similar' to experimental cases

with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa PRC97(2018)014606

“Worsening” the VMC wave function

Minimize expectation value of $H = T + AV18 + IL7$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- * single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- * central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- * pair correlation operators U_{ij} reflect influence of v_{ij} (AV18)
- * triple correlation operators U_{ijk} reflect the influence of V_{ijk} (IL7)

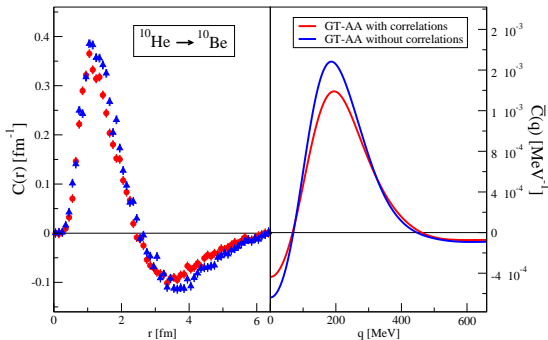
In an **uncorrelated** wave function

U_{ij} from pion-exchange and U_{ijk} are turned off

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399

Wiringa, PRC43(1991)1585

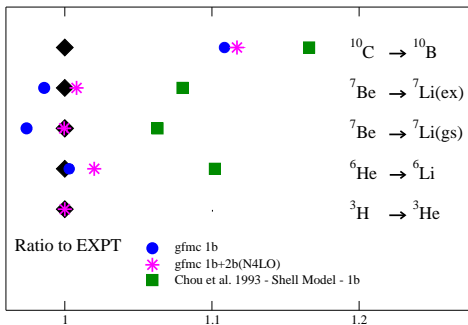
Sensitivity to 'pion-exchange-like' correlations



- * no 'pion-exchange-like' correlation operators U_{ij}
- * **yes** 'pion-exchange-like' correlation operators U_{ij}
- * $\sim 10\%$ increase in the matrix elements corresponds

with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa PRC97(2018)014606

Single Beta Decay Matrix Elements in $A = 6-10$



gfmc (1b) and gfmc (1b+2b); shell model (1b)

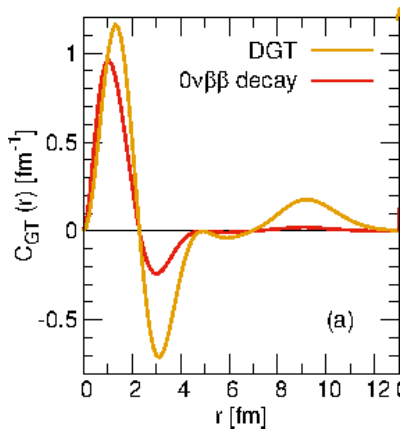
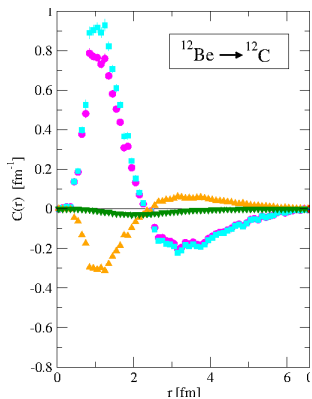
Pastore *et al.* PRC97(2018)022501

A. Baroni *et al.* PRC93(2016)015501 & PRC94(2016)024003

Based on $g_A \sim 1.27$ no quenching factor

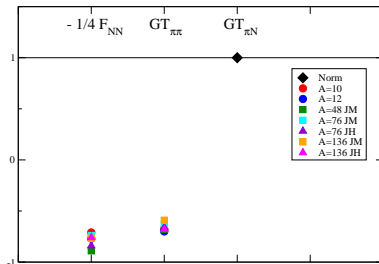
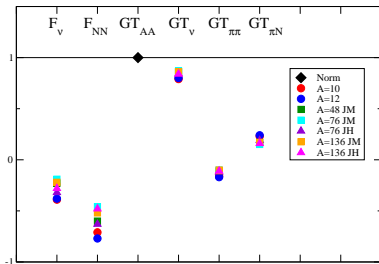
* data from TUNL, Suzuki *et al.* PRC67(2003)044302, Chou *et al.* PRC47(1993)163

Comparison with calculations of larger nuclei



J. Menendez arXiv:1712.08691

Comparison with calculations of larger nuclei



JM = Javier Menendez private communication

JH = Hyvärinen *et al.* PRC91(2015)024613

* Relative size of the matrix elements is approximately the same in all nuclei

* Short-range terms approximately the same in all nuclei

with Mereghetti & Dekens & Cirigliano & Carlson & Wiringa PRC97(2018)014606

Neutrinoless Double Beta Decay: Summary and Outlook

We studied **correlations** and **many-body currents** in single beta and neutrinoless double beta decays (NLDBD) in $A \leq 12$ nuclei

- * In single beta decays the calculations based on $g_A \sim 1.27$ are in good agreement with the data and axial two-body currents provide a negligible contribution $\sim 2\%$
- * In the neutrino-scattering Quasi Elastic kinematic region electroweak two-body are found to increase calculations based on one-body operators alone
- * In NLDBD we tested the neutrino-exchange potentials as well as contributions of one-pion and contact- range
- * Lack of correlations in the wave functions produces a $\sim 10\%$ increase in the NLDBD matrix elements

Summary and Outlook

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

- * Two-body currents can give $\sim 30 - 40\%$ contributions and improve on theory/EXPT agreement
- * Calculations of $\beta -$ and $\beta\beta -$ decay m.e.'s in $A \leq 12$ indicate two-body physics (currents and correlations) is required
- * Short-Time-Approximation to evaluate ν -A scattering in $A > 12$ nuclei is in excellent agreement with exact calculations and data
- * We are developing a coherent picture for neutrino-nucleus interactions *

Factorization: Short-Time Approximation

$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle$$

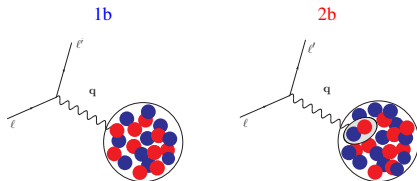
$$R_{\alpha}(q, \omega) = \int dt \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) e^{i(H-\omega)t} O_{\alpha}(\mathbf{q}) | 0 \rangle$$

At short time, expand $P(t) = e^{i(H-\omega)t}$ and keep up to 2b-terms

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

and

$$O_i^{\dagger} P(t) O_i + O_i^{\dagger} P(t) O_j + O_i^{\dagger} P(t) O_{ij} + O_{ij}^{\dagger} P(t) O_{ij}$$



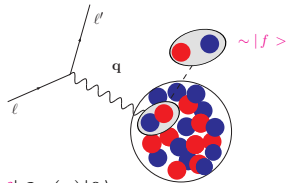
WITH

Carlson & Gandolfi (LANL) & Schiavilla (ODU+JLab) & Wiringa (ANL)

Factorization up to two-body operators: The Short-Time Approximation (STA)

In STA:

Response functions are given by the scattering off pairs of fully interacting nucleons that propagate into a correlated pair of nucleons



$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle$$

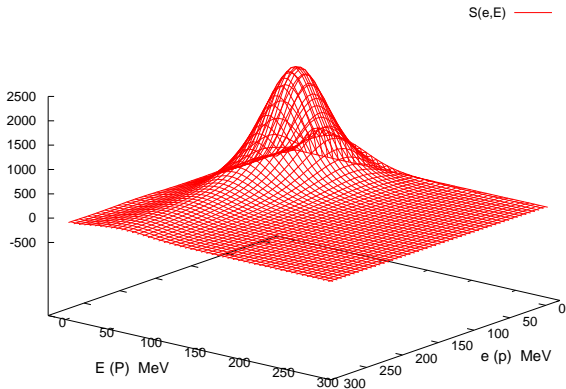
$$O_{\alpha}(\mathbf{q}) = O_{\alpha}^{(1)}(\mathbf{q}) + O_{\alpha}^{(2)}(\mathbf{q}) = 1b + 2b$$

$$|f\rangle \sim |\psi_{p,P,J,M,L,S,T,M_T}(r,R)\rangle = \text{correlated two-nucleon w.f.}$$

- * We retain **two-body physics** consistently **in the nuclear interactions** and **electroweak currents**
- * STA can be implemented to accommodate for more two-body physics, *e.g.*, pion-production induced by e and ν

$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

The Short-Time Approximation

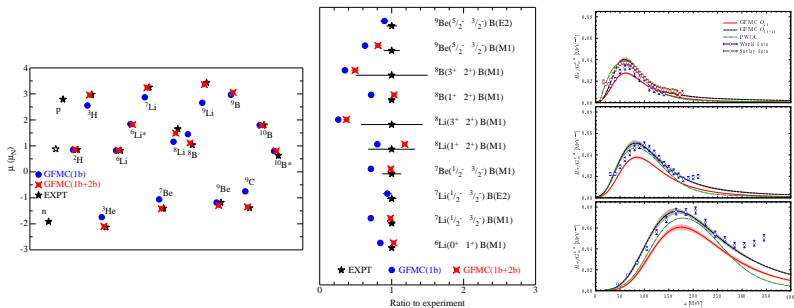


Transverse “response-density” $1b + 2b$ for ${}^4\text{He}$

$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_P d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

* Preliminary results *

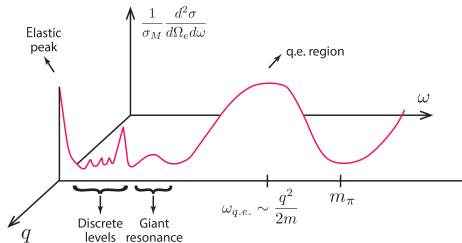
EM Moments, EM Decays and e -scattering off nuclei



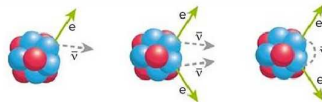
Electromagnetic data are explained when
two-body correlations and currents are accounted for!

Pastore *et al.* PRC87(2013)035503 – Lovato *et al.* PRC91(2015)062501

Towards a coherent and unified picture of neutrino-nucleus interactions



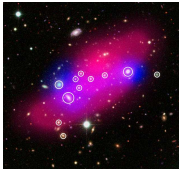
- * $\omega \sim \text{few MeV}, q \sim 0$: β -decay, $\beta\beta$ -decays
- * $\omega \lesssim \text{tens MeV}$: Nuclear Rates for Astrophysics
- * $\omega \sim 10^2 \text{ MeV}$: Accelerator neutrinos, ν -nucleus scattering



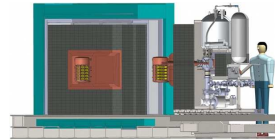
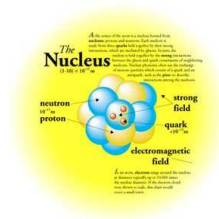
Standard β Decay Double β Decay Neutrinoless Double β Decay

Understand Nuclei to Understand the Cosmos

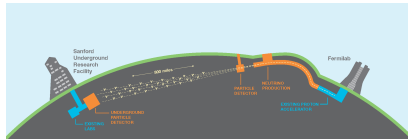
Jefferson Lab



ESA, XMM-Newton, Galdadello, CFHTL



Majorana Demonstrator

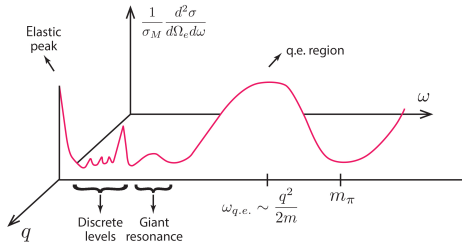


LBNF

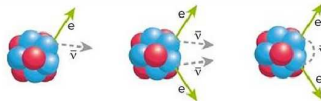
Thank you!

saori.pastore at gmail.com

Nuclear Physics for Neutrinoless Double Beta Decay: Kinematics



- $\Rightarrow \omega \sim \text{few MeV}, q \sim 0$: EM decay, β -decay, $\beta\beta$ -decays \Leftarrow
- $\Rightarrow \omega \sim \text{few MeV}, q \sim \text{hundreds of MeVs}$: $0\nu\beta\beta$ -decays \Leftarrow
- * $\omega \sim 10^2 \text{ MeV}$: Accelerator neutrinos, ν -nucleus scattering

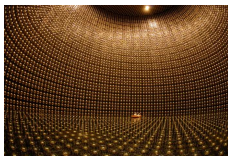
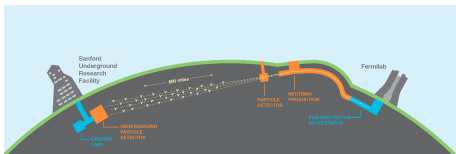


Standard β Decay

Double β Decay

Neutrinoless Double β Decay

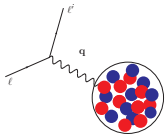
Nuclei for Accelerator Neutrinos' Experiments



LBNF

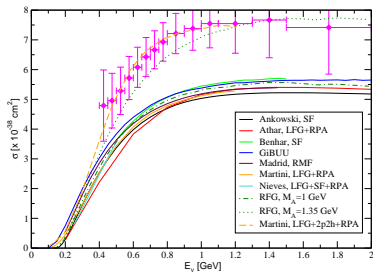
T2K

Neutrino-Nucleus scattering



$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{2E_\nu} \right)$$

CCQE on ^{12}C

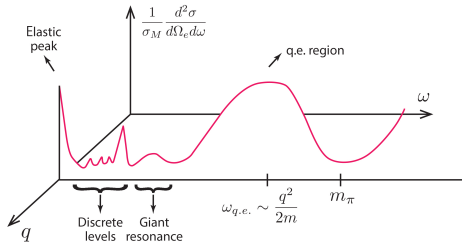


Alvarez-Ruso [arXiv:1012.3871](https://arxiv.org/abs/1012.3871)

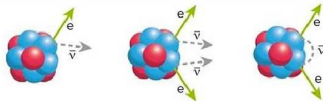
* Nuclei of ^{12}C , ^{40}Ar , ^{16}O , ^{56}Fe , ... *

are the DUNE, MiniBoone, T2K, Minerva ... detectors' active material

Nuclear Structure and Dynamics



- * $\omega \sim \text{few MeV}, q \sim 0$: EM decay, β -decay, $\beta\beta$ -decays
- * $\omega \lesssim \text{tens MeV}$: Nuclear Rates for Astrophysics
- * $\omega \sim 10^2 \text{ MeV}$: Accelerator neutrinos, ν -nucleus scattering

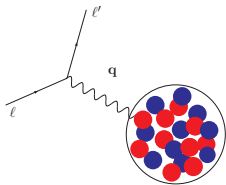


Standard β Decay

Double β Decay

Neutrinoless Double β Decay

The Microscopic (or *ab initio*) Description of Nuclei



Develop a **comprehensive theory** that describes **quantitatively** and **predictably** **all** nuclear structure and reactions

- * Accurate understanding of **interactions between nucleons**, *p*'s and *n*'s
- * and between *e*'s, *v*'s, **DM**, ..., with nucleons, nucleons-pairs, ...

$$H\Psi = E\Psi$$

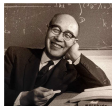
$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A)$$



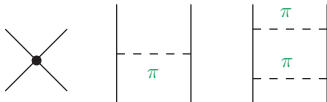
Erwin Schrödinger

Nuclear Force These Days

- * 1930s Yukawa Potential
- * 1960–1990 Highly sophisticated meson exchange potentials
- * 1990s– Highly sophisticated Chiral Effective Field Theory based potentials



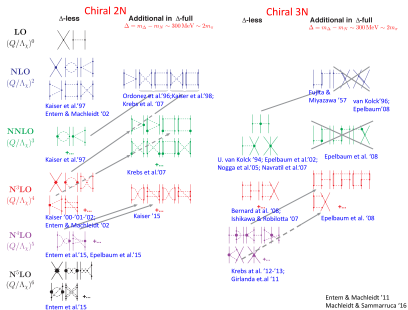
Hideki Yukawa



Steven Weinberg

- * Contact terms: short-range
- * One-pion-exchange: range $\sim \frac{1}{m_\pi}$
- * Two-pion-exchange: range $\sim \frac{1}{2m_\pi}$

Nuclear Interactions and the role of the Δ



Courtesy of Maria Piarulli

* N3LO with Δ nucleon-nucleon interaction constructed by Piarulli *et al.* in PRC91(2015)024003-PRC94(2016)054007-arXiv:1707.02883 with Δ 's fits ~ 2000 (~ 3000) data up 125 (200) MeV with $\chi^2/\text{datum} \sim 1$;

* N2LO with Δ 3-nucleon force fits ^3H binding energy and the nd scattering length

$$v_{12} = \sum_p v_{12}^p(r) O_{12}; \quad O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, (\mathbf{L} \cdot \mathbf{S})^2] \otimes [1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

+ operators 4 terms breaking charge independence

Phenomenological aka Conventional aka Traditional aka Realistic Two- and Three- Nucleon Potentials

NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk}$$

K_i : Non-relativistic kinetic energy, m_n - m_p effects included

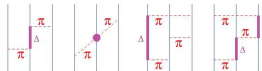
Argonne v_{18} : $v_{ij} = v_{ij}^T + v_{ij}^\pi + v_{ij}^I + v_{ij}^S = \sum_p v_p(r_{ij}) O_{ij}^p$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with $\chi^2/\text{d.o.f.}=1.1$

Wiringa, Stoks, & Schiavilla, PRC 51, (1995)

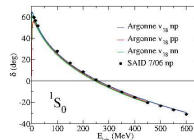
Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^R$

- Urbana has standard 2π P -wave + short-range repulsion for matter saturation
- Illinois adds 2π S -wave + 3π rings to provide extra $T=3/2$ interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei



Pieper, Pandharipande, Wiringa, & Carlson, PRC 64, 014001 (2001)

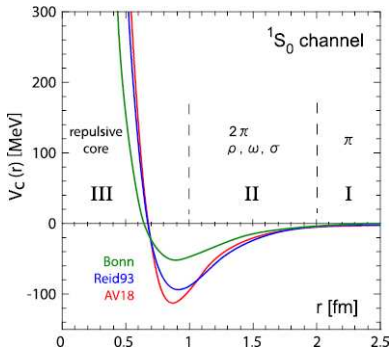
Pieper, AIP CP 1011, 143 (2008)



Courtesy of Bob Wiringa

- * AV18 fitted up to 350 MeV, reproduces phase shifts up to ~ 1 GeV *
- * IL7 fitted to 23 energy levels, predicts hundreds of levels *

Nucleon-nucleon potential



Aoki *et al.* [Comput.Sci.Disc.1\(2008\)015009](#)

CT = Contact Term* - short-range;

OPE = One Pion Exchange - range $\sim \frac{1}{m_\pi}$;

TPE = Two Pion Exchange - range $\sim \frac{1}{2m_\pi}$

* in practice CT's in r -space are coded with representations of a δ -function (*e.g.*, a Gaussian function), or special functions such as Wood-Saxon functions

ρ, ω, σ -exchange

The One Boson Exchange (OBE) Lagrangians

scalar

$$-g^{S0} \bar{\psi} \psi \phi^{S0} \qquad -g^{S1} \bar{\psi} \tau \psi \cdot \vec{\phi}^{S1}$$

pseudo-scalar

$$-ig^{PS0} \bar{\psi} \gamma_5 \psi \phi^{PS0} \qquad -ig^{PS1} \bar{\psi} \gamma_5 \tau \psi \cdot \vec{\phi}^{PS1}$$

vector

$$-g^{V0} \bar{\psi} \gamma^\mu \psi \phi_{\mu}^{V0} \qquad -g^{V1} \bar{\psi} \gamma^\mu \tau \psi \cdot \vec{\phi}_{\mu}^{V1}$$

tensor

$$\frac{-g^{T0}}{2m^{T0}} \bar{\psi} \sigma^{\mu\nu} \psi \partial_\nu \phi_{\mu}^{T0} \qquad \frac{-g^{T1}}{2m^{T1}} \bar{\psi} \sigma^{\mu\nu} \tau \psi \cdot \partial_\nu \vec{\phi}_{\mu}^{T1}$$

slide from my 15 mins HUGS talk...

CD Bonn Potential

	Mass (MeV)	I	J^π	$\frac{g^2}{4\pi}$	$\frac{g^T}{g_V}$	
π^\pm	139.56995	1	0^-	13.6		<i>PS1</i>
π^0	134.9764	1	0^-	13.6		<i>PS1</i>
η	547.3	0	0^-	0.4		<i>PS0</i>
ρ^\pm, ρ^0	769.9	1	1^-	0.84	6.1	<i>V1; T1</i>
ω	781.94	0	1^-	20.0	0.0	<i>V0; T0</i>
σ	400-1200	0	0^+			<i>S0</i>

R.Machleidt, Phys.Rev. C63, 014001 (2001)

$$O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}, \mathbf{L} \cdot \mathbf{S}] \otimes [1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

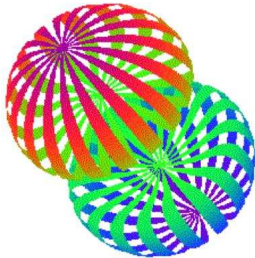
vs

$$O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2] \otimes [1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]; S_{12} \text{ from } 2\pi - \text{exchange}$$

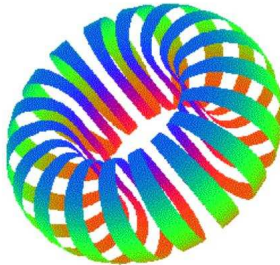
slide from my 15 mins HUGS...

Nucleon-Nucleon Potential and the Deuteron

$M = \pm 1$



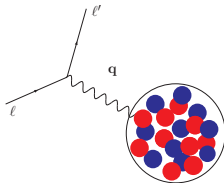
$M = 0$



Constant density surfaces for a polarized deuteron in the $M = \pm 1$ (left) and $M = 0$ (right) states

Carlson and Schiavilla [Rev.Mod.Phys.70\(1998\)743](#)

Quantum Monte Carlo Methods



Solve numerically the many-body problem

$$H\Psi = E\Psi$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A)$$

Ψ are spin-isospin vectors in $3A$ dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

${}^4\text{He}$: 96

${}^6\text{Li}$: 1280

${}^8\text{Li}$: 14336

${}^{12}\text{C}$: 540572

Variational Monte Carlo (VMC)

Minimize expectation value of $H = T + \text{AV18} + \text{IL7}$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- * single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- * central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- * pair correlation operators U_{ij} reflect influence of v_{ij} (AV18)
- * triple correlation operators U_{ijk} reflect the influence of V_{ijk} (IL7)

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399

Wiringa, PRC43(1991)1585

Green's function Monte Carlo (GFMC)

Ψ_V can be further improved by “filtering” out the remaining excited state contamination

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n \psi_n$$

$$\Psi(\tau \rightarrow \infty) = a_0 \psi_0$$

In practice, we evaluate a “mixed” estimates

$$\langle O(\tau) \rangle = \frac{f \langle \Psi(\tau) | O | \Psi(\tau) \rangle_i}{\langle \Psi(\tau) | \Psi(\tau) \rangle} \approx \langle O(\tau) \rangle_{\text{Mixed}}^i + \langle O(\tau) \rangle_{\text{Mixed}}^f - \langle O \rangle_V$$

$$\langle O(\tau) \rangle_{\text{Mixed}}^i = \frac{f \langle \Psi_V | O | \Psi(\tau) \rangle_i}{f \langle \Psi_V | \Psi(\tau) \rangle_i} ; \quad \langle O(\tau) \rangle_{\text{Mixed}}^f = \frac{f \langle \Psi(\tau) | O | \Psi_V \rangle_i}{f \langle \Psi(\tau) | \Psi_V \rangle_i}$$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)

Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)

Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

GFMC Energy calculation: An example

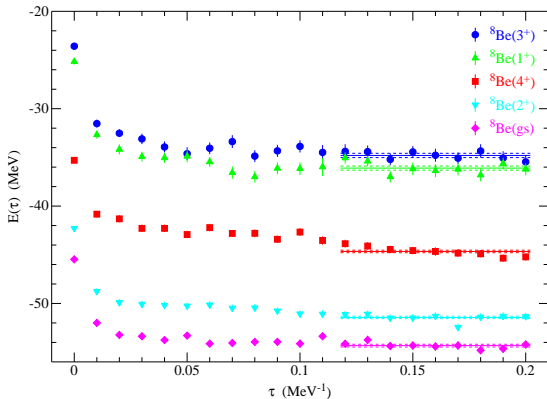
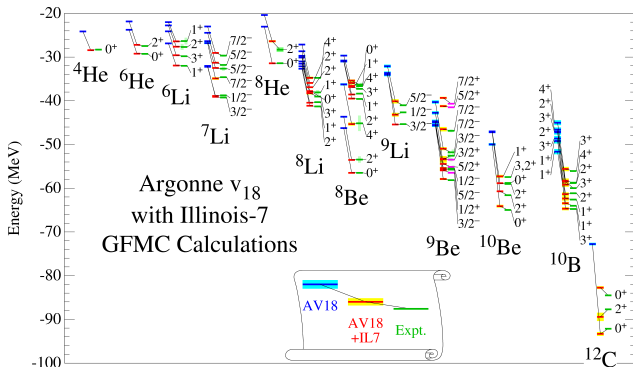


Fig. 6 (Wiringa, et al.)

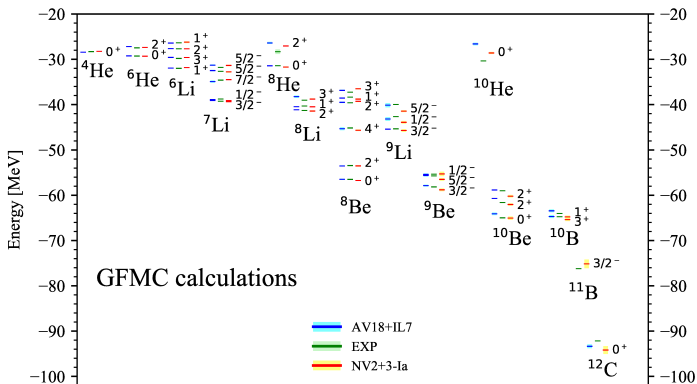
Wiringa *et al.* PRC62(2000)014001

Spectra of Light Nuclei



Carlson *et al.* Rev.Mod.Phys.87(2015)1067

Spectra of Light Nuclei



M. Piarulli *et al.* - arXiv:1707.02883

- * one-pion-exchange physics dominates *
- * it is included in both chiral and “conventional” potentials *

Three-body forces

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

$$V_{ijk} \sim (0.2 - 0.9) v_{ij} \sim (0.15 - 0.6) H$$

$$v_{\pi} \sim 0.83 v_{ij}$$

¹⁰B VMC code output

$$T_i + V_{ij} = -38.2131 (0.1433) \quad + V_{ijk} = -46.7975 (0.1150)$$

$$T_i = 290.3220 (1.2932) \quad V_{ij} = -328.5351 (1.1983) \quad V_{ijk} = -8.5844 (0.0892)$$

Two-body physics dominates!

(Very) Incomplete List of Credits and Reading Material

- * Pieper and Wiringa; [Ann.Rev.Nucl.Part.Sci.51\(2001\)53](#)
- * Carlson *et al.*; [Rev.Mod.Phys.87\(2015\)1067](#)
- * van Kolck *et al.*; [PRL72\(1994\)1982-PRC53\(1996\)2086](#)
- * Kaiser, Weise *et al.*; [NPA625\(1997\)758-NPA637\(1998\)395](#)
- * Epelbaum, Glöckle, Meissner*; [RevModPhys81\(2009\)1773](#) and references therein
- * Entem and Machleidt*; [PhysRept503\(2011\)1](#) and references therein

* NN Potentials suited for Quantum Monte Carlo calculations *

- * Pieper and Wiringa; [Ann.Rev.Nucl.Part.Sci.51\(2001\)53](#)
- * Gezerlis *et al.* and Lynn *et al.*;
[PRL111\(2013\)032501,PRC90\(2014\)054323,PRL113\(2014\)192501](#);
- * Piarulli *et al.*; [PRC91\(2015\)024003-PRC94\(2016\)054007-arXiv:1707.02883](#)

Summary: Nuclear Interactions

- * The Microscopic description of Nuclei is very successful
- * Nuclear two-body forces are constrained by large database of nucleon-nucleon scattering data
- * Intermediate- and long-range components are described in terms of one- and two-pion exchange potentials
- * Short-range parts are described by contact terms or special functions
- * Due to a cancellation between kinetic and two-body contribution, three-body potentials are (small but) necessary to reach (excellent) agreement with the data
- * Calculated spectra of light nuclei are reproduced within 1 – 2% of expt data
- * Two-body one-pion-exchange contributions dominate and are crucial to explain the data

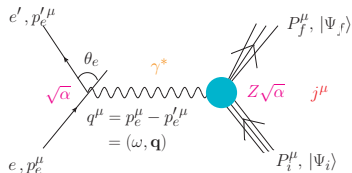
Neutrinos (Fundamental Symmetries) and Nuclei

Topics (5 hours)

- * Nuclear Theory for the Neutrino Experimental Program ✓
- * Microscopic (or *ab initio*) Description of Nuclei ✓
- * “Realistic” Models of Two- and Three-Nucleon Interactions ✓
- * “Realistic” Models of Many-Body Nuclear Electroweak Currents
- * Short-range Structure of Nuclei and Nuclear Correlations
- * Quasi-Elastic Electron and Neutrino Scattering off Nuclei
- * Validation of the theory against available data

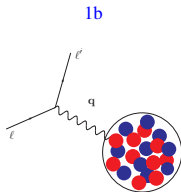


Electromagnetic Probes as tool to test theoretical models



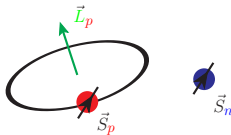
- * coupling constant $\alpha \sim 1/137$ allows for a perturbative treatment of the **EM** interaction; single photon γ exchange suffices
- * calculated x-sections factorize into a part $\propto |\langle \Psi_f | j^\mu | \Psi_i \rangle|^2$ with **j^μ nuclear EM currents** and a part completely specified by the electron kinematic variables
- * EXPT data are (in most cases) known with great accuracy providing stringent constraints on theories
- * For light nuclei, the many-body problem can be solved exactly or within controlled approximations

Nuclear Currents: One Body Component



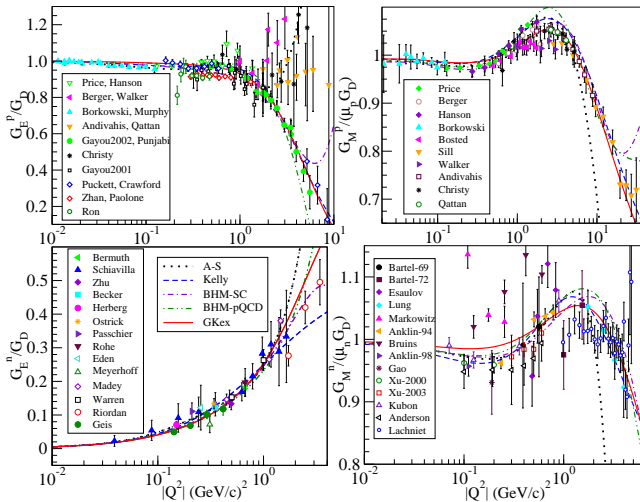
$$\rho = \sum_{i=1}^A \rho_i + \dots,$$
$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \dots$$

- * Nuclear currents given by the sum of p 's and n 's currents, **one-body currents** (1b)



- * Nucleonic electroweak form factors are taken from experimental data, and, in principle, from LQCD calculations where data are poor or scarce (e.g., nucleonic axial form factor)
- * A description based on 1b operators alone fails to reproduce “basic” observables (magnetic moments, np radiative capture)
- * corrections from two-body meson-exchange currents are required to explain, e.g., radiative capture Riska&Brown 1972

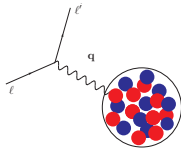
Electromagnetic Nucleonic Form Factors



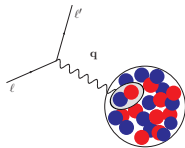
González-Jiménez Phys.Rept.524(2013)1-35

Nuclear Currents: Two-Body Component

1b



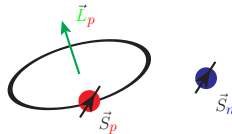
2b



$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots,$$

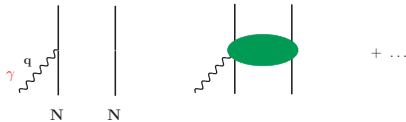
$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

* Nuclear currents given by the sum of p 's and n 's currents, **one-body currents (1b)**



* **Two-body currents (2b)** essential to satisfy current conservation

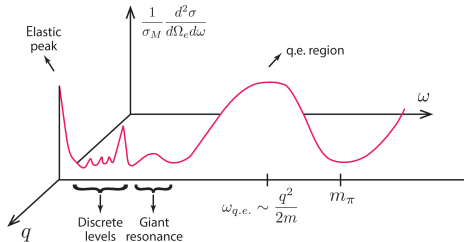
* We use **MEC (SNPA)** or **χ EFT currents**



$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + \mathbf{v}_{ij} + V_{ijk}, \rho]$$

$$\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \text{ classically}$$

Electromagnetic Reactions



- * $\omega \sim \text{few MeV}$, $q \sim 0$: EM-decays
- * $\omega \sim 10^2 \text{ MeV}$: e -nucleus scattering

A coherent and accurate picture of the way electrons interact with nuclei in a wide range of energy and momenta **exists**, provided that **two-body correlations** and **two-body currents** are accounted for!

Electromagnetic Currents from Nuclear Interactions

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

- 1) Longitudinal component fixed by current conservation
- 2) Plus transverse “phenomenological” terms

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V)$$

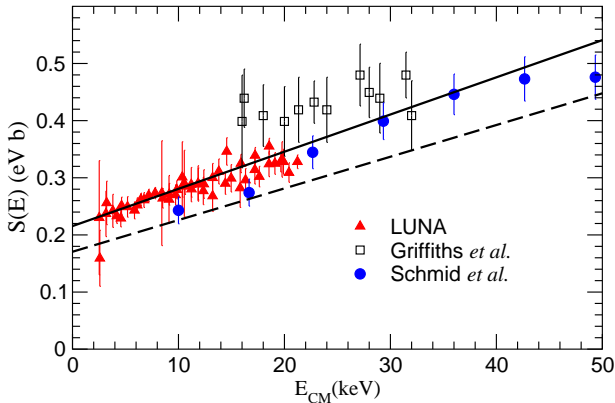
transverse

Villars, Myiazawa (40-ies), Chemtob, Riska, Schiavilla ...
 see, *e.g.*, [Marcucci *et al.* PRC72\(2005\)014001](#) and references therein

Currents from nuclear interactions

Satisfactory description of a variety of nuclear em properties in $A \leq 12$

${}^2\text{H}(p,\gamma){}^3\text{He}$ capture



Marcucci *et al.* PRC72, 014001 (2005)

Currents from χ EFT - Time-Ordered-Perturbation Theory

The relevant degrees of freedom of nuclear physics are bound states of QCD

- * non relativistic nucleons \mathbf{N}
- * pions π as mediators of the nucleon-nucleon interaction
- * non relativistic Delta's Δ with $m_\Delta \sim m_N + 2m_\pi$

Transition amplitude in time-ordered perturbation theory

$$T_{fi} = \langle N'N' | H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} | NN \rangle^*$$

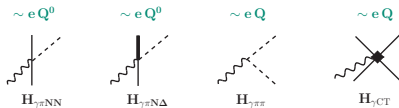
H_0 = free π , \mathbf{N} , Δ Hamiltonians

H_1 = interacting π , \mathbf{N} , Δ , and external electroweak fields Hamiltonians

$$T_{fi} = \langle N'N' | T | NN \rangle \propto \mathbf{v}_{ij}, \quad T_{fi} = \langle N'N' | T | NN; \gamma \rangle \propto (A^0 \boldsymbol{\rho}_{ij}, \mathbf{A} \cdot \mathbf{j}_{ij})$$

* $A^\mu = (A^0, \mathbf{A})$ photon field

External Electromagnetic Field



“Minimal” Electromagnetic Vertices

- * EM H_1 obtained by minimal substitution in the π - and N -derivative couplings
(same as doing $\mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A}$, minimal coupling)

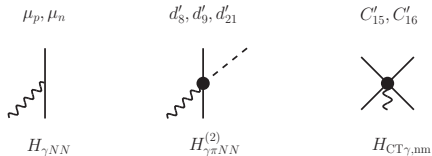
$$\nabla\pi_{\mp}(\mathbf{x}) \rightarrow [\nabla \mp ie\mathbf{A}(\mathbf{x})]\pi_{\mp}(\mathbf{x})$$

$$\nabla N(\mathbf{x}) \rightarrow [\nabla - iee_N\mathbf{A}(\mathbf{x})]N(\mathbf{x}), \quad e_N = (1 + \tau_z)/2$$

* same LECs as the Strong Vertices *

- * This is equivalent to say that the currents are conserved,
i.e., the continuity equation is satisfied

External Electromagnetic Field



“Non-Minimal” Electromagnetic Vertices

- * EM H_1 involving the tensor field $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$

LECs are **not** constrained by the strong interaction
there are **additional LECs** fixed to EM observables

- * $H_{\gamma NN}$ obtained by non-relativistic reduction of the covariant single nucleon currents constrained to $\mu_p = 2.793$ n.m. and $\mu_n = -1.913$ n.m.
- * $H_{\gamma\pi NN}$ involves $\nabla\pi$ and ∇N and **3 new LECs** (2 of them “mimicking” Δ)
- * $H_{CT2\gamma}$ involves **2 new LECs**

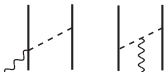
* These are the so called the “transverse” currents

EM Currents \mathbf{j} from Chiral Effective Field Theory

LO : $\mathbf{j}^{(-2)} \sim eQ^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim eQ^{-1}$



N²LO : $\mathbf{j}^{(-0)} \sim eQ^0$



* Note that \mathbf{j}_π satisfies the continuity equation with v_π (can be done analytically)

$$v_\pi(\mathbf{k}) = -\frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$$\mathbf{j}_\pi(\mathbf{k}_1, \mathbf{k}_2) = -ie \frac{g_A^2}{F_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \boldsymbol{\sigma}_1 \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{\omega_{k_2}^2} + 1 \Leftrightarrow 2$$

$$+ ie \frac{g_A^2}{F_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \frac{\mathbf{k}_1 - \mathbf{k}_2}{\omega_{k_1}^2 \omega_{k_2}^2} \boldsymbol{\sigma}_1 \cdot \mathbf{k}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{k}_2$$

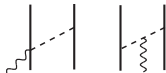
* LO = one-body current *

EM Currents \mathbf{j} from Chiral Effective Field Theory

LO : $\mathbf{j}^{(-2)} \sim eQ^{-2}$



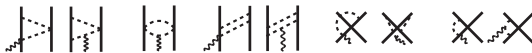
NLO : $\mathbf{j}^{(-1)} \sim eQ^{-1}$



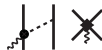
N²LO : $\mathbf{j}^{(-0)} \sim eQ^0$



N³LO : $\mathbf{j}^{(1)} \sim eQ$



unknown LEC's →



No three-body currents at this order!

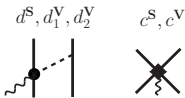
- * Analogue expansion exists for the Time Component (Charge Operator) ρ
- * Two-body corrections to the one-body Charge Operator appear at N3LO

Pastore *et al.* PRC78(2008)064002 & PRC80(2009)034004 & PRC84(2011)024001

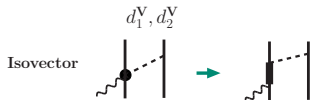
- * analogue expansion exists for the Axial nuclear current - Baroni *et al.* PRC93 (2016)015501 *

also derived by Park+Min+Rho NPA596(1996)515, Kölling+Epelbaum+Krebs+Meissner PRC80(2009)045502 & PRC84(2011)054008

Electromagnetic LECs



d^S , d_1^V , and d_2^V could be determined by $\pi\gamma$ -production data on the nucleon



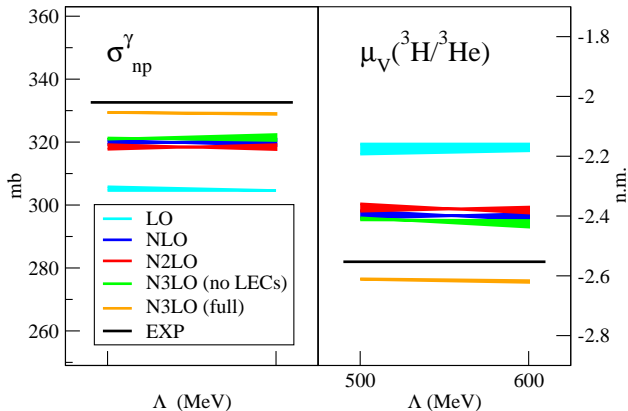
$d_2^V = 4\mu^* h_A / 9m_N (m_\Delta - m_N)$ and
 $d_1^V = 0.25 \times d_2^V$
 assuming Δ -resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

- * Isoscalar sector:
 - * d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- * Isovector sector:
 - * c^V from EXPT $n\text{pd}\gamma$ xsec.
 - or
 - * c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m.

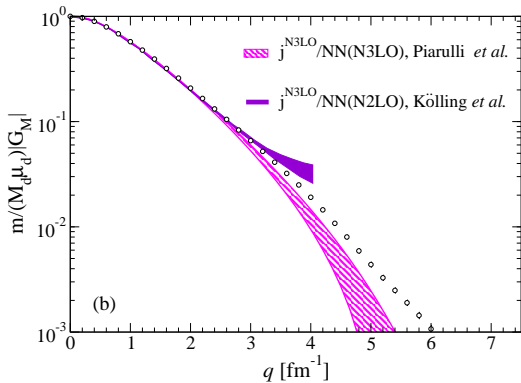
Low-energy observables and ground state properties

np capture x-section/ μ_V of $A = 3$ nuclei



$$\text{Observable} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

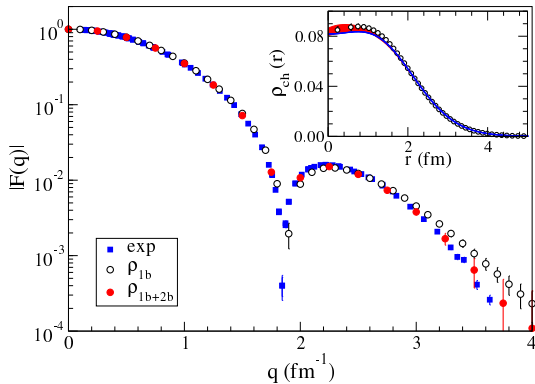
Deuteron magnetic form factor



$$\text{Observable} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

PRC86(2012)047001 & PRC87(2013)014006

^{12}C Charge form factor

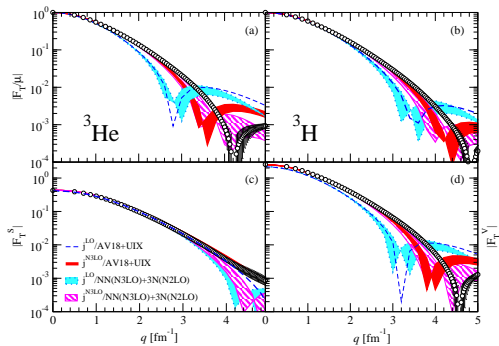


$$\propto \langle \Psi_f | \rho | \Psi_i \rangle$$

Lovato *et al.*

PRL111(2013)092501

^3He and ^3H magnetic form factors

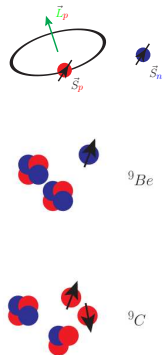
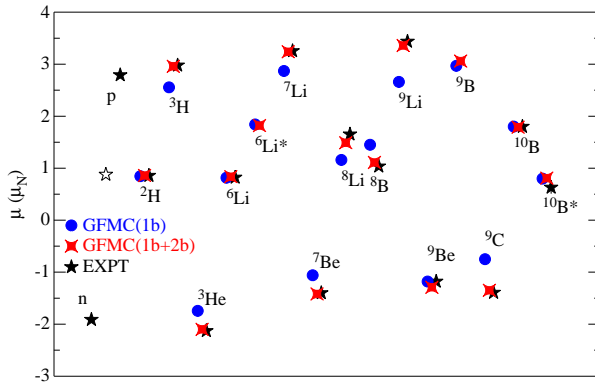


1b/1b+2b with AV18+UIX – 1b/1b+2b with χ -potentials NN(N3LO)+3N(N2LO)

$$\text{Observable} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

Piarulli *et al.* PRC87(2013)014006

Magnetic Moments of Nuclei

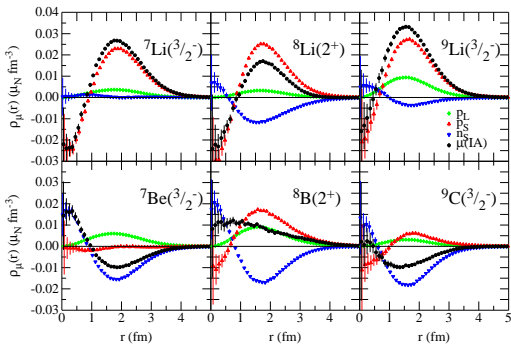


m.m.	THEO	EXP
${}^9\text{C}$	-1.35(4)(7)	-1.3914(5)
${}^9\text{Li}$	3.36(4)(8)	3.4391(6)

chiral truncation error based on [EE et al.](#) error algorithm, [Epelbaum, Krebs, and Meissner EPJA51\(2015\)53](#)

[Pastore et al. PRC87\(2013\)035503](#)

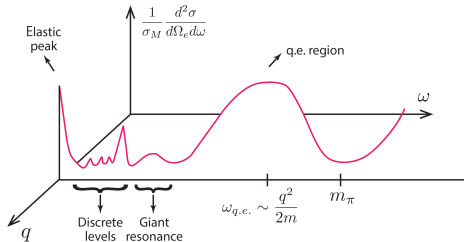
One-body magnetic densities



1b magnetic moment operator

$$\mu_{1b} = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Electromagnetic Reactions



- * $\omega \sim \text{few MeV}, q \sim 0$: EM-decays
- * $\omega \sim 10^2 \text{ MeV}$: e -nucleus scattering

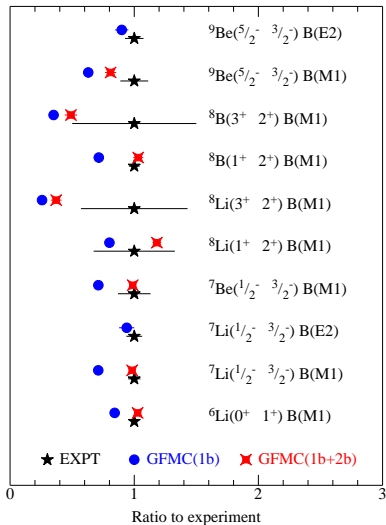
A coherent and accurate picture of the way electrons interact with nuclei in a wide range of energy and momenta exists, provided that **two-body correlations and two-body currents** are accounted for!

Electromagnetic Transitions in Light Nuclei

- * **2b** electromagnetic currents bring the THEORY in agreement with the EXPT
- * $\sim 40\%$ **2b**-current contribution found in ${}^9\text{C}$ m.m.
- * $\sim 60 - 70\%$ of total **2b**-current component is due to one-pion-exchange currents
- * $\sim 20-30\%$ **2b** found in M1 transitions in ${}^8\text{Be}$

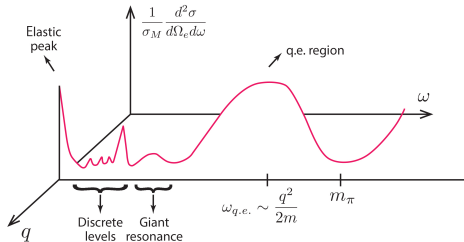
One M1 prediction: ${}^9\text{Li}(1/2 \rightarrow 3/2)^*$
+ a number of B(E2)s

*2014 TRIUMF proposal Ricard-McCutchan *et al.*



Pastore *et al.* PRC87(2013)035503 & PRC90(2014)024321, Datar *et al.* PRL111(2013)062502

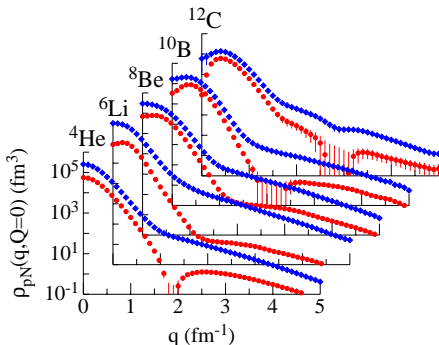
Electromagnetic Reactions



- * $\omega \sim \text{few MeV}, q \sim 0$: EM-decays
- * $\omega \sim 10^2 \text{ MeV}$: e -nucleus scattering

A coherent and accurate picture of the way electrons interact with nuclei in a wide range of energy and momenta **exists**, provided that **two-body correlations** and **two-body currents** are accounted for!

Back-to-back np and pp Momentum Distributions



Wiringa *et al.* - [PRC89\(2014\)024305](#)

Nuclear properties are strongly affected by correlations!

Triple coincidence reactions $A(e, e' np \text{ or } pp)A - 2$ measurements at JLab on ^{12}C indicate that at high values of relative momenta (400 – 500 MeV), $\sim 90\%$ of the pairs are in the form of np pairs and $\sim 5\%$ in pp pairs

Inclusive (e, e') scattering

* inclusive xsecs *

$$\frac{d^2\sigma}{dE'd\Omega_{e'}} = \sigma_M [v_L R_L(q, \omega) + v_T R_T(q, \omega)]$$

$$R_\alpha(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by $O_L = \rho$

Transverse response induced by $O_T = \mathbf{j}$

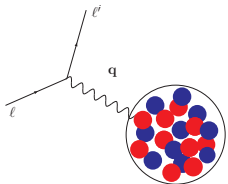
* Sum Rules *

Exploit integral properties of the response functions + closure to avoid explicit calculation of the final states

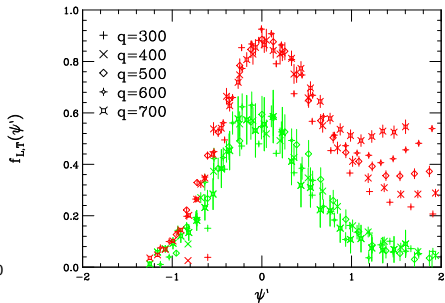
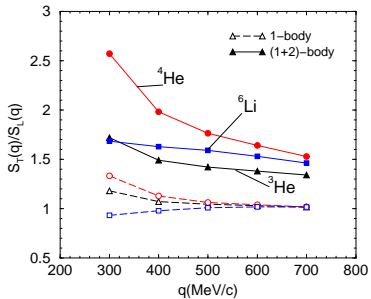
$$S(q, \tau) = \int_0^\infty d\omega K(\tau, \omega) R_\alpha(q, \omega)$$

* Coulomb Sum Rules *

$$S_\alpha(q) = \int_0^\infty d\omega R_\alpha(q, \omega) \propto \langle 0 | O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q}) | 0 \rangle$$

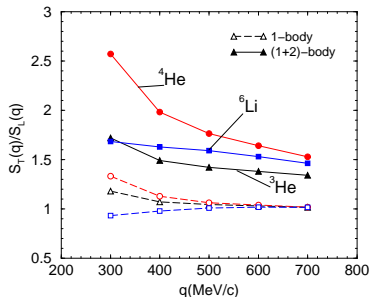


Sum Rules and the role of two-body currents



Carlson, Jourdan, Schiavilla, and Sick PRC65(2002)024002

Sum Rules and Two-Body Physics



- $S_T(q) \propto \langle 0 | \mathbf{j}^\dagger \mathbf{j} | 0 \rangle$
- $\mathbf{j} = \mathbf{j}_{1b} + \mathbf{j}_{2b}$
- enhancement of the transverse response is due to interference between **1b** and **2b** contributions **AND** presence of **correlations** in the wave function

PRC65(2002)024002

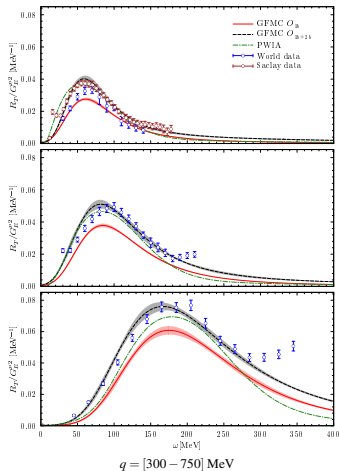


$$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} \rangle > 0$$



$$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} v_\pi \rangle \propto \langle v_\pi^2 \rangle > 0$$

Recent Developments on ^{12}C

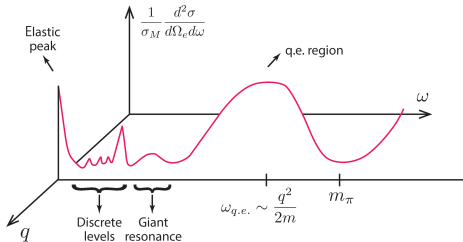


~ 100 million core hours

Lovato, Gandolfi *et al.* PRC91(2015)062501 + arXiv:1605.00248

Two-body correlations and currents essential to explain the data!

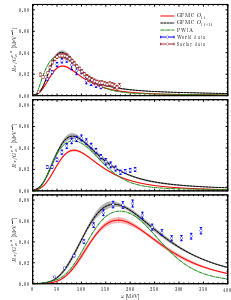
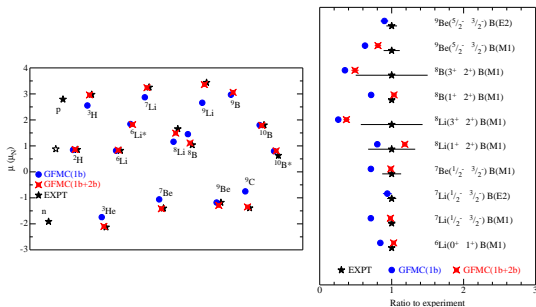
Electromagnetic Reactions



- * $\omega \sim \text{few MeV}, q \sim 0$: EM-decays
- * $\omega \sim 10^2 \text{ MeV}$: e -nucleus scattering

A coherent and accurate picture of the way electrons interact with nuclei in a wide range of energy and momenta **exists**, provided that **two-body correlations** and **two-body currents** are accounted for!

EM Moments, EM Decays and e -scattering off nuclei



Electromagnetic data are explained when
two-body correlations and **currents** are accounted for!

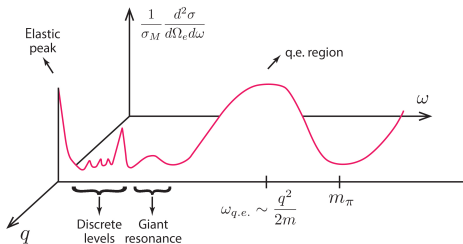
Pastore *et al.* PRC87(2013)035503 – Lovato *et al.* PRC91(2015)062501

Two-body Currents: Summary

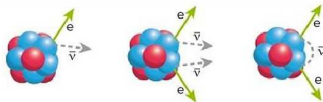
- * Two-body correlations and currents are essential to explain the data
 - * Two-body currents provide up to $\sim 40\%$ contributions to the magnetic moments of nuclei (ground state observable)
 - * Two-body currents enhance the transverse response up $\sim 50\%$ (dynamical observable)
 - * One-pion-exchange currents provide $\sim 0.8 \mathbf{j}_{ij}$

Neutrinos and Nuclei

Towards a coherent and unified picture of neutrino-nucleus interactions



- * $\omega \sim \text{few MeV}, q \sim 0$: β -decay, $\beta\beta$ -decays
- * $\omega \lesssim \text{tens MeV}$: Nuclear Rates for Astrophysics
- * $\omega \sim 10^2 \text{ MeV}$: Accelerator neutrinos, ν -nucleus scattering



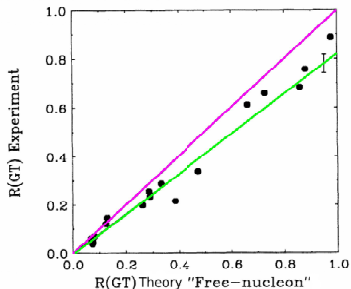
Standard β Decay

Double β Decay

Neutrinoless Double β Decay

Neutrinos and Nuclei: Challenges and Opportunities

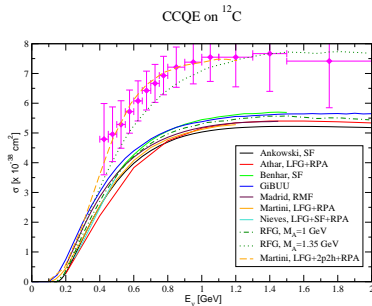
Beta Decay Rate



in $3 \leq A \leq 18 \rightarrow g_A^{\text{eff}} \simeq 0.80 g_A$

Chou *et al.* [PRC47\(1993\)163](#)

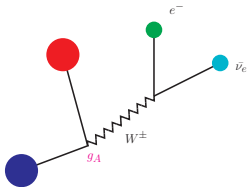
Neutrino-Nucleus Scattering



Alvarez-Ruso [arXiv:1012.3871](#)

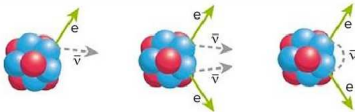
Standard Beta Decay

The “ g_A problem”
and
the role of two-body correlations and two-body currents



* Matrix Element $\langle \Psi_f | GT | \Psi_i \rangle \propto g_A$ and Decay Rates $\propto g_A^2$ *

$$(Z, N) \rightarrow (Z+1, N-1) + e + \bar{\nu}_e$$



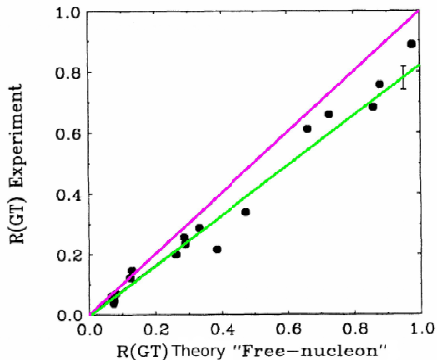
Standard β Decay

Double β Decay

Neutrinoless Double β Decay

“Anomalies” $q \sim 0$: The “ g_A problem”

Gamow-Teller Matrix Elements Theory vs Expt



in $3 \leq A \leq 18 \rightarrow g_A^{\text{eff}} \simeq 0.80 g_A$

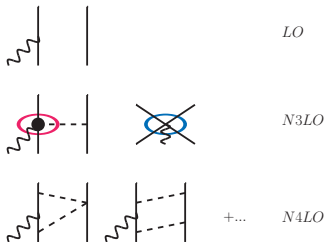
Chou *et al.* [PRC47\(1993\)163](#)

Missing Physics: 1. Correlations and/or 2. Two-body currents

Nuclear Interactions and Axial Currents

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

so far results are available with **AV18+IL7** ($A \leq 10$)
and SNPA or chiral currents (*a.k.a.* hybrid calculations)



A. Baroni *et al.* PRC93(2016)015501

H. Krebs *et al.* Ann.Phys.378(2017)

* c_3 and c_4 are taken them from Entem and Machleidt PRC68(2003)041001 & Phys.Rep.503(2011)1

* c_D fitted to GT m.e. of tritium Baroni *et al.* PRC94(2016)024003

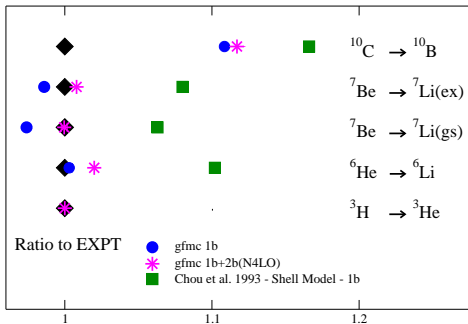
* cutoffs $\Lambda = 500$ and 600 MeV

* include also N4LO 3b currents (tiny)

* derived by Park *et al.* in the '90 used (mainly at tree-level) in many calculations

* pion-pole at tree-level derived by Klos, Hoferichter *et al.* PLB(2015)B746

Single Beta Decay Matrix Elements in $A = 6-10$



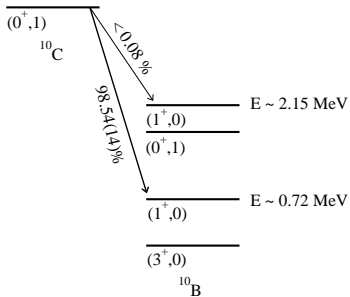
gfm (1b) and gfm (1b+2b); shell model (1b)

Pastore *et al.* PRC97(2018)022501

A. Baroni *et al.* PRC93(2016)015501 & PRC94(2016)024003

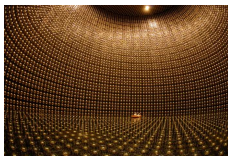
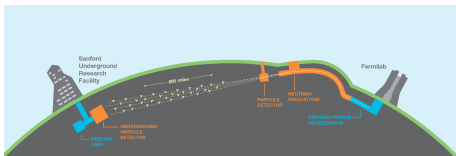
Based on $g_A \sim 1.27$ no quenching factor

* data from TUNL, Suzuki *et al.* PRC67(2003)044302, Chou *et al.* PRC47(1993)163

^{10}B 

- * In ^{10}B , ΔE with same quantum numbers $\sim 1.5 \text{ MeV}$
- * In $A = 7$, ΔE with same quantum numbers $\gtrsim 10 \text{ MeV}$

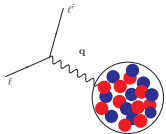
Nuclei for Accelerator Neutrinos' Experiments



LBNF

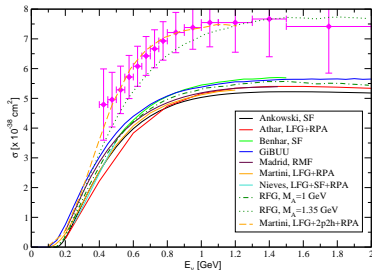
T2K

Neutrino-Nucleus scattering



$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{2E_\nu} \right)$$

CCQE on ^{12}C

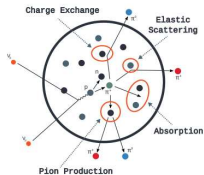


Alvarez-Ruso [arXiv:1012.3871](https://arxiv.org/abs/1012.3871)

* Nuclei of ^{12}C , ^{40}Ar , ^{16}O , ^{56}Fe , ... *

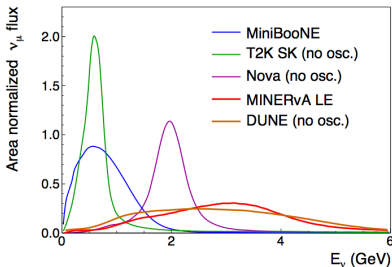
are the DUNE, MiniBoone, T2K, Minerva ... detectors' active material

Nuclei for Accelerator Neutrinos' Experiments: More in Detail



Tomasz Golan

Neutrino Flux



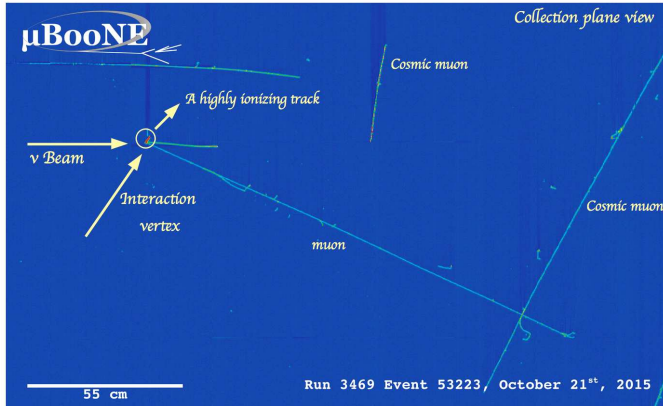
Phil Rodrigues

- * Oscillation Probabilities depend on the initial neutrino energy E_{ν}
- * Neutrinos are produced via decay-processes, E_{ν} is unknown!

$$P(\nu_{\mu} \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{2E_{\nu}} \right)$$

- * E_{ν} is reconstructed from the final state observed in the detector
- * **!! Accurate theoretical neutrino-nucleus cross sections are vital !!**
to E_{ν} reconstruction

$e - A$ and $\nu - A$ Scattering



μ Boone

Inclusive (e, ν scattering)

* inclusive xsecs *

$$\frac{d^2\sigma}{dE_f d\Omega_{e'}} = \sigma_M [v_L R_L(q, \omega) + v_T R_T(q, \omega)]$$

$$R_\alpha(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by $O_L = \rho$

Transverse response induced by $O_T = \mathbf{j}$

... 5 nuclear responses in ν -scattering...

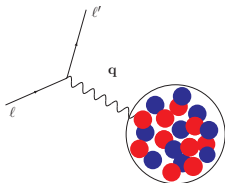
* Sum Rules *

Exploit integral properties of the response functions + closure to avoid explicit calculation of the final states

$$S(q, \tau) = \int_0^\infty d\omega K(\tau, \omega) R_\alpha(q, \omega)$$

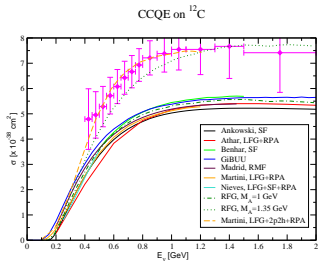
* Coulomb Sum Rules *

$$S_\alpha(q) = \int_0^\infty d\omega R_\alpha(q, \omega) \propto \langle 0 | O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q}) | 0 \rangle$$



Recent Developments on ^{12}C : Inclusive QE Scattering

Charge-Current Cross Section

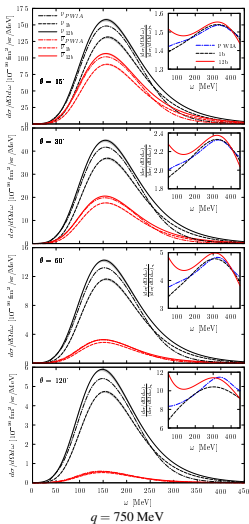


Alvarez-Ruso [arXiv:1012.3871](https://arxiv.org/abs/1012.3871)

CHALLENGES:

1. How do we describe electroweak-scattering off $A > 12$ without losing two-body physics (correlations and two-body currents)?
2. How to incorporate (more) exclusive processes?

NC Inclusive Xsec



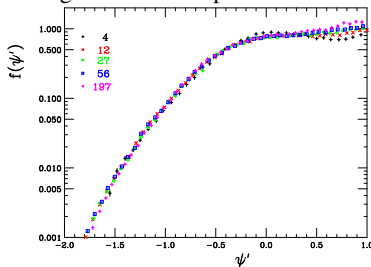
Lovato & Gandolfi *et al.* PRC97(2018)022502

~ 100 million core hours

Scaling properties of the Response Functions

Inclusive xsec depends on a single (scaling) function of ω and q

Scaling 2nd kind: independence form A



Donnelly and Sick - PRC60(1999)065502

1. Rely on observed scaling properties of inclusive xsecs, universal behavior of nucleon/ A momentum distributions, and exhibited locality of nuclear properties to build approximate response functions for $A > 12$ nuclei
2. From exact *ab initio* calculations we know that **two-body correlations** and **two-body currents** are crucial
3. Build a model that retains **two-body physics**

Factorization: Short-Time Approximation

$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle$$

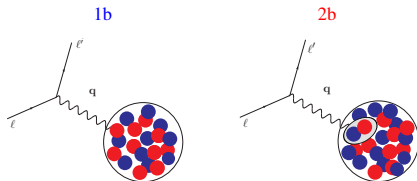
$$R_{\alpha}(q, \omega) = \int dt \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) e^{i(H-\omega)t} O_{\alpha}(\mathbf{q}) | 0 \rangle$$

At short time, expand $P(t) = e^{i(H-\omega)t}$ and keep up to 2b-terms

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

and

$$O_i^{\dagger} P(t) O_i + O_i^{\dagger} P(t) O_j + O_i^{\dagger} P(t) O_{ij} + O_{ij}^{\dagger} P(t) O_{ij}$$



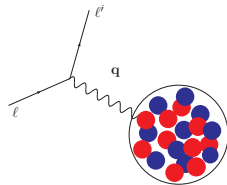
WITH

Carlson & Gandolfi (LANL) & Schiavilla (ODU+JLab) & Wiringa (ANL)

Factorization up to one body - The Plane Wave Impulse Approximation

In PWIA:

Response functions given by incoherent scattering off single nucleons that propagate freely in the final state (plane waves)



$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle$$

$$O_{\alpha}(\mathbf{q}) = O_{\alpha}^{(1)}(\mathbf{q}) = 1b$$

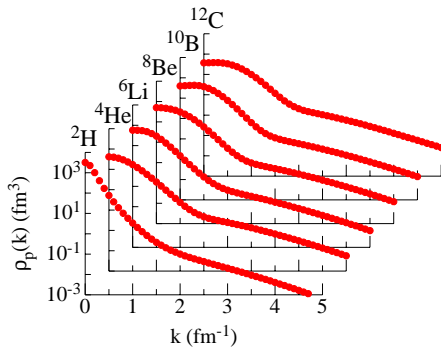
$$|f\rangle \sim e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}} = \text{free single nucleon w.f.}$$

* PWIA Longitudinal Response in terms of the p -momentum distribution $n_p(\mathbf{k})$ *

$$R_L^{\text{PWIA}}(q, \omega) = \int d\mathbf{k} n_p(\mathbf{k}) \delta\left(\omega - \frac{(\mathbf{k}+\mathbf{q})^2}{2m_N} + \frac{\mathbf{k}^2}{2m_N}\right)$$

$$O_L^{(1)}(\mathbf{q}) = e \sum_{i=1}^A \frac{1 + \tau_{i,z}}{2} e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

Proton Momentum Distributions



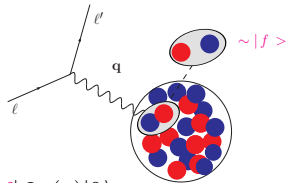
Wiringa *et al.* - PRC89(2014)024305

1-body momentum distributions <http://www.phy.anl.gov/theory/research/momenta/>
2-body momentum distributions <http://www.phy.anl.gov/theory/research/momenta2/>

Factorization up to two-body operators: The Short-Time Approximation (STA)

In STA:

Response functions are given by the scattering off pairs of fully interacting nucleons that propagate into a correlated pair of nucleons



$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle$$

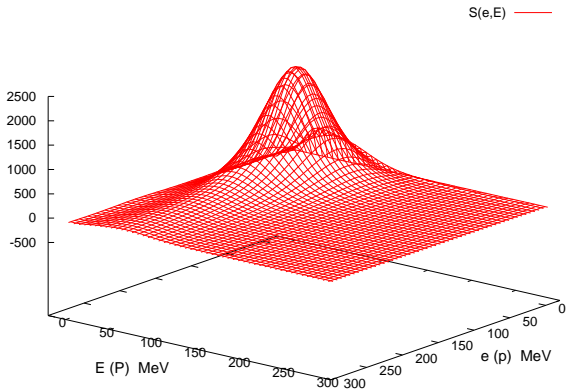
$$O_{\alpha}(\mathbf{q}) = O_{\alpha}^{(1)}(\mathbf{q}) + O_{\alpha}^{(2)}(\mathbf{q}) = 1b + 2b$$

$$|f\rangle \sim |\Psi_{p,P,J,M,L,S,T,M_T}(r,R)\rangle = \text{correlated two-nucleon w.f.}$$

- * We retain **two-body physics** consistently **in the nuclear interactions** and **electroweak currents**
- * STA can be implemented to accommodate for more two-body physics, *e.g.*, pion-production induced by e and ν

$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

The Short-Time Approximation



Transverse “response-density” $1b + 2b$ for ${}^4\text{He}$

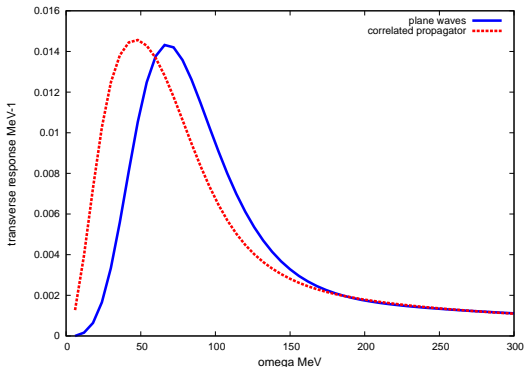
$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_P d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

* Preliminary results *

STA Transverse Response

$$q = 300 \text{ MeV}$$

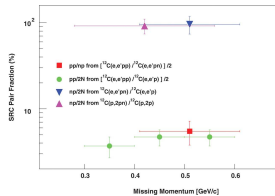
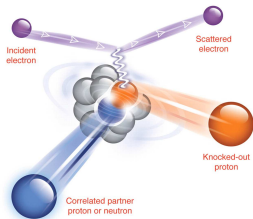
Plane Wave Propagator vs Correlated Propagator



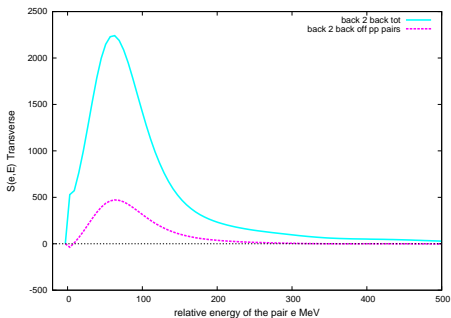
$$R_{\alpha}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) d\Omega_p d\Omega_p dP dp [p^2 P^2 \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | \mathbf{p}, \mathbf{P} \rangle \langle \mathbf{p}, \mathbf{P} | O_{\alpha}(\mathbf{q}) | 0 \rangle]$$

* Preliminary results *

STA back to back scattering



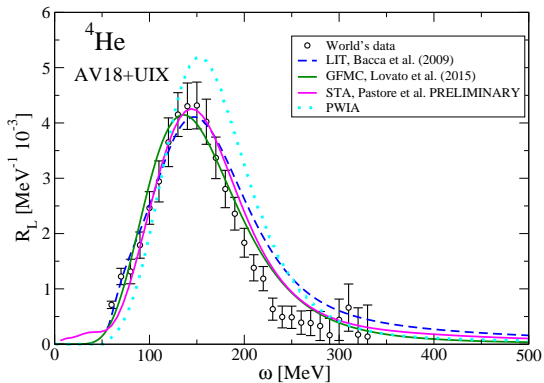
JLab, Subedi *et al.* Science320(2008)1475



$q = 500$ MeV, $E = 69$ MeV *pp* vs *tot*

* Preliminary results *

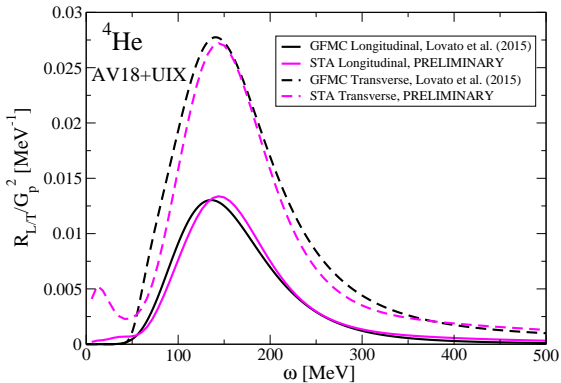
The Short-Time Approximation



Longitudinal Response function at $q = 500$ MeV

* Preliminary results *

The Short-Time Approximation



Longitudinal vs Transverse Response Function at $q = 500$ MeV

* Preliminary results *

Currents and Correlations: Summary

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

- * Two-body currents can give $\sim 30 - 40\%$ contributions and improve on theory/EXPT agreement
- * Calculations of $\beta -$ and $(\beta\beta - \text{decay})$ m.e.'s in $A \leq 12$ indicate two-body physics (currents and correlations) is required
- * Short-Time-Approximation to evaluate ν -A scattering in $A > 12$ nuclei is in excellent agreement with exact calculations and data
- * We are developing a coherent picture for neutrino-nucleus interactions *