Electroweak Physics

Lecture II:
Status of Electroweak Physics

Acknowledgement:
Slides from M. Gruenewald, P. Renton, F. Teubert
Review

- Introduction to electromagnetic and weak interactions
- Motivation for Electroweak Unification
- Introduced nomenclature for electroweak studies
- Described electron-positron collisions and implications of the data
Helicity Conservation

Extreme Relativistic Limit (ERL):  $E \gg mc^2$  \quad $E = pc$  \quad $\gamma \gg 1$

**Massless limit**  
**Helicity = chirality**

\[
P_L \equiv \frac{(1 - \gamma^5)}{2} \quad P_R \equiv \frac{(1 + \gamma^5)}{2}
\]

\[
P_i P_j = \delta_{ij} P_j \quad \sum_i P_i = I \quad P_{L,R} u \equiv u_{L,R}
\]

\[
J_{\mu}^{EM} = q \bar{u} \gamma_\mu u = q(\bar{u}_L + \bar{u}_R) \gamma_\mu (u_L + u_R)
\]

\[
J_{\mu}^{EM} = q \bar{u}_R \gamma_\mu u_R + q \bar{u}_L \gamma_\mu u_L
\]

But  \quad $\bar{u}_L \gamma_\mu u_R = \bar{u}_R \gamma_\mu u_L = 0$

What are the implications?

For particle-antiparticle collisions

$e^-_L + e^+_R$ or $e^-_R + e^+_L$

June 2, 2005  
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Angular Distribution

Start with spin 1 (forward or backward) along axis of collision: what is the probability of getting +1 or -1 along $\theta$?

\[
d_{j\lambda j'}(\theta) \equiv \langle j\lambda' | e^{-i\theta J_y} | j\lambda \rangle
\]

\[
d_{11}(\theta) = d_{-1-1}(\theta) = \frac{1}{2}(1 + \cos \theta)
\]

\[
d_{1-1}(\theta) = d_{-11}(\theta) = \frac{1}{2}(1 - \cos \theta)
\]

\[
\sum (d_{j\lambda j'}(\theta))^2 = 1 + \cos^2 \theta
\]
Z Decays

$$e^+ e^- \rightarrow Z^0 \rightarrow l^+ l^-, q\bar{q}$$

$$J^Z_\mu \sim g_R \bar{u}_R(e) \gamma_\mu u_R(e) + g_L \bar{u}_L \gamma_\mu u_L$$

Even if electrons and positrons are unpolarized, the Z's are produced polarized

$$P_Z = \frac{N_+ - N_-}{N_+ + N_-} = \frac{g_R^2 - g_L^2}{g_R^2 + g_L^2}$$

$$A_{++} \text{: probability of } J = +1 \text{ Z boson producing a } J = +1 \text{ final state}$$

$$A_{++} = \frac{g_R^e g_\mu^L}{2} (1 + \cos \theta)$$

$$\sigma \propto \sum A_{ij}^2 \left[ (g_R^e)^2 + (g_L^e)^2 \right] \left[ (g_R^\mu)^2 + (g_L^\mu)^2 \right] (1 + \cos^2 \theta) + \left[ (g_R^e)^2 - (g_L^e)^2 \right] \left[ (g_R^\mu)^2 - (g_L^\mu)^2 \right] 2 \cos \theta$$

$$\sigma \propto (1 + \cos^2 \theta) + 2 P_e P_\mu \cos \theta$$

$$P_e = \frac{(g_R^e)^2 - (g_L^e)^2}{(g_R^e)^2 + (g_L^e)^2}$$

$$P_\mu = \frac{(g_R^\mu)^2 - (g_L^\mu)^2}{(g_R^\mu)^2 + (g_L^\mu)^2}$$
Forward Backward Asymmetry

\[ P_f = \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2} \approx 2 \frac{g_{Vf}}{g_{Af}} \approx 1 - 4 \sin^2 \theta_W \]

\[ A_{FB} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)} = \frac{3}{4} P_e P_f \]

\[ A^0_{FB} = \frac{3}{4} A_e A_f \]

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Electroweak Phys.
Left-Right Asymmetry

AFB is the product of 2 small numbers.
Can they be disentangled?

Polarize the electron beam and measure Z production

\[ P_b = \frac{N_+ - N_-}{N_+ + N_-} \]
Fraction of beam polarized along or against the momentum

\[ A_{LR} = \frac{N_{Z-} - N_{Z+}}{N_{Z-} + N_{Z+}} = \frac{(1 - P_b)g_L^2 - (1 + P_b)g_R^2}{(1 - P_b)g_L^2 + (1 + P_b)g_R^2} = P_b P_e \]

All final states can be used!

This was the motivation for the SLAC Linear Collider:

Could compete with a factor of 10 to 100 less luminosity
### Tau Polarization

Instead of polarizing the initial state,

Analyze the final state: need a polarization filter

**For tau leptons, use the weak decay!**

\[ \Gamma_\tau = \frac{G_F^2 m_\tau^5}{192 \pi} \]

- Lifetime \( \sim \) few ps
- Travels a few mm

V-A interaction reveals tau polarization

**Pion lab energy distribution is related trivially to the rest frame angular distribution**
Perturbation Theory

From Feynman rules: Construct all possible diagrams
Consistent with standard conservation laws

Amplitude is sum of all possible states: Feynman’s path integral formulation of QM

Problem: Total amplitude diverges

- Feynman rules with electric charge
- Calculate $\sigma_1(e)$ for a test process
- Measure $\sigma_1(e)$ and extract $e$
- Calculate $\sigma_2(e)$ for another process
Charge Renormalization

Start with

\[ M_1 \sim \frac{e^2}{q^2} \]

Add

\[ \Sigma_{\gamma\gamma}(q^2) \]

(It is infinite)

\[ M_1 \sim i \frac{e^2}{q^2} + i \frac{e^2}{q^2} \frac{i \Sigma_{\gamma\gamma}(q^2)}{q^2} \]

\[ e^2 \rightarrow e^2(1 - \Pi_{\gamma\gamma}(q^2)) \]

Introduce parameter \( \Sigma_{\gamma\gamma}(p^2) \) (Also infinite)

\[ M_2 \sim \frac{e^2}{p^2}(1 - \Pi_{\gamma\gamma}(p^2)) \]

\[ M_2 \sim \frac{e^2}{p^2}(1 - [\Pi_{\gamma\gamma}(p^2) - \Pi_{\gamma\gamma}(q^2)]) \quad \text{Finite!} \]
Running Couplings

Fine structure constant: 1/137 at low energy, 1/128 at Z pole

Not all Quantum Field Theories behave this way:
The ones that do are renormalizable theories

**Electroweak theory:** t’Hooft and Veltman
**QCD:** Gross, Politzer and Wilzcek

**Diagram:**
- Total charge enclosed is less than q
- Total charge depends on relative distance
- Effective charge increases with decreasing distance:
- Higher order terms in perturbative expansion
The shift $\Delta \alpha$ can be determined analytically for lepton loops and by a dispersion integral over the $e^+e^-$ annihilation cross section for light quarks $(u,d,s,c,b)$.

$$\Delta \alpha_{\text{lepton}} = \sum_{l=e,\mu,\tau} \frac{\alpha}{3\pi} \left( \log \frac{m_Z^2}{m_l^2} - \frac{5}{3} \right) + \ldots$$

$$\alpha(m_Z^2) = \frac{\alpha}{1-\Delta \alpha}$$

**Optical theorem**

$$\Delta \alpha_{\text{hadron}} = -\frac{\alpha}{3\pi} \int_{4m_{\pi}^2}^{\infty} \frac{m_Z^2 \, ds'}{s'[s'-m_Z^2]} \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$

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Electroweak Physics: Lecture II
Electroweak Input Parameters

For electroweak interactions, there are three parameters needed:

1. Scale of electromagnetism (electric charge)
2. Scale of the weak interaction (Vector boson mass)
3. Weak mixing angle

Parameters are chosen from experimental measurements:

1. Low energy Thomson Scattering
2. The muon lifetime
3. The mass of the Z boson

Z mass know to 23 parts per million!
LEP at CERN
Main energy systematics

**Tides** and other ground motion:

Use tidal model and beam position monitors to correct for orbit changes

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**Trains**

Use NMR probes and *thermal* model to extrapolate energy during fills

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Fermilab 1 May 2002

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David Strom – Oregon
Indirect Evidence for the Top

Measure any asymmetry on the $Z$ pole: function of weak mixing angle

The answer differs from what you would get at tree level

$\Pi_{WW} - \Pi_{ZZ} \propto m_t^2 - m_b^2$
Electroweak Precision Data

Very high $Q^2$ physics at LEP, SLC, and the Tevatron:
More than 1000 measurements with (correlated) uncertainties
Combined to 17 precision electroweak observables

Z boson physics (LEP-1, SLD):
5  Z lineshape and leptonic forward-backward asymmetries
2  Polarised leptonic asymmetries $P_\tau$, $A_{LR}(FB)$
1  Inclusive hadronic charge asymmetry
6  Heavy quark flavour results (Z decays to b and c quarks)

W boson & top quark physics – ongoing at Tevatron's Run-II:
2  W boson mass and width (LEP-2, Tevatron)
1  Top quark mass (Tevatron)

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Top Physics

Tevatron: only source of top quarks in the world!
Primarily top-pair production

\[ p\bar{p} \rightarrow t\bar{t} X, \quad t\bar{t} \rightarrow b\bar{b} W^+ W^- \]
\[ W^- \rightarrow q\bar{q}, l\bar{\nu} \]

qq annihilation (85%)

 gluon fusion (15%)
Lepton+jets most promising channel:
Charged lepton, 2 b-quark jets
2 other jets, only 1 neutrino
Invariant mass $M(\text{top}) = M(\text{Wb})$
W-Pairs at LEP

W Leptonic Branching Ratios

ALEPH
DELPHI
L3
OPAL

LEP $W \rightarrow e\nu$

ALEPH
DELPHI
L3
OPAL

LEP $W \rightarrow \mu\nu$

ALEPH
DELPHI
L3
OPAL

LEP $W \rightarrow \tau\nu$

LEP $W \rightarrow l\nu$

ALEPH, DELPHI, L3 final, OPAL prelim.

Subsequent maximisation of discrepancy:

W-tau branching fraction $\sim 2.9\sigma$ above W-e/\mu average
Standard Model Analysis

SM: Each observable calculated as a function of:

$\Delta \alpha_{\text{had}}, \alpha_s(M_Z), M_Z, M_{\text{top}}, M_{\text{Higgs}}$ (and $G_F$)

$\Delta \alpha_{\text{had}}$: hadronic vacuum polarisation [0.02761±0.00036]

$\alpha_s(M_Z)$: given by $\Gamma_{\text{had}}$ and related observables

$M_Z$: constrained by LEP-1 lineshape

Precision requires 1$^{\text{st}}$ and 2$^{\text{nd}}$ order electroweak and mixed radiative correction calculations (QED to 3$^{\text{rd}}$)

$M_{\text{top}}, M_{\text{Higgs}}$ enter through electroweak corrections ($\sim 1\%$)

Calculations by programs TOPAZ0 and ZFITTER
Heavy Particle Masses $W$ and $T$

Direct measurements:
- Tevatron and LEP2

Z-Pole measurements:
- Constrain electroweak radiative corrections
- Allow to predict $M_W$ and $M_{top}$ within SM

Good agreement:
- Successful SM test

Both data sets prefer a light Higgs boson

$M_{Higgs} < 280 \text{ GeV (95\%CL)}$
Standard Model Analysis

$M_{\text{Higgs}} = 126^{+73}_{-48}$ GeV

Incl. theory uncertainty:
$M_{\text{Higgs}} < 280$ GeV (95\%CL)

does not include:

Direct search limit (LEP-2):
$M_{\text{Higgs}} > 114$ GeV (95\%CL)

Renormalise probability
for $M_H > 114$ GeV to 100%:
$M_{\text{Higgs}} < 300$ GeV (95\%CL)

Theory uncertainty:
Dominated by two-loop calculations for $\sin^2\Theta_{\text{eff}}$
## Standard Model Analysis

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Fit</th>
<th>Fit to 17 high-$Q^2$ observables plus $\Delta\alpha_{\text{had}}$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta\alpha_{\text{had}}^{(5)}(m_Z)$</td>
<td>0.02761 ± 0.00036</td>
<td>$\chi^2/\text{ndof} = 18.3/13 (14.7%)$</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>91.1874</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4965</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>41.481</td>
</tr>
<tr>
<td>$R_l$</td>
<td>20.767 ± 0.025</td>
<td>20.739</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,l}$</td>
<td>0.01714 ± 0.00095</td>
<td>0.01642</td>
</tr>
<tr>
<td>$A_{l}(P_\tau)$</td>
<td>0.1465 ± 0.0032</td>
<td>0.1480</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21630 ± 0.00066</td>
<td>0.21562</td>
</tr>
<tr>
<td>$R_z$</td>
<td>0.1723 ± 0.0031</td>
<td>0.1723</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,b}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1037</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0742</td>
</tr>
<tr>
<td>$A_{b}$</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
</tr>
<tr>
<td>$A_{c}$</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
</tr>
<tr>
<td>$A_{l}(\text{SLD})$</td>
<td>0.1513 ± 0.0021</td>
<td>0.1480</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{fb}})$</td>
<td>0.2324 ± 0.0012</td>
<td>0.2314</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>80.425 ± 0.034</td>
<td>80.390</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.133 ± 0.069</td>
<td>2.093</td>
</tr>
<tr>
<td>$m_\tau$ [GeV]</td>
<td>178.0 ± 4.3</td>
<td>178.4</td>
</tr>
</tbody>
</table>

Decided in favour of leptons by $M_W$

A_{\text{fb}}(b) has largest pull: 2.8

Predict observables measured in reactions with low-$Q^2$:

$$Q^2 \ll M_W^2$$
Comparison of all Z-Pole Asymmetries

Effective electroweak mixing angle:

$$\sin^2 \Theta_{\text{eff}} = \frac{1 - g_V / g_A}{4}$$

$$= 0.23153 \pm 0.00016$$

$$\chi^2 / \text{ndof} = 11.8 / 5 \ [3.8\%]$$

Subsequent observation:

0.23113$\pm$0.00021 leptons

0.23222$\pm$0.00027 hadrons

3.2 $\sigma$ difference

But is really:

$A_l$(SLD) vs. $A_{fb}$b(LEP)

3.2 $\sigma$ difference

$$A_{fb}$$

$$A_{l}(P_\tau)$$

$$A_{l}(\text{SLD})$$

$$Q_{fb}^{\text{had}}$$

Average

$$0.23153 \pm 0.00016$$

$$\chi^2 / \text{d.o.f.} = 11.8 / 5$$

$$m_{l} = 178.0 \pm 4.3 \text{ GeV}$$

$$\Delta Q_{\text{had}}^{(5)} = 0.02761 \pm 0.00036$$
Summary

• The electroweak theory has been tested to extraordinary precision
• Typical scale is 0.1%
• No significant deviations, but some tantalizing hints
• The Higgs boson is expected to be light
• Should we pack up and go home? Some answers in Lecture III