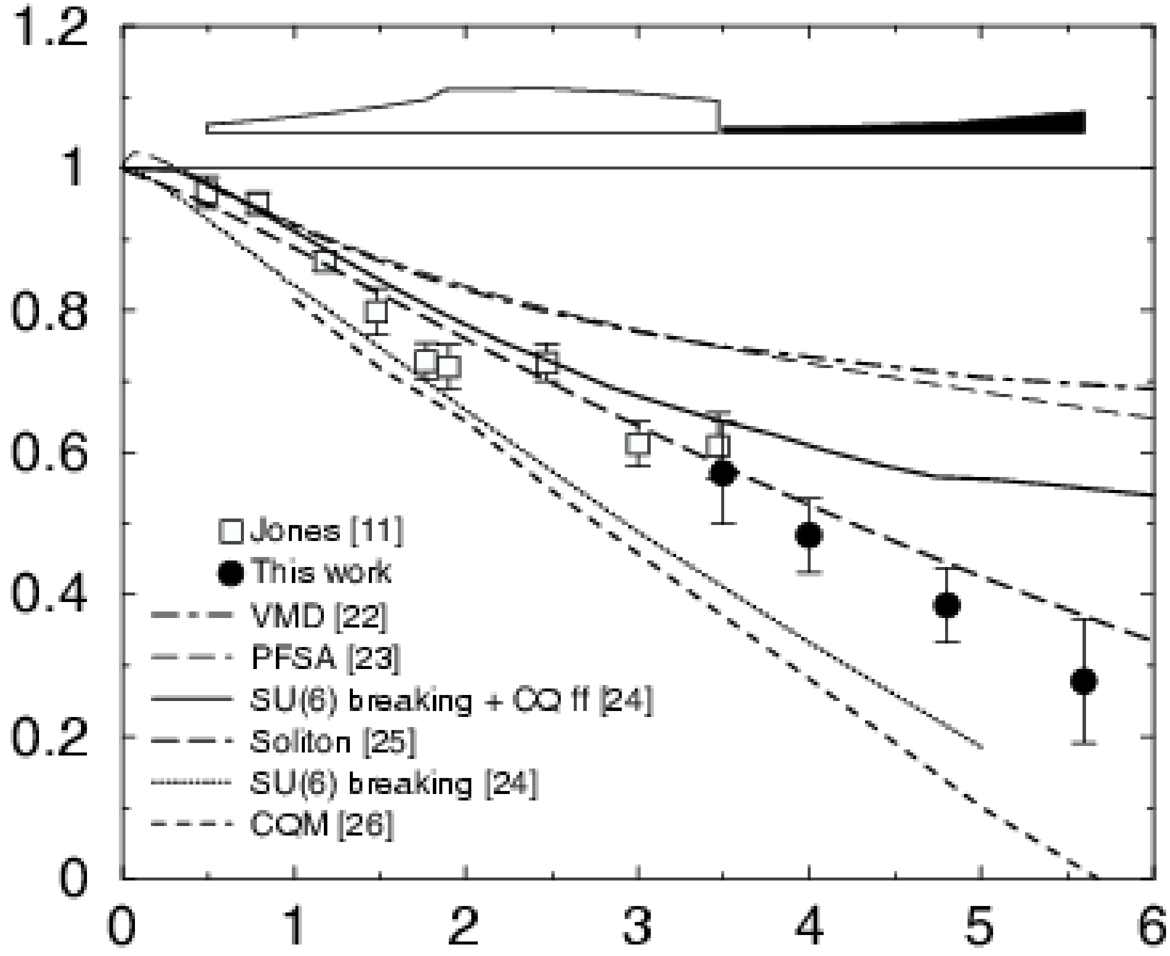


O. Gayou et al, PRL (2002)

PQCD Prediction for the Proton's Pauli Form Factor

Xiangdong Ji
University of Maryland

Collaborators: Andrei Belitsky (Maryland)
Jian-Ping Ma (ITP, Beijing)
Feng Yuan (Maryland)

Outline

- Hard exclusive processes and Brodsky-Farrar counting rule
- A generalized counting rule
- F_2 and orbital angular momentum (OAM)
- Form factors and their interpretation
- F_2 at large Q^2 and pQCD predictions
- Implications

Hard Exclusive Processes

- Definition
  - Initial & final states with fixed (small) numbers of hadrons (+ leptons + gauge bosons)
  - All kinematical invariants (*s, t, u*,…) going to infinity at the same rate.

  Examples:

  form factors, $\gamma^* p \to \gamma + p$*, p+p → p+p, etc*

- Cross section
  - Decays algebraically 1/s.
  - Ruled out string theory as the fundamental theory of strong interactions (there is, however, a reincarnation through AdS/CFT…)

Brodsky-Farrar Counting Rule

- Based on dimensional analysis

$$\frac{d\sigma}{dt} \sim s^{-2+\sum_H (n_H - 1)}$$

Brodsky-Farrar(1973)
Matveev et al. (1973)

where n_H is the minimal number of partons in "hadron" H
Valid for any renormalizable field theory

- Examples

p+p → p+p d♦/dt ~ s🕘10

♑+p → ◘+p d♦/dt ~ s🕘7

♑+D → n+p d♦/dt ~ s🕘11

Counting Rule in QCD

- Can be justified through QCD factorization. The rule is modified in general by the QCD running coupling (logarithms).
- QCD factorization is nontrivial!
  - Chernyak & Zhitnitsky, Efremov & Radyushkin, Brodsky&Lepage, Duncan&Muller, Botts&Sterman, etc.
- Counting rule works **ONLY** for the leading, *hadron helicity conserving* amplitudes
  - Hadrons are made of quarks
  - Hadron helicity = sum of quark helicities

    (leading light-cone wave functions)
  - Total quark helicity is conserved in pQCD.

Hadron-Helicity Non-Conserving Ampliutde??

- Assume QCD factorization!
- Hadron-helicity change can occur only through
  - Quark mass effects. (not important)
  - **Quark orbital angular momentum** (OAM) in the hadron wave function
  - **Gluon component** in the hadron wave function.

The last two often come together to guarantee gauge invariance. We will mostly focus on OAM.

Generalized Counting Rule

- Hep-ph/0301141 (Ji, Ma & Yuan)
- We write down general structure of the light-cone wave functions for hadrons with **OAM**.
- We derive the asymptotic behavior of the amplitudes at large transverse momentum.
- From which we derive the generalized counting rule for the cross section

$$\frac{d\sigma}{dt} \sim s^{-2+\sum_H (n_H + |l_{zH}| - 1)}$$

Applications

- p+p→p+p, there are 5 amplitudes

$$M(++\to++)\sim M(+-\to+-)\sim M(-+\to+-)\sim 1/s^4$$

$$M(++\to+-)\sim 1/s^{9/2}\quad(\ell z=1)$$

$$M(--\to++)\sim 1/s^{5}\quad(\ell z=2)$$

  – New mechanism to explain the oscillation in cross section?

- ♑+p → □+n (doable at Jlab!)

$$M(\gamma+p_{\uparrow}\to\pi^{+}+n_{\uparrow})\sim 1/s^{5/2}$$

$$M(\gamma+p_{\uparrow}\to\pi^{+}+n_{\downarrow})\sim 1/s^{3}\quad(\ell z=1)$$

Nucleon's Pauli Form Factor F_2

- Define as the matrix elements of electromagnetic current

$$\langle p'|j^{\mu}|p\rangle = U(p')\left[F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2M}\right]U(p)$$

- Choose the Breit frame, F_2 is a helicity flip amplitude,

$$F_2 \sim \langle p'\downarrow|J^{+}|p\uparrow\rangle$$

Brodsky& Drell: PRD (1980)

Role of Orbital Angular Momentum

- **Vital**!

  Image there is no OAM:

  – *F_2* 🕓*0,*

  – *&* 🕓*0 (would have been discovered in 1933)*

- Models consistent with the QCD picture:

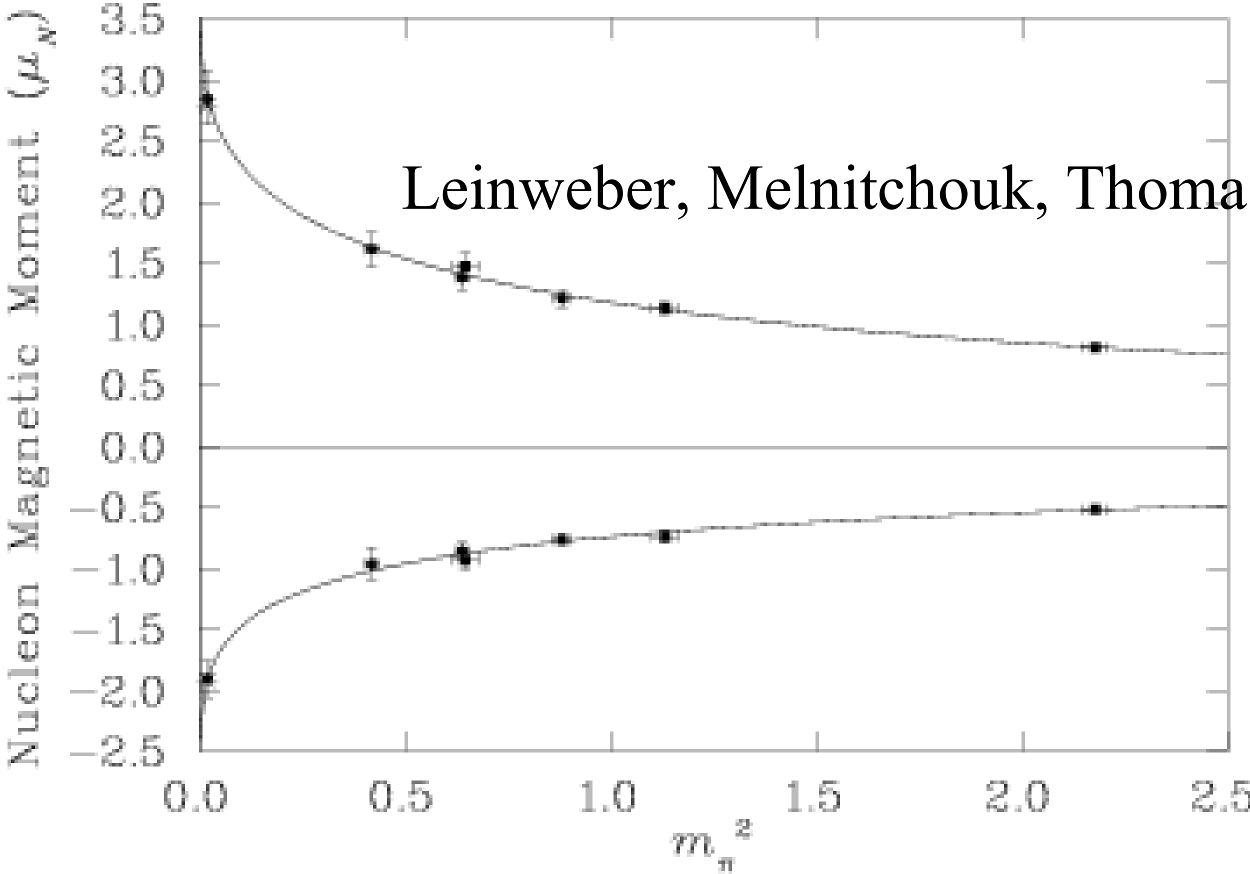
  – Meson cloud model

  In which the anomalous mag. mom. is generated by orbital motion of pions around a bare nucleon.

  – MIT bag model

  & is proportional to the lower component of the quark wave function which is a p-wave.

Magnetic Moment from Lattice QCD



Leinweber, Melnitchouk, Thomas

Large & reflects the chiral physics from the pion cloud.

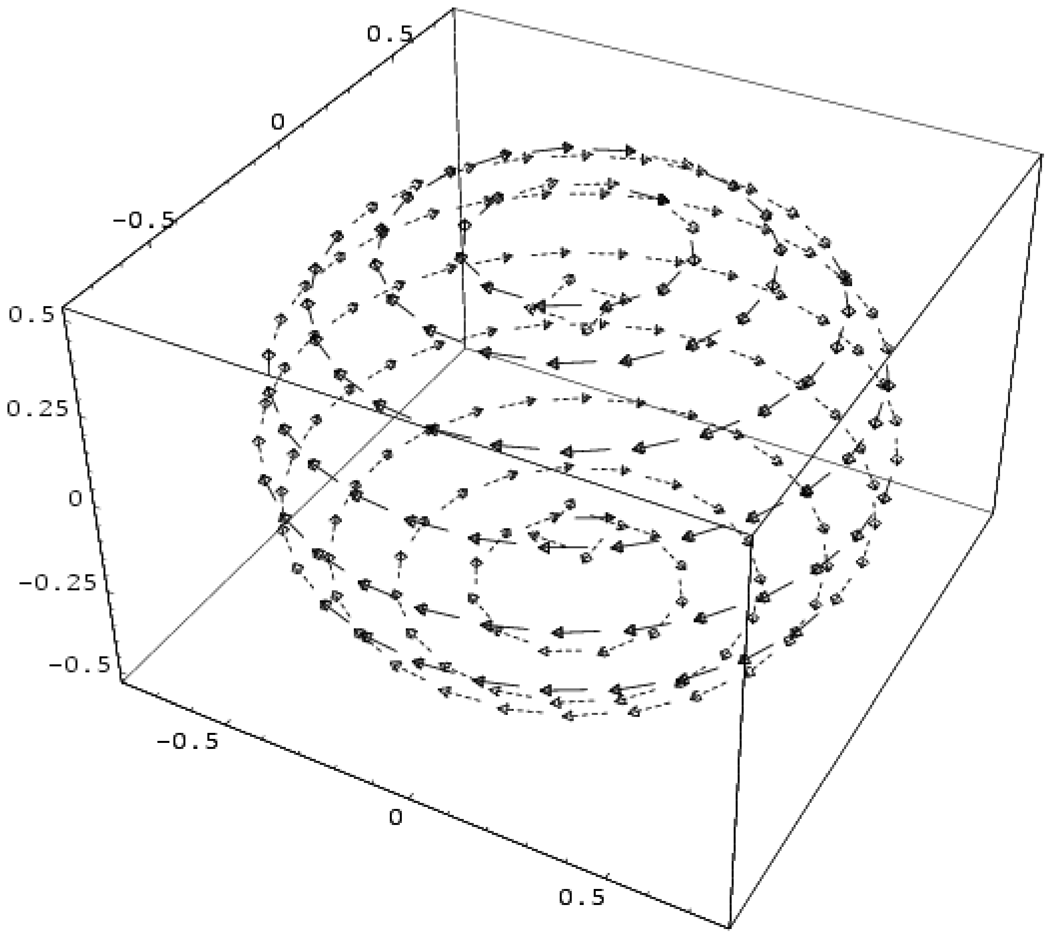
F_2 & Electric Current Distribution

- Electric current in the nucleon

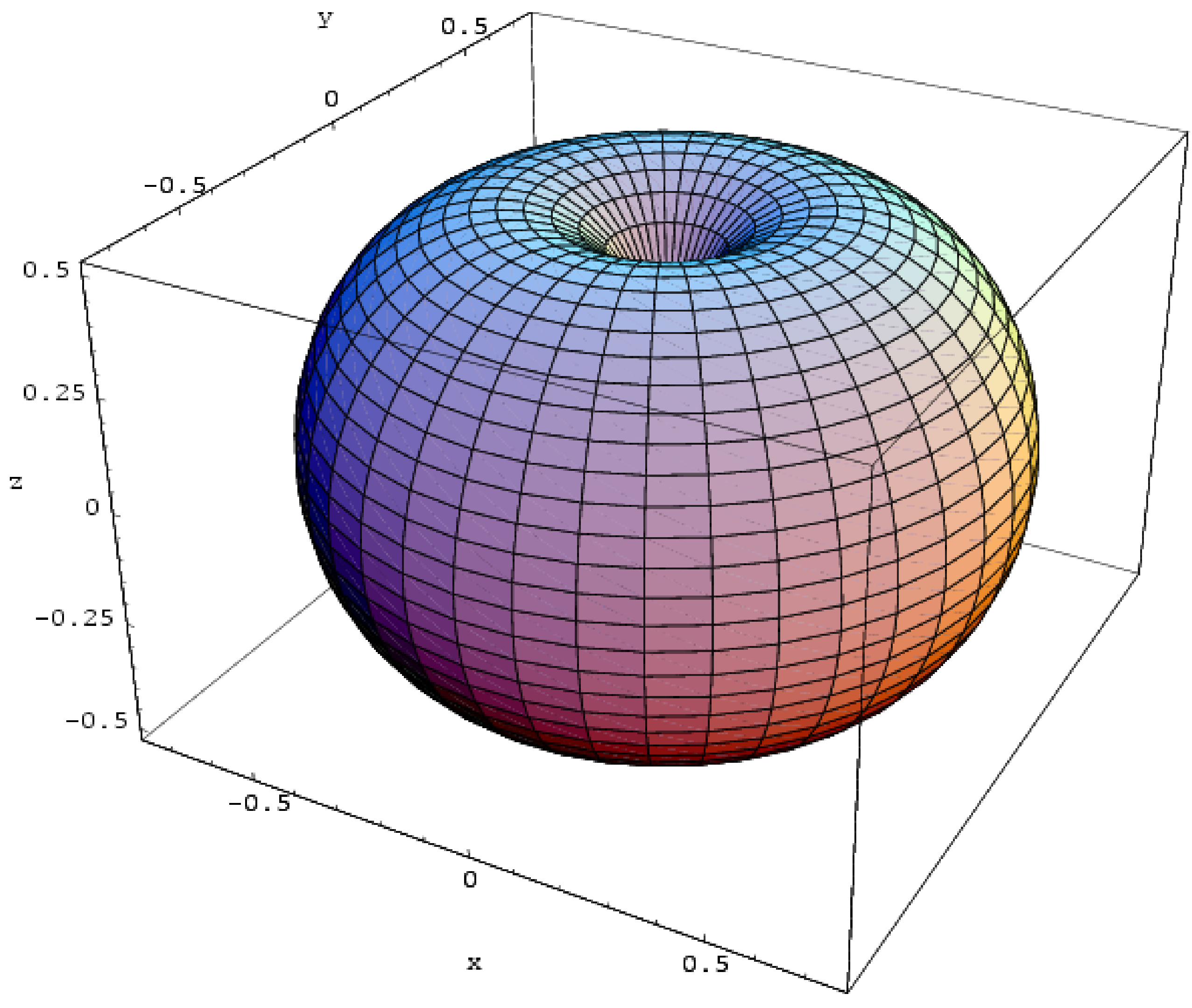
$$\vec{j}(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} i(\vec{s}\times\vec{q}) \frac{1}{2M} (F_1(q^2) + F_2(q^2))$$

  - The current from F_1 is related to the static charge distribution in the nucleon, and might be interpreted as from quark magnetization.
  - F_2 –term generates the current *directly from the quark orbital motion.*

Electric current produced by the quark orbital motion in the proton.



Surface of the constant current density

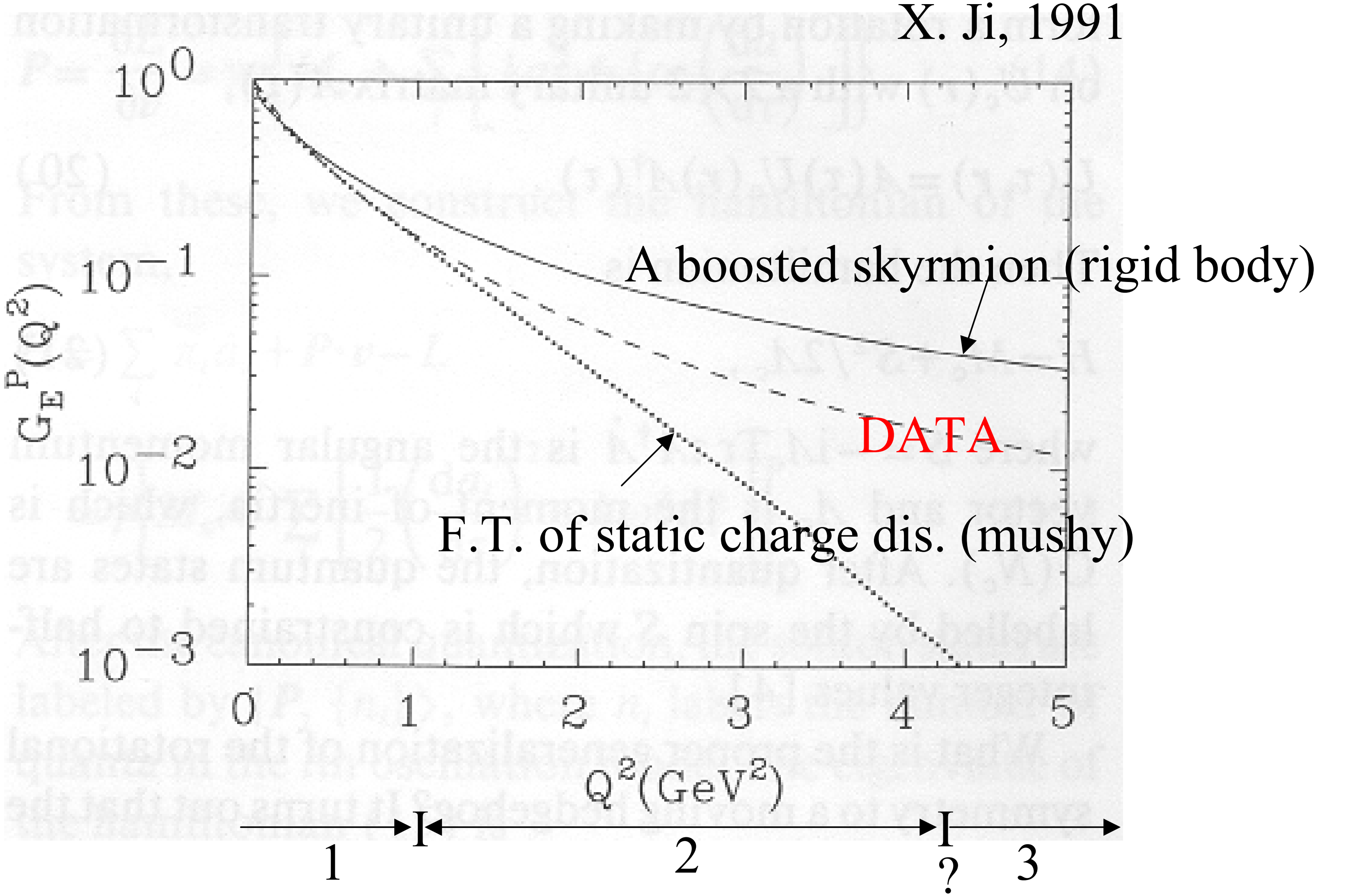


Physical Interpretation of Nucleon Form Factors

- Complicated by relativity!
  - One cannot localize a particle, and hence define the static size R, to a precision better than 1/M!
  - For non-relativistic systems, RM⑦1. But for the nucleon, RM ~ 4.
- Resolution and nucleon structure
  1. For photon momentum Q ~ 1/R, one probes the static distributions.
  2. For Q ~ M, the form factor is a result of internal structure and *mechanisms to keep the nucleon intact after absorbing a large momentum transfer*.
  3. For Q ⑦ M, the form factor is entirely dominated by the reaction mechanisms.

Electric Form Factor For A Skyrmion

X. Ji, 1991



Jlab F_2 Data

- Orbital angular momentum plays a key role!

  (also, Ralston, Miller,…)
- Data at $Q^2 \sim 3\text{-}5\ GeV^2$ cannot be straightforwardly explained as static property.
  - When calculated in static models like bag model, skyrme model, etc. sensitively depends on how to boost the wave function. (factor of 2 to 10 effects)
  - In light-cone models where boost is easy (such as G. Miller's, nucl-th/0304076), it is not clear how to translate the wave function to the rest frame!

Going down from high Q^2…

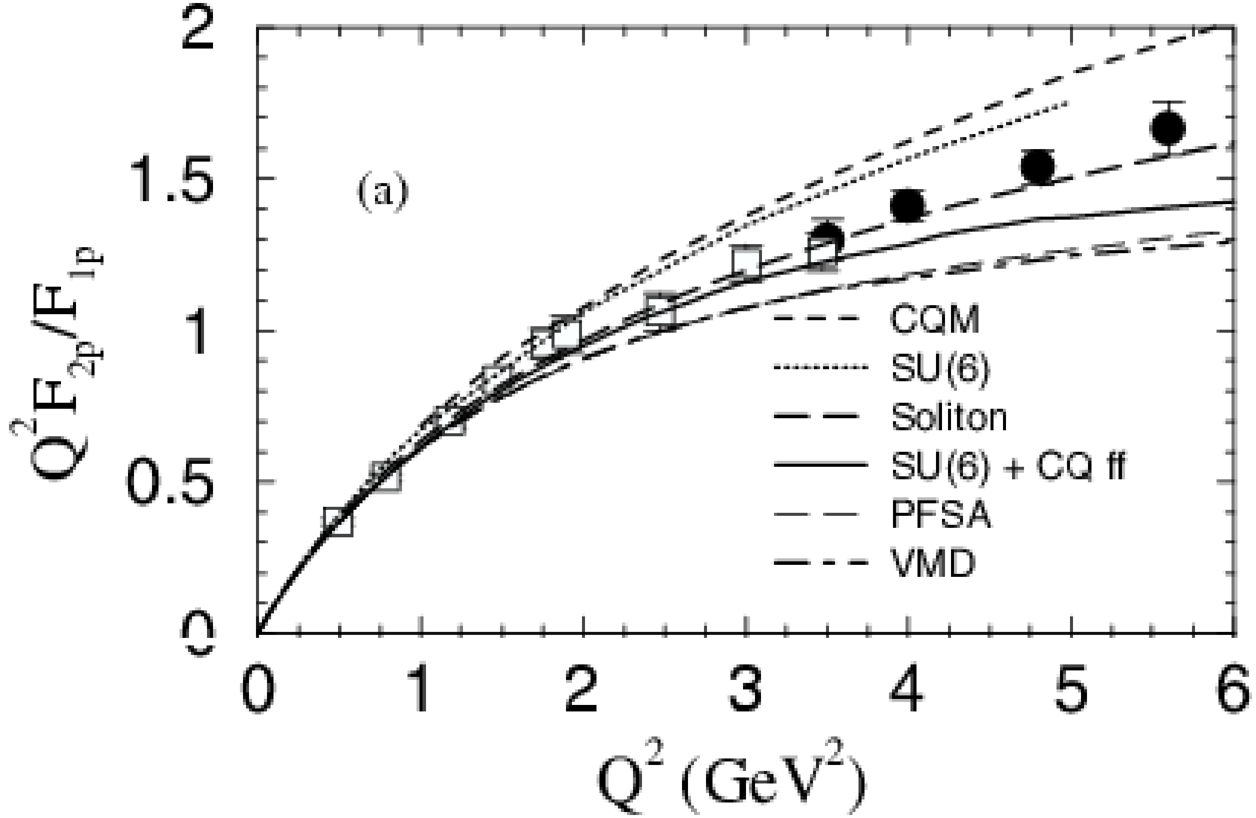
- Reaction mechanism dominates the interpretation of F2, not the current distribution in the nucleon!
- S. Brodsky (SLAC) and collaborators predicted 20 years ago in pQCD

$$\frac{F_2(Q^2)}{F_1(Q^2)} \sim \frac{m_q M}{Q^2}$$

No OAM was included

Brodsky & Farrar (1975)
Brodsky & Lepage (1980)

- It disagrees with the recent Jlab data…
  – It is not applicable because the data is at too low Q^2.



2
1.5
1
0.5
0
0
1
2
3
4
5
6
(a)
CQM
SU(6)
Soliton
SU(6) + CQ ff
PFSA
VMD
Q^2F_{2p}/F_{1p}
Q^2 (GeV^2)

Quark OAM in the Proton

- Consider the light-cone wave functions of the proton with 3 quarks. There are 6 independent LC amplitudes:

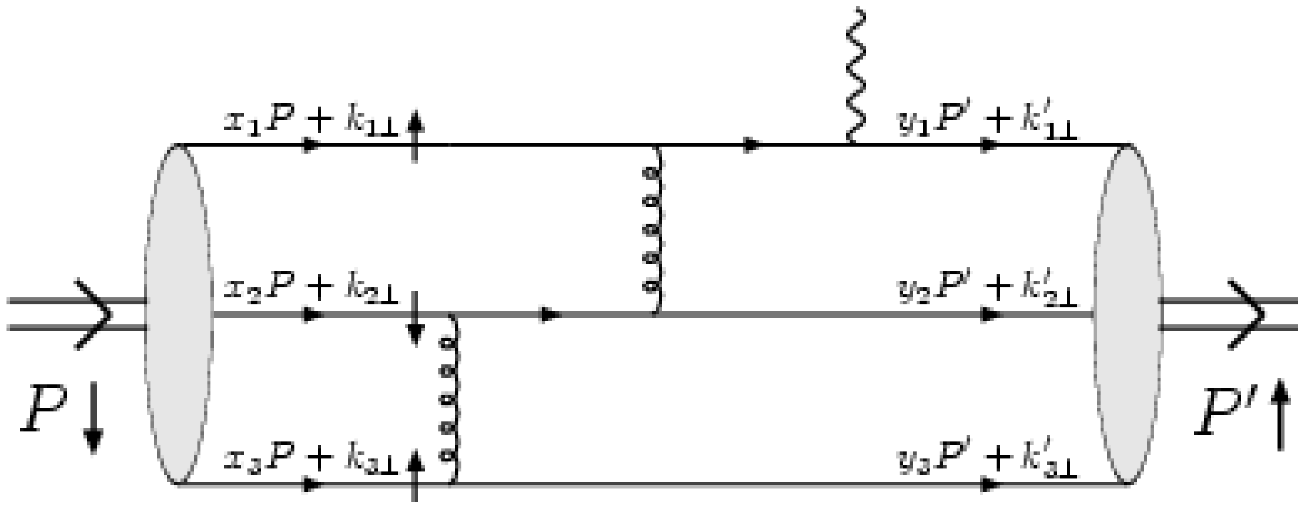
$$|P\uparrow\rangle = |P\uparrow\rangle_{-3/2} + |P\uparrow\rangle_{-1/2} + |P\uparrow\rangle_{1/2} + |P\uparrow\rangle_{3/2}$$

total quark helicity

$$\begin{aligned}
|P\uparrow\rangle_{1/2} &= \int d[1]d[2]d[3]\left(\underline{\tilde{\psi}^{(1)}(1,2,3)} + i(k_1^x k_2^y - k_1^y k_2^x)\underline{\tilde{\psi}^{(2)}(1,2,3)}\right) \\
&\quad \times \frac{\epsilon^{abc}}{\sqrt{6}} u_{a\uparrow}^\dagger(1)\left(u_{b\downarrow}^\dagger(2)d_{c\uparrow}^\dagger(3) - d_{b\downarrow}^\dagger(2)u_{c\uparrow}^\dagger(3)\right)|0\rangle\ , \\
|P\uparrow\rangle_{-1/2} &= \int d[1]d[2]d[3]\left((k_1^x + ik_1^y)\underline{\tilde{\psi}^{(3)}(1,2,3)} + (k_2^x + ik_2^y)\underline{\tilde{\psi}^{(4)}(1,2,3)}\right) \\
&\quad \times \frac{\epsilon^{abc}}{\sqrt{6}}\left(u_{a\uparrow}^\dagger(1)u_{b\downarrow}^\dagger(2)d_{c\downarrow}^\dagger(3) - d_{a\uparrow}^\dagger(1)u_{b\downarrow}^\dagger(2)u_{c\downarrow}^\dagger(3)\right)|0\rangle\ .
\end{aligned}$$

A pQCD Calculation of $F_2(Q)$

Belitsky, Ji & Yuan, hep-ph/0212234



$$F_2(Q^2) = \int [dx_i][dy_i]\left[x_3\Phi_4(x_1,x_2,x_3)T_\Phi(x_i,y_i) + x_1\Psi_4(x_2,x_1,x_3)T_\Psi(x_i,y_i)\right]\Phi_3(y_i)$$

L_z=1 L_z=0

Result

We predict that F_2 goes like ♋$_s^2(\ln^2 Q^2)/Q^6$ and so

$$\boxed{\frac{F_2(Q^2)}{F_1(Q^2)} \sim \frac{1}{Q^2} \log^2 (Q^2/\Lambda^2)}$$

The power behavior confirms the Brodsky et al. prediction!
And also the generalized power counting of Ji, Ma & Yuan.
But it is accurate to logarithms!

Brodsky, Hiller & Hwang

$$\frac{F_2(Q^2)}{F_1(Q^2)} = \frac{\mu_A}{1+(Q^2/c)\ln^b(1+Q^2/a)}$$

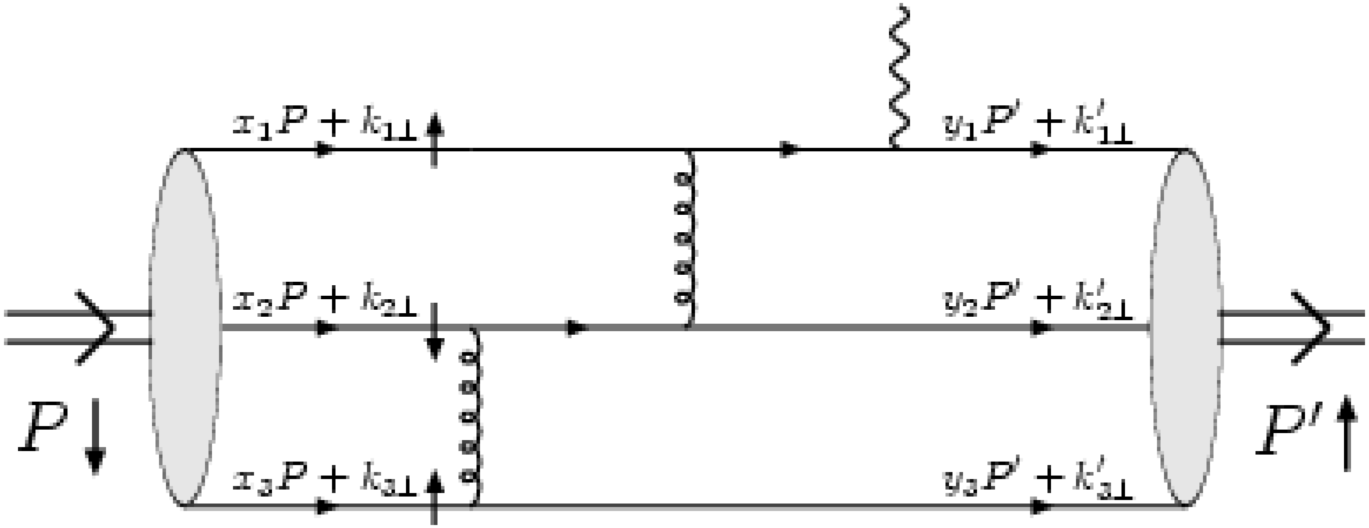
(b = 📫 0.6)

Physics of Logs:

--- it has to do with the hard scattering!

- The pQCD hard scattering must change the direction of three valence quarks.
- For F_2, it also must create one unit of OAM!
- The small x quarks contribute little to the linear momentum, but can contribute to the OAM just as easily as the large x quarks!
  - Therefore, one needs to count the number of small x quarks, ⇩dx/x
- However, for x<☹/Q, the quark is *de-localized* and its contribution is strongly suppressed because of the color. (Sudakov suppression)

$$\int dx/x \sim \log Q$$

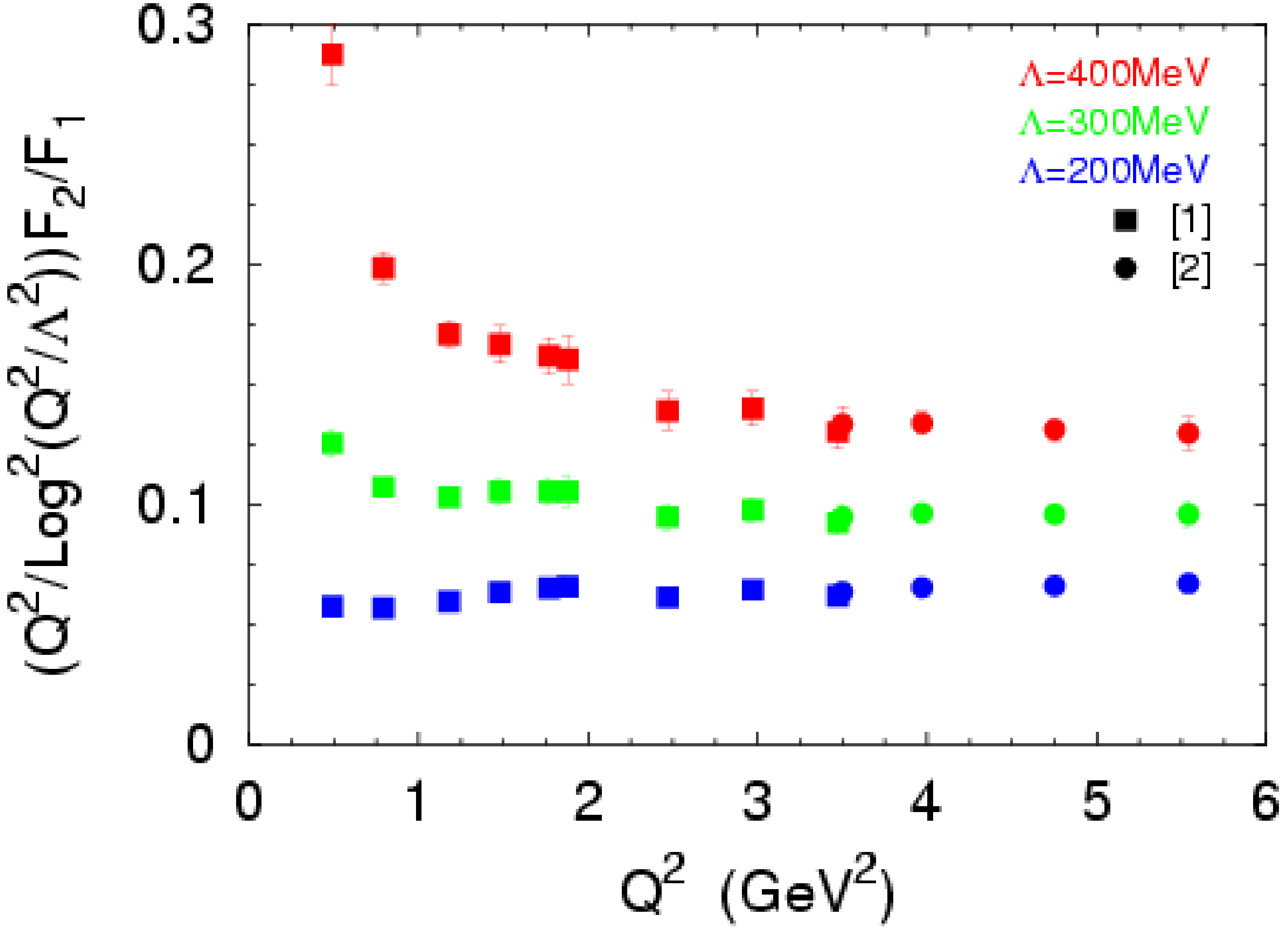


Lz=1

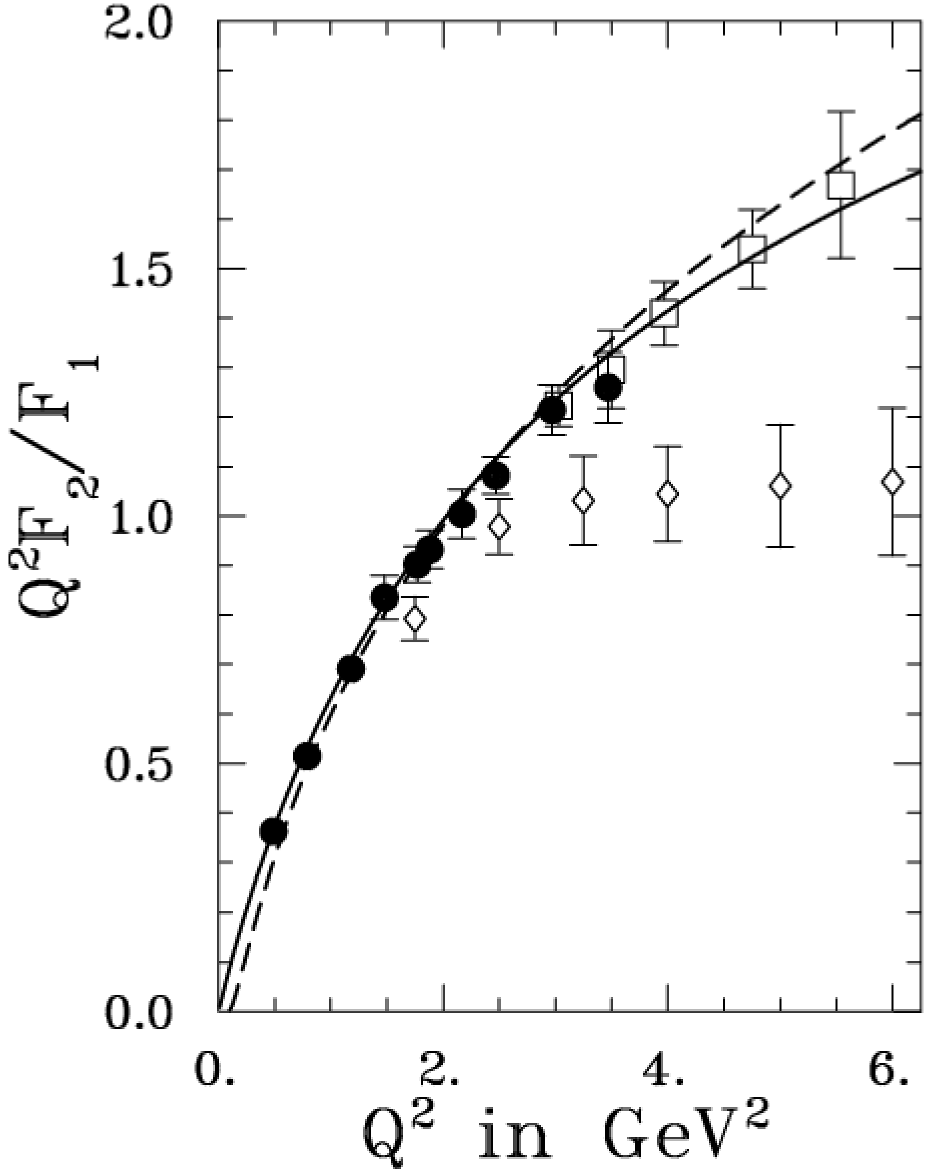
Lz=0

The hard part must create one unit of OAM. This can happen as easily for small-x quarks as for large-x ones.

Scaling: the combination becomes independent of Q^2 in the large Q^2 limit



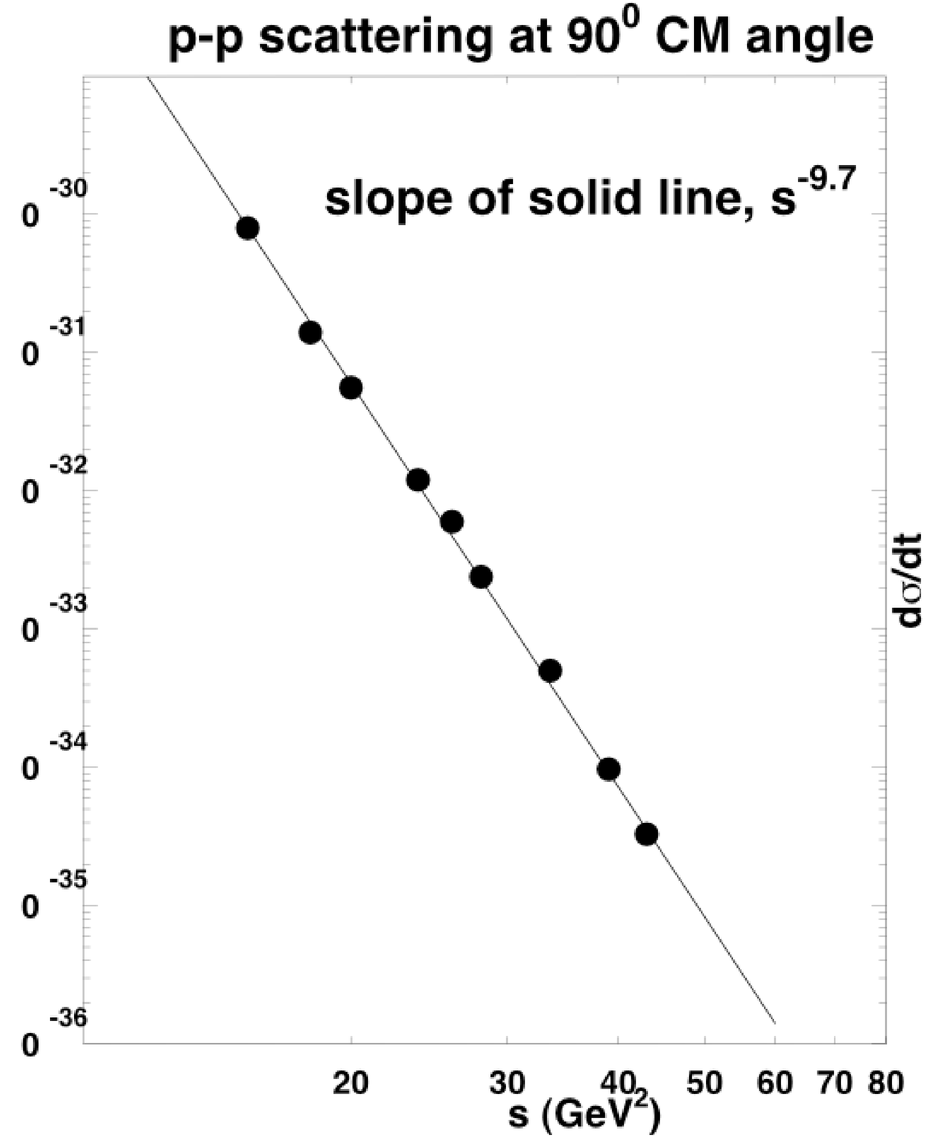
Taking into account
the quark orbital motion, the
Maryland group preduct a solid
curve which matches the Jlab data
very well.



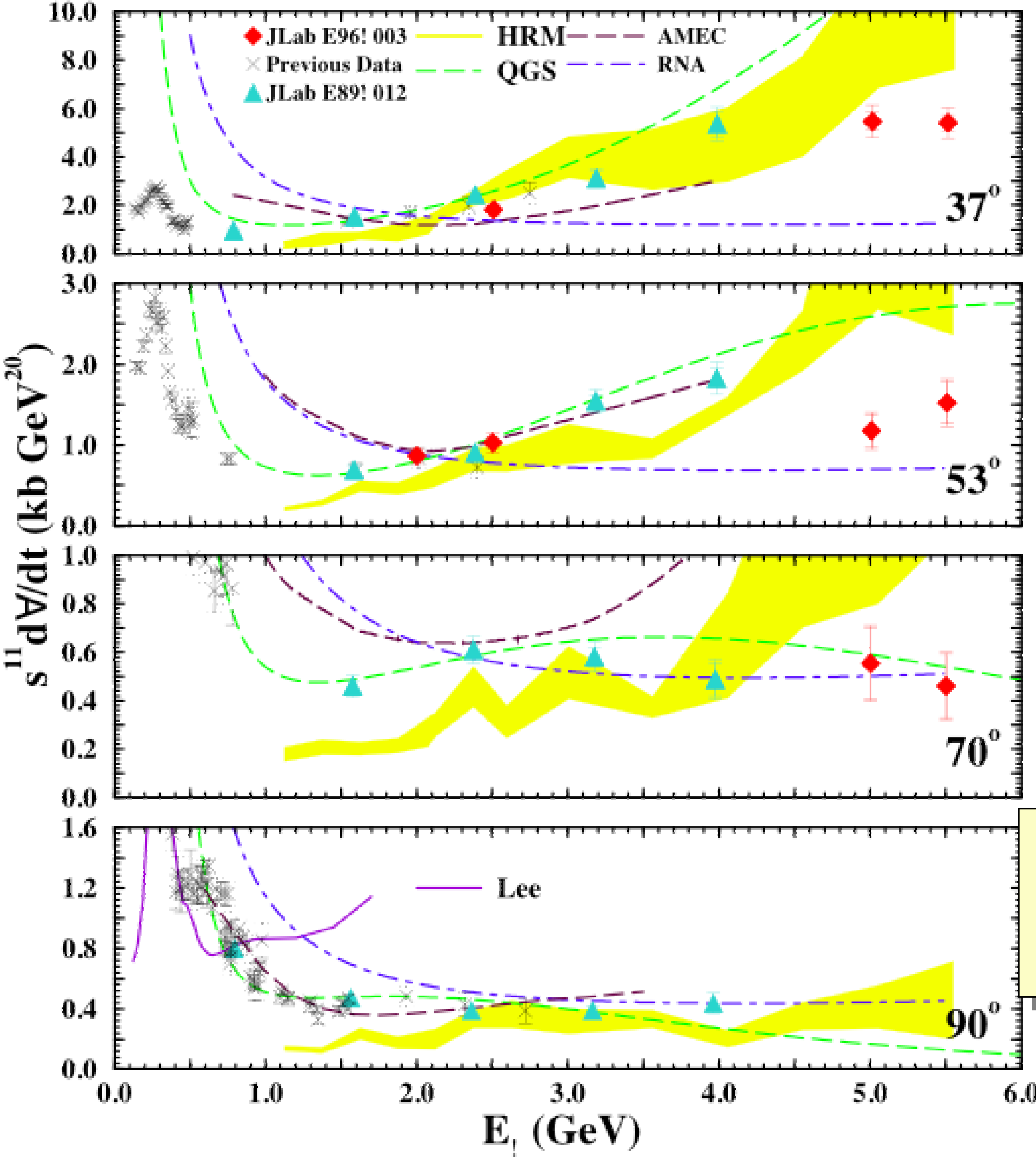
Implications?

- ***Precocious*** scaling!
  - pQCD works better than it has the right to.

    New data at higher Q^2 ?
  - Seen in many other examples
    - Inclusive DIS (duality)
    - $p+p \rightarrow p+p$
    - ♑+D $\rightarrow$ n+p
  - Can only be understood if we know how to calculate corrections reliably (loop corrections, higher-twist).



p-p scattering at 90^0 CM angle
slope of solid line, $s^{-9.7}$
0^{-30}
0^{-31}
0^{-32}
0^{-33}
0^{-34}
0^{-35}
0^{-36}
20
30
40
50
60
70
80
s (GeV^2)
dσ/dt



E. Schulte et al. PRL 87, 102302 (2001)

Conclusion

- In QCD, helicity-changing hard exclusive processes can only happen through quark OAM and gluons. A generalized scaling law can be derived, assuming factorization.
- $F_2(Q)$ is related to quark OAM, and at small Q, to the spatial current distribution in the nucleon.
- At large Q, F_2 is related to both nucleon structure and the hard reaction mechanism.
- JLab data at $Q^2>3$ GeV^2 is consistent with asymptotic pQCD prediction, in which the small-x quarks carry a substantial amount of OAM.