Partonic view of radiative corrections to elastic electron-proton scattering

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Topics

- Review motivation
- Radiative corrections to elastic $eN$ scattering—partonic calculation
- Applications to Rosenbluth determination of form factors
- Radiative corrections to polarizations and $e^+p$ scattering

Collaborators

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See it on the ArXiv: hep-ph/0403058
Introduction

Problem!

$F_2$, or $G_E$, for the proton, measured different ways gives different results.
Rosenbluth separation method

Cross section for one-photon exchange (w/ no polarization)

\[
\frac{d\sigma}{d\Omega_{Lab}} = \frac{\sigma_{NS}}{\epsilon(1 + \tau)} \left( \tau |G_M(Q^2)|^2 + \epsilon |G_E(Q^2)|^2 \right)
\]

where

\[
\tau \equiv \frac{Q^2}{4M^2}, \quad \frac{1}{\epsilon} \equiv 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}
\]

(Note: forward direction, \(\theta \to 0\), means \(\epsilon \to 1\).)

Method: fix \(Q^2\), vary angle (vary \(\epsilon\)), adjusting incoming energy as needed, and plot reduced cross section

\[|G_M|^2 + \frac{\epsilon}{\tau} |G_E|^2\]

vs. \(\epsilon\).
Get (one-photon theorist’s view):

\[ G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \]

\[ \text{data} \]

\( \{ \text{gives } G_E^2 \} \)

\( G_M \text{ alone} \)

\[ \varepsilon \]

**FIG. 1:** Rosenbluth plot
Polarization transfer method

Polarized electron beam $\Rightarrow$ sideways and longitudinal proton polarization

$$e^+ + p \rightarrow e + p^-$$

Measure sideways/longitudinal polarization ratio.

Get form factor ratio from,

$$\frac{P_x}{P_z} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \frac{G_E}{G_M},$$

which follows from a lowest order (one-photon exchange) calculation.

Results:
Polarization measurements

Two methods, two results.

(figure from Arrington, PRC 2003)
Dream solution

Dream: There are radiative corrections to the Rosenbluth experiment that are important and not yet included.

Further: the unincluded corrections are linear in $\epsilon$, with positive slope.
Still further dreaming: The extra radiative corrections are not strongly $Q^2$ dependent. Since contributions from $G_E^2$ terms are smaller at high $Q^2$, have

\[ G_M^2 + \frac{e}{\tau} G_E^2 \]

data gives $G_E^2$

Low $\tau$ (Low $Q^2$)

\[ G_M^2 + \frac{e}{\tau} G_E^2 \]

data gives $G_E^2$

High $\tau$ (High $Q^2$)

I.e., the dream-solution Rosenbluth-extracted $G_E$ shrinks more at high $Q^2$ than at low $Q^2$.  

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Numerical note:

Take $Q^2 = 6$ Gev$^2$, and find

$$\frac{G_E^2}{\tau G_M^2} = \frac{4M^2}{Q^2 \mu_P^2} = 7.6\%$$

If $G_E$ and $G_M$ both scale the same way.

Typically, radiative corrections are a few percent (a few times $\alpha$, in this case, not just $\alpha/\pi$), and $\epsilon$ dependent. Thus, the radiative corrections are of the same size as the Rosenbluth measurement needed to determine $G_E$ (at high $Q^2$).
Where should we look?

Radiative Correction Diagrams:

Bremsstrahlung

Elastic scattering–Vertex Corrections

Elastic Scattering–Box Diagrams
Comments

- Electrons are well understood. Corrections involving just electrons well done.
- Bremsstrahlung involves soft, long wavelength, photons. Compositeness of proton should not be an issue for bremsstrahlung.
- Box diagrams involve photons of all wavelengths. Contributions where one photon is soft are easy and give Coulomb phase correction times lowest order.
- Box contributions where both photons are hard require treating proton as structured, composite, system. Not done in “old days.”

So: There is an opening. Study two-photon exchange (box) contributions.
Preliminary: what has been done

- "Old days" E.g., Tsaï [1961] or Mo & Tsaï [1968] box diagram evaluation

Did minimal calculation to give the IR divergent terms correctly. (Box diagram IR divergences needed to cancel bremsstrahlung IR divergences.)

- Intermediate hadron only proton.
- Note IR divergences come from $q_1 \approx 0$ or $q_2 \approx 0$. Hence, set (e.g.) $q_2 = 0$ everywhere “safe.” Meaning: $q_2 = 0$ in $q_1$ propagator, and in numerator.
- Not accurate when both photons hard. Quote: “assume the noninfrared parts of these diagrams to be negligible.” Honest and totally o.k. if true.
Improved by Maximon and Tjon [2000] ($q_2 \to 0$ in fewer places).

Better hadronic evaluation: Blunden, Melnitchouk, and Tjon [2003]

- Intermediate hadron only proton
- Include form factors for proton, within integral
- But need form factors when proton, not just photon, is off-shell.
- At higher momentum transfers, need resonances in intermediate state if pursuing hadronic calculation.

Also 2003, Possibility proof for parton calc.: Guichon & Vanderhaeghen

- But not *ab initio* calculation.
Partonic Evaluation of Box Diagrams.

Start with notation for $ep \rightarrow ep$ amplitudes

\[
\begin{array}{c}
\begin{array}{c}
\bar{u}(k_2)\gamma_\mu u(k_1) \times \bar{u}(p_2) \left[ \gamma^\mu G_M' - \frac{(p_1 + p_2)^\mu}{2M} F_2' \right] u(p_1) \\
\end{array} \\
\end{array}
\]

Basic: extra structure from the multiple photon exchange,

\[
\mathcal{M}^N = \frac{Ze^2}{Q^2} \left\{ \bar{u}(k_2)\gamma_\mu u(k_1) \times \bar{u}(p_2) \left[ \gamma^\mu G_M' - \frac{(p_1 + p_2)^\mu}{2M} F_2' \right] u(p_1) \\
+ \bar{u}(k_2)\gamma_5 \bar{u}(k_1) \times \bar{u}(p_2) \left[ \gamma^\mu \gamma_5 G_A' \right] u(p_1) \right\}
\]

Alternatively, equivalent to

\[
\mathcal{M}^N = \frac{Ze^2}{Q^2} \sigma(k_2)\gamma_\mu u(k_1) \times \pi(p_2) \left[ \gamma^\mu \bar{G}_M - \frac{(p_1 + p_2)^\mu}{2M} \bar{F}_2 + \frac{(p_1 + p_2)^\mu (\slashed{y}_1 + \slashed{y}_2)}{4M^2} \bar{F}_3 \right] u(p_1)
\]
Form factors above have contributions from $1\gamma$ and $2\gamma$ exchanges,

\[ G'_M = G_M^{(1\gamma)} + G_M^{(2\gamma)} = G_M + \delta G'_M \]
\[ G'_E = G_E + \delta G'_E \]
\[ G'_A = \text{zero} + \delta G'_A \]

$G_{M,E}$ are standard form factors defined from matrix element of e.m. current.

Cross section (to LO and $O(\epsilon^2) \times$ LO),

\[ \frac{d\sigma}{d\Omega_{Lab}} = \frac{\sigma_{NS}}{\epsilon(1 + \tau)} \left( \tau |G'_M|^2 + \epsilon |G'_E|^2 + 2\sqrt{\tau(1 + \tau)(1 - \epsilon^2)} \text{ Re } G'_M G'_A \right) \]

with

\[ \sigma_{NS} = \frac{4\alpha^2 \cos^2(\theta/2)}{Q^4} \frac{E^3_2}{E_1} \]

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Partonic calculations

- Main contributions come from “handbag” diagrams (one active quark).
- “Cat’s ears” diagrams, where photons interact with different quarks, important for getting overall IR divergence correct. However, contributions when both photons are hard is suppressed at higher $Q^2$.

- Calculate box (and crossed box) diagram at quark level, then embed in proton using generalized parton distribution (GPD).
Box diagrams for $eq \to eq$, with massless quarks

\[
M^q = M^q_{LO} + \frac{e^2}{Q^2} \bar{u}(k_2) \gamma_{\mu} u(k_1) \times \bar{u}(p_{q_2}) \left[ \gamma^{\mu} e_q^2 f_1 + P_q^\mu K e_q^2 f_3 \right] u(p_{q_1})
\]

for $P_q \equiv (p_{q_1} + p_{q_2})/2$ and $K \equiv (k_1 + k_2)/2$.

Calculation same as $e\mu \to e\mu$, which has been done analytically, can be found in the literature (e.g., van Nieuwenhuizen [1971]), and has been verified locally.

- Boxes have IR divergence, which must cancel or disappear in end; control by temporarily putting in photon mass $\lambda$.
- Separate soft (IR divergent) and hard parts by criterion of Grammer and Yennie.
We have both real and imaginary parts of $\tilde{f}_1$ and $\tilde{f}_3$.

For here, just display imaginary parts,

$$\text{Im } \tilde{f}_1^{soft} = \frac{e^2}{4\pi} \ln \left( \frac{s}{\lambda^2} \right)$$

$$\text{Im } \tilde{f}_1^{hard} = \frac{e^2}{4\pi} \left\{ \frac{Q^2}{2\hat{u}} \ln \left( \frac{s}{Q^2} \right) - \frac{1}{2} \right\}$$

$$\text{Im } \tilde{f}_3 = -\frac{e^2}{4\pi} \frac{1}{\hat{u}} \left\{ \frac{s - \hat{u}}{2\hat{u}} \ln \left( \frac{s}{Q^2} \right) + 1 \right\}$$

($\hat{s}$ and $\hat{u}$ are Mandelstam variables for the subprocess $e\bar{q} \rightarrow e\bar{q}$).

And also

$$\text{Re } \tilde{f}_1^{soft} = \frac{e^2}{4\pi^2} \left\{ \ln \left( \frac{\lambda^2}{\sqrt{-su}} \right) \ln \frac{s}{-\hat{u}} + \frac{\pi^2}{2} \right\}$$
Soft contributions

- ∃ low energy theorem: sum of soft contributions from partonic calculations equals soft contributions from nucleonic calculation.

- Works because there are also soft contributions from cat’s ears diagrams.

Pictorial explanation of low energy theorem:

Say that right-hand photon is the soft one.

\[ \sum_{q_i} \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array} + \begin{array}{c}
\text{Diagram 4} \\
\text{Diagram 5} \\
\text{Diagram 6}
\end{array} = \begin{array}{c}
\text{Diagram 7}
\end{array} \]

LHS equivalent to one hard photon, with form factor, and one soft photon on nucleon.
• Repeat: hard parts from cat’s ears diagrams are subleading in $Q^2$ because of momentum mismatches in integrals.

• For imaginary parts, consequence of low energy theorem is that all amplitudes multiplied by same Coulomb phase. ∴ contribution of soft parts to coming calculation of $A_n$ is zero.

• Real parts and bremsstrahlung: next page.
• The IR divergence in the box is cancelled by an IR divergence from bremsstrahlung, specifically an interference between bremsstrahlung from the electron and bremsstrahlung from the proton. Write

$$\sigma_{1\gamma+2\gamma,soft} = \sigma_{1\gamma} \left( 1 + \delta_{2\gamma}^{\text{soft}} + \delta_{\text{brems}}^{\text{ep}} \right)$$

• Because of low energy theorem, take $\delta_{2\gamma}^{\text{soft}}$ from nucleonic calculation,

$$\delta_{2\gamma}^{\text{soft}} = \frac{e^2}{2\pi^2} \left\{ \text{Nucleonic} \left[ \ln \left( \frac{\lambda^2}{\sqrt{-s\,u}} \right) \ln \frac{\hat{s}}{-\hat{u}} \right] + \frac{\pi^2}{2} \right\}$$

• Take bremsstrahlung from Maximon and Tjon [2000]

• Compare numerically to corresponding Mo and Tsai correction: essentially the same (to 0.1% level) except for the $\pi^2/2$ term.

• Thus, since data generally presented with Mo-Tsai correction done, soft corrections give a constant factor $(1 + \pi \alpha)$ plus terms that are quite small.
Hard contributions.

Embed partonic calculation in a nucleon.

- Set-up for generalized parton distributions: remove a quark from the proton, and replace it with a quark of different momentum and possibly different helicity.

\[
\mathcal{M}^N_{h,\lambda_2,\lambda_1} = \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[ \mathcal{M}^q_{h,+1/2} + \mathcal{M}^q_{h,-1/2} \right] \bar{u}_{\lambda_2}(p_2) \left[ \gamma^+ H^q + \frac{i\sigma^{+\nu} q_\nu E^q}{2M} \right] u_{\lambda_1}(p_1) \\
+ \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[ \mathcal{M}^q_{h,+1/2} - \mathcal{M}^q_{h,-1/2} \right] \sigma_{\mu\nu}(x) \bar{u}_{\lambda_2}(p_2) \gamma^+ \gamma^5 H^q u_{\lambda_1}(p_1)
\]

- Work in light-front frame, \( q^+ \propto q^0 + q^3 = 0 \).
- Arguments of GPD’s are \( H^q(x, \xi = 0, Q^2) \), etc.
The $2\gamma$ corrections to the nucleon form factors become,

$$
\delta G'_M = \frac{1 + \epsilon}{2\epsilon} A - \frac{1 - \epsilon}{2\epsilon} C,
$$

$$
\delta G'_E = \sqrt{\frac{1 + \epsilon}{2\epsilon}} B,
$$

$$
\delta G'_A = \frac{t}{s - u} \frac{1 + \epsilon}{2\epsilon} (A - C),
$$

where the characteristic integrals are,

$$
A = \int_{-1}^{1} \frac{dx}{x} \frac{(s - \hat{u}) \hat{f}_1^{\text{hard}} - \hat{s} \hat{u} \hat{f}_3}{s - u} \sum_q e_q^2 (H^q + E^q)
$$

“electric GPD”

$$
B = \int_{-1}^{1} \frac{dx}{x} \frac{(s - \hat{u}) \hat{f}_1^{\text{hard}} - \hat{s} \hat{u} \hat{f}_3}{s - u} \sum_q e_q^2 (H^q - \tau E^q)
$$

“magnetic GPD”

$$
C = \int_{-1}^{1} \frac{dx}{x} \hat{f}_1^{\text{hard}} \text{sgn}(x) \sum_q e_q^2 \hat{H}^q.
$$

“axial GPD”

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Final inputs

- $G_{Ep}/G_{Mp}$ from polarization transfer data,

$$G_{Ep} = (1 - 0.13(Q^2 - 0.04)) \frac{G_{Mp}}{\mu_p}$$

- $G_{Mp}$ from analytic fit of Brash et al., solid line in
• For GPD’s use gaussian-valence model like Radyushkin and Diehl et al.,

\[ H^q(x, 0, Q^2) = q_v(x) \exp \left( -\frac{(1 - x)Q^2}{4x\sigma} \right) \]

\[ \bar{H}^q(x, 0, Q^2) = \Delta q_v(x) \exp \left( -\frac{(1 - x)Q^2}{4x\sigma} \right) \quad \text{[used } \sigma = 0.8 \text{ GeV}^2\text{]} \]

\[ E^q(x, 0, Q^2) = \frac{\kappa q}{N_q} (1 - x)^2 q_v(x) \exp \left( -\frac{(1 - x)Q^2}{4x\sigma} \right) \]

• Valence quark distributions are from Martin, Stirling, Roberts, and Thorne (MRST2002 NNLO fit at baseline \( Q_0^2 = 1 \text{GeV}^2 \)),

\[ u_v = 0.262 x^{-0.69} (1 - x)^{3.50} (1 + 3.83\sqrt{x} + 37.65x) \]

\[ \Delta u_v = 0.505 x^{-0.33} (1 - x)^{3.428} (1 + 2.179\sqrt{x} + 14.57x) \]

\[ d_v = 0.061 x^{-0.65} (1 - x)^{4.03} (1 + 49.05\sqrt{x} + 8.65x) \]

\[ \Delta d_v = -0.0185 x^{-0.73} (1 - x)^{3.864} (1 + 35.47\sqrt{x} + 28.97x) \]
Rosenbluth Plot Results

Plot reduced cross section, normalized to dipole form factor, vs. $\epsilon$.

Recall,

$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{\tau \sigma_{NS}}{\epsilon(1 + \tau)} \sigma_R$$

$$\sigma_R = \left(G_M^2 + \frac{\epsilon}{\tau} G_E^2\right) \left(1 + \delta_{2\gamma}^{so} + \delta_{brems}^{ep} - \delta_{MT}\right)$$

$$+ (1 + \epsilon) G_M \text{Re} A + \frac{\sqrt{2\epsilon(1 + \epsilon)}}{\tau} G_E \text{Re} B + (1 - \epsilon) G_M \text{Re} C$$

Plots show

$$R \equiv \frac{\sigma_R}{\mu_p^2 G_{dipole}^2}; \quad G_{dipole} \equiv \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2}$$

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Reduced Xsctn for ep elastic scattering

\[ Q^2 = 4 \text{ GeV}^2 \]

\[ \frac{\sigma_R}{(\mu_p G_{\text{dipole}})^2} \]

- **dashed (red):** LO result, w/ Brash et al. \( G_{Mp} \)
- **dotted (green):** our full result, w/ Brash et al. \( G_{Mp} \)
- **full curve (black):** our full result, w/ Brash et al. \( G_{Mp} \times 0.995 \).
Reduced Xsctn for ep elastic scattering

\[ \sigma_R / (\mu_p G_{dipole})^2 \] vs. \( Q^2 = 5 \text{ GeV}^2 \)

- \( R_{1\gamma} \)
- \( R_{1\gamma+2\gamma} \)
- \( R_{1\gamma+2\gamma} - G_{Mp} \text{ rescaled} \)

\( \sigma_R \) vs. \( \epsilon \)
Reduced Xsctn for ep elastic scattering

\[ Q^2 = 6 \text{ GeV}^2 \]

\[ \sigma_R \left/ \left( \mu_p G_{\text{dipole}} \right)^2 \right. \]

- \( R_{1\gamma} \)
- \( R_{1\gamma+2\gamma} \)
- \( R_{1\gamma+2\gamma} - G_{M_p} \) rescaled

\( \bullet \) data
Re: Rosenbluth plots

- Polarization transfer determined form factors do not fit data, if just Mo-Tsai (e.g.) are only radiative corrections applied.
- Including hard two-photon exchange corrections changes the slope in $\epsilon$ and reconciles the Rosenbluth and polarization transfer data. Dependence in $\epsilon$ not linear.
- Should do reanalysis of extraction of $G_{Mp}$ and $G_{Ep}$ from data using full HO corrections. Beyond today's scope. Did show that reducing present good $G_{Mp}$ fit by (1/2)% could improve fit to data.
**Polarization Results**

Analyzing powers and polarizations:

\[
\sigma_R A_x = -(2h_e) \sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}} \left\{ G_E G_M + \sqrt{\frac{1+\epsilon}{2\epsilon}} G_M \text{Re} B + G_E \text{Re} C \right\} = \sigma_R P_x
\]

\[
\sigma_R A_y = \sqrt{\frac{2\epsilon(1+\epsilon)}{\tau}} \left\{ G_E \text{Im} A - \sqrt{\frac{1+\epsilon}{2\epsilon}} G_M \text{Im} B \right\} = \sigma_R P_y
\]

\[
\sigma_R A_z = -(2h_e) \sqrt{1-\epsilon^2} \left\{ G_M^2 + G_M (\text{Re} A + \text{Re} C) \right\} = -\sigma_R P_z
\]

\[ h_e = \text{electron helicity} = \pm 1/2. \]
Longitudinal Polarization

$P_z^{(1\gamma+2\gamma)}$

$P_z^{(1\gamma)}$

$Q^2 = 5 \text{ GeV}^2$

$P_z$ vs $\varepsilon$

$\varepsilon$ values from 0 to 1.
Sideways Polarization

\( Q^2 = 5 \text{ GeV}^2 \)

- \( P_x (1 \gamma + 2 \gamma) \)
- \( P_x (1 \gamma) \)
Polarization ratio with radiative corrections

\[ \frac{P_x}{P_z}(1\gamma) \]

\[ \frac{P_x}{P_z}(1\gamma+2\gamma) \]

\[ Q^2 = 5 \text{ GeV}^2 \]
Normal Polarization or Analyzing Power

$E_{e,\text{Lab}} = 6 \text{ GeV}$

$P_y (%)$, full

$\theta_{\text{CM}}$

$P_y (%)$

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Normal Polarization or Analyzing Power

$E_{e,\text{Lab}} = 3 \text{ GeV}$

$P_y (%)$, full

(Cut off at ends when $-t = M^2$ or $-u = M^2$.)
Electron-Positron Ratio Results

\[ \frac{e^+ p}{e^- p} \text{ cross section ratio} \]

\[ \sigma_{e^+p}/\sigma_{e^-p} \]

- \( Q^2 = 4 \text{ GeV}^2 \)
- \( Q^2 = 5 \text{ GeV}^2 \)
- \( Q^2 = 6 \text{ GeV}^2 \)

(data figure from Arrington)
Closing Comments

- Presented a partonic calculation of the two-photon exchange corrections to elastic electron-proton scattering.
- Valid for high $Q^2$, say $Q^2 >> M^2$

- In comparing to data, used $G_{EP}/G_{MP}$ from polarization measurements.
- Find that in Rosenbluth plot two-photon exchange corrections give additional slope, sufficient to reconcile Rosenbluth and polarization data.
- Detail: Soft photon corrections shifted the data but did not introduce a slope (compared to existing Mo-Tsai corrections). Change in slope came from hard (both photons energetic) corrections.
more closing comments

- For sideways and longitudinal polarization, corrections small.
- For normal direction, predict $O(1/2\%)$ polarization
- Predict $O(\text{few}\%)$ effects in positron-proton/electron-proton cross section ratio.

The End