The shape of the nucleon

November 19, 2004

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JLab and W&M

Outline:

I. Background

II. The model

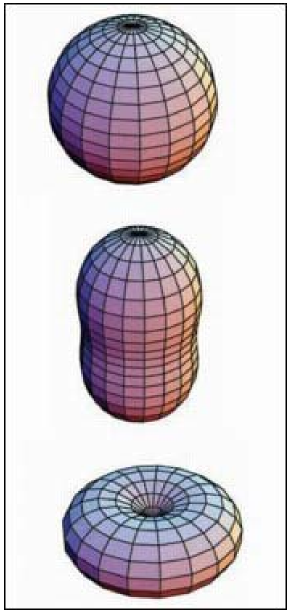
II. Parameters and results

III. Discussion and implications

*FG and Peter Agbakpe, JLAB-THY-04-39 (to be submitted next week)

Background

- Beautiful new data from JLab on the form factors
- Lots of excitement about the decrease in the ratio $R_p = G_{Ep}/G_{Mp}$ with Q^2
- Enter Jerry Miller, John Ralston, and the workshop of May 2002
  - Deformed protons !!!!
  - lack of helicity conservation !!!!
  - USA Today, Science News !!!
  - Great discovery !!!!
- Whats going ON !!!?



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*G. Miller, Phys. Rev. C 68, 022201(R) (2003)

Physicists thrown for a loop ; New insight on protons changes the shape of things; [FINAL Edition]

Dan Vergano. USA TODAY. McLean, Va.: Sep 23, 2002. pg. D.07

USA TODAY

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Science and education

The humble proton, an atomic particle with mysteries long thought solved, turns out to have a hidden secret, scientists report.

Experimental results released this year by the Department of Energy's Jefferson Lab in Newport News, Va., have upturned the normally placid world of nuclear physics with the suggestion that protons, the positively charged particles found in the center of every atom, aren't round. Instead, they seem somewhat elliptical.

The round proton has been a staple of textbooks for 40 years, tied to the theory that protons and neutrons are built of three smaller particles called "quarks" slowly bubbling inside their interiors.
What difference does it make whether protons are round or elliptical? Plenty, physicists say. Adjustments in protons and neutrons could affect scientific understanding of the magnetic "spin" of atoms. Scientists hope to use "spintronics" in future computers and tiny "nano-scale" devices. Understanding the fundamental shape of particles will affect those application's success.

At a Jefferson Lab meeting in May, about 60 nuclear physicists met to debate the "crisis," in the words of physicist John Ralston of the University of Kansas, over the odd shape of the proton.

At the May meeting, a consensus emerged that Einstein's theory of special relativity, which explains how things moving near the speed of light have smooshed lengths and increased mass, seemed the likeliest explanation for the weird experiment results.

"The new thing we've figured out is that quarks are moving around inside the proton at relativistic (near speed of light) speeds," says physicist Gerald Miller of the University of Washington- Seattle. Quarks moving at those speeds simply elongate the particle's electromagnetic shape, Miller says. In a paper in the journal Physical Review C, he outlines how quarks moving at high speeds, about 90% of the speed of light, stretch out protons.

"The proton is the simplest thing around, and it is not spherical," says physicist Charles Glashausser of the Rutgers University campus in Piscataway, N.J. The neutron, the uncharged partner-particle to the proton in the nucleus of atoms, also is built of quarks, he notes.

WHAT'S GOING ON?

- A spin 1/2 object cannot have a quadrupole deformation!

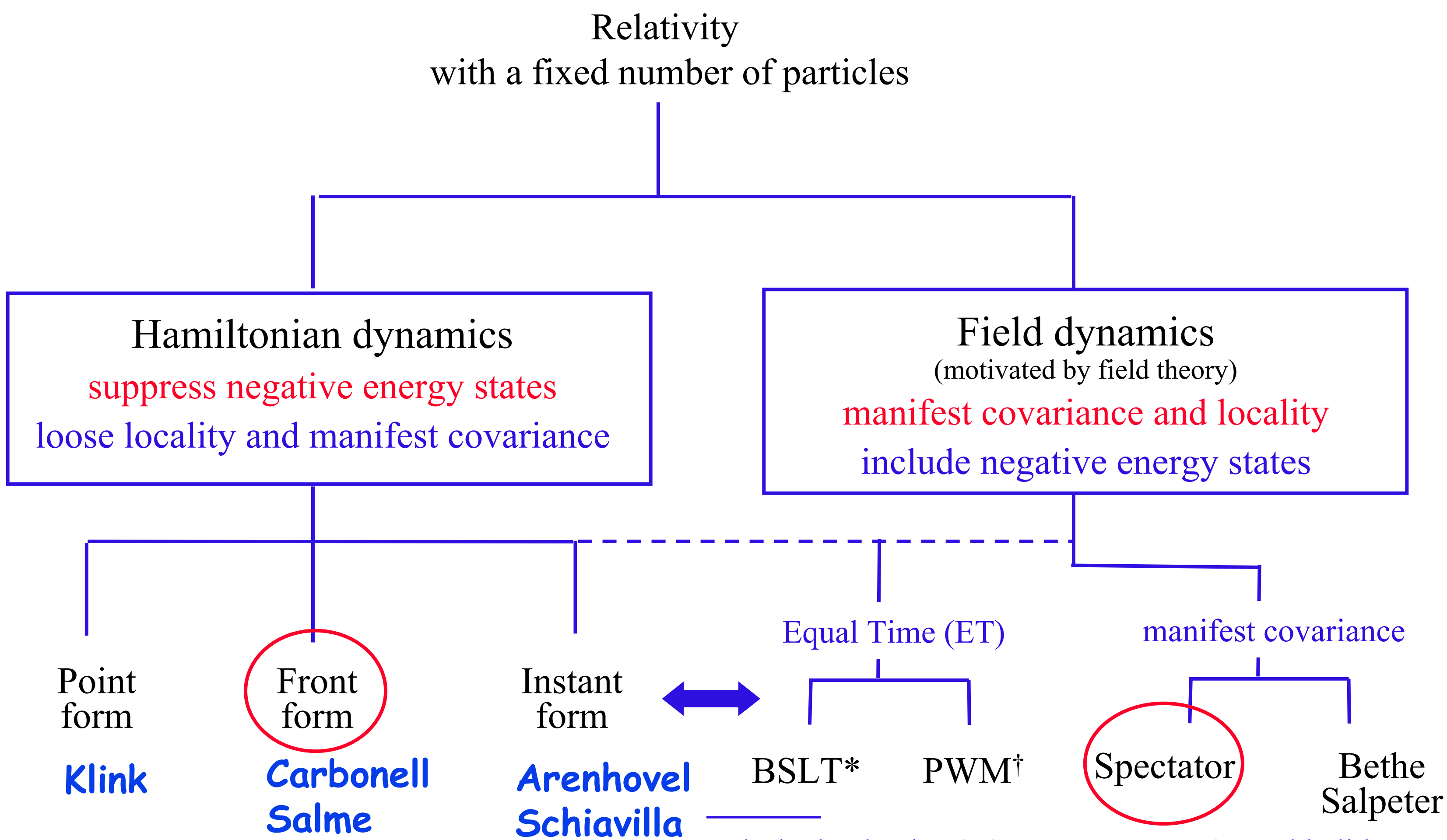
  (*Unless it is deformed in the sense of the transition nuclei, described by Bohr and Mottelson.* In this case there will be rotational bands and large electromagnetic transition rates, not seen.*)

- Talking about such things could make the lab look foolish, or we could waste our time arguing about what we mean. It matters (confusion and ethics).
- Can the data be explained using standard physics?
- To remove the danger of making a mistake about shapes, we MUST be sure rotational invariance is treated exactly.

  (The light-front formalism does not.)

***See: Buchmann and Henley, PRC 63,015202 (2001)**

There are many choices of relativistic theory



*Blankenbecler & Sugar, Logunov & Tavkhelidze
†Phillips, Wallace, and Mandelzweig

Hamiltonian dynamics: Dirac classifications of 1947

Some of the Poincaré transformations are kinematic; others involve the dynamics

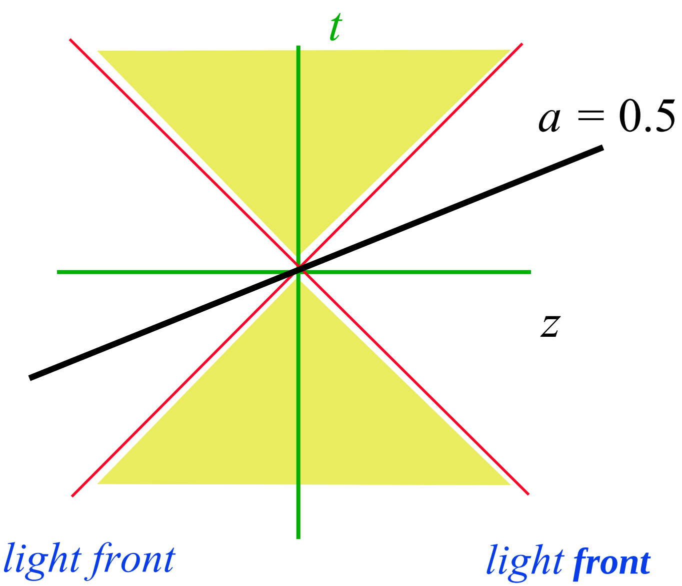
- Plane forms

$$t - a\,z = 0$$
$$-1 \le a \le 1$$

$a \le 1$: instant form — 6+4

$a = 1$: front form — 7+3

Limit not continuous

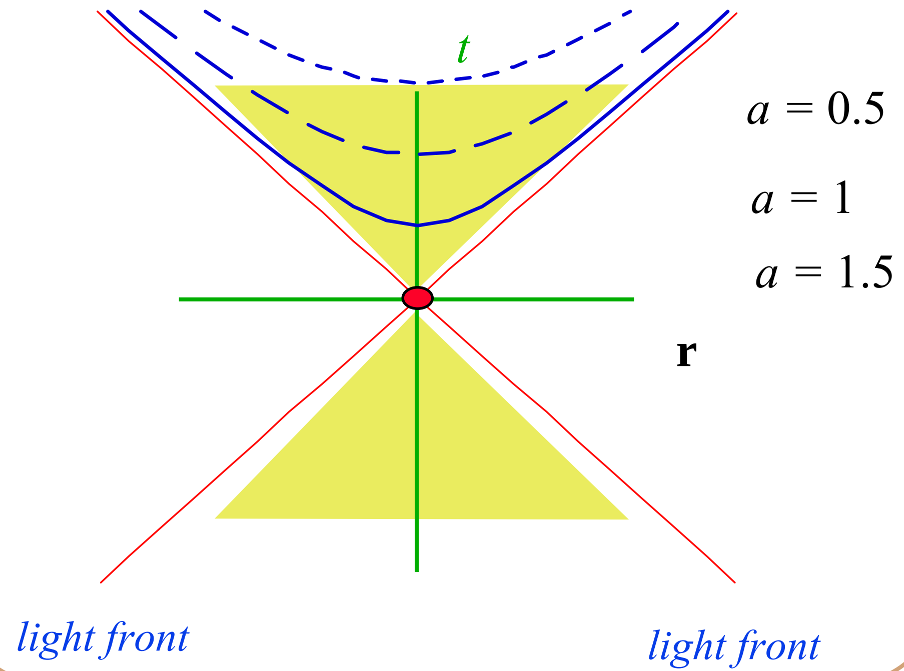


- Hyperbolic forms

$$t = \sqrt{(\mathbf{r}^2 + a^2)}$$

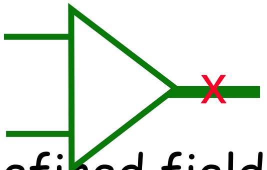
$a = 0$: point form on the light cone

$a = \infty$: instant form



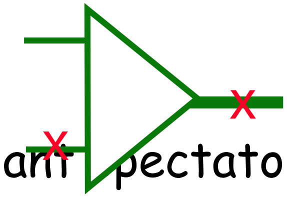
Field dynamics has a connection to field theory

- The Bethe-Salpeter amplitude is a well defined field theoretic matrix element:



$$\Psi(x_1,x_2)=\langle 0\,|\,T\big(\psi(x_1)\,\psi(x_1)\big)\,|\,d\rangle$$

- The covariant spectator amplitude is *also* a well defined field theoretic amplitude:



$$\Psi(x_1)=\langle N\,|\,\psi(x_1)\,|\,d\rangle$$

- Equations for the Bethe-Salpeter and the covariant spectator* amplitudes can be derived from field theory
  - Both are manifestly covariant under *all* Poincaré transformations
  - Both incorporate negative energy (antiparticle) states

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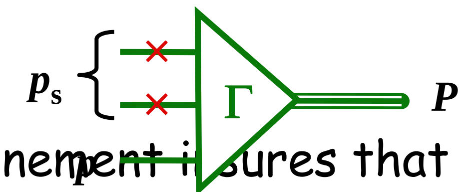
*O. W. Greenberg's "n-quantum approximation"

The Model

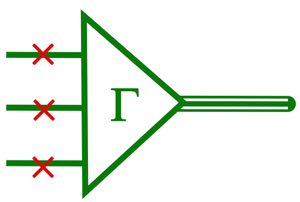
The nucleon wave function (1)

- The nucleon consists of 3 constituent quarks (CQ) with a size, mass, and form factor given by the dressing of the quark in the sea of gluons and $q\bar{q}$ pairs.
- Using the spectator theory, the nucleon is described by a 3-CQ vertex function with two of the CQ on shell



$$\Psi_\alpha = \left(\frac{1}{m - \not{p} - i\varepsilon}\right)_{\alpha\beta} \Gamma_\beta(P, p_s)$$

- Confinement insures that this vertex function is zero when all three quarks are on shell (i.e. there is no 3q scattering)



= 0

Hence, model Ψ directly

The nucleon wave function (2)

- Spin-isospin structure of the NR nucleon wave function
  - Fermion antisymmetry comes from the color factor
  - Assume a simple, fully symmetric S state spatial wave function
  - Then spin-flavor wave function is fully symmetric
  - In nonrelativistic theory, the proton wave function is

$$\xi_{23}^{00} = \frac{1}{\sqrt{2}}\left(u_2 d_3 - d_2 u_3\right)$$

$$\xi_{23}^{1m} = \begin{cases} \xi_{23}^{11} = u_2 u_3 \\ \xi_{23}^{10} = \frac{1}{\sqrt{2}}\left(u_2 d_3 + d_2 u_3\right) \\ \xi_{23}^{1-1} = d_2 d_3 \end{cases}$$

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}\left\{\Phi_F^0\left(+\tfrac{1}{2}\right)\Phi_S^0\left(+\tfrac{1}{2}\right) + \Phi_F^1\left(+\tfrac{1}{2}\right)\Phi_S^1\left(+\tfrac{1}{2}\right)\right\}$$

$$\Phi_F^0\left(+\tfrac{1}{2}\right) = u_1\,\xi_{23}^{00} \qquad \Phi_F^1\left(+\tfrac{1}{2}\right) = \sqrt{\tfrac{2}{3}}\,d_1\,\xi_{23}^{11} - \sqrt{\tfrac{1}{3}}\,u_1\,\xi_{23}^{10}$$

hence, suppressing the 1,2,3 quark labels

$$\Phi_F^0\left(+\tfrac{1}{2}\right)\Phi_S^0\left(+\tfrac{1}{2}\right) = u^\uparrow\,\tfrac{1}{2}\left[u^\uparrow d^\downarrow - u^\downarrow d^\uparrow - d^\uparrow u^\downarrow + d^\downarrow u^\uparrow\right]$$

$$\Phi_F^1\left(+\tfrac{1}{2}\right)\Phi_S^1\left(+\tfrac{1}{2}\right) = \tfrac{2}{3}\,d^\downarrow u^\uparrow u^\uparrow - \tfrac{1}{3}\,d^\uparrow\left(u^\uparrow u^\downarrow + u^\downarrow u^\uparrow\right) - \tfrac{1}{3}\,u^\downarrow\left(u^\uparrow d^\uparrow + d^\uparrow u^\uparrow\right)$$

$$+\tfrac{1}{6}\,u^\uparrow\left(u^\uparrow d^\downarrow + d^\downarrow u^\uparrow + u^\downarrow d^\uparrow + d^\uparrow u^\downarrow\right)$$

and

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left\{2u^\uparrow u^\uparrow d^\downarrow + 2u^\uparrow d^\downarrow u^\uparrow + 2d^\downarrow u^\uparrow u^\uparrow - u^\uparrow u^\downarrow d^\uparrow - u^\uparrow d^\uparrow u^\downarrow - d^\uparrow u^\uparrow u^\downarrow - d^\uparrow u^\downarrow u^\uparrow - u^\downarrow u^\uparrow d^\uparrow - u^\downarrow d^\uparrow u^\uparrow\right\}$$

The nucleon wave function (3)

- Spin-isospin structure of the NR nucleon wave function (cont'd)
  - introduce a matematically compact form

$$|fs\rangle = \frac{1}{\sqrt{2}}\left\{\left[\xi_S^0\,\chi^s\right]\left[\xi_F^0\,\chi^f\right] - \frac{1}{3}\left[\sigma\cdot\xi_S^1\,\chi^s\right]\left[\tau\cdot\xi_F^1\,\chi^f\right]\right\} \qquad \chi^{\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} \quad \chi^{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix}$$

scalar spin
scalar flavor
diquark

axial vector spin
vector flavor
diquark

- Relativistic wave function, including spin-flavor structure

$$\Psi_\alpha = \frac{1}{\sqrt{2}}\left\{u_\alpha(P,s)\left[\xi^0\,\chi^f\right]\psi_{00}(P,p_s) + \frac{1}{3}\left(\gamma^5\,\not{\eta}\right)_{\alpha\beta}u_\beta(P,s)\left[\tau\cdot\xi^1\,\chi^f\right]\psi_{11}(P,p_s)\right\}$$

(0,0) diquark
scalar spin - scalar flavor

(1,1) diquark
axial vector spin - vector flavor

- when P = 0, this reduces to the nonrelativistic form

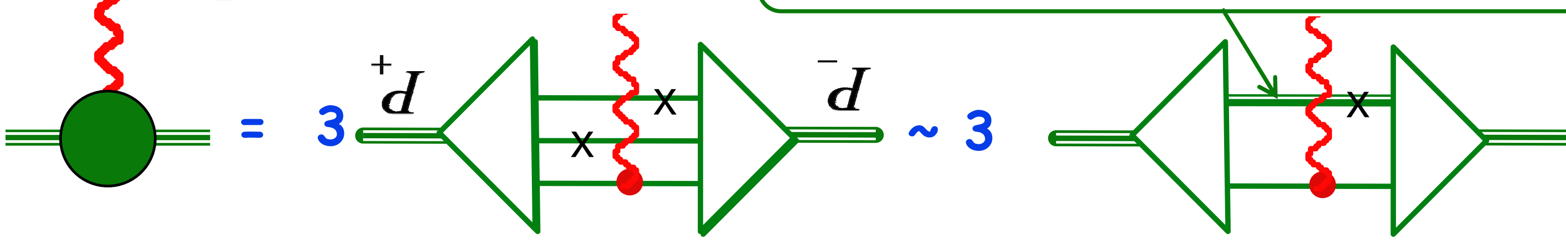
$\longrightarrow$

Relativistic impulse approximation for the form factors (1)

In the spectator theory, the photon couples to the off-shell quark, and because of the symmetry, the coupling to all three quarks is 3 times the coupling to one

$$P_{\pm} = P \pm \tfrac{1}{2} q \qquad Q^2 = -q^2$$

approximate the two spectator quarks by a single diquark with a fixed mass



$$\mathrm{J}_I^{\mu} = \bar{u}(P_+, \lambda') \, \frac{3}{2} \int \frac{d^3 p_s}{(2\pi)^3 2E_s(p_s)} \Big\{ j_I^{\mu} \, \psi_{00}(P_+, p_s) \psi_{00}(P_-, p_s) - \frac{1}{9} \gamma^{\nu} \gamma^5 \tau_j j_I^{\mu} \tau_j \gamma^5 \gamma^{\nu'} \Delta_{\nu\nu'} \psi_{11}(P_+, p_s) \psi_{11}(P_-, p_s) \Big\} u(P_-, \lambda)$$

quark currents with form factors

integrate over the (on-shell) spectator three momentum

sum over the vector diquark, using

$$\sum_{\eta} \eta_{\nu} \eta_{\nu'} \equiv \Delta_{\nu\nu'} = -g_{\nu\nu'} + \frac{p_{s\nu} p_{s\nu'}}{m_s^2}$$

Relativistic Impulse Approximation for the form factors (2)

- Do the Dirac and isospin algebra to get

$$F_1(Q^2) = \frac{3}{2}\int \frac{d^3\kappa}{(2\pi)^3 2E_s(\kappa)}\left\{\phi_0^+\phi_0^- j_1 + \frac{1}{3}\phi_1^+\phi_1^-\left(j_3 R_1 - j_4 R_2 \tfrac{1}{4}Q_0^2\right)\right\}$$

$$F_2(Q^2) = \frac{3}{2}\int \frac{d^3\kappa}{(2\pi)^3 2E_s(\kappa)}\left\{\phi_0^+\phi_0^- j_2 - \frac{1}{3}\phi_1^+\phi_1^-\left(j_3 R_2 + j_4 R_3\right)\right\}$$

where all variables are dimensionless $p_s = m_s\kappa,\ E_s(\kappa) = \sqrt{1+\kappa^2}$, $Q_0 = Q/M$,

and the wave functions are $\psi_{00}(P_\pm, p_s) = \frac{1}{m_s^2}\phi_0^\pm$

$$\phi_0^\pm \equiv \frac{N_0}{\left(\beta_1 - 2 + \Theta^\pm\right)\left(\beta_2 - 2 + \Theta^\pm\right)}$$

$$\phi_1^\pm = \sqrt{\frac{6}{2+\left(\Theta^\pm\right)^2}}\,\phi_0^\pm$$

Why this factor?

and

$$\left(1+\tfrac{1}{4}Q_0^2\right)R_1 = 3 + 2\kappa^2 + \tfrac{1}{4}Q_0^2\left(1-\kappa^2\sin^2\theta\right)$$

$$\left(1+\tfrac{1}{4}Q_0^2\right)R_1 = 2 + \kappa^2\left(3-\cos^2\theta\right)$$

$$R_3 = 1 + \kappa^2\left(1+\cos^2\theta\right) - \tfrac{1}{4}Q_0^2 R_2$$

with $\Theta^\pm = \dfrac{M^2 + m_s^2 - (P_\pm - p_s)^2}{Mm_s} = 2E_s(\kappa)\sqrt{1+\tfrac{1}{4}Q_0^2} \pm \kappa Q_0\cos\theta$

Normalization and shape

- To find the shape and normalization, compute

$$J_I^0(q=0)=\bar{u}(P,\lambda')\,\frac{3}{2}\int\frac{d^3p_s}{(2\pi)^3 2E_s(p_s)}\Big\{\left(\tfrac{1}{6}+\tfrac{1}{2}\tau_3\right)\gamma^0\ \psi_{00}^2(P,p_s)$$

$$-\frac{1}{9}\gamma^\nu\gamma^5\tau_j\left(\tfrac{1}{6}+\tfrac{1}{2}\tau_3\right)\gamma^0\tau_j\gamma^5\gamma^{\nu'}\Delta_{\nu\nu'}\psi_{11}^2(P,p_s)\Big\}u(P,\lambda)$$

with $P=\{M,\mathbf{0}\}$, $u(P,\lambda)=\begin{pmatrix}1\\0\end{pmatrix}\chi_\lambda$. Then the densities are $\bar{u}(P,\lambda')\gamma^0 u(P,\lambda)=\delta_{\lambda'\lambda}$

and $-\bar{u}(P,\lambda')\gamma^\nu\gamma^5\gamma^0\gamma^5\gamma^{\nu'}\Delta_{\nu\nu'}\bar{u}(P,\lambda)$

$$=\bar{u}(P,\lambda')\left\{-\gamma^\nu\gamma^0\gamma_\nu+\frac{\not{p}_s\gamma^0\not{p}_s}{m_s^2}\right\}\bar{u}(P,\lambda)=\left(2+E_s^2(\kappa)+\kappa^2\right)\delta_{\lambda'\lambda}=\left(1+2E_s^2(\kappa)\right)\delta_{\lambda'\lambda}$$

Hence,

$$J_I^0(q=0)=\frac{3}{2}\int\frac{d^3p_s}{(2\pi)^3 2E_s(p_s)}\left\{\left(\tfrac{1}{6}+\tfrac{1}{2}\tau_3\right)\delta_{\lambda'\lambda}\ \psi_{00}^2(P,p_s)+\overbrace{\frac{1}{3}\tau_j\left(\tfrac{1}{6}+\tfrac{1}{2}\tau_3\right)\tau_j}^{\frac{1}{6}(1-\tau_3)}\left(1+2E_s^2(\kappa)\right)\frac{\psi_{00}^2(P,p_s)}{\left(1+2E_s^2(\kappa)\right)}\right\}$$

$$J_I^0(q=0)=\tfrac{1}{2}\left(1+\tau_3\right)\delta_{\lambda'\lambda}\int\frac{d^3p_s}{(2\pi)^3 2E_s(p_s)}\ \psi_{00}^2(P,p_s)$$

$$=\tfrac{1}{2}\left(1+\tau_3\right)\delta_{\lambda'\lambda}\int\frac{d^3\kappa}{(2\pi)^3 2E_s(\kappa)}\frac{N_0^2}{\left(\beta_1-2+2E_s(\kappa)\right)^2\left(\beta_2-2+2E_s(\kappa)\right)^2}$$

spherical shape

correct normalization

Quark form factors include vector dominance and a pion cloud

- The quark currents are

$$j_I^\mu = j_1 + j_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M},$$

with 4 form factors:

$j_1 = f_{1+} + \tau_3 f_{1-}$ and $j_1 = f_{1+} + \tau_3 f_{1-}$

here

+ is isoscalar, and

– is isovector, and

$$f_{1\pm} = \tfrac{1}{3} f_{1u} \mp \tfrac{1}{6} f_{1d}$$

$$f_{2\pm} = \tfrac{1}{2}\left(\mu_u f_{2u} \pm \mu_d f_{2d}\right)$$

the quark form factors have 9 parameters:

VDM terms

pion cloud

$$f_{1+} = e_+ \left(\frac{1-\lambda}{1 + Q_0^2 / \Lambda_{1+}^2} + \lambda \right) \quad f_{1-} = e_- \left(\frac{1-\lambda-\lambda_\pi}{1 + Q_0^2 / \Lambda_{1-}^2} + \frac{\lambda_\pi}{\left(1 + Q_0^2 / \Lambda_\pi^2\right)^2} + \lambda \right) \quad f_{2\pm} = \mu_\pm \left(\frac{1}{1 + Q_0^2 / \Lambda_{2\pm}^2} \right)$$

for Model II (following):

isoscalar charge $f_{1+} = e_+ \left(\dfrac{0.823}{1 + Q_0^2 / 3.23} + 0.177 \right)$

isovector charge $e_- \left(\dfrac{0.579}{1 + Q_0^2 / 5.81} + \dfrac{0.244}{\left(1 + Q_0^2 / 0.742\right)^2} + 0.177 \right)$

isoscalar magnetic $f_{2+} = \left(\dfrac{\mu_+}{1 + Q_0^2 / 0.386} \right)$

isovector magnetic $f_{2-} = \left(\dfrac{\mu_-}{1 + Q_0^2 / 1.23} \right)$

The Parameters and Results

Results: Overview

- The form factors data is fit with 11 parameters:
  - 1 asympotic value of the f_1 quark form factors, λ
  - 2 nucleon wave function parameters β_1 and β_2
  - 2 quark anomalous moments μ_u and μ_d
  - 2 pion cloud parameters, the strength λ_π , and its range, Λ_π
  - 4 vector monopole dominance scales for the four quark form factors, $\Lambda_{i\pm}$
  - 11 total
- The the model automatically normalizes $G_{Ep}(0) = 1$ and $G_{En}(0) = 0$, by adjusting the normalization constant (not a parameter) N_0
- The 2 quark anomalous moments are determined exactly from the nucleon anomalous moments
- The other 9 parameters are determined my minimizing χ^2.

The parameters $r_p^2(\exp) = 0.780(25), \quad r_n^2(\exp) = -0.113(7)$

- Model I fit to data

$$\chi^2/\text{datum} = 1.18, \quad \lambda = 0.230, \quad N_0^2 = 36.39, \quad r_p^2 = 0.800, \quad r_n^2 = -0.065$$

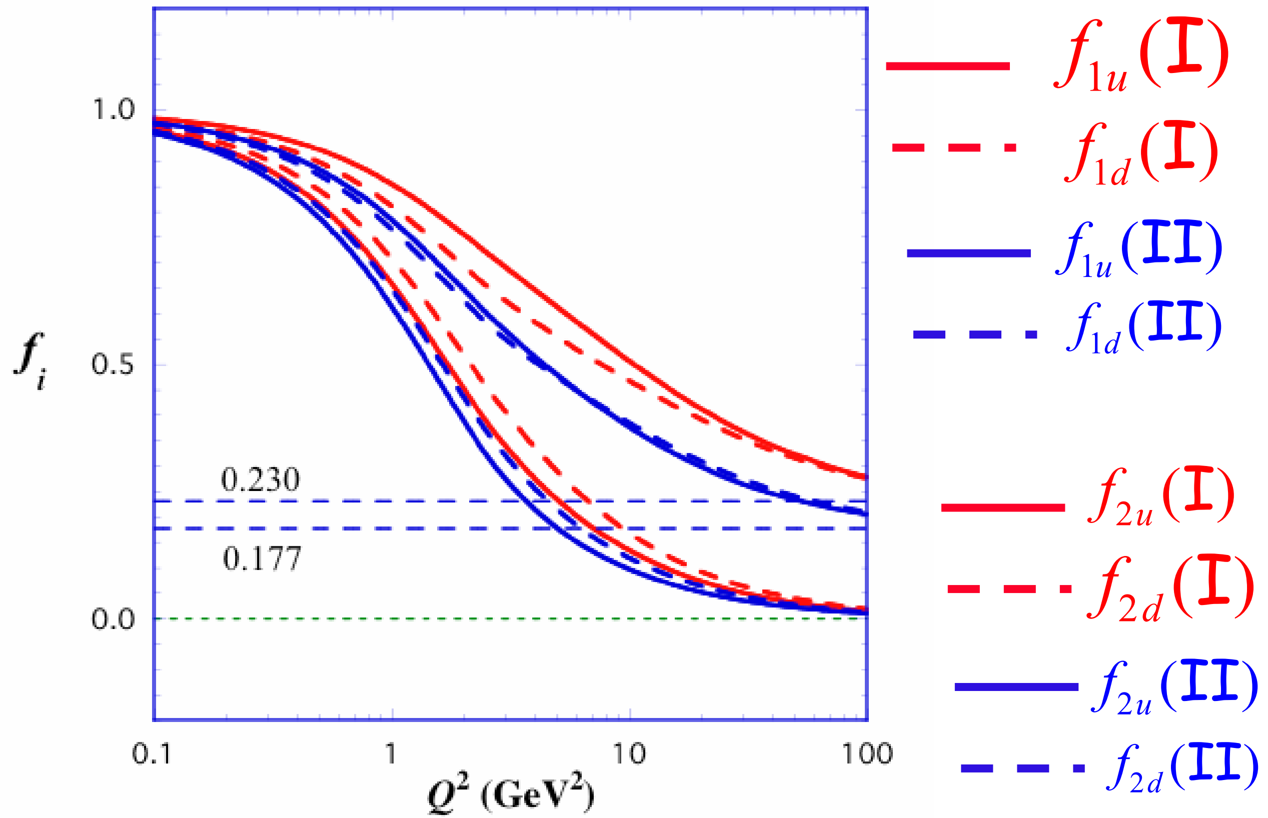
$\beta_1 \; \beta_2$	$\mu_u \; \mu_d$	$\Lambda_{1+}^2 \; \Lambda_{1-}^2$	$\Lambda_{2+}^2 \; \Lambda_{2-}^2$	$\lambda_\pi \; \Lambda_\pi^2$
0.057	1.125	7.69	0.362	0.245
1.982	–0.837	10.67	1.82	2.15

- **Model II fit to radii**

$$\chi^2/\text{datum} = 1.94, \quad \lambda = 0.177, \quad N_0^2 = 60.87, \quad r_p^2 = 0.771, \quad r_n^2 = -0.106$$

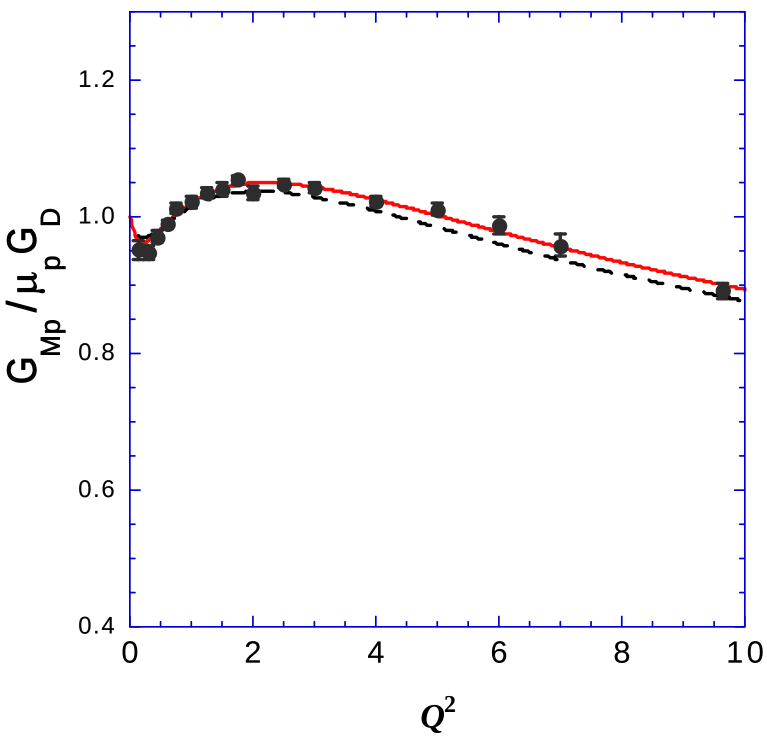
$\beta_1 \; \beta_2$	$\mu_u \; \mu_d$	$\Lambda_{1+}^2 \; \Lambda_{1-}^2$	$\Lambda_{2+}^2 \; \Lambda_{2-}^2$	$\lambda_\pi \; \Lambda_\pi^2$
0.069	1.126	3.23	0.386	0.244
1.543	–0.825	5.81	1.23	0.742

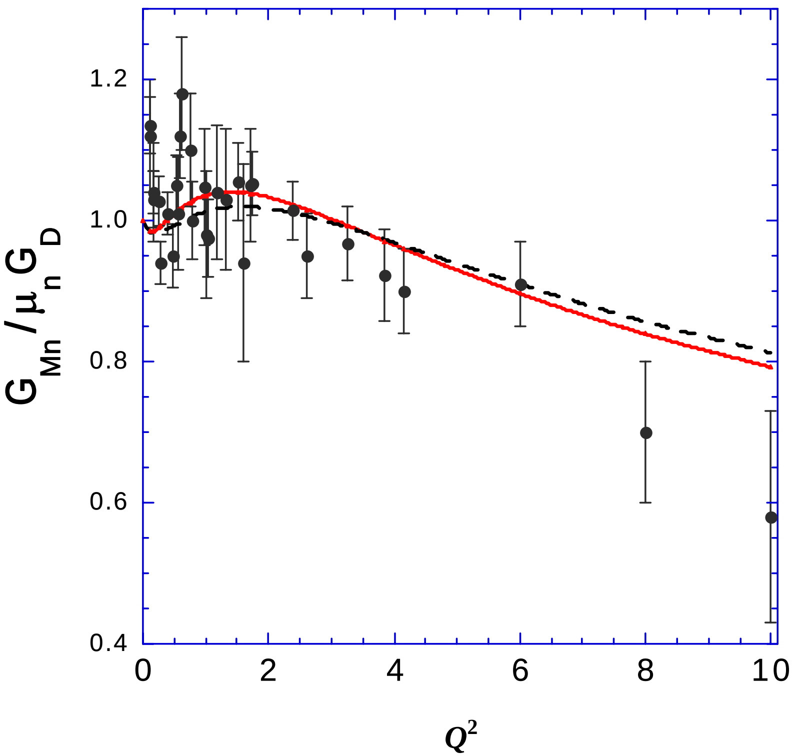
The quark form factors



The magnetic form factors

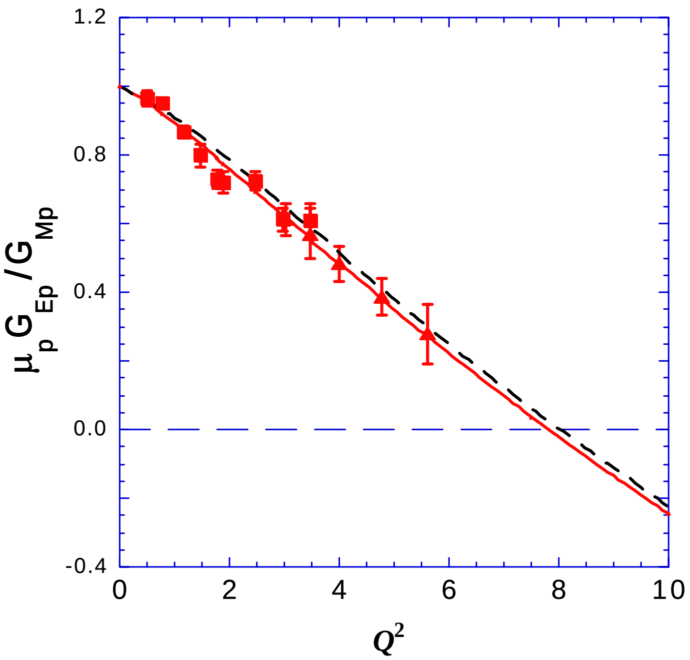
- Both magnetic form factors are similar in shape. Better data for G_{Mn} a must!

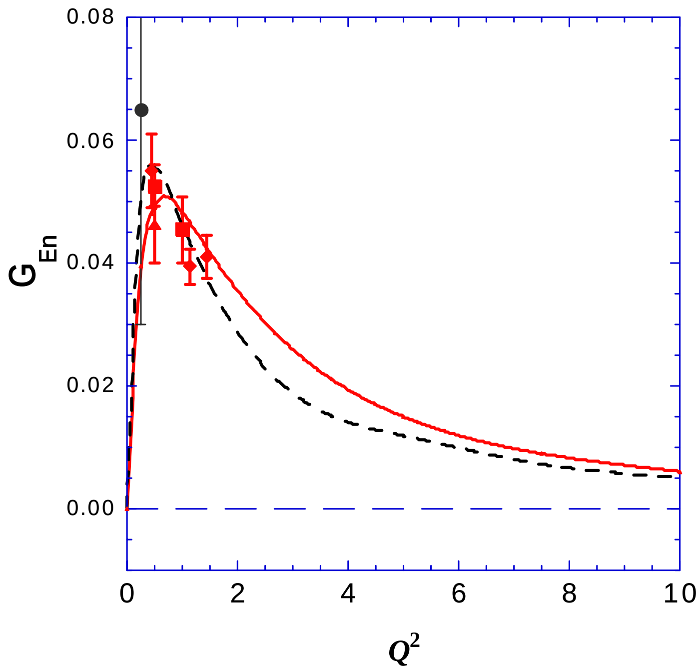




The charge form factors

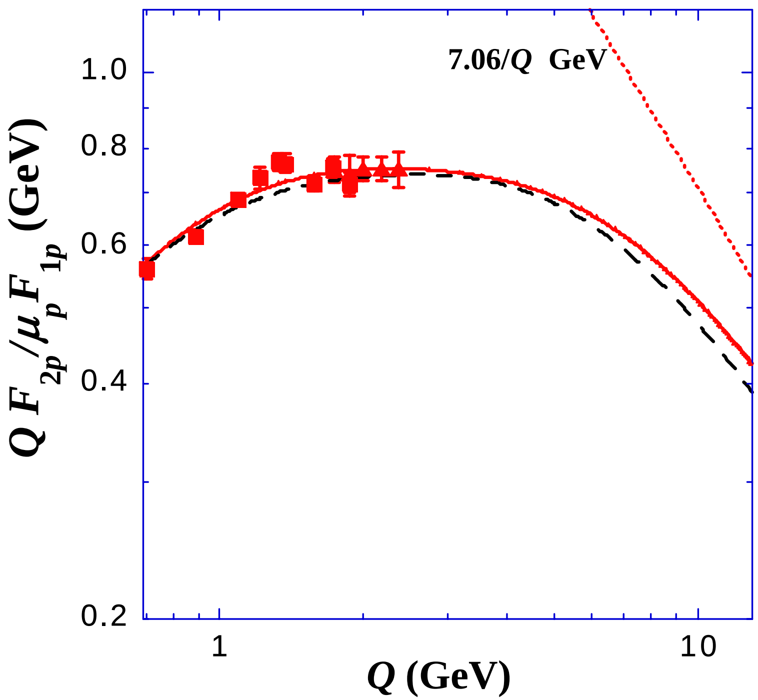
- The fits to the charge from factors are both excellent.
- Prediction: G_{Ep} will change sign at $Q^2 \sim 8$ GeV^2

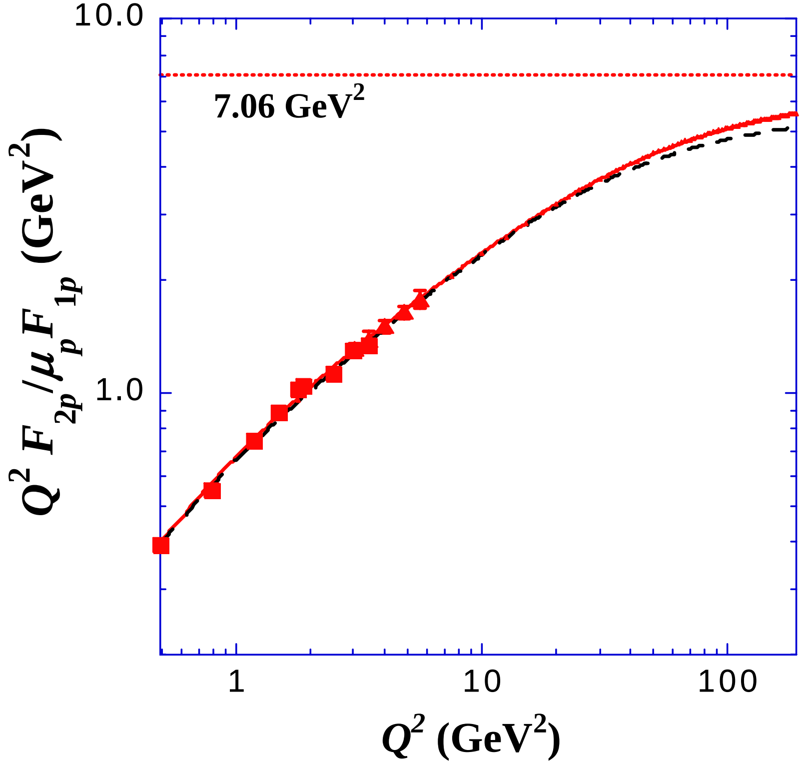




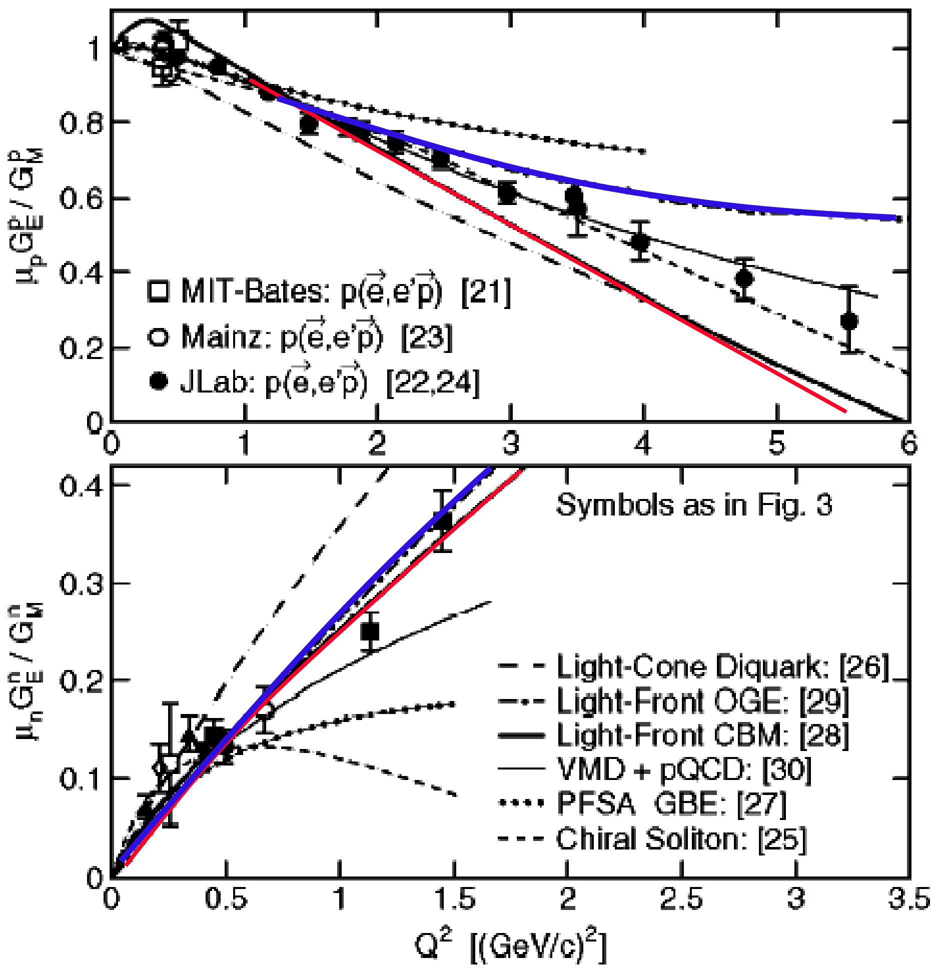
The F2/F1 puzzle for the proton

- QF_{2p}/F_{1p} is indeed flat, but this is an accident; The asymptotic value will not be reached until $Q^2 \sim 100$ GeV^2.





Comparisons (light front theory)



G. A. Miller,
Phys. Rev. *C* **66**, 032201(R) (2002)

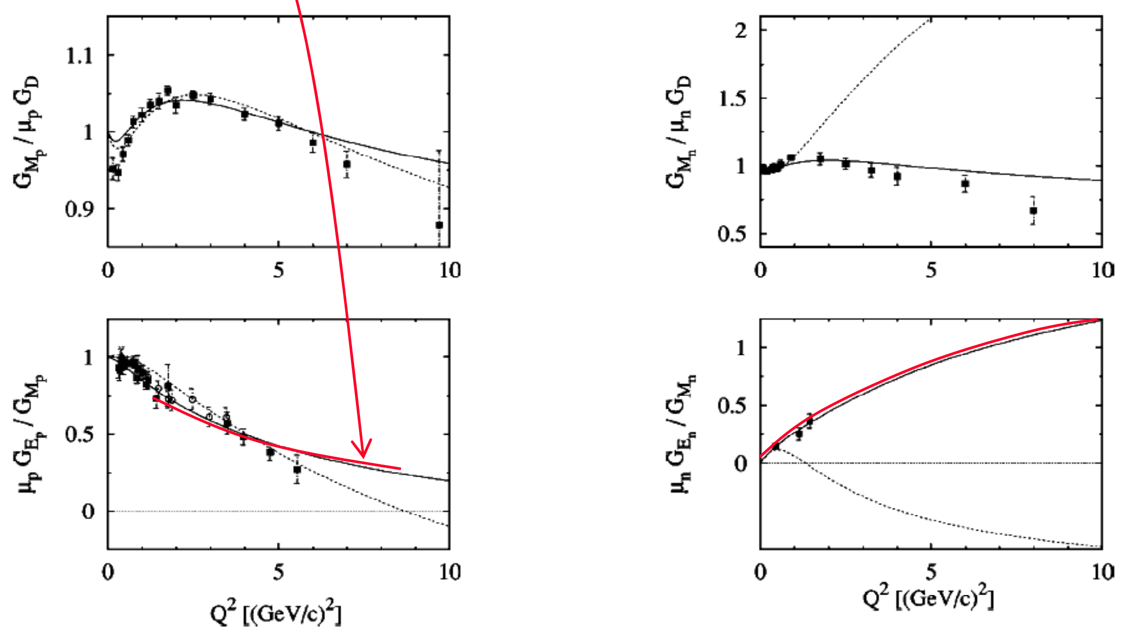
F. Cardarelli and S. Simula,
Phys. Rev. *C* 62, 065201 (2000)

Comparisons (vector dominance)

- Bijker and Iachello, Phys. Rev. *C* 69, 068201 (2004)

  Original model (IJL, 1973) fails, and new fit gives different prediction for G_{Ep} !!

- Lomon, Phys. Rev. *C* 66, 045501 (2002), gives similar result for G_{EP}



Discussion and Implications

- The data do not require the proton to be deformed!
  - This is forbidden by quantum mechanics (unless there are rotational bands).
  - This model is a counter example to claims that the data cannot be explained by a spherical proton
- The data do give interesting new information about the CQ form factors, and tell us about the proton wave function.
- The model is so simple that it can be used to study many phenomena near the quark-hadron transition
- Predictions
  - G_{Ep} will change sign near $Q^2 \sim 8$ GeV^2
  - G_{Mn} will be larger than the older data suggests
- What does this problem teach us about over-selling JLab's science?

END