The Role of Higher Twist and Positivity Constraints in Determining Polarized Parton Densities

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New QCD fits to the inclusive polarized DIS data

- two sets of polarized PD (in both the \(\overline{\text{MS}}\) and the JET schemes)
- JLab Hall A neutron data
- very recent COMPASS data on \(A_1^d\)

Role of higher twist in determining polarized PD

Factorization scheme dependence of the results

Impact of positivity constraints on polarized PD

Summary

*LSS: hep-ph/0503140*
one of the best tools to study the structure of nucleon

\[ Q^2 = -q^2 = 4E E' \sin^2 (\theta/2) \]

\[ x = \frac{Q^2}{2M \nu} \quad \nu = E - E' \]

DIS regime \( \Longrightarrow \quad Q^2 \gg M^2, \quad \nu \gg M \)

unpolarized SF

polarized SF

pQCD
As in the unpolarized case the main goal is:

- to test QCD
- to extract from the DIS data the **polarized** PD

\[
\begin{align*}
\Delta q(x, Q^2) &= q_+(x, Q^2) - q_-(x, Q^2) \\
\Delta \bar{q}(x, Q^2) &= \bar{q}_+(x, Q^2) - \bar{q}_-(x, Q^2) \\
\Delta G(x, Q^2) &= G_+(x, Q^2) - G_-(x, Q^2)
\end{align*}
\]

where "+" and "-" denote the helicity of the parton, along or opposite to the helicity of the parent nucleon, respectively.
The knowledge of the polarized PD will help us:

- to make predictions for other processes like polarized **hadron-hadron** reactions, etc.
- more generally, to answer the question how the helicity of the nucleon is divided up among its constituents:

\[
S_z = \frac{1}{2} = \frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + L_z(Q^2)
\]

\[
\Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}
\]

the parton polarizations \( \Delta q_a \) and \( \Delta G \) are the first moments

\[
\Delta q_a(Q^2) = \int_0^1 dx \Delta q_a(x, Q^2) \quad \Delta G(Q^2) = \int_0^1 dx \Delta G(x, Q^2)
\]

of the helicity densities: \( \Delta u(x, Q^2), \Delta \bar{u}(x, Q^2) \ldots, \Delta G(x, Q^2) \ldots \)
DIS Cross Section Asymmetries

Measured quantities

\[ A_{||} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\downarrow\uparrow}}{d\sigma^{\downarrow\uparrow} + d\sigma^{\uparrow\uparrow}}, \]

\[ A_{\perp} = \frac{d\sigma^{\downarrow\downarrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\downarrow\downarrow} + d\sigma^{\uparrow\downarrow}}. \]

\( (A_{||}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2) \)

where \( A_1, A_2 \) are the virtual photon-nucleon asymmetries.

At present, \( A_{||} \) is much better measured than \( A_{\perp} \)

If \( A_{||} \) and \( A_{\perp} \) are measured

\( \Rightarrow \frac{g_1}{F_1} \)

If only \( A_{||} \) is measured

\( \Rightarrow \frac{A_{||}^N}{D} \approx (1 + \gamma^2) \frac{g_1}{F_1} \)

\( \gamma^2 = 4M_N^2x^2/Q^2 \quad \text{- kinematic factor} \)

**NB.** \( \gamma \) cannot be neglected in the SLAC, HERMES and JLab kinematic regions
The data on $A_1$ are really the experimental values of the quantity

\[
\frac{A_1^N}{D} = (1 + \gamma^2) \frac{g_1^N}{F_1^N} + (\eta - \gamma)A_2^N = A_1^N + \eta A_2^N
\]

\[\gamma \approx \eta \text{ and } A_2 \text{ small}
\]

very well approximated with even when $\gamma(\eta)$ can not be neglected
An important difference between the kinematic regions of the unpolarized and *polarized* data sets

A lot of the present data are at moderate $Q^2$ and $W^2$:

$$Q^2 \approx 1 - 5 \text{ GeV}^2, \ 4 < W^2 < 10 \text{ GeV}^2$$

While in the determination of the PD in the unpolarized case we can cut the low $Q^2$ and $W^2$ data in order to eliminate the less known non-perturbative HT effects, it is impossible to perform such a procedure for the present data on the spin-dependent structure functions without losing too much information.

$O(1/Q^2)$

**HT corrections** should be *important in polarized DIS!*
Theory

In QCD

\[ g_1(x, Q^2) = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{HT} \]

\[ g_1(x, Q^2)_{LT} = g_1(x, Q^2)_{pQCD} \]

\[ g_1(x, Q^2)_{HT} = h(x, Q^2)/Q^2 + h^{\text{TMC}}(x, Q^2)/Q^2 \]

dynamical HT power corrections \((\tau = 3, 4)\) ➞ non-perturbative effects (model dependent)

target mass corrections which are calculable \(J. \text{ Blumlein, A. Tkabladze}\)

In NLO pQCD

\[ g_1(x, Q^2)_{pQCD} = \frac{1}{2} \sum_q e_q^2 [(\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G \otimes \frac{\delta C_G}{N_f}] \]

\(\delta C_q, \delta C_G\) — Wilson coefficient functions

\(N_f (= 3)\) - a number of flavours

polarized PD evolve in \(Q^2\) according to \textbf{NLO DGLAP} eqs.
Test of QCD and determination of PPD

\[(\Delta q_i, \Delta q_i, \Delta G)(x, Q_0^2; a_k) \xrightarrow{\text{DGLAP eqs.}} (\Delta q_i, \Delta q_i, \Delta G)(x, Q^2; a_k)\]

\[\frac{\chi^2}{\Delta g_1(x_i, Q_j^2)^2} = \sum_{i,j} \left[ \frac{g_1(x_i, Q_j^2)_{\text{exp}} - g_1(x_i, Q_j^2; a_k)_{\text{pQCD}}}{\Delta g_1(x_i, Q_j^2)_{\text{exp}}} \right]^2\]

\[\rightarrow a_k \pm \Delta a_k\]
Methods of analysis

- Fit to $g_1/F_1$ data - `$g_1/F_1` fit $\Rightarrow$ PD($g_1/F_1$) or Set 1

\[
\chi^2 = \left[ \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right]_{\text{exp}} \iff \frac{g_1(x, Q^2)_{LT}}{F_1(x, Q^2)_{LT}} + \frac{h^{g_1/F_1}(x)}{Q^2}
\]

\[
(g_1)_{QCD} = (g_1)_{LT} + (g_1)_{HT}
\]

\[
(F_1)_{QCD} = (F_1)_{LT} + (F_1)_{HT}
\]

$\Rightarrow h^{g_1/F_1} \approx 0 \Rightarrow \frac{(g_1)_{HT}}{(g_1)_{LT}} \approx \frac{(F_1)_{HT}}{(F_1)_{LT}}$

The HT corrections to $g_1$ and $F_1$ approximately compensate each other in the ratio $g_1/F_1$ and the PPD extracted this way are less sensitive to HT effects.

LSS: EPJ C23 (2002) 479
hep-ph/0309048
Our predictions for the JLAB experimental values of $g_1^n/F_1^n$ using the LSS'2001 NLO(JET) polarized PD

significant improvement of the precision of the data

- LSS 2001 ($Q^2 = 5 \text{ GeV}^2$)

• **Fit to $g_1/F_1$ data** - Gluck et al. (GRSV); Leader et al. (LSS)

\[
\frac{g_1(x, Q^2)}{F_1(x, Q^2)} \exp \overset{\chi^2}{\leftrightarrow} \frac{g_1(x, Q^2)_{LT}}{F_1(x, Q^2)_{LT}} ; \quad \chi^2_{dof} = 0.884
\]

\[
\frac{\left[ g_1(x, Q^2) \right]}{\left[ F_1(x, Q^2) \right]} \exp \overset{\chi^2}{\leftrightarrow} \frac{g_1(x, Q^2)_{LT}}{F_1(x, Q^2)_{LT}} + \frac{h_{g_1/F_1}(x)}{Q^2} \Rightarrow h_{g_1/F_1}(x) \approx 0
\]

• **Fit to $g_1$ data** - SMC; Blumlein, Bottcher (BB); AAC

\[
g_1(x, Q^2)_{\exp} = \frac{g_1(x, Q^2)_{LT}}{F_1(x, Q^2)_{\exp}} \overset{\chi^2}{\leftrightarrow} \frac{g_1(x, Q^2)_{LT}}{240.7}
\]

\[
g_1(x, Q^2)_{\exp} \overset{\chi^2}{\leftrightarrow} g_1(x, Q^2)_{LT} ; \quad \chi^2_{dof} = 0.886
\]

\[
g_1(x, Q^2)_{\exp} \overset{\chi^2}{\leftrightarrow} g_1(x, Q^2)_{LT} + h_{g_1(x) / Q^2} ; \quad \chi^2_{dof} = 0.886
\]

Important from $g_1/F_1$ fit
Fit to $g_1$ data - `$g_1$+HT` fit $\Rightarrow$ PD( $g_1$+HT) or Set 2

$$\left[ \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right]_{\text{exp}} \quad F_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{\text{exp}} \quad \chi^2 \quad g_1(x, Q^2)_{LT} + h_{g_1}^2(x)/Q^2$$

HT corrections to $g_1$ cannot be compensated because the HT corrections to $F_1$ ($F_2$ and $R$) are absorbed in the phenomenological parametrizations of the data on $F_2$ and $R$.

Input PD $\Delta f_i(x, Q_0^2) = A_i x^{\alpha_i} f_i^{MRST}(x, Q_0^2)$  $Q_0^2 = 1 \text{GeV}^2$, $A_i, \alpha_i$ - free par.

$h^p(x_i), h^n(x_i)$ - 10 parameters ($i = 1, 2, \ldots 5$) to be determined from a fit to the data

$\Rightarrow$ 8-2(SR) = 6 par. associated with PD; positivity bounds imposed by MRST'02 unpol. PD

$g_A = (\Delta u + \Delta \bar{u})(Q^2) - (\Delta d + \Delta \bar{d})(Q^2) = F - D = 1.2670 \pm 0.0035$

$a_8 = (\Delta u + \Delta \bar{u})(Q^2) + (\Delta d + \Delta \bar{d})(Q^2) - 2(\Delta s + \Delta \bar{s})(Q^2) = 3F - D = 0.585 \pm 0.025$

*Flavor symmetric sea convention:* $\Delta u_{\text{sea}} = \Delta \bar{u} = \Delta d_{\text{sea}} = \Delta \bar{d} = \Delta s = \Delta \bar{s}$
The sum rule (1) reflects the isospin SU(2) symmetry, whereas the relation (2) is a consequence of the SU(3) flavour symmetry treatment of the hyperon β-decays.

While isospin symmetry is not in doubt, there is some question about the accuracy of assuming SU(3)$_f$ symmetry in analyzing hyperon β-decays. The results of the recent KTeV experiment at Fermilab on the β-decay of $\Xi^0$, $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$, however, are all consistent with exact SU(3)$_f$ symmetry. Taking into account the experimental uncertainties one finds that SU(3)$_f$ breaking is at most of order 20%.
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$

$\beta$-decay

SU(3)$_f$ prediction for the form factor ratio $g_1/f_1$

$$\frac{g_1}{f_1} = g_A = 1.2670 \pm 0.0035$$

Experimental result

$$\frac{g_1}{f_1} = 1.32^{+0.21}_{-0.17} \pm 0.05$$

A good agreement with the exact SU(3)$_f$ symmetry!

From exp. uncertainties $\Rightarrow$ SU(3) breaking is at most of order 20%

NA48 exp. at CERN $\Rightarrow$ will improve the stat. error (~ 500 $\Rightarrow$ 6238 events)
RESULTS OF ANALYSIS

- $\Delta u + \Delta \bar{u}$, $\Delta d + \Delta \bar{d}$ well determined
- $\Delta s + \Delta \bar{s}$ reasonably well determined and negative if accept for $a_8$ its SU(3) symmetric value $a_8 = 3F-D = 0.58$
- $\Delta G$ not well constrained

$PD(g_1^{\text{NLO}} + HT) \leftrightarrow PD(g_1^{\text{NLO}} / F_1^{\text{NLO}})$

$\chi^2_{DF,NLO} = 0.872 \leftrightarrow \chi^2_{DF,NLO} = 0.874$

In $g_1$ data fit HT corrections are important!

The two sets of polarized PD are very close to each other, especially for u and d quarks.
Higher twist effects

- The size of HT corrections to $g_1$ is **NOT** negligible
- The shape of HT depends on the target
- Thanks to the very precise JLab Hall A data the higher twist corrections for the neutron target are now much better determined at large $x$.

\[
\int_0^1 dx \, h^g_1(x) = \frac{4}{9} M^2 (d_2 + f_2)
\]

Our result is in agreement with the instanton model predictions (Balla et al., NP B510, 327, 1998) but disagrees with the renormalon calculations (Stein, NP 79, 567, 1999).
Main goal

To extract correctly PPD including the data in the preasymptotic region \((Q^2: 1 – 5 \text{ GeV}^2, W^2 > 4 \text{ GeV}^2)\) Mainly to study the HT effects. The data in the resonance region are also included

The analysis is performed in Bjorken \(x\)-space

in n-space of the Nachtmann moments of \(g_1\)

Data set

\(g_1(p,n,d)\) \(g_1^p\)

LT + HT approximations

NLO, \(O(1/Q^2)\) NLO \(\oplus\) SGR (soft gluon resummation) \(O(1/Q^2) + O(1/Q^4)\)

- Not easy to compare directly the results of the two analyses
- Is the quark-hadron duality satisfied in the polarised case?

\((A.Fantoni et al., hep-ph/0501180)\)
Effect of COMPASS $A_1^d$ data (hep-ph/0501073) on polarized PD and HT

- The statistical accuracy at small $x$: $0.004 < x < 0.03$ is considerably improved
- $\Delta u_\gamma(x)$ and $\Delta d_\gamma(x)$ do NOT change in the exp. region
- $x|\Delta s(x)|$ and $x \Delta G(x)$ decrease, but the corresponding curves lie within the error bands

LSS'05: hep-ph/0503140
The new values are in good agreement with the old ones. The COMPASS data are in the DIS region – their effect on HT is negligible.

\[ \Delta G/G = 0.06 \pm 0.31\text{(stat)} \pm 0.06\text{(sys)} \text{ at } \langle x \rangle = 0.13 \pm 0.08 \]

**LSS'05 result**

\[ \Delta G/G = 0.058 \text{ Set 1/NLO}(\text{MS}) \]
\[ \begin{align*}
y_{\text{Set 2/NLO}(\text{MS})} & = 0.095 \\
& \text{ for } x=0.13, \quad Q^2=2 \text{ GeV}^2
\end{align*} \]

\[ G(x,Q^2) \] is the NLO MRST'02 unpolarized gluon density.

**Effect of the COMPASS data on the HT values**

- The new values are in **good agreement** with the old ones.
- The COMPASS data are in the DIS region – their effect on HT is **negligible**.
Factorization scheme dependence

- NLO polarized PD in MS and JET schemes

In NLO QCD the **valence quarks** and **gluons** should be the **same** in both schemes, while

$$\Delta s(x, Q^2)_{JET} = \Delta s(x, Q^2)_{MS} + \frac{\alpha_s}{2\pi} (1 - x) \otimes \Delta G(x, Q^2)_{MS}$$

\[ n=1: \quad \Delta \Sigma_{JET} = \Delta \Sigma(Q^2)_{MS} + 3 \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2)_{MS} \]

\( \Delta \Sigma_{JET} \) is a **\( Q^2 \) independent** quantity

\[ \Delta \Sigma_{JET}(DIS) \leftrightarrow \Delta \Sigma(Q^2\sim \Lambda^2_{QCD}) \]

\( Q^2 = 1 \text{ GeV}^2 \)

<table>
<thead>
<tr>
<th>Fit</th>
<th>( \Delta \Sigma(Q^2)_{MS} )</th>
<th>( \Delta G(Q^2)_{JET} )</th>
<th>( \Delta \Sigma_{JET} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSS01</td>
<td>0.21±0.10</td>
<td>0.68±0.32</td>
<td>0.37±0.07</td>
</tr>
<tr>
<td>LSS05</td>
<td>0.19±0.06</td>
<td>0.29±0.32</td>
<td>0.29±0.08</td>
</tr>
</tbody>
</table>

Our numerical results for PPD are in a good agreement with pQCD
Impact of positivity constraints on polarized PD

LSS'01  \[ \Delta f(x) \leq f(x)_{\text{Bar.}} \]  LSS'05 (Set 1)  \[ \Delta f(x) \leq f(x)_{\text{MRST02}} \]

Bar.: Barone et al., EPJ C12 (2000) 243

MRST02: EPJ C28 (2003) 455

At large $x$: $s(x)_{\text{Bar.}} > s(x)_{\text{MRST02}}$  $G(x)_{\text{Bar.}} < G(x)_{\text{MRST02}}$
\( \Delta u_v \) and \( \Delta d_v \) of the two sets are closed to each other

\( \Delta s \) and \( \Delta G \) are **significantly** different

\( \Delta s \) and \( \Delta G \) are **weakly** constrained from the data, especially for high \( x \). That is why the role of positivity constraints is very **important** for their determination in this region.

Flavour symmetric sea convention:

\[
\Delta u_{\text{sea}} = \Delta \bar{u} = \Delta d_{\text{sea}} = \Delta \bar{d} = \Delta s = \Delta \bar{s}
\]
NLO QCD PPD (MS) obtained by different groups

\( x \Delta s \) and \( x \Delta G \) are weakly constrained from the present data on inclusive DIS

**GRSV:** Glück et al., hep-ph/0011215

**BB:** Blümlein, Böttcher, hep-ph/0203155

**AAC:** Goto et al., hep-ph/0312112

**LSS’05:** Leader et al., hep-ph/0503140

\( x \Delta u_\nu \) and \( x \Delta d_\nu \) well consistent
Impact of positivity constraints on $x\Delta s(x, Q^2)$

GRSV: Glück et al., hep-ph/0011215  
BB: Blümlein, Böttcher, hep-ph/0203155  
AAC: Goto et. al., hep-ph/0312112  
LSS’05: Leader et al., hep-ph/0503140

$$|x \Delta f(x, Q_0^2)| \leq x f(x, Q_0^2)_{GRV}$$

$$|x \Delta f(x, Q_0^2)|_{LSS} \leq x f(x, Q_0^2)_{MRST'02}$$

GRSV, BB and AAC have used the GRV unpolarized PD for constraining their PPD, while LSS have used those of MRST'02.

As a result, $x|\Delta s(x)|$ (LSS) for $x > 0.1$ is larger than the magnitude of the polarized strange sea densities obtained by the other groups.
Role of unpolarized PD in determining PPD at large $x$

- At large $x$ the unpolarized GRV and MRST'02 gluons are practically **the same**, while $x_s(x)_{\text{GRV}}$ is much smaller than that of MRST'02.

- For the adequate determination of $x_\Delta s$ and $x_\Delta G$ at large $x$, the role of the corresponding **unpolarized** PD is very important.

- Usually the sets of unpolarized PD are extracted from the data **in the DIS region** using cuts in $Q^2$ and $W^2$ chosen in order to minimize the higher twist effects.

- The latter have to be determined with good accuracy at large $x$ in the preasymptotic ($Q^2, W^2$) region too.
\[ \chi^2 / \text{DF} \]

<table>
<thead>
<tr>
<th>Fit</th>
<th>LO HT=0</th>
<th>NLO HT=0</th>
<th>LO+HT</th>
<th>NLO+HT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2 )</td>
<td>249.8</td>
<td>212.5</td>
<td>153.8</td>
<td>149.8</td>
</tr>
<tr>
<td>DF</td>
<td>185-8</td>
<td>185-6</td>
<td>185-16</td>
<td>185-16</td>
</tr>
<tr>
<td>( \chi^2 / \text{DF} )</td>
<td>1.41</td>
<td>1.19</td>
<td>0.910</td>
<td>0.886</td>
</tr>
</tbody>
</table>

- \( \alpha_s(Q^2) \) is large
- HT effects are large

LO QCD approximation - NOT reasonable in the preasymptotic region

\( \alpha_s(Q^2) \) is large

HT effects are large

Dependence of \( \chi^2 \) on HT corrections

\[ \chi^2 / \text{DF} \]
\[
\begin{bmatrix}
g_1 \\
F_1
\end{bmatrix}_{\text{exp}} \quad \chi^2_{DF} \quad \frac{g_1^{LO}}{F_1^{LO}} \quad 0.92
\]

\[
\chi^2_{DF} (NLO) = 0.87
\]

- at large \( Q^2 \): \( 2x(F_1)_{\text{exp}} \approx (F_2)_{\text{exp}} \)
- preasymt. region: \( 2x(F_1)_{\text{exp}} < (F_2)_{\text{exp}} \) (25-30%)

E04-113, Semi-Sane exp. at JLab Hall C

\[
\Delta u - \Delta d = \frac{1}{2} (\Delta q_3 - \Delta u_V + \Delta d_V)
\]

In LO: \( \Delta q_3(x, Q^2) = 6g_1^{(p-n)}(x, Q^2)_{\text{exp}} \)

In preas. region: \( \Delta \tilde{q}_3(x, Q^2) = 6 \left[ g_1^{(p-n)}(x, Q^2)_{\text{exp}} - \frac{h^{(p-n)}(x)}{Q^2} \right] \)

If \( x \in [0.1 - 0.4], \quad Q^2 = 2 \text{ GeV}^2 \)

HT contribution is about 24-34% (LSS’05)
SUMMARY

- Two sets of polarized PD in both the MS and the JET schemes are extracted from the world DIS data including the new JLab and COMPASS data in a good agreement with the pQCD predictions.

- While the HT corrections to $g_1$ and $F_1$ compensate each other in $g_1/F_1$, the HT($g_1$) are important in the analysis of the $g_1$ data.

- Impact of JLab data on PPD and HT PPD unchanged, HT for a neutron target much better determined at high $x$.

- Impact of COMPASS data on PPD $\Delta u_\nu$ and $\Delta d_\nu$ unchanged, $|\Delta s|$ and $\Delta G$ decrease.

- $\Delta s$ and $\Delta G$ are not well determined from the data the effect of the positivity conditions used to constrain them is essential, especially at high $x$.

- A more precise determination of unpolarized PD in the preasymptotic region is very important.