Transverse Force on Quarks in DIS

or: What can we learn from $g_2(x)$?

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Outline

- impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \xrightarrow{FT} q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \xrightarrow{FT} \perp$ distortion of PDFs for $\perp$ polarized target
- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow \quad \perp \text{SSA in} \quad \gamma N \longrightarrow \pi + X$$

- Quark gluon correlations: $g_2(x) \rightarrow d_2 \rightarrow \perp$ force on quarks in DIS
  - basic idea
  - simple estimates/data
- Chirally odd:
  - transversity distribution for unpolarized target
  - Boer-Mulders function
  - chirally odd quark-gluon correlations ($e(x)$)
- Summary

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
$\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$ [MB, PRD62, 071503 (2000)]

$$\boxed{\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2) \equiv \mathcal{H}(x,\mathbf{b}_\perp), \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \equiv \tilde{\mathcal{H}}(x,\mathbf{b}_\perp) \end{aligned}}$$

Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\,\gamma^+ q(x^-)|\, P,\uparrow\rangle \quad = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\,\gamma^+ q(x^-)|\, P,\downarrow\rangle \quad = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

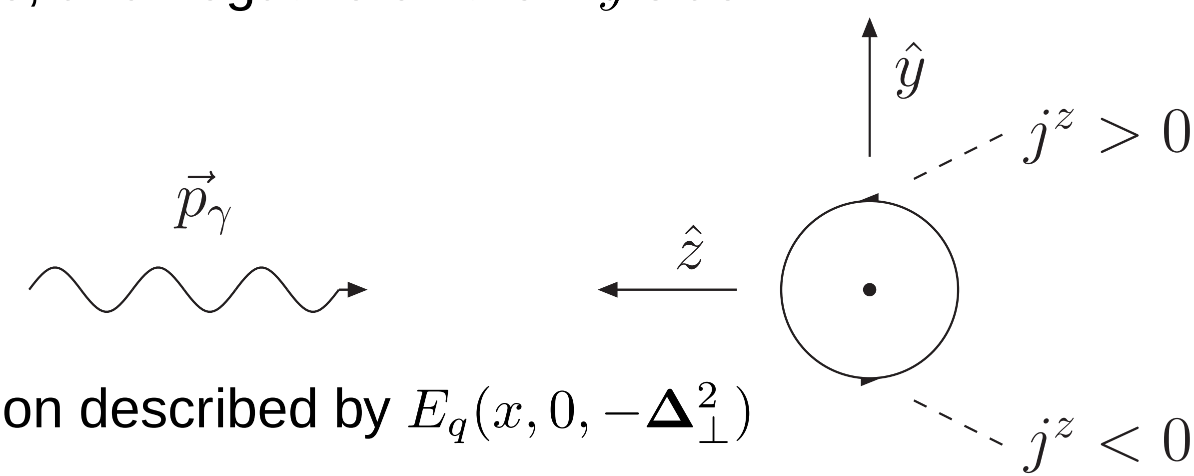
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{L}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\hookrightarrow$ j^+ larger than j^0 when quark current is directed towards the γ^*; suppressed when they move away from γ^*

$\hookrightarrow$ For quarks with positive angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

Transversely Deformed PDFs and $E(x,0,-\mathbf{\Delta}_\perp^2)$

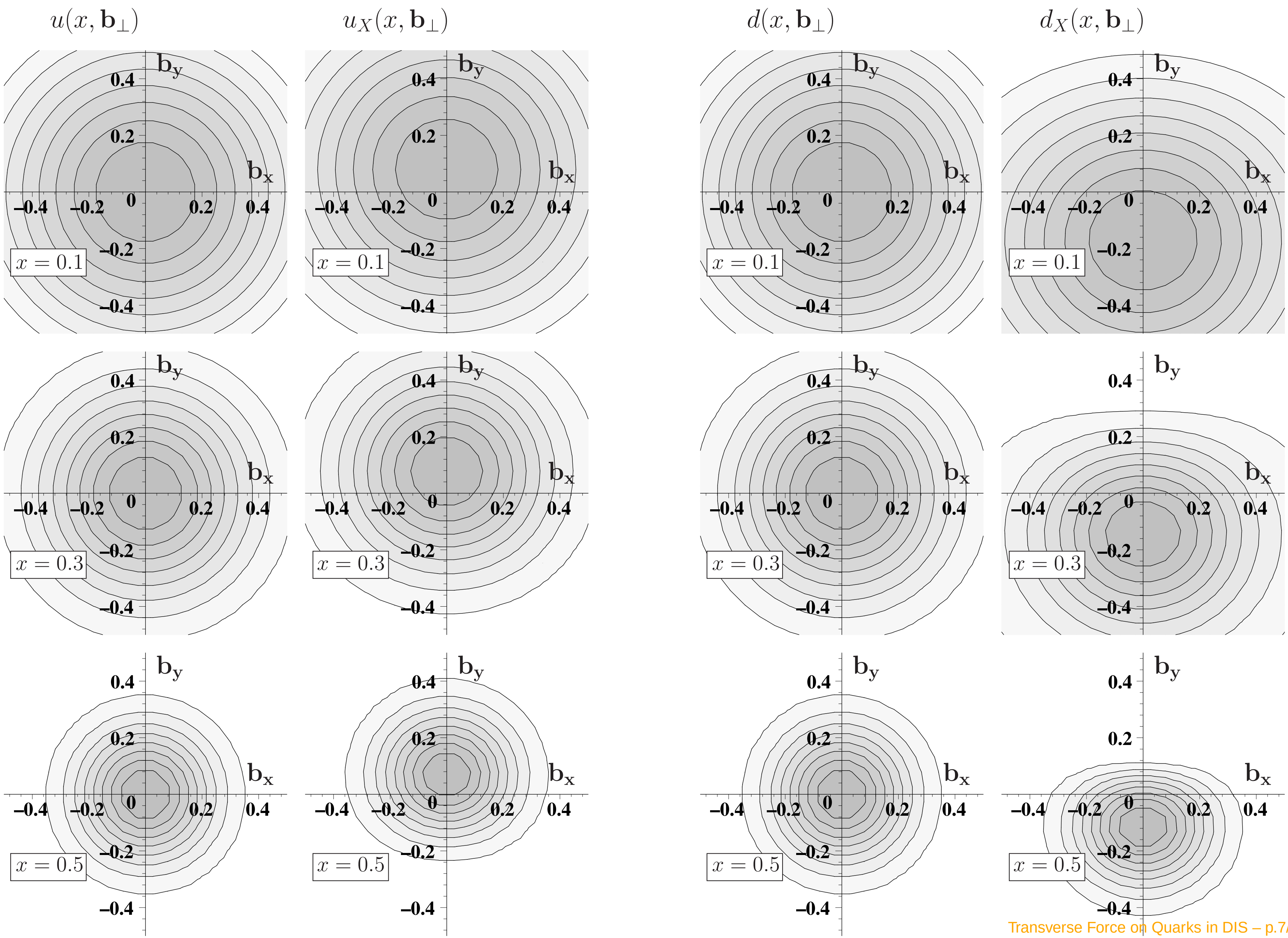
- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

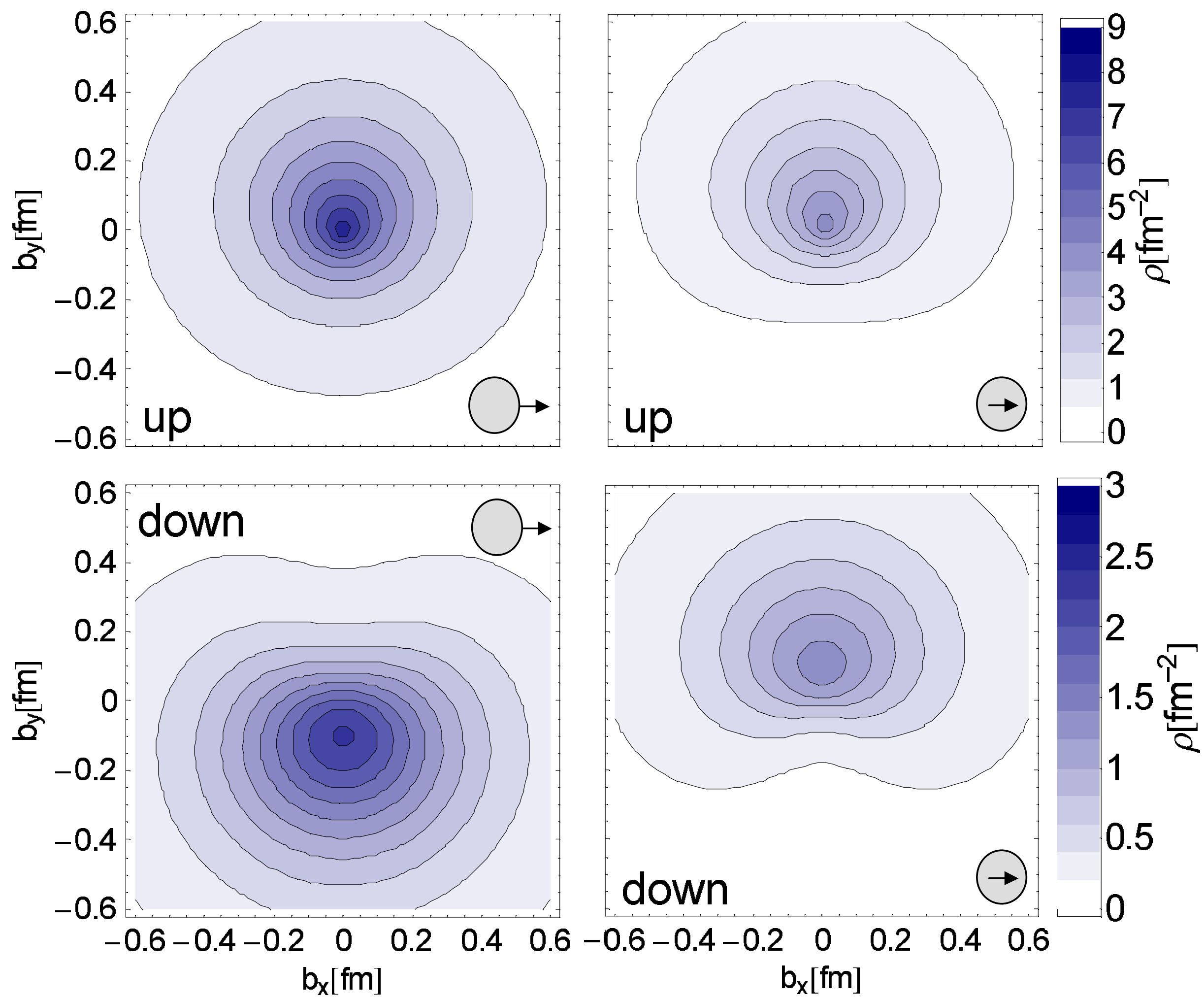
- $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

$\hookrightarrow$ very significant deformation of impact parameter distribution

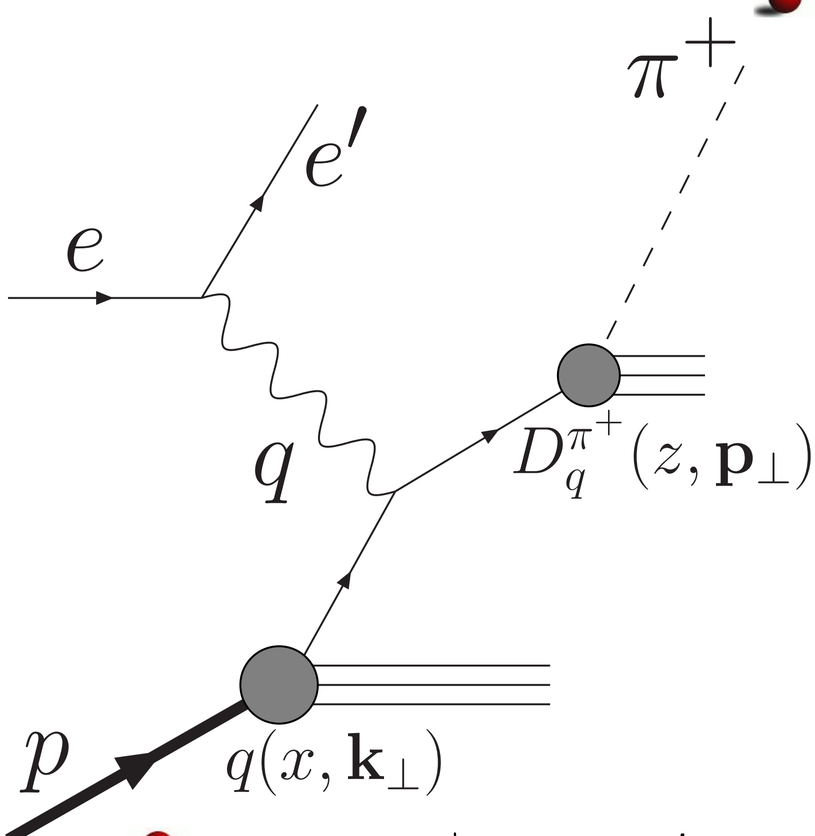

$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$

IPDs on the lattice (P.Hägler et al.)

- lowest moment of distribution of unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):



SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$
- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $f_{q/p}(x,\mathbf{k}_\perp)$
- Why interesting?
  - $\perp$ asymmetry involves nucleon helicity flip
  - quark density chirally even (no quark helicity flip)
  - $\hookrightarrow$ 'helicity mismatch' requires orbital angular momentum (OAM)
  - $\hookrightarrow$ (like κ), Sivers requires matrix elements between **wave function components that differ by one unit of OAM** (Brodsky, Diehl, ..)
  - Sivers requires nontrivial final state interaction phases

$\perp$ Single-Spin Asymmetry (Sivers)

- treat FSI to lowest order in g

$\hookrightarrow$

$$\left\langle k_q^i \right\rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{b}_\perp}{2\pi} \frac{b^i}{|\mathbf{b}_\perp|^2} \left\langle p,s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0) \rho_a(\mathbf{b}_\perp) \right| p,s \right\rangle$$

with $\rho_a(\mathbf{b}_\perp) = \int dr^- \rho_a(r^-, \mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of density-density correlations

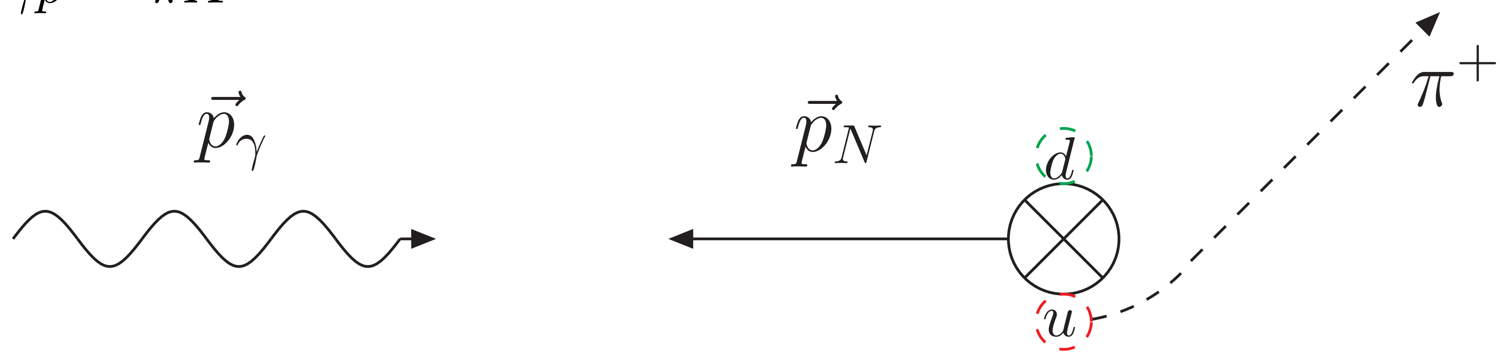
- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y \bar{u}(0)\gamma^+ \frac{\lambda_a}{2} u(0) \rho_a(\mathbf{b}_\perp) \rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \rightarrow \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign “determined” by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES results (also consistent with COMPASS $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

Quark-Gluon Correlations (Introduction)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- 'usually', contribution from g_2 to polarized DIS X-section kinematically suppressed by $\frac{1}{\nu}$ compared to contribution from g_1

$$\sigma_{TT} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- for transversely polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- $\int dx x^2 g_1(x) \propto \langle PS|\bar{\psi}(0)\gamma^+\gamma_5 D^+ D^+\psi(0)|PS\rangle$ completely symmetric in Lorentz indices
- $\int dx x^2 g_T(x) \propto \langle PS|\bar{\psi}(0)\gamma^i\gamma_5 D^+ D^+\psi(0)|PS\rangle$
  - use equations of motion $\gamma^\mu D_\mu \psi = 0$ to relate to $\langle PS|\bar{\psi}(0)\gamma^+\gamma_5 D^+ D^j\psi(0)|PS\rangle$
  - decompose into term symmetric in Lorentz indices, and thus $\propto \int dx x^2 g_1(x)$, and one involving $\left[D^+, D^j\right] = igG^{+j}$

$\hookrightarrow$ $3\int x^2 g_2(x) + 2\int x^2 g_1(x) \propto \langle P,S\,|\bar{q}(0)G^{+y}(0)\gamma^+ q(0)|\,P,S\rangle$

  - 'Wandzura Wilczek' approximation: $\int x^2 g_2(x) \approx \int x^2 g_2^{WW}(x) \equiv -\frac{2}{3}\int x^2 g_1(x)$

Quark-Gluon Correlations (QCD analysis)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu \gamma_5 \psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 g_2^{int}(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)gG^{+y}(0)\gamma^+ q(0) \right| P,S \right\rangle$$

- $\sqrt{2}G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- matrix elements of $\bar{q}B^x\gamma^+ q$ and $\bar{q}E^y\gamma^+ q$ are sometimes called color-electric and magnetic polarizabilities
  $2M^2\vec{S}\chi_E = \left\langle P,S \left| \vec{j}_a \times \vec{E}_a \right| P,S \right\rangle$ & $2M^2\vec{S}\chi_B = \left\langle P,S \left| j_a^0 \vec{B}_a \right| P,S \right\rangle$
  with $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{g}{6MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

- QED: $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ correlator between quarkdensity $\bar{q}\gamma^+ q$ and Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\sqrt{2}F^{+y}.$$

for charged paricle moving with $\vec{v} = (0, 0, -1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ yields volume integral (plane wave states!) of γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0, 0, -1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **average color Lorentz force** (ensemble average) acting on the struck quark in SIDIS in the instant **right after being hit by the virtual photon**

Quark-Gluon Correlations (Interpretation)

- d_2 a measure for the **average color Lorentz force** acting on the struck quark in SIDIS in the instant **right after being hit by the virtual photon**

$$\langle F^y(0)\rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

- alternative approach: consider $2p^+\frac{d}{dt}\langle \mathbf{p}^y\rangle$, where $\langle \mathbf{p}^y\rangle \equiv \langle PS|\,\bar{q}\gamma^+\left(-iD^y - gA^y\right)q\,|PS\rangle\,/2p^+$
- Using equations of motion, one finds

$$\begin{aligned} 2p^+\frac{d}{dt}\langle \mathbf{p}^y\rangle &= \sqrt{2}\,\langle PS|\,\bar{q}\gamma^+ gG^{y+}q\,|PS\rangle + `\langle PS|\,\bar{q}\gamma^+\gamma^-\gamma^i D^i D^j q\,|PS\rangle' \\ &\to \sqrt{2}\,\langle PS|\,\bar{q}\gamma^+ gG^{y+}q\,|PS\rangle \end{aligned}$$

where only terms are kept that dominate for a quark that moves with infinite momentum.

Quark-Gluon Correlations (Interpretation)

- Interpretation of d_2 with the transverse FSI force in DIS also consistent with to average $k_\perp$ in SIDIS (Qiu, Sterman)

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P,S \left| \bar{q}(0) \int_0^\infty dx^- G^{+y}(x^-)\gamma^+ q(0) \right| P,S \right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

- matrix element defining d_2 same as the integrant in the QS-integral:
  - $\langle k_\perp^y \rangle = \int_0^\infty dt F^y(t)$ (use $\mathrm{d}x^- = \sqrt{2}\mathrm{d}t$)

$\hookrightarrow$ first integration point $\longrightarrow F^y(0)$

$\hookrightarrow$ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$
  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \left| \bar{q}(0) g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0) \right| P,S \right\rangle$

$\hookrightarrow$ different linear combination of χ_E and χ_M

$\hookrightarrow$ combine with data for $g_2 \Rightarrow$ disentangle electric and magnetic force

Quark-Gluon Correlations (Estimates)

- What should one expect (magnitude)?
  - if all spectators were to pull in the same direction, force should be on the order of the QCD string tension $\sigma \approx (0.45GeV)^2 \approx 0.2GeV^2$
  - however, expect significant cancellation for FSI force, from spectators ‘pulling’ in different directions
  - $\hookrightarrow$ expect FSI force to be suppressed compared to string tension by about one order of magnitude (more?)
  - $\hookrightarrow$ $|d_2| \sim 0.02$ (electric charge factors taken out)
- What should one expect (sign)?
  - signs of $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction respectively for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$
  - $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton
  - d_2 negative/positive for u/d quarks in proton
  - large N_C: $d_2^{u/p} = -d_2^{d/p}$
  - consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$

Quark-Gluon Correlations (data/lattice)

- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$ (with large errors)

$\hookrightarrow$ using $M^2 \approx 5\frac{GeV}{fm}$ this implies

$$\langle F_u^y(0)\rangle \approx -50\frac{MeV}{fm} \qquad \langle F_d^y(0)\rangle \approx 28\frac{MeV}{fm}$$

- signs consistent with impact parameter picture
- SLAC data ($5GeV^2$): $d_2^p = 0.007 \pm 0.004$, $d_2^n = 0.004 \pm 0.010$
- combined JLab/SLAC data ($1GeV^2$) $\chi_E^n = 0.033 \pm 0.029$, $\chi_B^n = -0.001 \pm 0.016$
- combined with SIDIS data for $\langle k^y\rangle$, should tell us about 'effective range' of FSI $R_{eff} \equiv \frac{\langle k^y\rangle}{F^y(0)}$
- would be interesting to study $\left\langle P,S \left|\bar{q}\left((D^+)^n G^{+\perp}\right)\gamma^+ q\right| P,S\right\rangle$

Chirally Odd GPDs

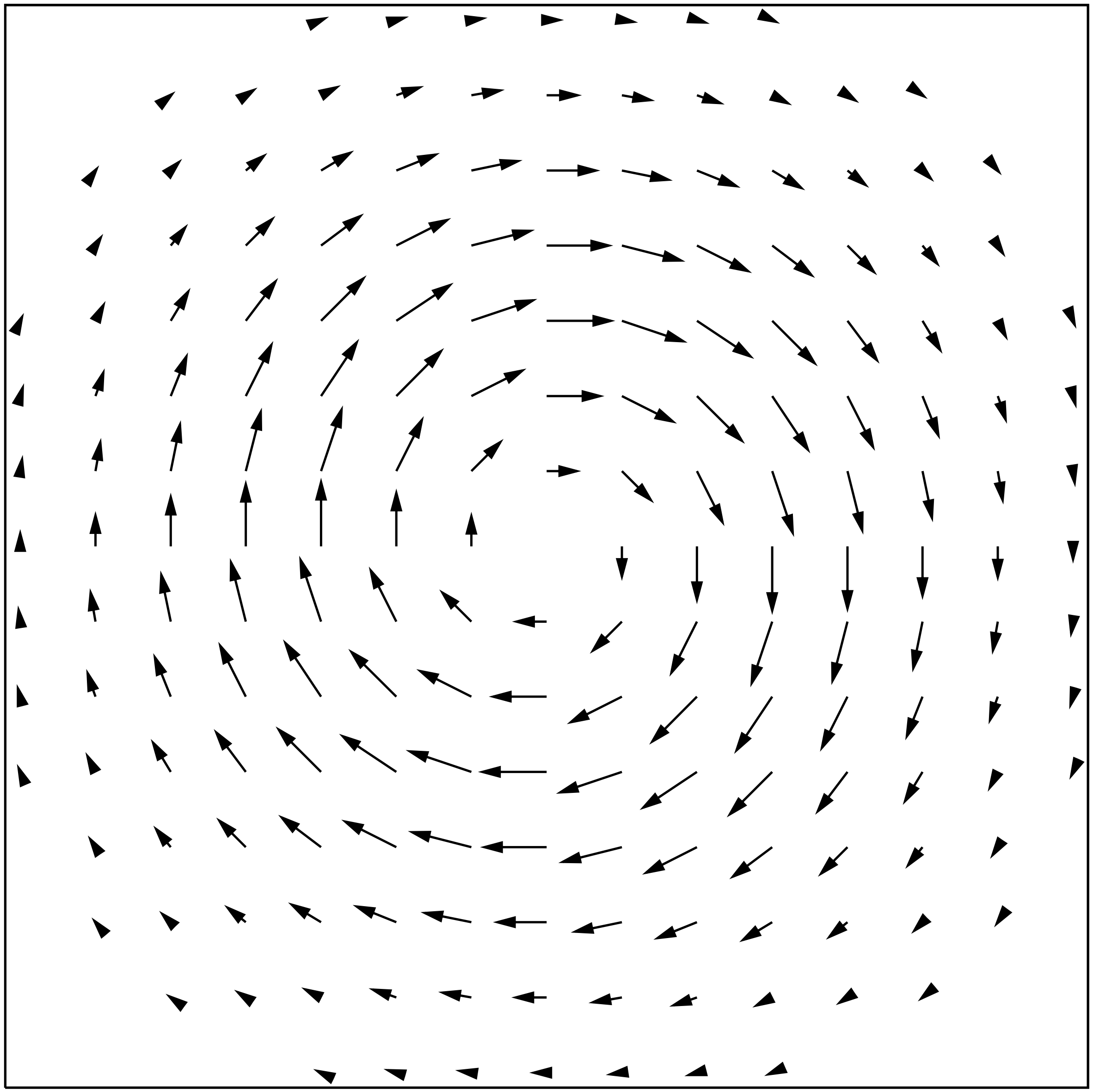
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u} \frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, EPJ C44, 87 (2005).
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \bar{E}_T^q(x, 0, -\mathbf{\Delta}_\perp^2)$$

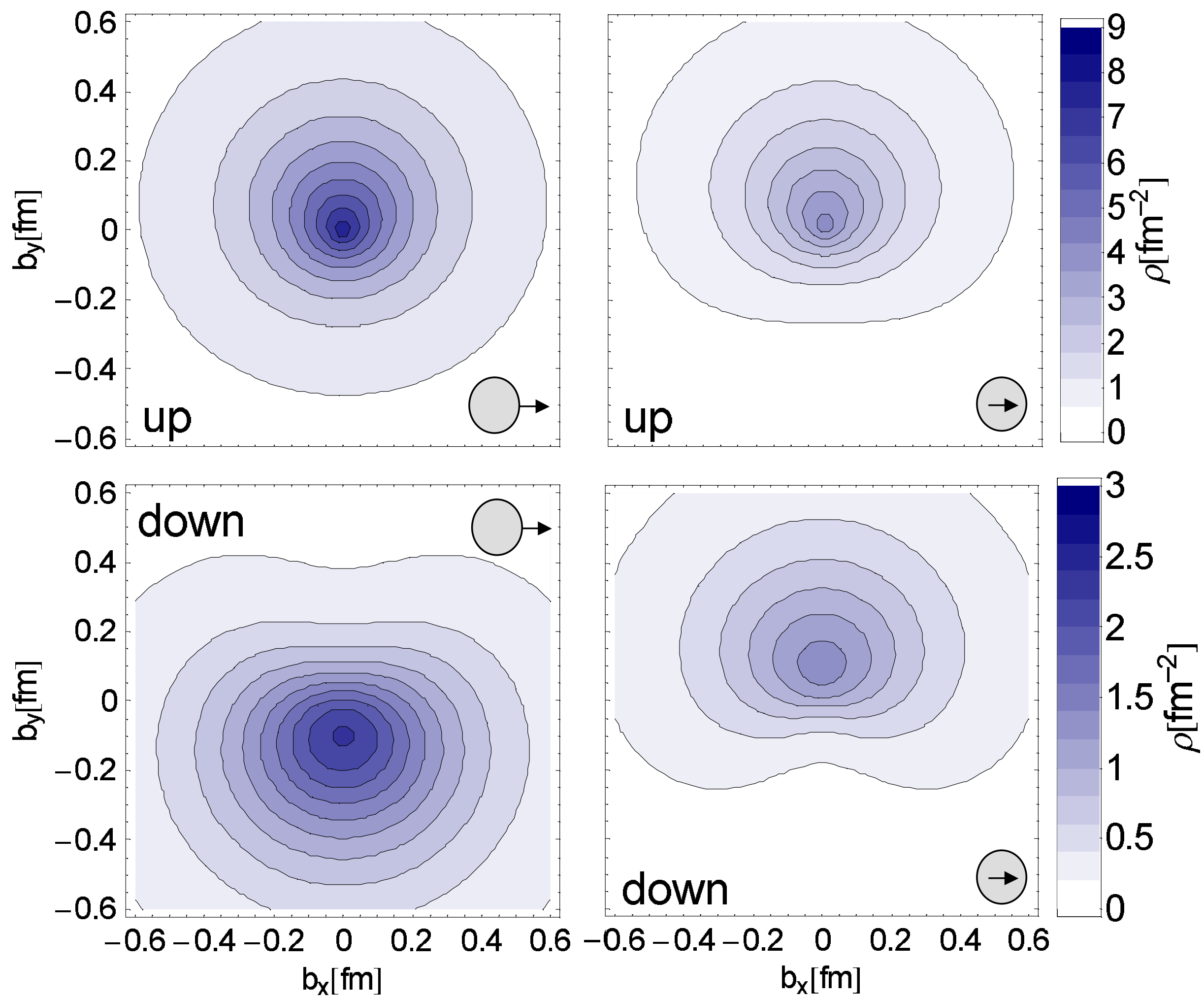
- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



IPDs on the lattice (Hägler et al.)

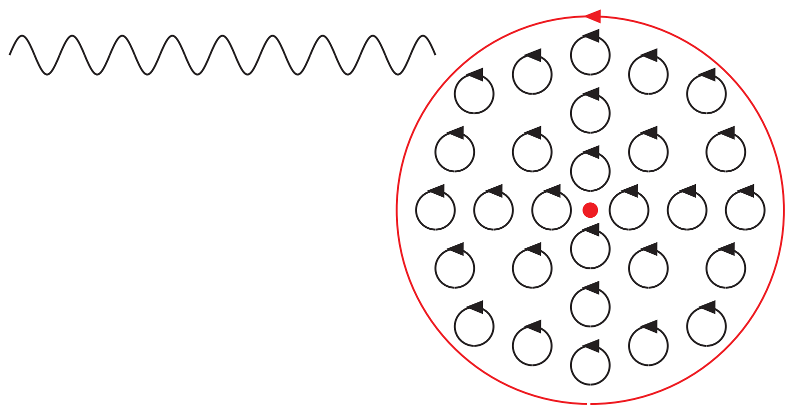
- lowest moment of distribution of unpol. quarks in ⊥ pol. proton (left) and of ⊥ pol. quarks in unpol. proton (right):



Transversity Distribution in Unpolarized Target (sign)

- Consider quark polarized out of the plane in ground state hadron

$\hookrightarrow$ expect counterclockwise net current $\vec{j}$ associated with the magnetization density in this state



- virtual photon ‘sees’ enhancement of quarks (polarized out of plane) at the top, i.e.

$\hookrightarrow$ virtual photon ‘sees’ enhancement of quarks with polarization up (down) on the left (right) side of the hadron

$\hookrightarrow$ $\bar{E}_T > 0$

Transversity Distribution in Unpolarized Target (sign)

[M.B.+B.Hannafious, PLB 658, 1130 (2008)]

- matrix element for $\bar{E}_T$ involves quark helicity flip

$\hookrightarrow$ requires interference between wave function components that differ by one unit of OAM (e.g. s-p interference)

$\hookrightarrow$ sign of $\bar{E}_T$ depends on rel. sign between s & p components

- bag model: p-wave from lower component

$$\Psi_m = \begin{pmatrix} i f \chi_m \\ -g(\vec{\sigma}\cdot\hat{\vec{x}})\chi_m \end{pmatrix},$$

(relative sign from free Dirac equation $g = \frac{1}{E}\frac{d}{dr}f$)

- $\bar{E}_T \propto -f\cdot g$. Ground state: f peaked at $r=0 \Rightarrow \bar{E}_T > 0$
- more general potential model: $\frac{1}{E} \longrightarrow \frac{1}{E-V_0(r)+m+V_S(r)}$

$\hookrightarrow$ sign of $\bar{E}_T$ same as in Bag model!

Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M} \right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in DY (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation

probing BM function in tagged SIDIS

- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

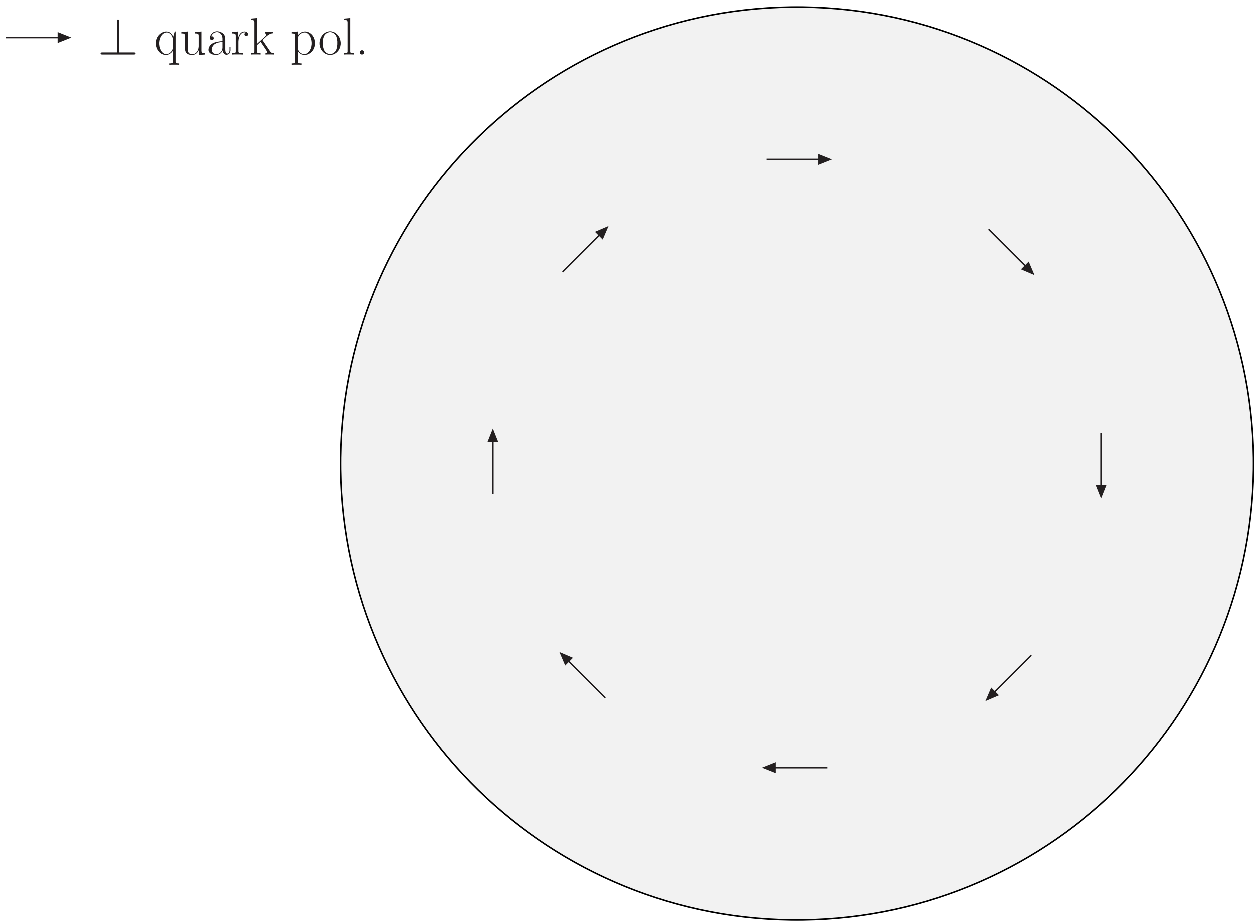
- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

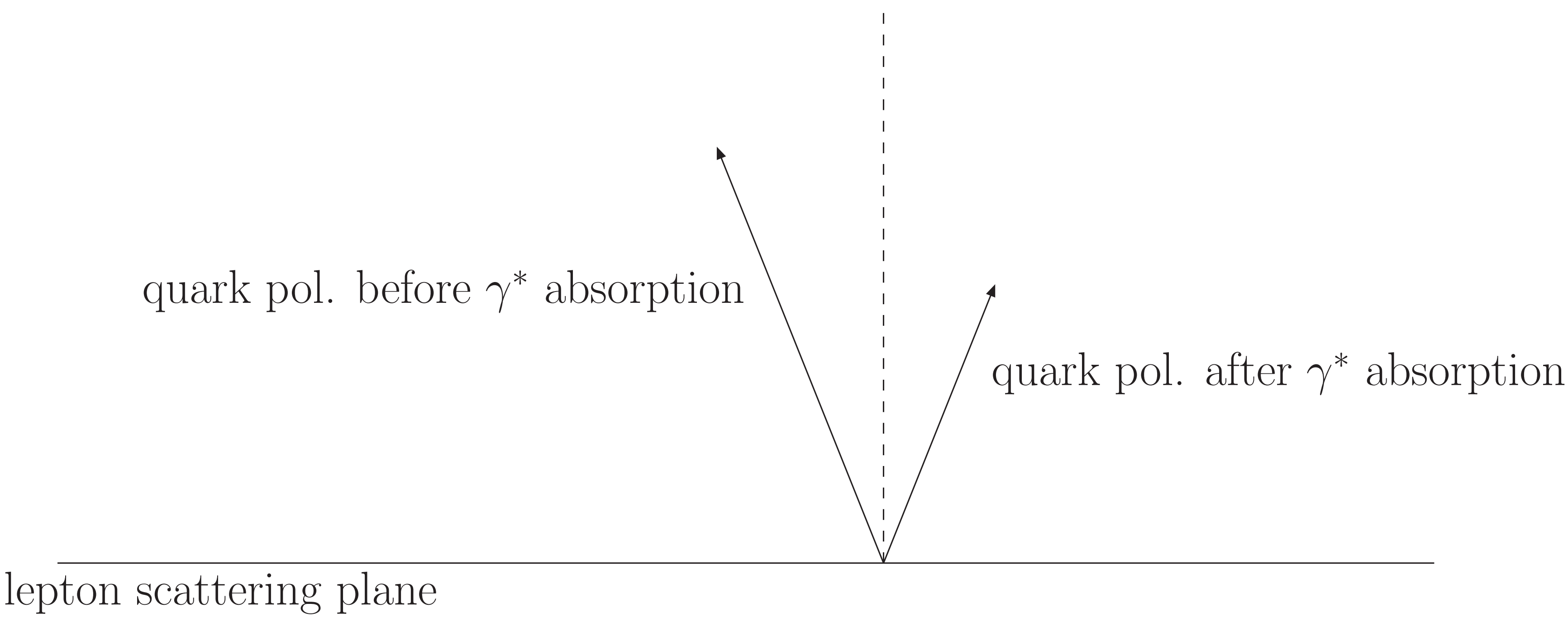
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



$\perp$ polarization and γ^* absorption

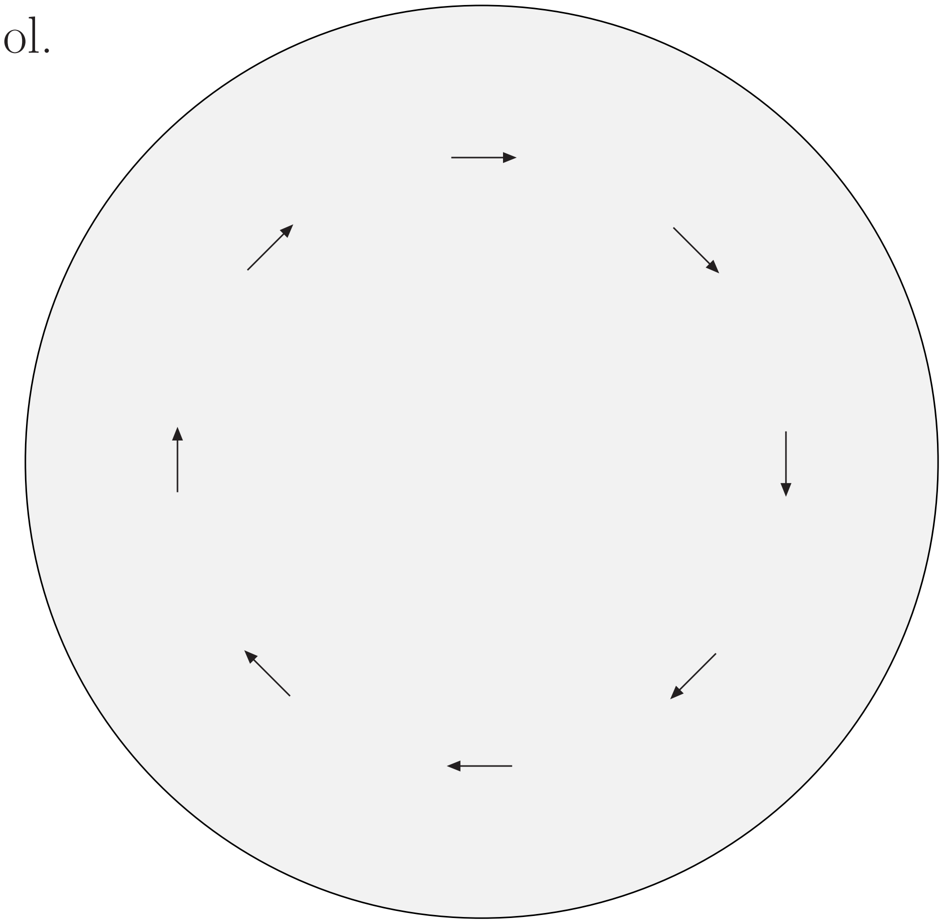
- QED: when the γ^* scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane



probing BM function in tagged SIDIS

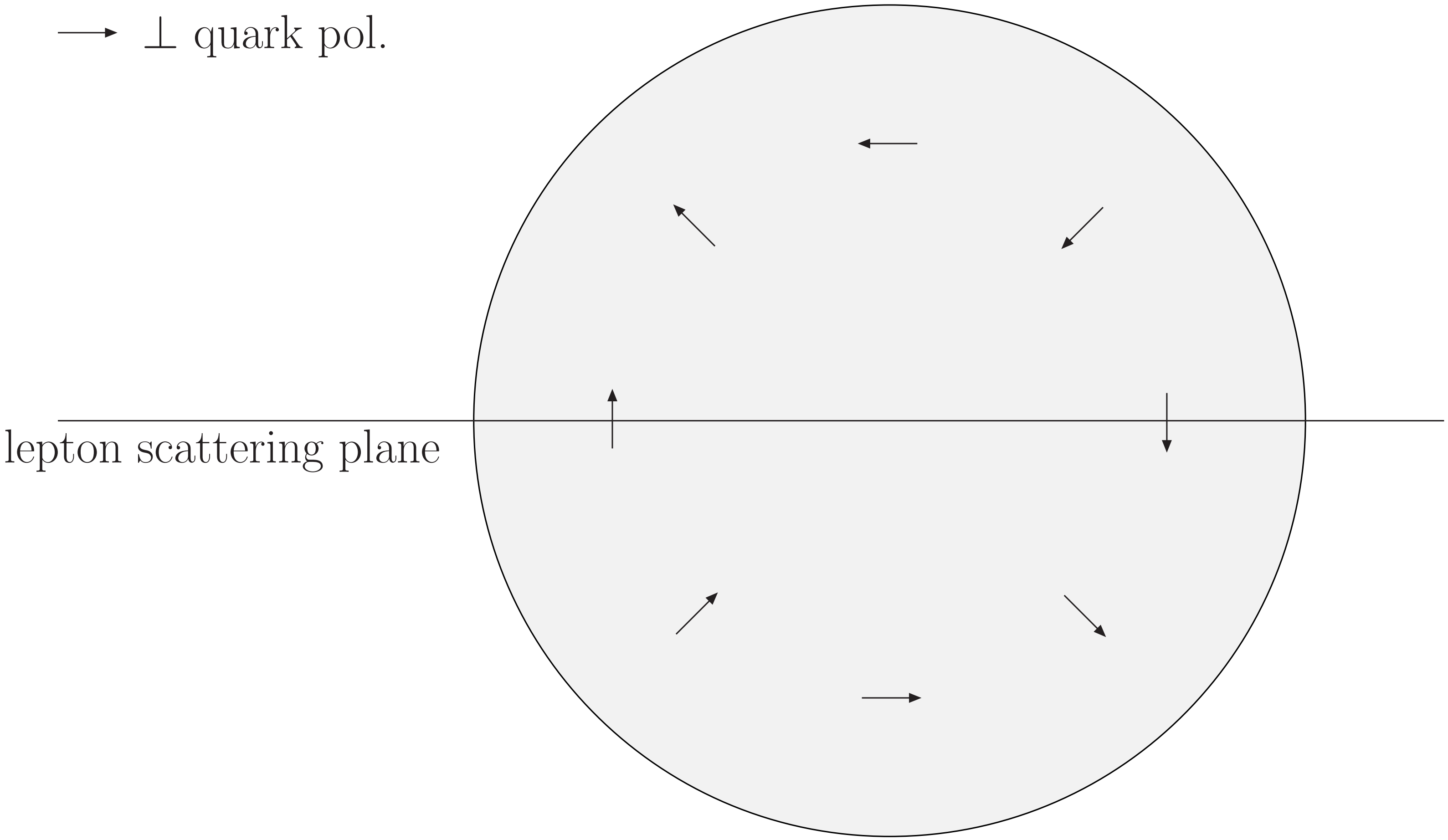
Primordial Quark Transversity Distribution

$\longrightarrow$ $\perp$ quark pol.



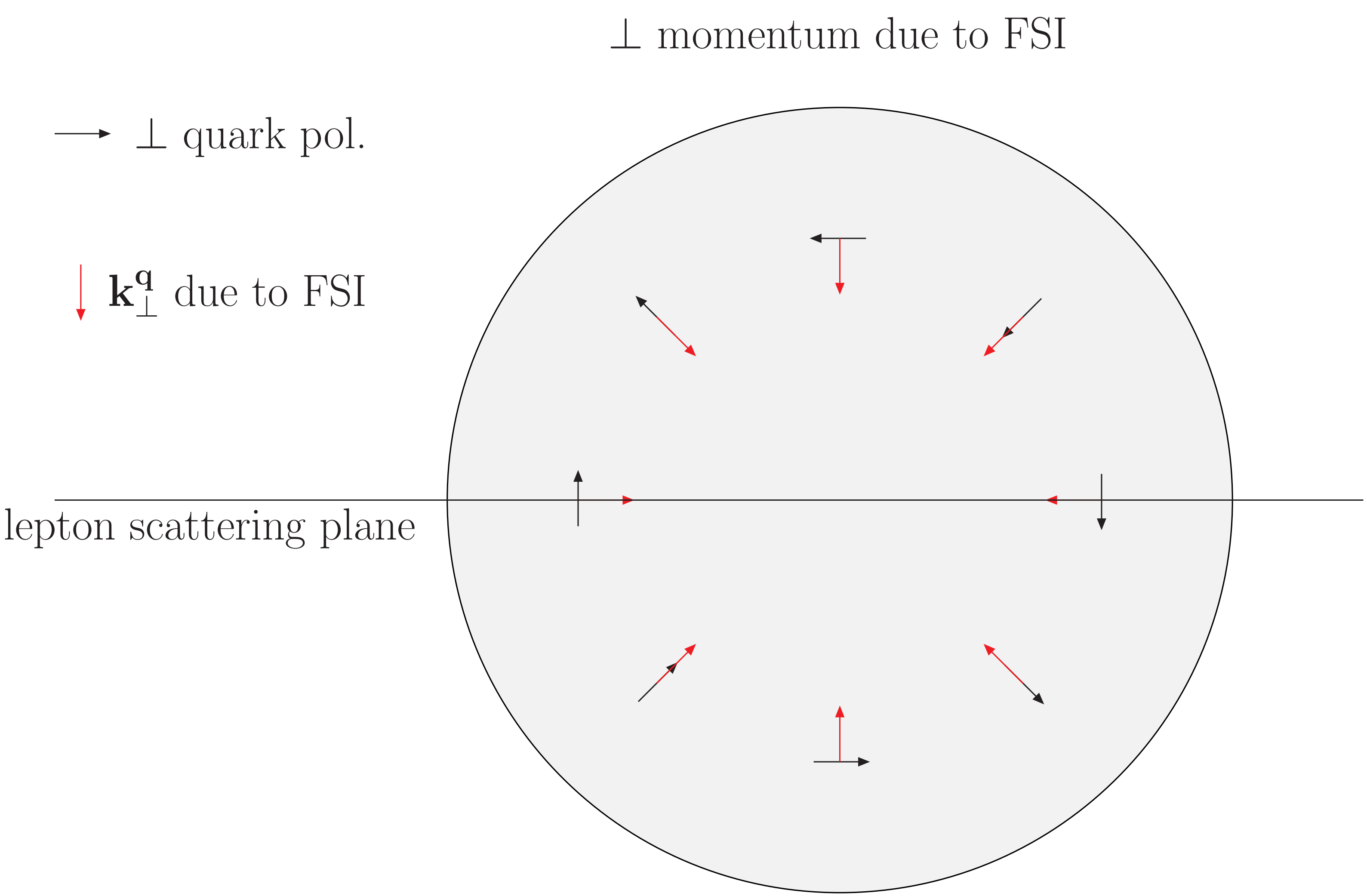
probing BM function in tagged SIDIS

Quark Transversity Distribution after γ^* absorption



quark transversity component in lepton scattering plane flips

probing BM function in tagged SIDIS

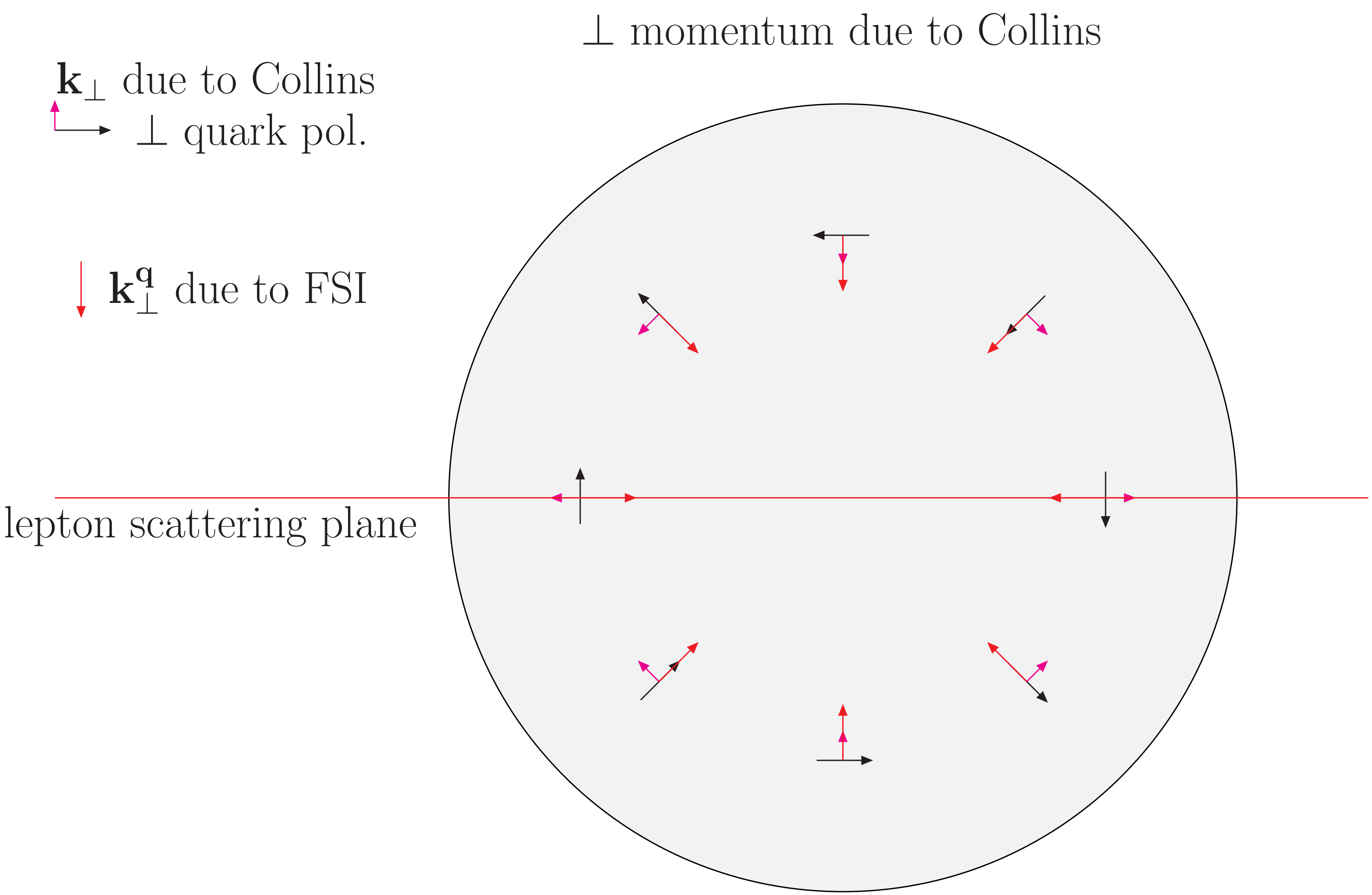


on average, FSI deflects quarks towards the center

Collins effect

- When a $\perp$ polarized struck quark fragments, the strucure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by **Collins fragmentation function**
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with ${}^3P_0$ 'vacuum' quantum numbers
  - $\hookrightarrow$ pion 'inherits' OAM in direction of $\perp$ spin of struck quark
  - $\hookrightarrow$ produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (BELLE)

probing BM function in tagged SIDIS



SSA of π in jet emanating from $\perp$ pol. q

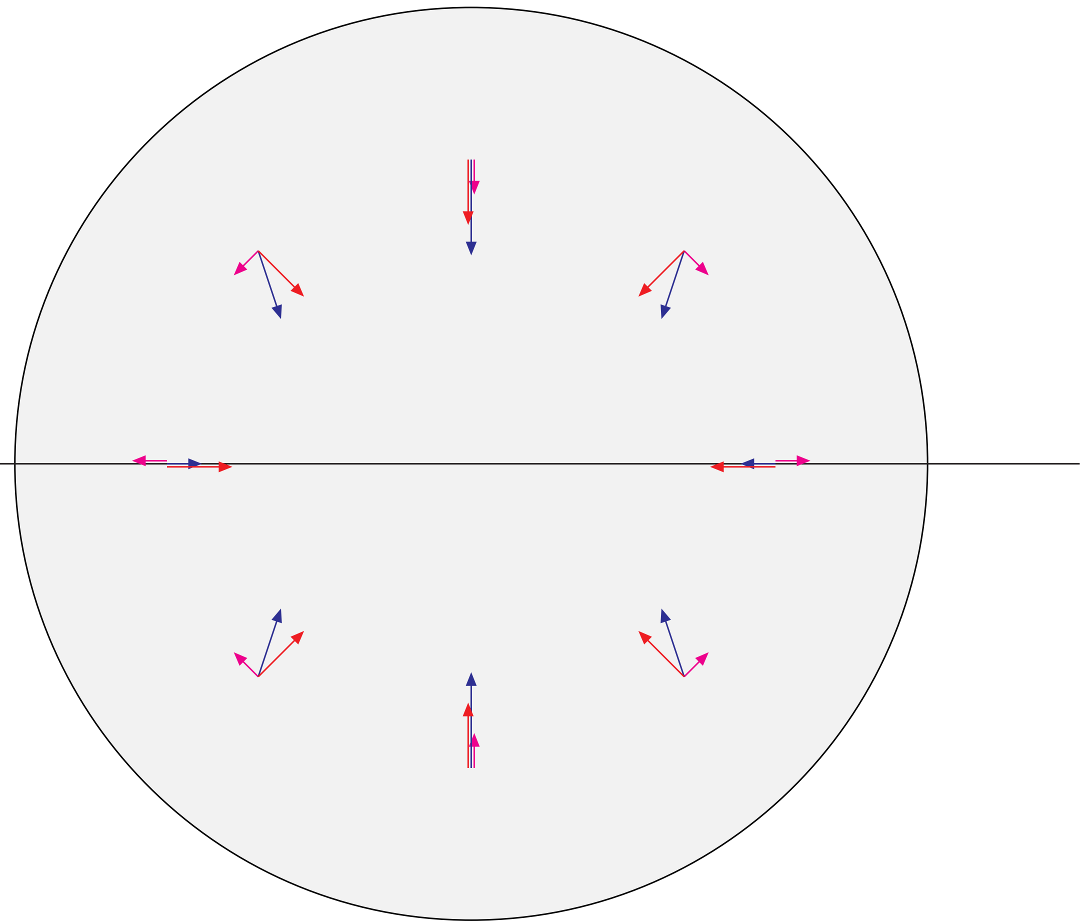
probing BM function in tagged SIDIS

net $\perp$ momentum (FSI+Collins)

$\mathbf{k}_\perp$ due to Collins

$\mathbf{k}_\perp^\mathbf{q}$ due to FSI

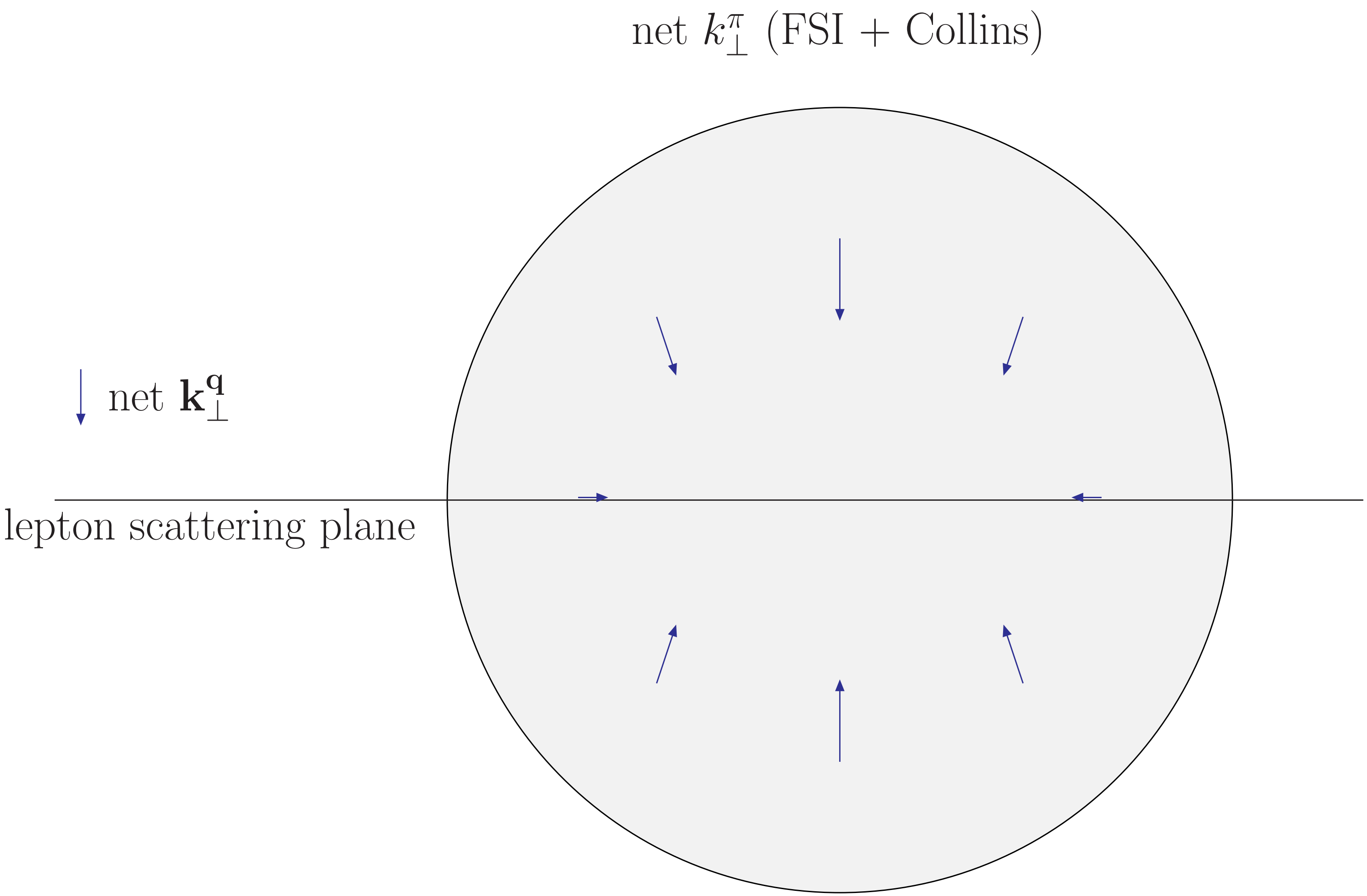
net $\mathbf{k}_\perp^\mathbf{q}$



lepton scattering plane

$\hookrightarrow$ in this example, enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

probing BM function in tagged SIDIS



$\hookrightarrow$ expect enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

Quark-Gluon Correlations (chirally odd)

- $\perp$ momentum for quark polarized in $+\hat{x}$-direction (unpolarized target)

$$\langle k_\perp^y \rangle = \frac{g}{2p^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- G^{+y}(x^-) \sigma^{+y} q(0) \right| P, S \right\rangle$$

- compare: interaction-dependent twist-3 piece of $e(x)$

$$\int dx x^2 e^{int}(x) \equiv e_2 = \frac{g}{4MP^{+2}} \left\langle P, S \left| \bar{q}(0) G^{+y}(0) \sigma^{+y} q(0) \right| P, S \right\rangle$$

$\hookrightarrow$ $\langle F^y \rangle = M^2 e_2$

$\hookrightarrow$ (chromodynamic lensing) $e_2 < 0$

Summary

- Interpretation of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as $\perp$ **force on active quark in DIS in the instant after being struck by the virtual photon**
- In combination with measurements of f_2
  - color-electric force $\frac{M^2}{4}\chi_E$
  - color-magnetic force $\frac{M^2}{2}\chi_M$
- expect d_2 to be significantly smaller than $\frac{\sigma}{M^2} \approx 0.2$
- $\kappa^{q/p} \leftrightarrow$ transverse distortion of impact parameter dependent PDFs $\leftrightarrow$ direction of FSI force

$\hookrightarrow$ opposite signs for $d_2^{u/p}$ and $d_2^{d/p}$

- combine with measurements of Sivers function to learn about **range of FSI**
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longrightarrow$ Boer-Mulders)

What is a Polarizability?

- Polarizability is the relative tendency of a charge distribution, like the electron cloud of an atom or molecule, to be distorted from its normal shape by an external electric field, which may be caused by the presence of a nearby ion or dipole (Wikipedia)
- It may be consistent with this original use of the term to enlarge the definition to encompass all observables that describe the ease with which a system can be distorted in response to an applied field or force
- Suppose one enlarges this definition to encompass 'how the color electric and magnetic field responds to the spin of the nucleon'

$\hookrightarrow$ many other obeservables also become 'polarizabilities', e.g.

  - Δq, as is describes how the quark spin responds to the spin of the nucleon
  - $\vec{\mu}_N$, as it describes how the magnetic field of the nucleon responds to the spin of the nucleon
  - $\vec{L}_q$, as it describes how the quark orbital angular momentum responds to the spin of the nucleon
  - as well as many other 'static' properties of the nucleon

$f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x,\mathbf{k}_\perp)_{SIDIS}$

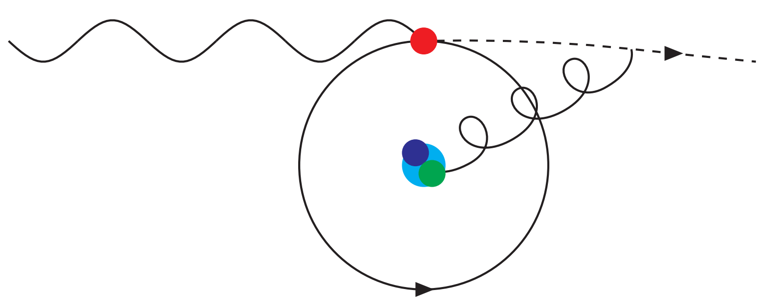
- Naively (time-reversal invariance) $f(x,\mathbf{k}_\perp) = f(x,-\mathbf{k}_\perp)$
- However, final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

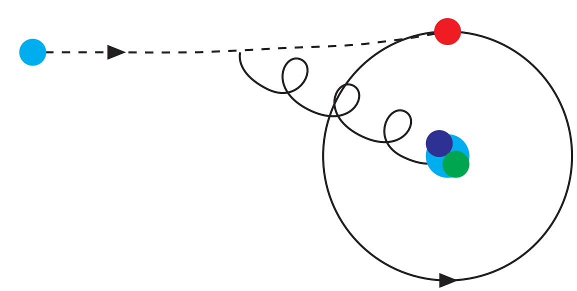
$\hookrightarrow$ FSI for knocked out q is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive



a)



b)

$\perp$ flavor dipole moments $\leftrightarrow$ Ji-relation

[M.B., PRD**72**, 094020 (2005)]

- two terms in $J_x^q \sim \int d^3r T^{tz} b^y - T^{ty} b^z$ equal by rot. inv.!

$\hookrightarrow$ identify $J_\perp^q$ with $\perp$ center of momentum ($\perp$COM)

$$J_y^q = M \sum_{i \in q} x_i b_i^y$$

- nucleon with $\perp$COM at $\mathbf{R}_\perp = \mathbf{0}_\perp$ and polarized in $\hat{x}$ direction:

$\hookrightarrow$ $\perp$ COM for quark flavor q at $y = \frac{1}{2M} \int dx\, x E^q(x,0,0)$

- additional $\perp$ displacement of the whole nucleon by $\frac{1}{2M}$ from boosting delocalized wave packet for $\perp$ polarized nucleon from rest frame to ∞ momentum frame (Melosh ...)

$\hookrightarrow$ when $\perp$ polarized nucleon wave packet is boosted from rest to ∞ momentum, $\perp$ flavor dipole moment for quarks with flavor q is

$$\sum_{i \in q} x_i b_i^y = \frac{1}{2M} \int dx\, x E^q(x,0,0) + \frac{1}{2M} \int dx\, x q(x) \qquad \text{(Ji relation)}$$