Quark-hadron duality in neutron spin structure

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Outline

- Brief introduction to inclusive scattering

- Review of Quark-Hadron duality:
  - Theoretical interpretations
  - Sample of related world data

- Hall A experiment E01-012
  - Experimental setup
  - Neutron (³He) “Spin duality” results

- More results from E01-012

- Summary
Inclusive electron scattering

\[ e = (E, \vec{k}) \quad \quad e' = (E', \vec{k}') \]

\[ q = (\nu, \vec{q}) \]

\[ p = (M, \vec{0}) \quad \quad W \]

Unpolarized case

\[
\frac{d^2 \sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right]
\]
Inclusive electron scattering

\( e = (E, \vec{k}) \)  \( e' = (E', \vec{k}') \)

\[ q = (\nu, \vec{q}) \]

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\( W \)

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Polarized case

\[ \frac{d^2 \sigma^{\uparrow \uparrow}}{d\Omega dE'} - \frac{d^2 \sigma^{\downarrow \downarrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{vE Q^2} \left[ (E + E' \cos \theta) g_1(x, Q^2) - 2 M g_2(x, Q^2) \right] \]

\[ \frac{d^2 \sigma^{\uparrow \Rightarrow}}{d\Omega dE'} - \frac{d^2 \sigma^{\downarrow \Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{vE Q^2} \sin \theta \left[ g_1(x, Q^2) + \frac{2ME}{\nu} g_2(x, Q^2) \right] \]
Inclusive electron scattering

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\[ q = (\nu, \vec{q}) \]

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- **Unpolarized case**

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- **Polarized case**

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\]

**4-momentum transfer squared**

\[ Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2} \]

**Invariant mass squared**

\[ W^2 = M^2 + 2M
\]

**Bjorken variable**

\[ x = \frac{Q^2}{2M \nu} \]
Deep inelastic scattering

\[ e = (E, \vec{k}) \]

\[ e' = (E', \vec{k}') \]

High \( Q^2 \) and \( W > 2 \text{GeV} \): fine resolution → we see partons

scaling → asymptotic freedom of the strong interaction

2004 Nobel Prize

D. J. Gross, H. D. Politzer and F. Wilczek
Scaling of $F_2$

$F_2 = \nu W_2$

H. W. Kendall, Rev. Mod. Phys. 63 (1991) 597

$x = 0.25$

1990 Nobel Prize

J. I. Friedman, H. W. Kendall and R. E. Taylor
Structure functions in the parton model

In the infinite-momentum frame, partons are point-like non-interacting particles:

$$\sigma_{\text{Nucleon}} = \sum_i \sigma_i$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^\uparrow(x) + q_i^\downarrow(x)]$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^\uparrow(x) - q_i^\downarrow(x)]$$
Structure functions in the parton model

In the infinite-momentum frame, partons are point-like non-interacting particles:

$$\sigma_{\text{Nucleon}} = \sum_i \sigma_i$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^\uparrow(x) + q_i^\downarrow(x)] = \frac{1}{2x} F_2(x)$$  \hspace{1cm} \text{Callan-Gross relation}$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^\uparrow(x) - q_i^\downarrow(x)]$$  \hspace{1cm} \text{No simple partonic description for } g_2$$
The resonance region

\[ e = (E, \vec{k}) \]

\[ e' = (E', \vec{k}') \]

Low \( Q^2 \) and \( W < 2 \text{ GeV} \): coarse resolution \( \rightarrow \) we don’t see individual partons.

- The nucleon goes through different excited states: the resonances
SCALING, DUALITY, AND THE BEHAVIOR OF RESONANCES
IN INELASTIC ELECTRON-PROTON SCATTERING*

E. D. Bloom and F. J. Gilman

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 June 1970)

We propose that a substantial part of the observed behavior of inelastic electron-proton scattering is due to a nondiffractive component of virtual photon-proton scattering. The behavior of resonance electroproduction is shown to be related in a striking way to that of deep inelastic electron-proton scattering. We derive relations between the elastic and inelastic form factors and the threshold behavior of the inelastic structure functions in the scaling limit.
Scaling curve seen at high $Q^2$ is an accurate average over the resonance region at lower $Q^2$. 

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

We propose that a substantial fraction of the energy loss in the inelastic electron-proton scattering is due to a nonuniformity of resonance positions. The behavior of resonances in the $p_{1/2}$ region is qualitatively different from that of deep inelastic electron-proton scattering. The behavior of the vector sum of elastic and inelastic form factors in the scaling limit is consistent with the predictions of a model.
Quark-hadron duality

High precision Hall C data allowed the confirmation of global duality and the observation of local duality for $F_2$.

What about spin-dependent structure functions?
Theoretical interpretations

**Lattice QCD**

Quark models

**OPE**

**pQCD**

\[ Q^2 = 0 \]

\[ Q^2 = \infty \]

**pQCD** (Carlson, Mukhopadhyay):

\[ Q^2 \text{ dependence of transition form factors vs. } x \text{ dependence of parton distribution functions} \]

In resonance

\[
g_1 = \frac{M_N^2 g_+^2}{\pi M_R \Gamma_R Q^6} \approx \frac{M_N^2}{\pi M_R \Gamma_R} \left( \frac{g_+^2}{M_R^2 - M_N^2} \right)^3 (1 - x)^3
\]

In DIS

\[
\lim_{x \to 1} g_1(x) \propto (1 - x)^3
\]
Theoretical interpretations

\[ \chiPT \quad Q^2 = 0 \quad \text{Lattice QCD} \quad \text{Quark models} \quad \text{OPE} \quad \text{pQCD} \quad Q^2 = \infty \]

Operator Product Expansion (Rujula, Georgi, Politzer):

\[ \Rightarrow \text{Higher twist corrections are small or cancel.} \]

\[ \Gamma_1(Q^2) = \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + O\left(\frac{1}{Q^6}\right) \]

\[ \text{Leading twist} \quad \text{Higher twists} \]
Theoretical interpretations

SU(6) symmetry breaking in the quark model (Close, Isgur and Melnitchouk):
- investigate several scenarios with suppression of spin-3/2, helicity-3/2 or symmetric wave function

\[ |N\rangle = \cos \theta_w |\psi_\rho\rangle + \sin \theta_w |\psi_\lambda\rangle \]

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>no $^410$</th>
<th>no $^210, ^410$</th>
<th>no $S_{3/2}$</th>
<th>no $\sigma_{3/2}$</th>
<th>no $\psi_{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^{np}$</td>
<td>2/3</td>
<td>10/19</td>
<td>1/2</td>
<td>6/19</td>
<td>3/7</td>
<td>1/4</td>
</tr>
<tr>
<td>$A^p_1$</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A^n_1$</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Existing data on “spin duality”

Hall B

$g_1^p(DIS) > 0$
Existing data on “spin duality”


Hall B

$g_1^p(\text{DIS}) > 0$

but $g_1^p(\Delta) < 0$ even at $Q^2$ as high as 3.5 GeV$^2$

Duality for $g_1^p$?
Existing data on “spin duality”


Hall B

Global duality

local duality
Existing data on “spin duality”

Hall C

Existing data on “spin duality”

Global duality

Hall C

Existing data on “spin duality”

Indication of duality for $g_1^{^3\text{He}}$ from Hall A (E94-010)
Neutron spin duality?

Onset of duality for $g_1^n$ is expected “sooner”

$g_1^n(DIS) < 0$ and $g_1^n(\Delta)$ is negative up to the its FF fall off

In order to improve our understanding of duality, we need to explore duality in:

- polarized SF vs. unpolarized SF
- proton vs. neutron

A dedicated experiment to study spin duality on the neutron was necessary.
The experiment 01-012

- Ran in Jan.-Feb. 2003

- Inclusive experiment: $^3\text{He}(\bar{e},e')X$
  
  ➤ Polarized electron beam:
  
  $70 < P_{\text{beam}} < 85\%$
  
  ➤ Hall A in standard equipment

  ➤ Pol. $^3\text{He}$ target (para and perp):
  
  $<P_{\text{targ}} > = 37\%$

- Measured polarized cross section differences and form $g_1$ and $g_2$ for $^3\text{He}$

⇒ Test of spin duality on the neutron ($^3\text{He}$)
The E01-012 Collaboration

Experimental setup

Both HRS in symmetric configuration at 25° and 32°
- Double the statistics
- Control the systematics

Particle ID = Cerenkov + EM calorimeter
- $\pi/e$ reduced by $10^4$
The polarized $^3$He target

$^3$He as neutron target

\[ P_n = 86\% \text{ and } P_p = -2.8\% \]
The polarized $^3$He target

Pressure ~ 14 atm under running conditions
High luminosity: $10^{36}$ s$^{-1}$cm$^{-2}$

$L_{tg} \sim 40$cm
The polarized $^3\text{He}$ target
The polarized $^3$He target

\[ S = \kappa \omega \]
The polarized $^3$He target

$P_{^3\text{He}} = \kappa_{\text{epr}} \Delta \nu$

$\frac{\partial \rho}{\partial B} + \delta B_{^3\text{He}}$
Unpolarized cross sections

Agreement between both HRS better than 2%

Statistical errors only

- left arm
- right arm
Asymmetries

Statistical errors only
Asymmetries

Statistical errors only
From constant \((E, \theta)\) to constant \(Q^2\)
The structure function $g_1$ in $^3\text{He}$

P. Solvignon et al., PRL 101, 182502 (2008)

Target mass corrections were applied on PDFs.
Spin duality on $^3$He and neutron

Use partial moments:

Integrate $g_{\text{res}}$ and $g_{\text{dis}}$ over the same $x$-range and at the same $Q^2$:

$$\tilde{\Gamma}^{\text{res}}_1 = \int_{x_{\text{min}}}^{x_{\text{max}}} g^{\text{res}}_1(x,Q^2) \, dx$$

$$\tilde{\Gamma}^{\text{dis}}_1 = \int_{x_{\text{min}}}^{x_{\text{max}}} g^{\text{dis}}_1(x,Q^2) \, dx$$

If $\tilde{\Gamma}^{\text{res}}_1 = \tilde{\Gamma}^{\text{dis}}_1$, duality is verified
Spin duality on $^{3}\text{He}$ and neutron

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Integrate $g_{\text{res}}^{1}$ and $g_{\text{dis}}^{1}$ over the same $x$-range and at the same $Q^{2}$:

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If $\tilde{\Gamma}_{1}^{\text{res}} = \tilde{\Gamma}_{1}^{\text{dis}}$ duality is verified

Neutron extraction using the effective polarization equation:

$$\tilde{\Gamma}_{1}^{^{3}\text{He}} = P_{n} \tilde{\Gamma}_{1}^{n} + 2P_{p} \tilde{\Gamma}_{1}^{p}$$

$P_{n}=86\%$

$P_{p}=-2.8\%$

Target mass corrections were applied on PDFs
Virtual photon-nucleon asymmetry

\[ A_1(x, Q^2) = \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{F_1(x, Q^2)} \]

with \( \gamma^2 = \frac{4M^2x^2}{Q^2} \)

In the parton model:

\[ A_1(x, Q^2) \approx \frac{g_1(x, Q^2)}{F_1(x, Q^2)} = \frac{\sum_i e_i^2 \Delta q_i(x, Q^2)}{\sum_i e_i^2 q_i(x, Q^2)} \]

If \( Q^2 \) dependence similar for \( g_1 \) and for \( F_1 \),

\( \Rightarrow \) weak \( Q^2 \) dependence of \( A_1 \)
$A_1$ for $^3\text{He}$
$A_1$ for $^3$He

P. Solvignon et al., PRL 101, 182502 (2008)

Large negative value in the $\Delta(1232)$ region
$A_1$ for $^3$He

P. Solvignon et al., PRL 101, 182502 (2008)

Large negative value in the $\Delta(1232)$ region

Still large negative value in the $\Delta(1232)$ region
$A_1$ for $^3$He

P. Solvignon et al., PRL 101, 182502 (2008)

Large negative value in the $\Delta(1232)$ region

Still large negative value in the $\Delta(1232)$ region

$A_1$ becomes positive in the $\Delta(1232)$ region due to the drop in the $\Delta FF$ and the rising of the DIS background
\( A_1 \) for \(^3\text{He}\)

P. Solvignon et al., PRL 101, 182502 (2008)

Large negative value in the \( \Delta(1232) \) region

Still large negative value in the \( \Delta(1232) \) region

\( A_1 \) becomes positive in the \( \Delta(1232) \) region due to the drop in the \( \Delta \text{FF} \) and the rising of the DIS background

No strong \( Q^2 \)-dependence is now observed
$A_1^n$ in the resonance region

\[
A_1^n = \frac{g_1^n - \gamma^2 g_2^n}{F_1^n}
\]

- Effective equation polarization cannot be used for a pt-to-pt neutron extraction in the resonance region

- Y. Kahn, W. Melnitchouk and S. Kulagin are including a $Q^2$-dependence in their convolution model (arXiv:0809.4308)

- Goal: test of quark-hadron duality on $A_1^n$ and possible access to high $x$ region
$g_1^n$ and $g_2^n$ in the resonance region

$g_1^p$ from Hall B

$g_2^p$ from MAID: its use is questionable for $Q^2 > 1\text{GeV}^2$

Convolution code: courtesy of Yonatan Kahn

neutron uncertainties will be improved by using fit of our data in the convolution
The $g_2$ structure function

Leading twist contribution determined entirely from $g_1$ through the Wandzura-Wilczek relation:

$$g_2 = g_2^{WW} + \bar{g}_2$$

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_0^x dy \frac{g_1(y, Q^2)}{y}$$
The $g_2$ structure function

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higher twist contribution
The structure function $g_2$ in $^3$He

P. Solvignon et al., in preparation
Burkhard-Cottingham sum rule on the neutron

\[ \Gamma_2(Q^2) = \int_0^1 dx \, g_2(x, Q^2) = 0 \]

Graph showing the data points for different experiments, including E94-010 (Res), E01-012 prel. (Res), SLAC E155x, and RSS (Res). The graph also includes a note indicating that the data is preliminary and from P. Solvignon et al., in preparation and RSS: K. Slifer et al., in preparation.
Higher moment $d_2$

\[ d_2(Q^2) = \int_0^1 x^2 \left[ 2 \ g_1(x, Q^2) + 3 \ g_2(x, Q^2) \right] \ dx \]
Higher moment $d_2$

\[ d_2(Q^2) = \int_0^1 x^2 \left[ 2 g_1(x, Q^2) + 3 g_2(x, Q^2) \right] dx \]

\[ d_2^n \]

\[ Q^2(GeV/c)^2 \]

Summary

E01-012 provides first precise data of Spin Structure Functions on neutron ($^3$He) in the resonance region for $1.0 < Q^2 < 4.0 \text{GeV}^2$

✓ Overlap between E01-012 resonance data and DIS data:
  first dedicated test of Quark-Hadron Duality for neutron and $^3$He SSF
✓ No strong $Q^2$-dependence in resonance $A_1^{^3\text{He}}$ for $Q^2 > 2.0 \text{ GeV}^2$
  ➞ DIS-like behavior

Preliminary extraction of $g_1^n$ and $g_2^n$ in the resonance region ⇒ $A_1^n$ will come soon

Preliminary results on the Burkhard-Cottingham sum rule and $d_2^n$ at moderate $Q^2$

and more to come ...
At JLab 12GeV