# THE TRANSVERSE ANGULAR MOMENTUM SUM RULE

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Based on a classical paper of Jaffe and Manohar who stressed the subtleties and warned that 'a careful limiting procedure has to be introduced'.

Despite all the care, there are flaws. With the J-M result one cannot have a sum rule for a transversely polarized nucleon.

With the correct version one can!

## OUTLINE OF TALK

1) A brief reminder of why the problem is non-trivial in the traditional approach

2) The incorrect result

3)A simple derivation of the correct result

What is the aim???

We consider a nucleon with 4-momentum $p^\mu$ and covariant spin vector $S$ corresponding to some specification of its spin state e.g. helicity, transversity or spin along the Z-axis i.e. a nucleon in state $|p, S\rangle$.

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We require an expression for the expectation value of the angular momentum in this state i.e. for $\langle p, S|\boldsymbol{J}|p, S\rangle$

i.e. we require an expression in terms of $p$ and $S$. This can then be used to relate the expectation value of $\boldsymbol{J}$ for the nucleon to the angular momentum carried by its constituents.

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Typically the angular momentum density involves the energy-momentum tensor density $T^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV \, [xT^{02}(x) - yT^{01}(x)]$$

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Hence, by the above:

$$O(x) = e^{iP.x} O(0) e^{-iP.x} \tag{4}$$

$= 0$ for ALL $x$

Clearly absurd!

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$$= \int dV x \langle p, S| e^{i\boldsymbol{P}.\boldsymbol{x}} T^{02}(0) e^{-i\boldsymbol{P}.\boldsymbol{x}} |p, S\rangle$$

Now the nucleon is in an eigenstate of momentum, so $\boldsymbol{P}$ acting on it just becomes $\boldsymbol{p}$. The numbers $e^{i\boldsymbol{p}.\boldsymbol{x}}e^{-i\boldsymbol{p}.\boldsymbol{x}}$ cancel out and we are left with:

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The solution is an old one: Build a wave packet, a superposition of physical plane wave states

In QM we use

$$\Psi_{\boldsymbol{p}_0}(\boldsymbol{x}) = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, e^{i\boldsymbol{p}.\boldsymbol{x}}$$

where $\psi(\boldsymbol{p}_0 - \boldsymbol{p})$ is peaked at $\boldsymbol{p} = \boldsymbol{p}_0$

We then calculate some physical quantity and at the end take the limit of a very sharp wave packet

In field theory we do essentially the same and build a physical wave packet state:

$$|\Psi(\boldsymbol{p}_0)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}\rangle$$

then an expectation value in the state $|\Psi(\boldsymbol{p}_0)\rangle$ will involve non-diagonal matrix elements

$$\langle \boldsymbol{p}'|\boldsymbol{J}|\boldsymbol{p}\rangle$$

What about the spin??? J-M use

$$|\Psi(\boldsymbol{p}_0, S)\rangle = \int d^3\boldsymbol{p}\, \psi(\boldsymbol{p}_0 - \boldsymbol{p})\, |\boldsymbol{p}, S\rangle$$

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But this is incorrect. The wave packet is not physical. Recall that for a physical nucleon

$$p.S = 0$$

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We have to first factor out the Dirac spinors

$$\bar{u}(p', S)[\gamma^{\mu} F_1 + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2] u(p, S)$$

This is the second problem—-Point 2

Now Jaffe and Manohar are generally very careful, but nonetheless there are errors in their derivation. They end up with the following expression for the matrix elements of the angular momentum operator:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0}\left[(3p_0^2 - m^2)\boldsymbol{s}_i - \frac{3p_0 + m}{p_0 + m}(\boldsymbol{p}.\boldsymbol{s})\boldsymbol{p}_i\right]$$

where $p^\mu = (p^0, \boldsymbol{p})$ and $\boldsymbol{s}_i$ are the components of the rest frame spin vector.

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Recall that the parton picture is supposed to be valid when the nucleon is viewed in a frame where it is moving very fast. In other words to derive a sum rule involving partons we must take the limit

$$p^0 \to \infty$$

if we consider longitudinal spin i.e $\boldsymbol{p} // \boldsymbol{s}$ one obtains:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{2} \boldsymbol{s}_i$$

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But for transverse polarization one gets:

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We will see in a moment that the result for transverse spin is incorrect.

The J-M reaction to our criticism—-very gracious and positive!

Dear Larry, Elliot, Bernard,

Better late than never. Aneesh and I finally found ourselves in the same place with the time to review the issues you raised by email and in your recent paper. We agree that there is an error in our eq. (6.9). It came from treating the quantity u(p', s )u(p, s) with insufficient care.

Thanks for taking care and finding this mistake. It's good to get it cleared up.

I have to add that I found your paper rather difficult to read. There is quite a bit of stuff that gets in the way of the relatively simple error...........

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We know how rotations affect states. If $|\boldsymbol{p}, m\rangle$ is a state with momentum $\boldsymbol{p}$ and spin projection $m$ in the rest frame of the particle, and if $\widehat{R}_i(\beta)$ is the operator for a rotation $\beta$ about the axis $i$, then

$$\widehat{R}_i(\beta)|\boldsymbol{p}, m\rangle = |\boldsymbol{R}_i(\beta)\boldsymbol{p}, m'\rangle\, D^s_{m'm}[R_i(\beta)]$$

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From the above we know what the matrix element of $\hat{R}_i(\beta)$ looks like. So we simply differentiate, multiply by $i$, and put $\beta = 0$.

Thus we have

$$\langle \boldsymbol{p}', m' | \boldsymbol{J}_i | \boldsymbol{p}, m \rangle = i \frac{\partial}{\partial \beta} \langle \boldsymbol{p}', m' | R_i(\beta) | \boldsymbol{p}, m \rangle |_{\beta=0}$$

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$$= \quad i \frac{\partial}{\partial \beta} \left[ \langle \boldsymbol{p}', m' | R_i(\beta) \boldsymbol{p}, n \rangle \, D^s_{nm}[R_i(\beta)] \right]_{\beta=0}$$

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One technical point: you have to know that the derivative of the rotation matrix for spin $s$ at $\beta = 0$ is just the spin matrix for that spin. (more correctly: the matrix generator of rotations for that spin) e.g. for spin 1/2 just $\sigma_i/2$.

## COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result (suppressing a delta-function term):

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Recall that the JM result for longitudinal spin was precisely:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s}|\boldsymbol{J}_i|\boldsymbol{p}, \boldsymbol{s}\rangle\rangle_{JM} = \frac{1}{2}\boldsymbol{s}_i$$

in complete agreement with our result.

But for transverse polarization JM had:

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which disagrees with our result and which, as we said, would imply no possibility of a transverse sum rule.

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With our correct result there is no fundamental distinction between the transverse and longitudinal cases.

## SUM RULES

Expand nucleon state as superposition of $n$-parton Fock states.

$$|\boldsymbol{p}, m\rangle \simeq \sum_n \sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1 \dots d^3\boldsymbol{k}_n \\ \psi_{\boldsymbol{p},m}(\boldsymbol{k}_1, \sigma_1, ...\boldsymbol{k}_n, \sigma_n) \\ \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 ... - \boldsymbol{k}_n)|\boldsymbol{k}_1, \sigma_1, ...\boldsymbol{k}_n, \sigma_n\rangle.$$

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There are two independent cases:

(a)Longitudinal polarization i.e. $\boldsymbol{s}$ along $OZ$. The sum rule for $\boldsymbol{J}_z$ yields the well known result

$$1/2 = 1/2\,\Delta\Sigma + \Delta G + \langle L_z^q\rangle + \langle L_z^G\rangle$$

(b) Transverse polarization i.e. $\boldsymbol{s} \perp \boldsymbol{p}$. The sum rule for $\boldsymbol{J}_x$ or $\boldsymbol{J}_y$ yields a a new sum rule

$$1/2 = 1/2 \sum_{q,\,\bar{q}} \int dx\, \Delta_T q(x) + \sum_{q,\,\bar{q},\,G} \langle L_{\boldsymbol{s}_T} \rangle$$

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Here $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are known as the quark transversity or transverse spin distributions in the nucleon.

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As mentioned no such parton model sum rule is possible with the J-M formula because, as $p \to \infty$, for $i = x, y$ the matrix elements diverge.

It is absolutely crucial to note that the sum rule involves a SUM of Quark and Antiquark densities.

Not realizing this has led to some misunderstandings.

What some people call the TENSOR CHARGE of the NUCLEON is the difference between quark and antiquark contributions.

Thus the transverse spin sum rule, although it involves the transverse spin or transversity quark and antiquark densities, does NOT involve the nucleon's transversity. The Tensor Charge operator is NOT related to the angular momentum.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are most directly measured in doubly polarized Drell-Yan reactions

$$p(\boldsymbol{s}_T) + p(\boldsymbol{s}_T) \to l^+ + l^- + X$$

where the asymmetry is proportional to

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They can also be determined from the asymmetry in semi-inclusive hadronic interactions like

$$p + p(\boldsymbol{s}_T) \to H + X$$

where $H$ is a detected hadron, typically a pion.

Also in SIDIS reactions with a transversely polarized target

$$\ell + p(\boldsymbol{s}_T) \to \ell + H + X.$$

The problem is that in these semi-inclusive reactions $\Delta_T q(x)$ always occurs multiplied by the Collins fragmentation function, about which we are at present gathering information.

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• This can be handled using wave packets but the calculations are long and unwieldy

• Using our knowledge of how states transform under rotations leads quickly and relatively painlessly to correct results

• The great success of the correct approach is that it allows derivation of a sum rule also for transversely polarized nucleons