Hadron Spectroscopy from QCD

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&

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the local team :

Robert Edwards, Balint Joo, David Richards, Christopher Thomas

work under the auspices of the

Hadron Spectrum Collaboration

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H-W.Lin *Washington*

J.Bulava *DESY*

N.Mathur *Tata*

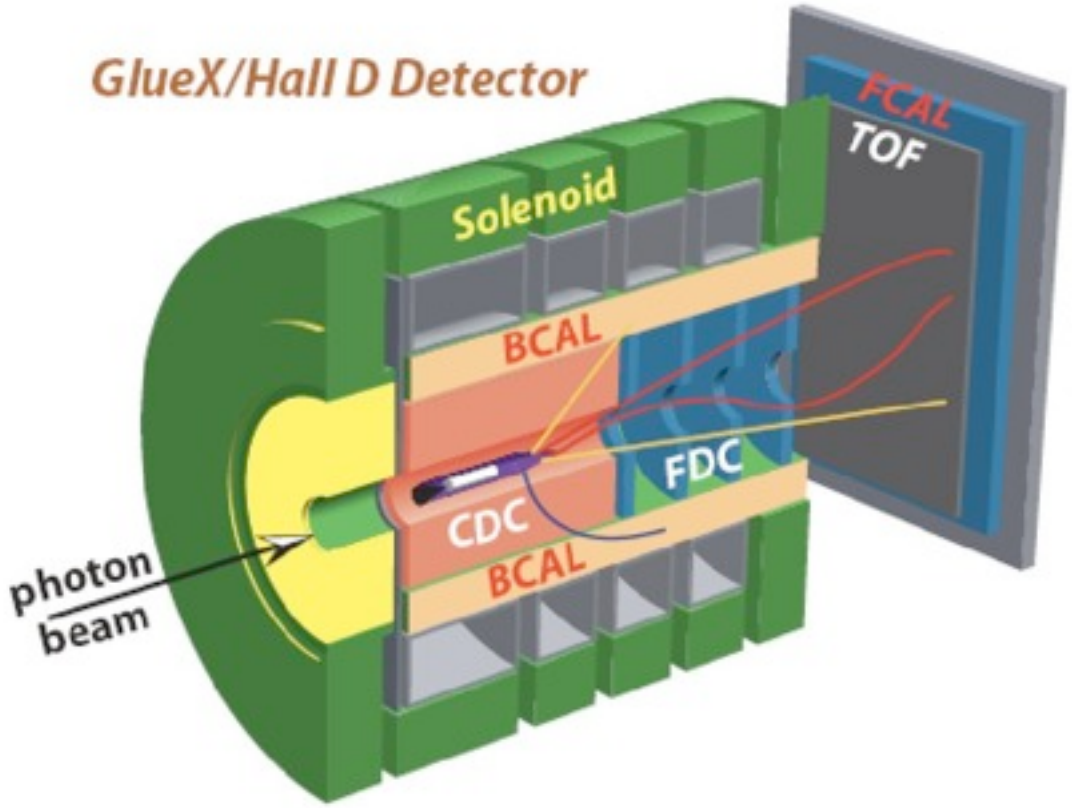
hadron spectroscopy at JLab

CLAS

- baryon resonance electroproduction
- baryon spectrum
- transition form-factors $N \xrightarrow{\gamma^\star} N^\star$

GlueX

- meson resonance photoproduction
- meson spectrum
- photocouplings $g\big(m \xrightarrow{\gamma} m^\star\big)$

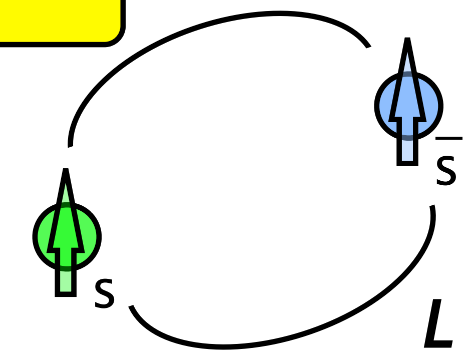


plus BES III, PANDA, Belle ...

hadron models

quark model - mesons as bound states of 'constituent quarks'

e.g. $q\bar{q}$ in a $^{2S+1}L_J$ eigenstate



$$|m\rangle \sim \int d^3\vec{q}\;\; \varphi(|\vec{q}|) Y_L(\hat{q}) \;\; |q_\sigma(\vec{q});\; \bar{q}_{\bar{\sigma}}(-\vec{q})\rangle$$

$$m_q \sim 300\,\mathrm{MeV}$$

φ(q) is bound state wavefunction from solution of Schrödinger equation with phenomenological potential

$^1D_2 \Rightarrow 2^{-+}$	$\pi_2(1670)$
$^3D_{1,2,3} \Rightarrow (1,2,3)^{--}$	$\cdots\;\; \rho_3(1690)$
$^1P_1 \Rightarrow 1^{+-}$	$b_1(1235)$
$^3P_{0,1,2} \Rightarrow (0,1,2)^{++}$	$a_0(980)\;\; a_1(1260)\;\; a_2(1320)$
$^3S_1 \Rightarrow 1^{--}$	$\rho(770)$
$^1S_0 \Rightarrow 0^{-+}$	$\pi(140)$ ✘

? why is the effective degree-of-freedom so heavy ***?***

? where is the dynamical glue ***?***

hadron models

following quantum numbers can't be made : $1^{-+},\ 0^{+-},\ 2^{+-}\ \ldots$

"exotics"

can arise via addition of

- ★ extra quark pairs — ***"multiquarks"***
- ★ excited glue — ***"hybrids"***

lattice QCD formalism - the one slide summary ?

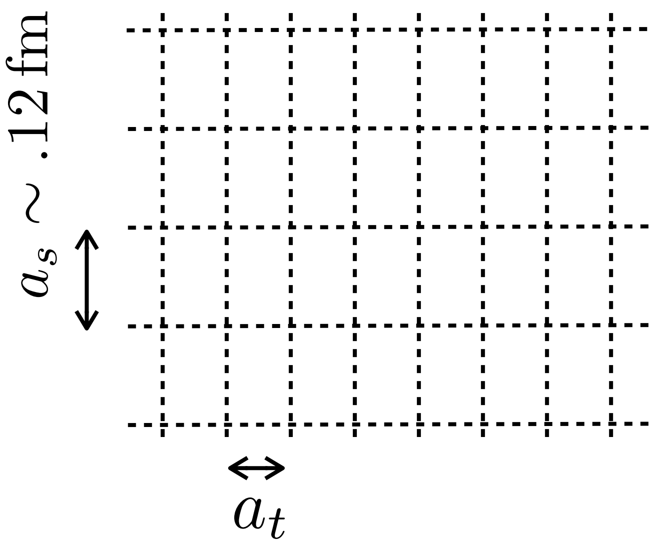
$$\mathcal{Z}_{\rm QCD} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \, e^{iS[\psi,\bar{\psi},A_\mu]}$$

quark field gluon field

'path' integral

'sum' over possible field configurations

discretise on a finite grid - in Euclidean space-time ($t \to i\,t$)



$a_t^{-1} \sim 5.6\,\mathrm{GeV}$

$\psi(x) \longrightarrow \psi_x$ quark fields on the sites

$A_\mu(x) \to U_{x,\mu} = e^{-aA_{x\mu}}$ gauge-fields on the links

$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U \, f(\psi,\bar{\psi},U)\, e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

do the fermion integral

$$\int \mathcal{D}U \, f(\mathcal{Q}^{-1}[U],U) \quad \det \mathcal{Q}[U] e^{-\tilde{S}_G[U]}$$

probability, $\mathcal{P}[U]$

Monte Carlo

$$\sum_{\{U\}} f(\mathcal{Q}^{-1}[U],U)$$

$\left\{\begin{matrix}16^3\\20^3\\24^3\end{matrix}\right\} \times 128$

$L \gtrsim 2\,\mathrm{fm}$

$N_F = 2 \oplus 1$
$(u, d \oplus s)$

$230\,\mathrm{MeV} < m_\pi < 700\,\mathrm{MeV}$

where's the approximation ?

finite lattice spacing, ***a***

repeat the calculations with smaller ***a*** & extrapolate

finite box size, ***L***

repeat the calculations with larger ***L*** & extrapolate

★ finite volume will turn out to be useful ★

unphysical quark mass

not really an approximation - just not reality

★ use quark mass dependence to learn about ***QCD*** ★

choice of discretised actions $Q[U],\ \tilde{S}_G[U]$

results should agree as ***a*** → ***0***

two-point functions & the energy spectrum

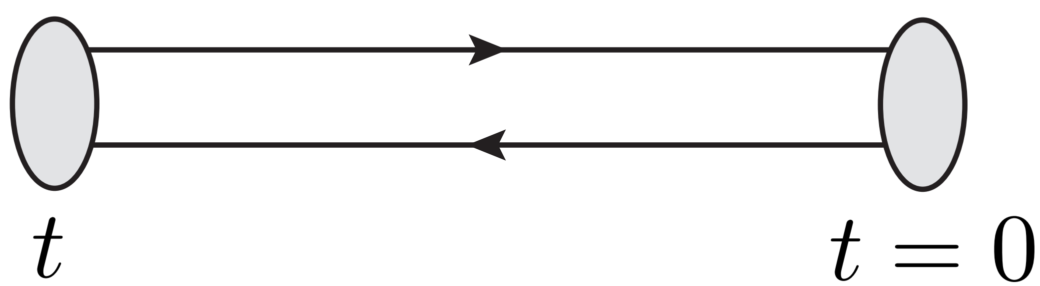
a two-point correlator :

$$C_{\mathbf{\Gamma},\mathbf{\Gamma}'}(\vec{p};t,0) = \langle 0 | \bar{\psi}_t \mathbf{\Gamma} \psi_t \; \bar{\psi}_0 \mathbf{\Gamma}' \psi_0 | 0 \rangle$$

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \cdot \int d^3y \, e^{-i\vec{p}\cdot\vec{y}} \, \bar{\psi}_{y,t} \mathbf{\Gamma} \psi_{y,t} \boxed{\int d^3x \, e^{i\vec{p}\cdot\vec{x}} \, \bar{\psi}_{x,0} \mathbf{\Gamma}' \psi_{x,0}} \cdot e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

meson operator at mom $\vec{p}$

$$\bar{\psi}_{y,t} \mathbf{\Gamma} \psi_{y,t} \; \bar{\psi}_{x,0} \mathbf{\Gamma}' \psi_{x,0}$$



two-point functions & the energy spectrum

relation to the spectrum :

complete set of ***QCD*** eigenstates*

$$1 = \sum_n |n\rangle\langle n|$$

$$\langle 0|\bar{\psi}_t \mathbf{\Gamma} \psi_t \; \bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n \langle 0|\bar{\psi}_t \mathbf{\Gamma} \psi_t|n\rangle\langle n|\bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n e^{-E_n t}\langle 0|\bar{\psi}_0 \mathbf{\Gamma} \psi_0|n\rangle\langle n|\bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n Z_n^{\mathbf{\Gamma}} Z_n^{\mathbf{\Gamma}'} e^{-E_n t}$$

in principle - contribution from all states with the right q.n.'s

fitting a sum of exponentials is unstable
- noisy data
- possibly degenerate states

** in a finite volume*

operator basis & variational solution

better - build a basis of operators, calculate a matrix of correlators

$$\begin{bmatrix} \langle 0|\mathcal{O}_1\mathcal{O}_1|0\rangle & \langle 0|\mathcal{O}_1\mathcal{O}_2|0\rangle & \cdots \\ \langle 0|\mathcal{O}_2\mathcal{O}_1|0\rangle & \langle 0|\mathcal{O}_2\mathcal{O}_2|0\rangle & \\ \vdots & & \ddots \end{bmatrix}$$

diagonalising this matrix gives the optimal linear combination of operators for each state

eigenvalues → energies
eigenvectors → Z

optimal combinations are ***orthogonal*** - deals with degenerate states

our operators - built out of covariant derivatives

$$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi \qquad D = \partial - A$$

use Clebsch-Gordans to build the desired q.n.'s, e.g. 1^{--}

$$\bar{\psi}\vec{\gamma}\psi \qquad D_{\pm,0} \sim \left\{ \begin{matrix} \frac{1}{\sqrt{2}}(D_x \pm i D_y) \\ D_z \end{matrix} \right\}$$

$$\bar{\psi}\overleftrightarrow{D}\psi$$

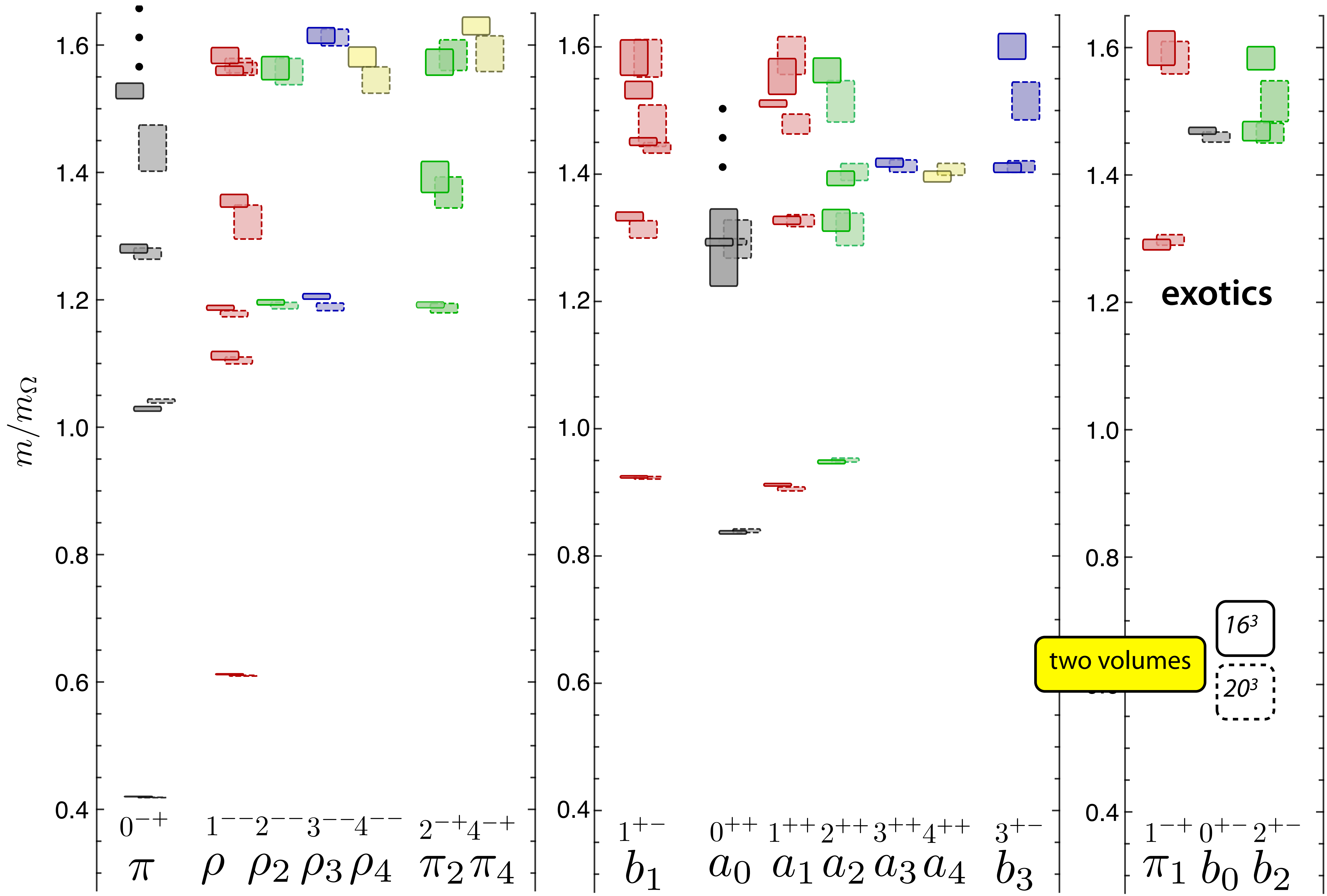
$$\langle 1, m_1; 1, m_2 | 1, m\rangle \; \bar{\psi}\gamma_5 \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \psi$$

$$\begin{matrix} \langle 1, m_3; 2, m_D | 1, m\rangle \\ \langle 1, m_1; 1, m_2 | 2, m_D\rangle \end{matrix} \; \bar{\psi}\gamma_{m_3} \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \psi$$

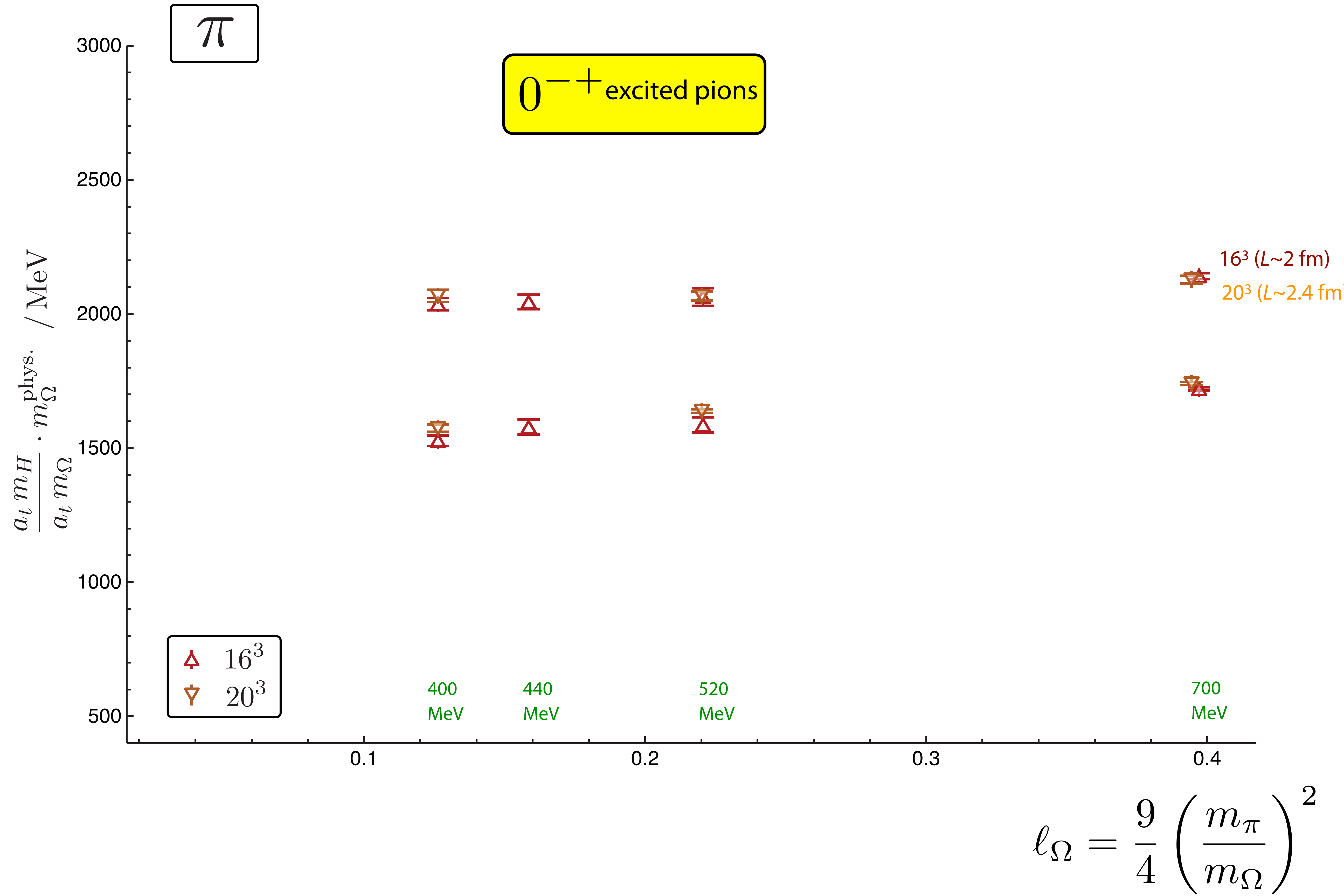
meson spectrum

$N_F = 3$ (s,s,s) $m_\pi \sim 700$ MeV

PRL 103:262001,2009



reducing the quark mass



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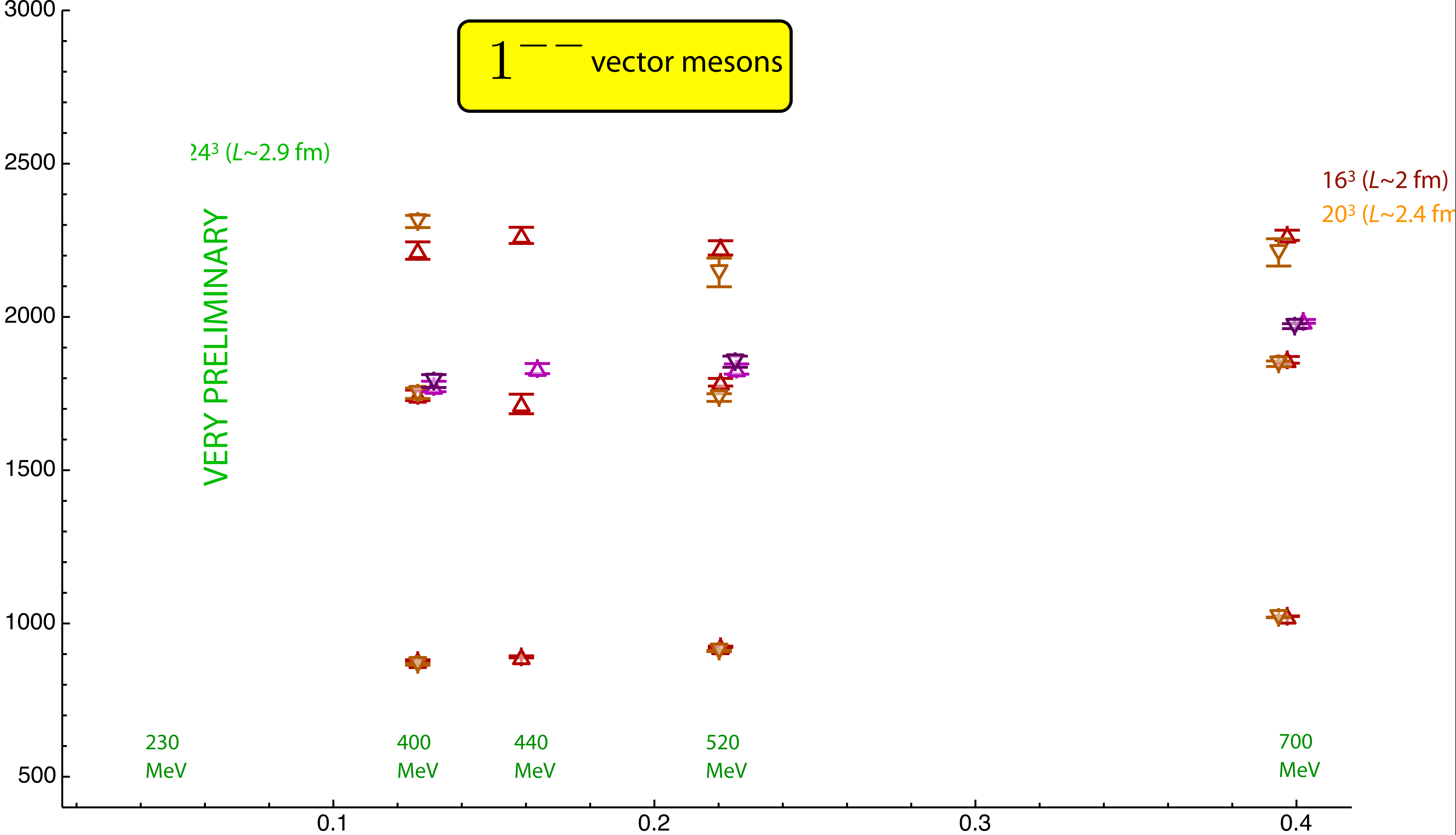

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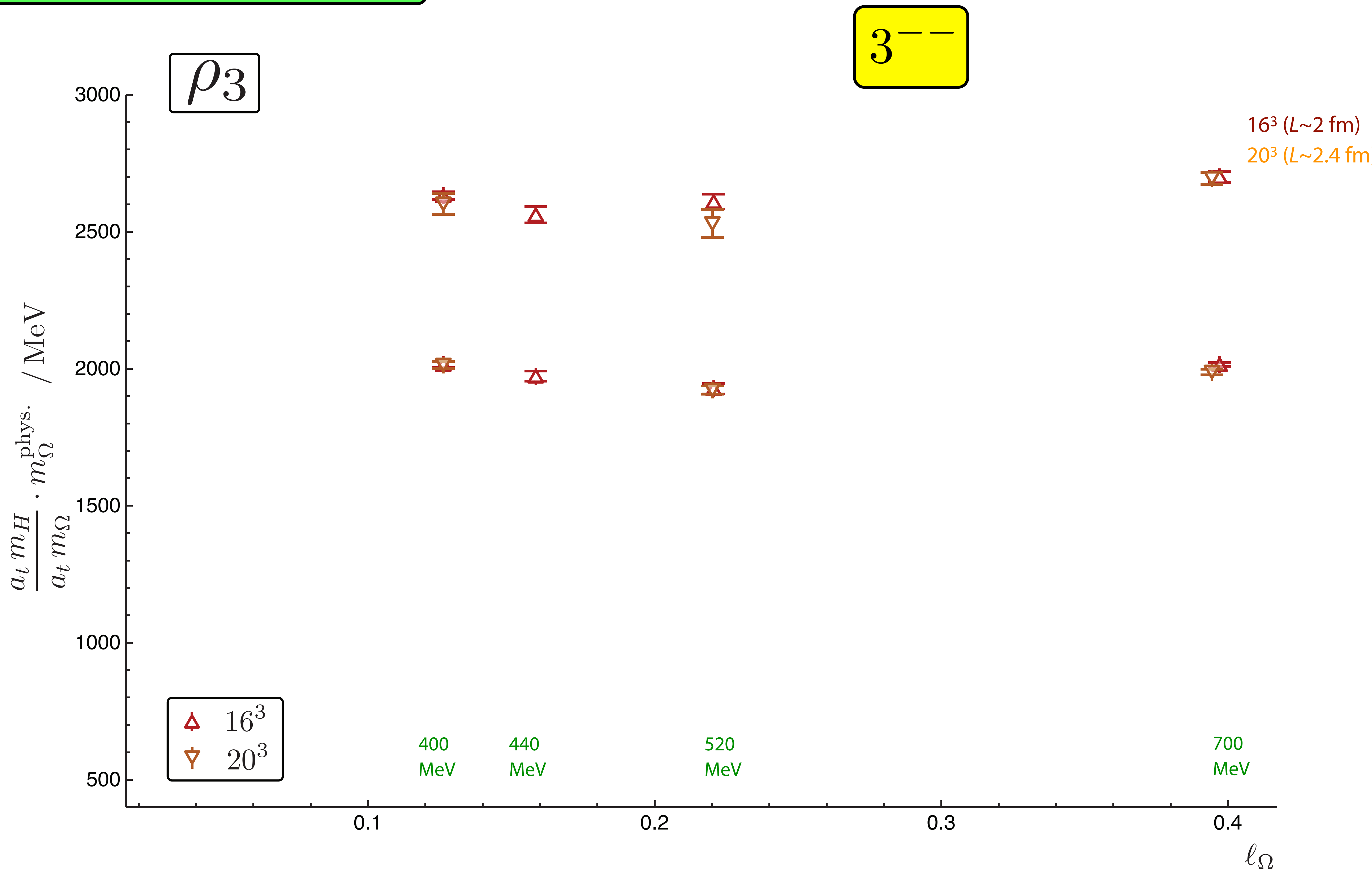
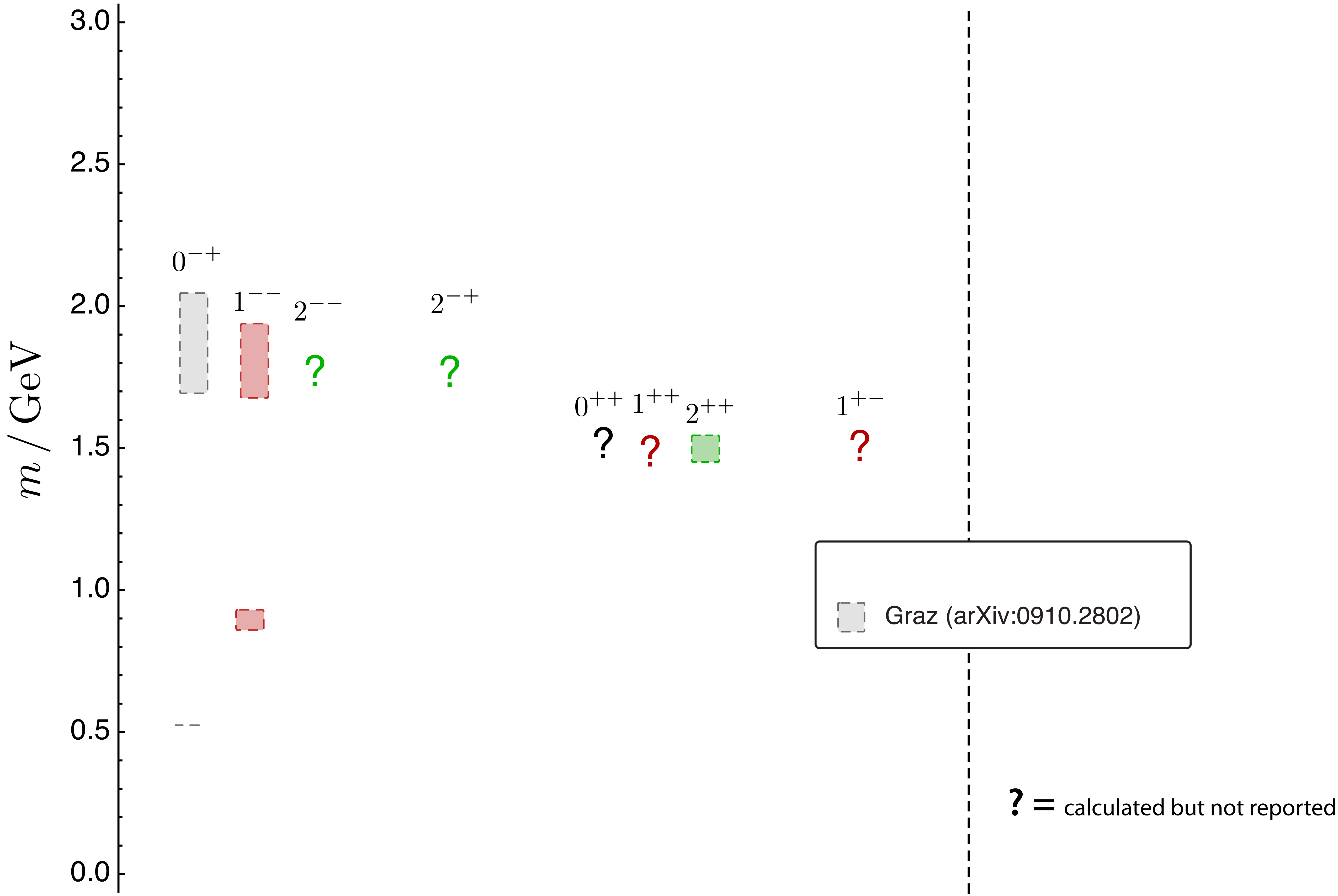


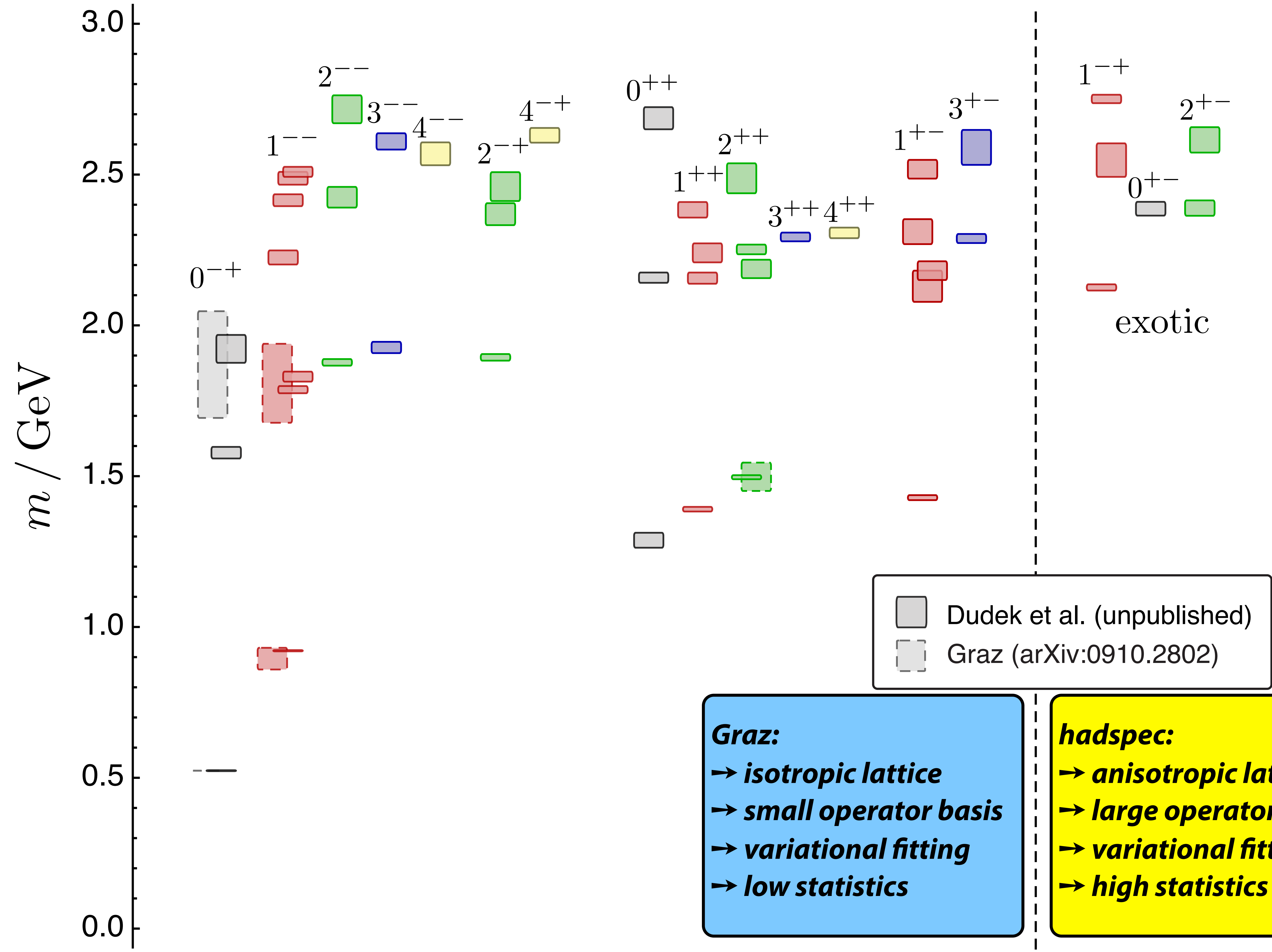
the competition

$N_F = 2$ (u,d) $m_\pi \sim 500$ MeV



the competition

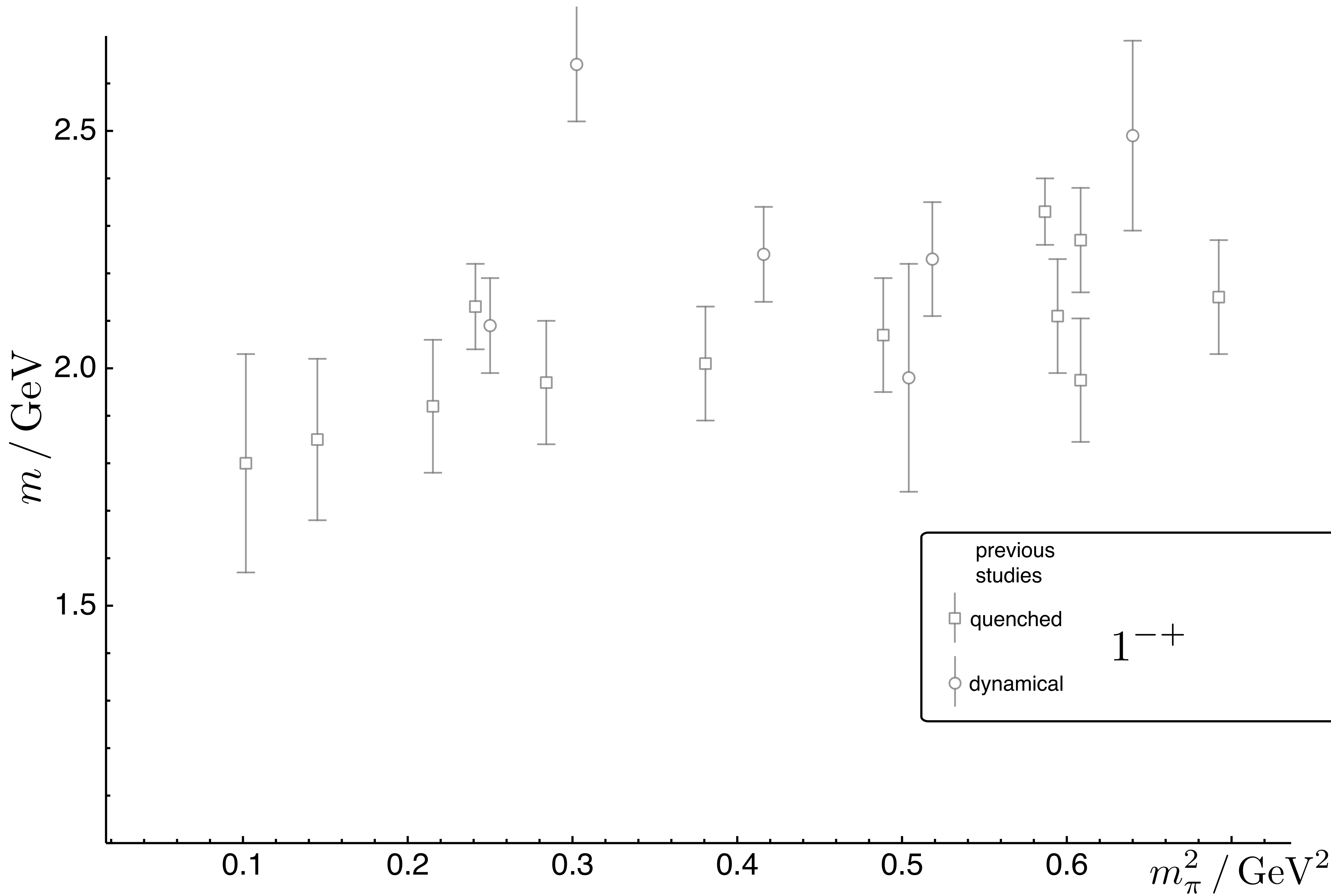
$N_F = 2+1$ (u,d,s) $m_\pi \sim 520$ MeV $V = 16^3$



Graz:
- → ***isotropic lattice***
- → ***small operator basis***
- → ***variational fitting***
- → ***low statistics***

hadspec:
- → ***anisotropic lattice***
- → ***large operator basis***
- → ***variational fitting***
- → ***high statistics***

exotics - world summary



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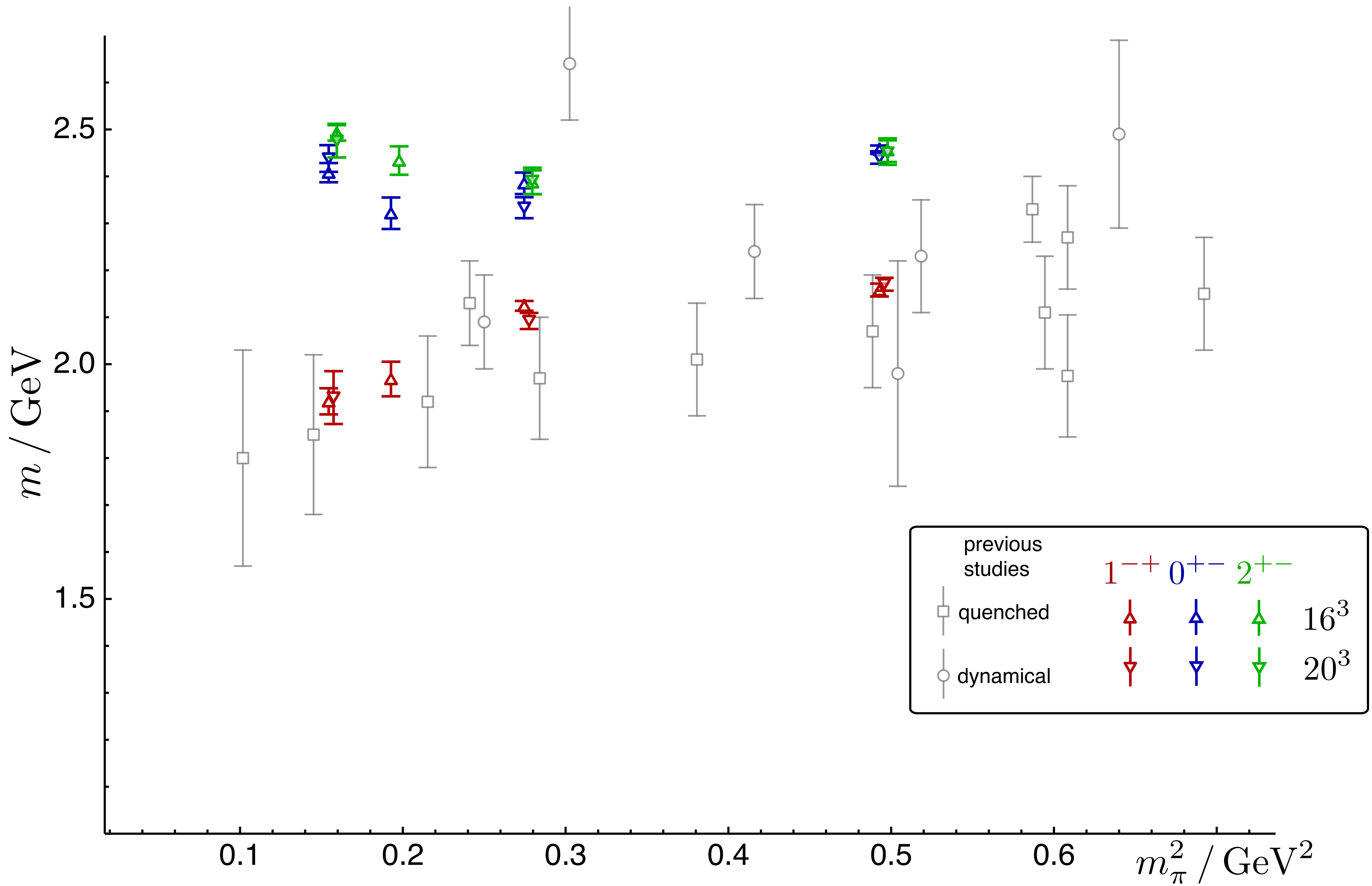
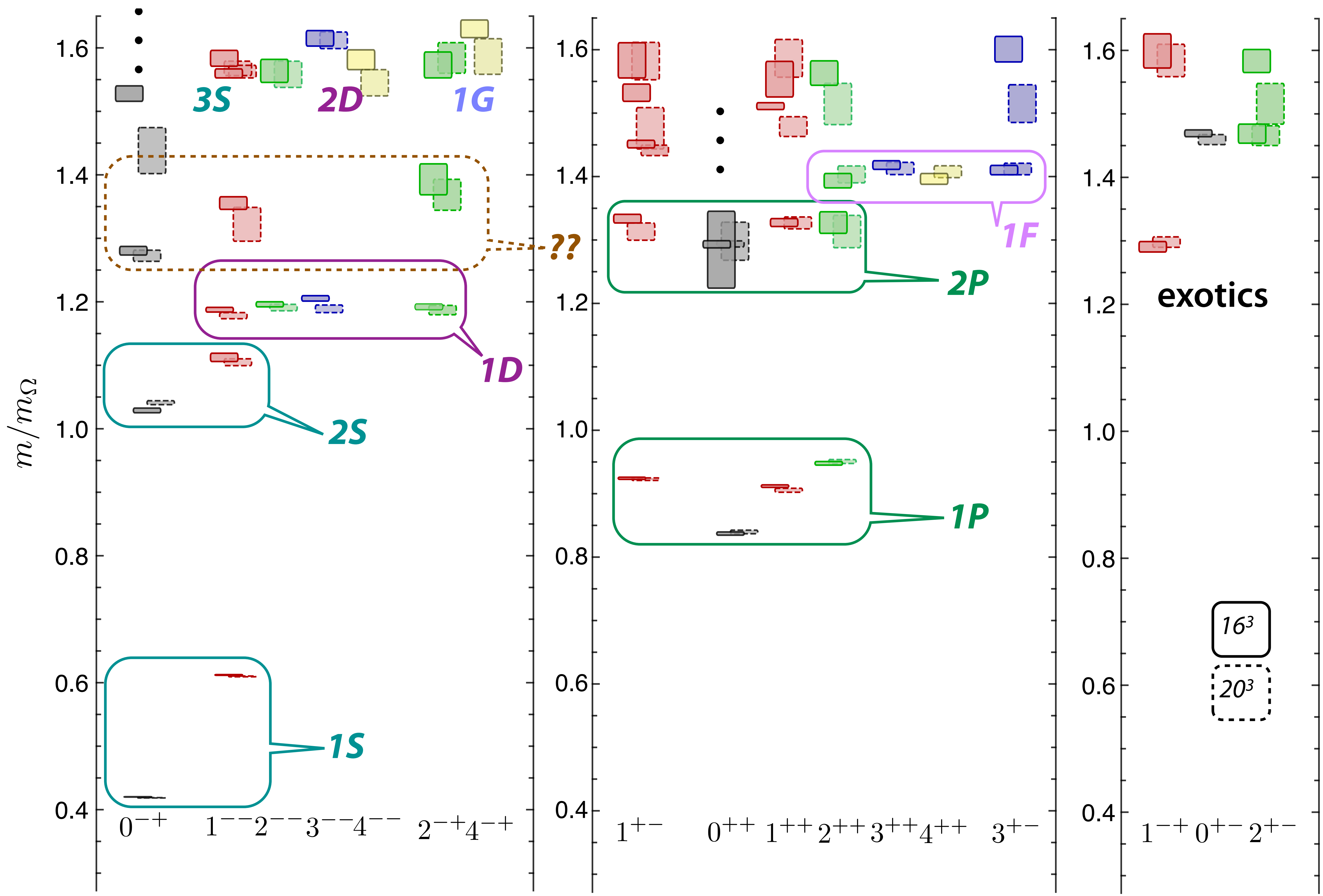


what do we learn ?

patterns in the meson spectrum ?

$N_F = 3$ (s,s,s) $m_\pi \sim 700$ MeV



interpretation of the meson spectrum

$N_F = 3$ (s,s,s) $m_\pi \sim 700$ MeV

$$1^{--}$$

look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$

non-rel.
quark model

ρ	$\bar{\psi}\vec{\gamma}\psi$	${}^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$\langle 1, m_3; 2, m_D \vert 1, m\rangle \langle 1, m_1; 1, m_2 \vert 2, m_D\rangle \; \bar{\psi}\gamma_{m_3} \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \psi$	${}^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	$\langle 1, m_1; 1, m_2 \vert 1, m\rangle \, \bar{\psi}\gamma_5 \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \psi$	✘

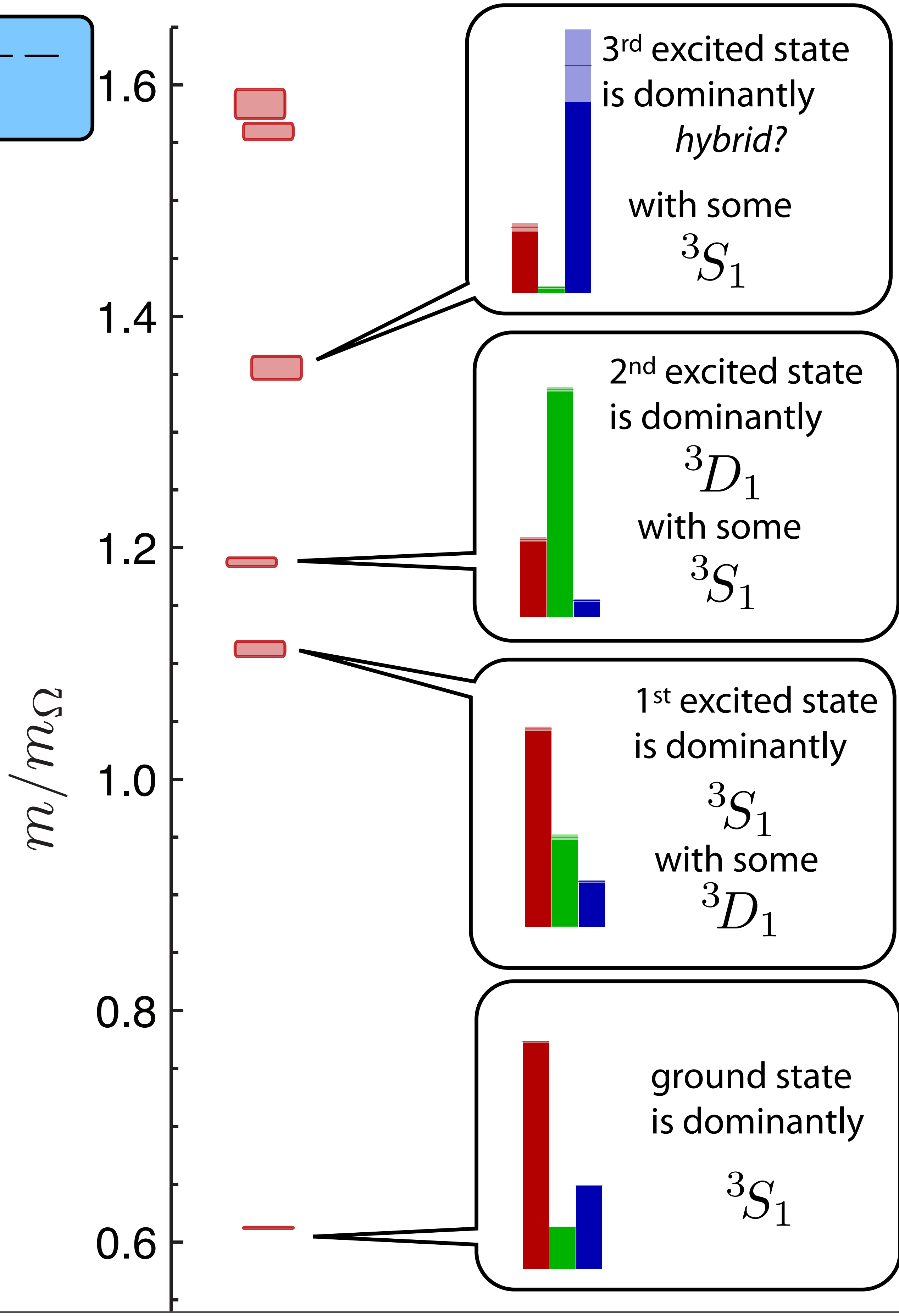
$[D, D] \sim F$

non-exotic hybrid ?

interpretation of the meson spectrum

$N_F = 3$ *(s,s,s)* $m_\pi \sim 700$ MeV

1^{--}



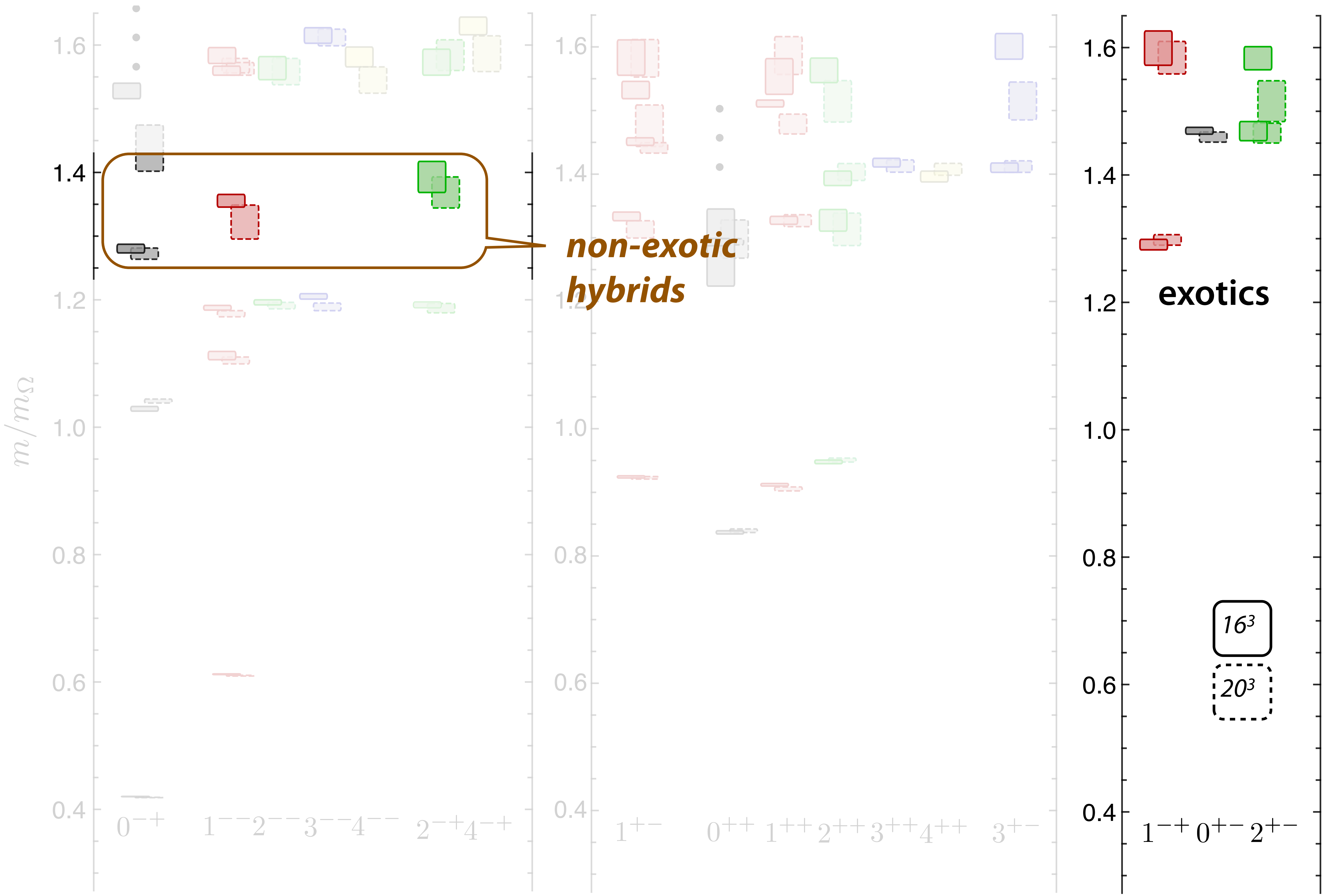
look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

build a ***bound state model*** phenomenology comparable to the quark model using non-perturbative ***QCD*** calculations

patterns in the meson spectrum ?

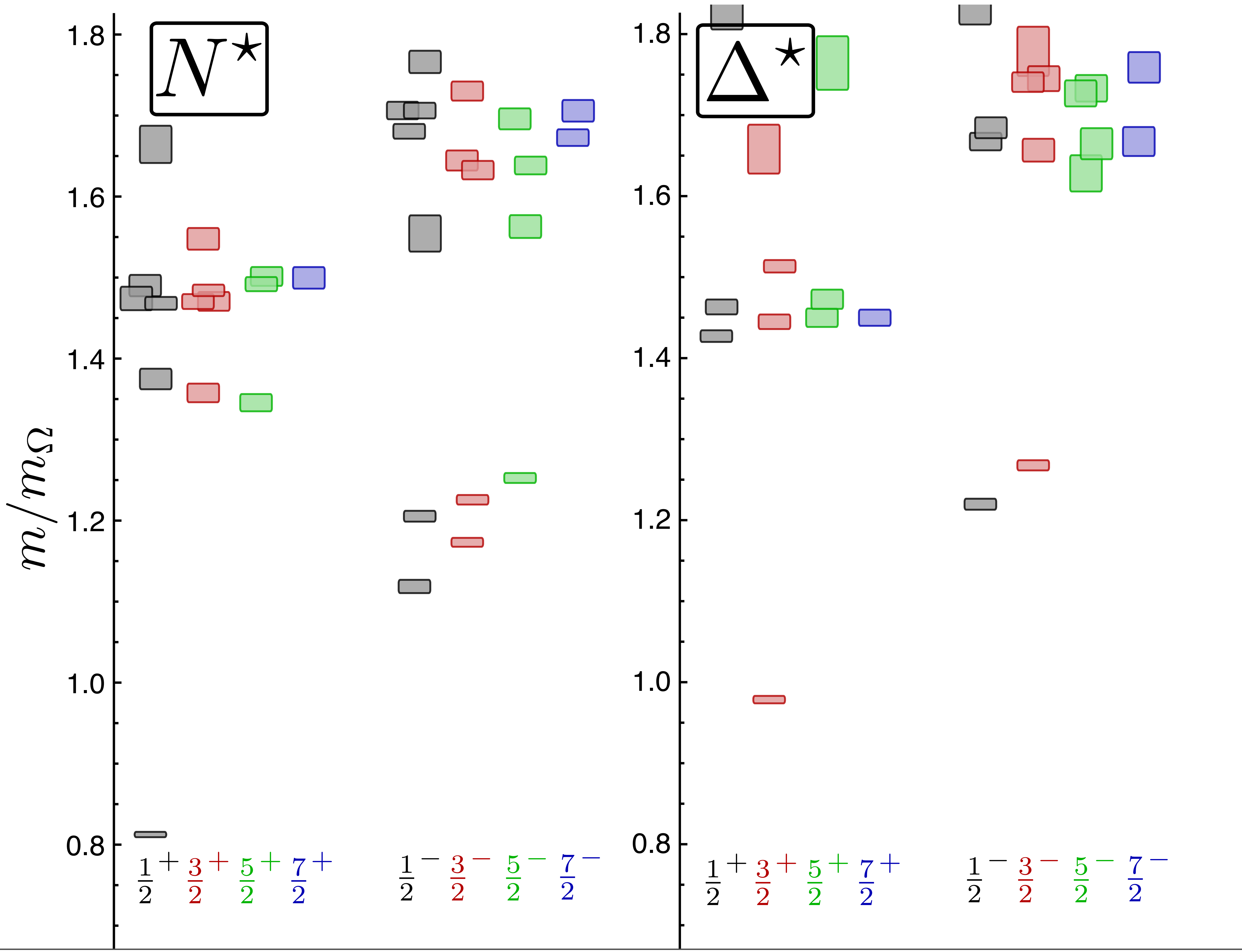
$N_F = 3$ (s,s,s) $m_\pi \sim 700$ MeV



baryons ?

baryons

$N_F = 2+1$ (u,d,s) $m_\pi \sim 520$ MeV $V=16^3$



baryons

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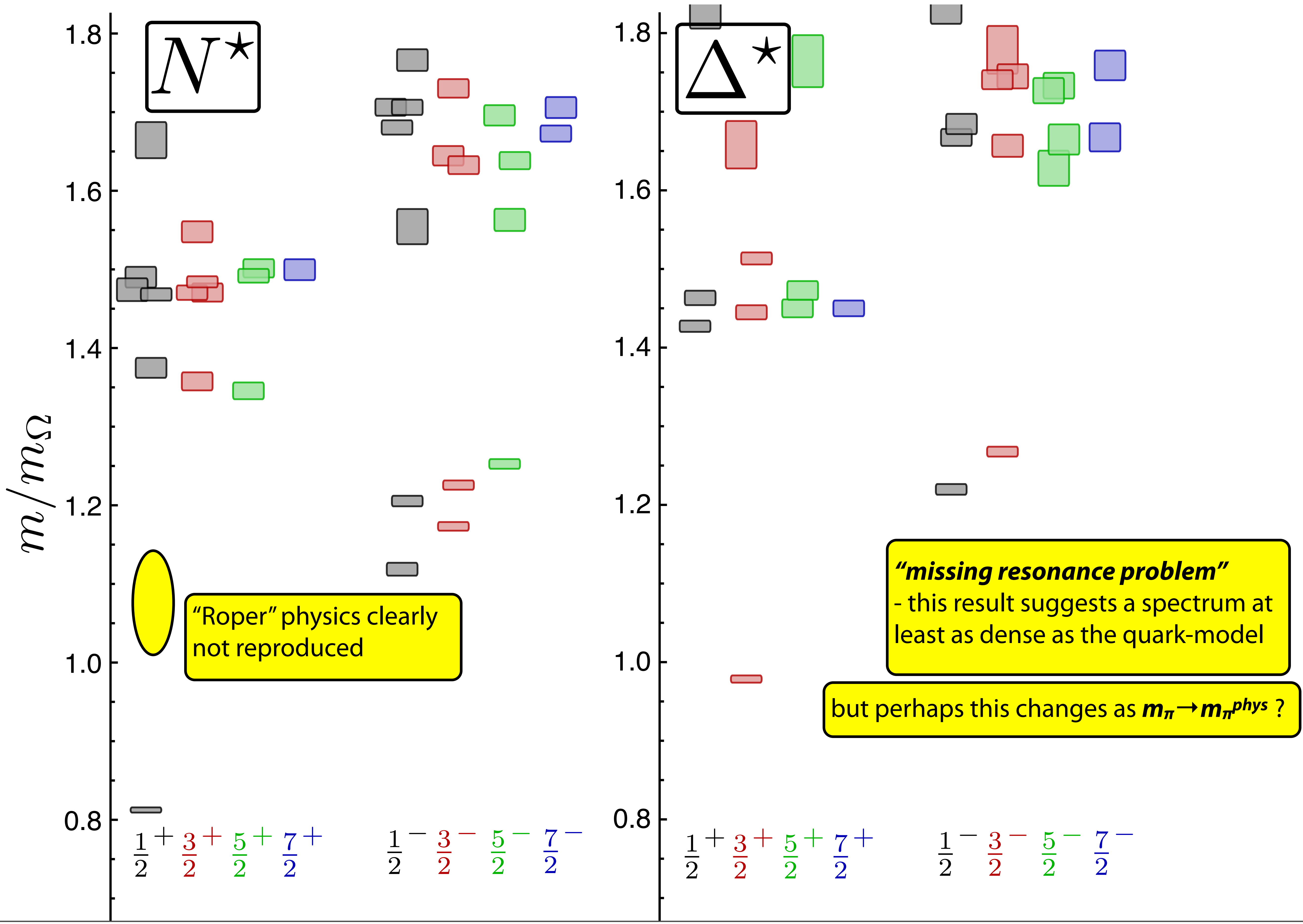


coupling to photons

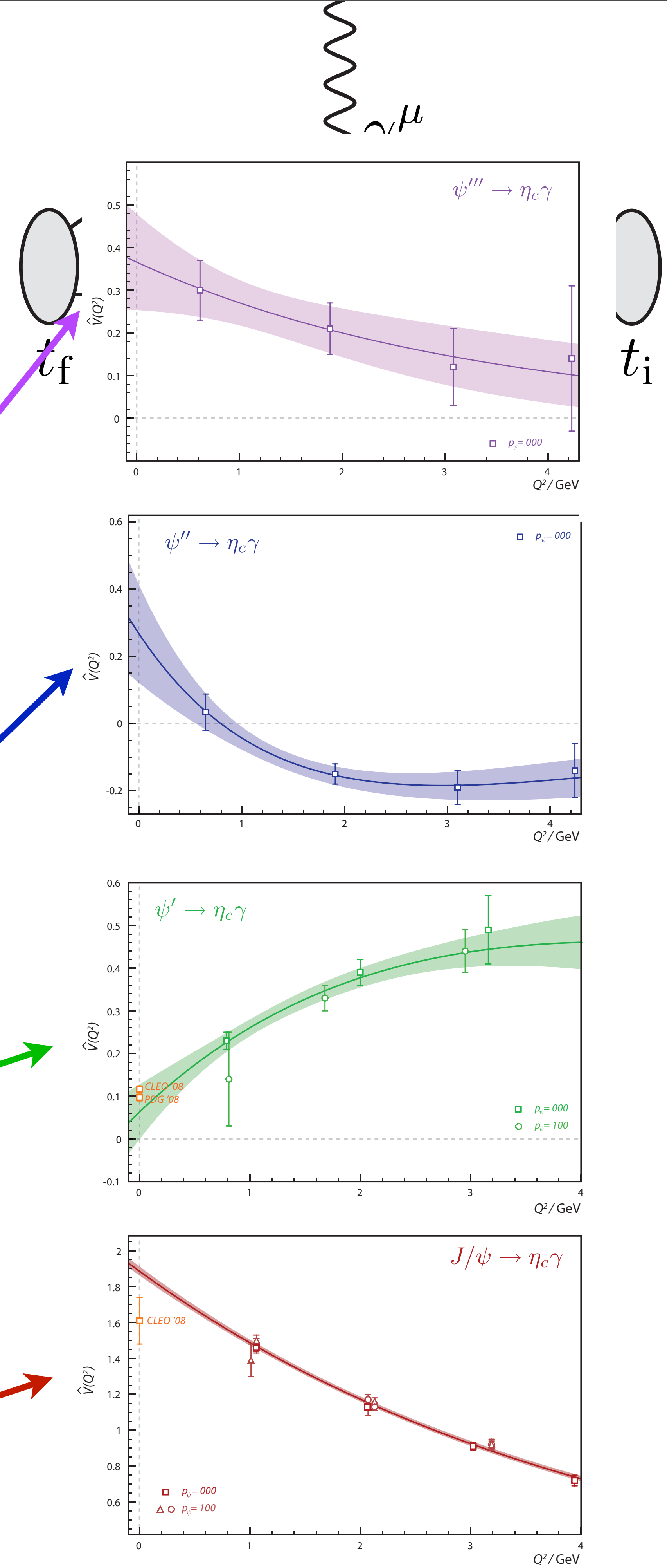
$$\langle 0 | \bar{\psi}\mathbf{\Gamma}\psi_{t_\mathrm{f}} \;\cdot\; \bar{\psi}\gamma^\mu\psi_t \;\cdot\; \bar{\psi}\mathbf{\Gamma}'\psi_{t_\mathrm{i}} | 0 \rangle$$

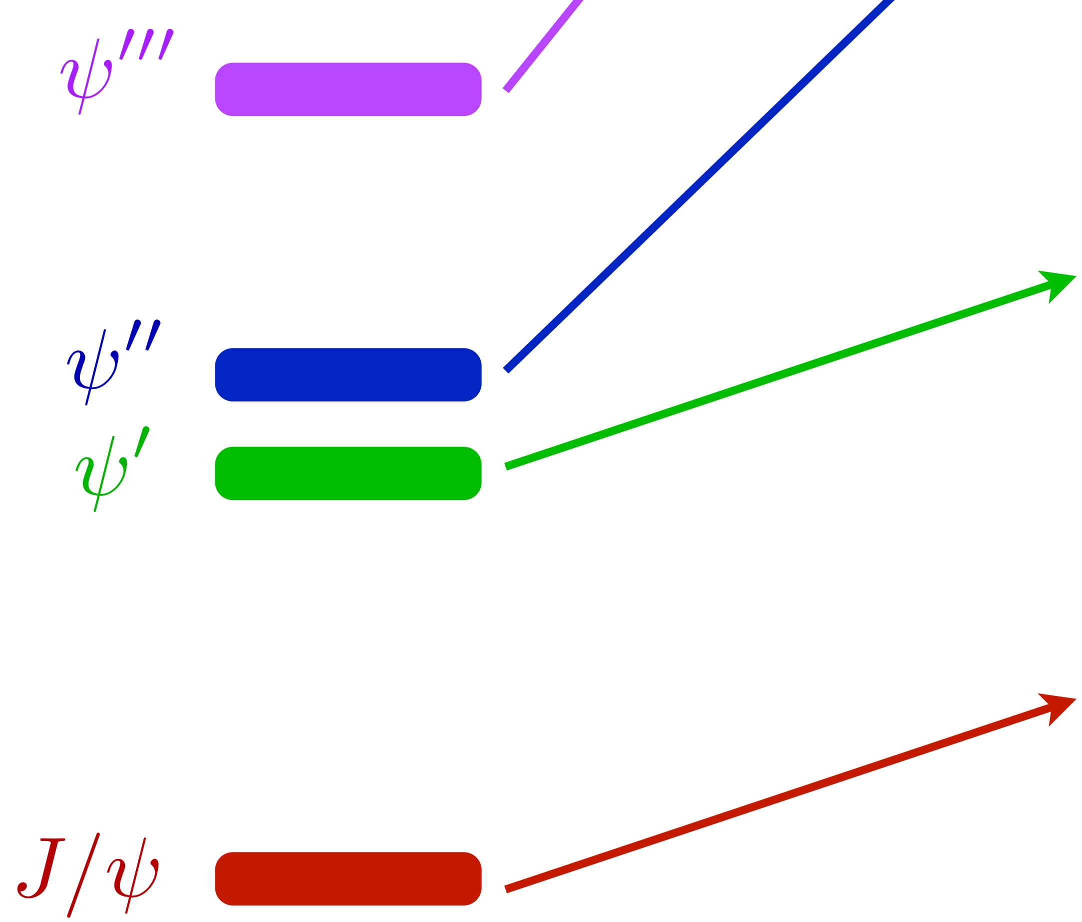
can use the optimal operator combinations from the spectrum study

project excited states from correlators

transition form-factors

don't have light-quark results yet - some results in charmonium :





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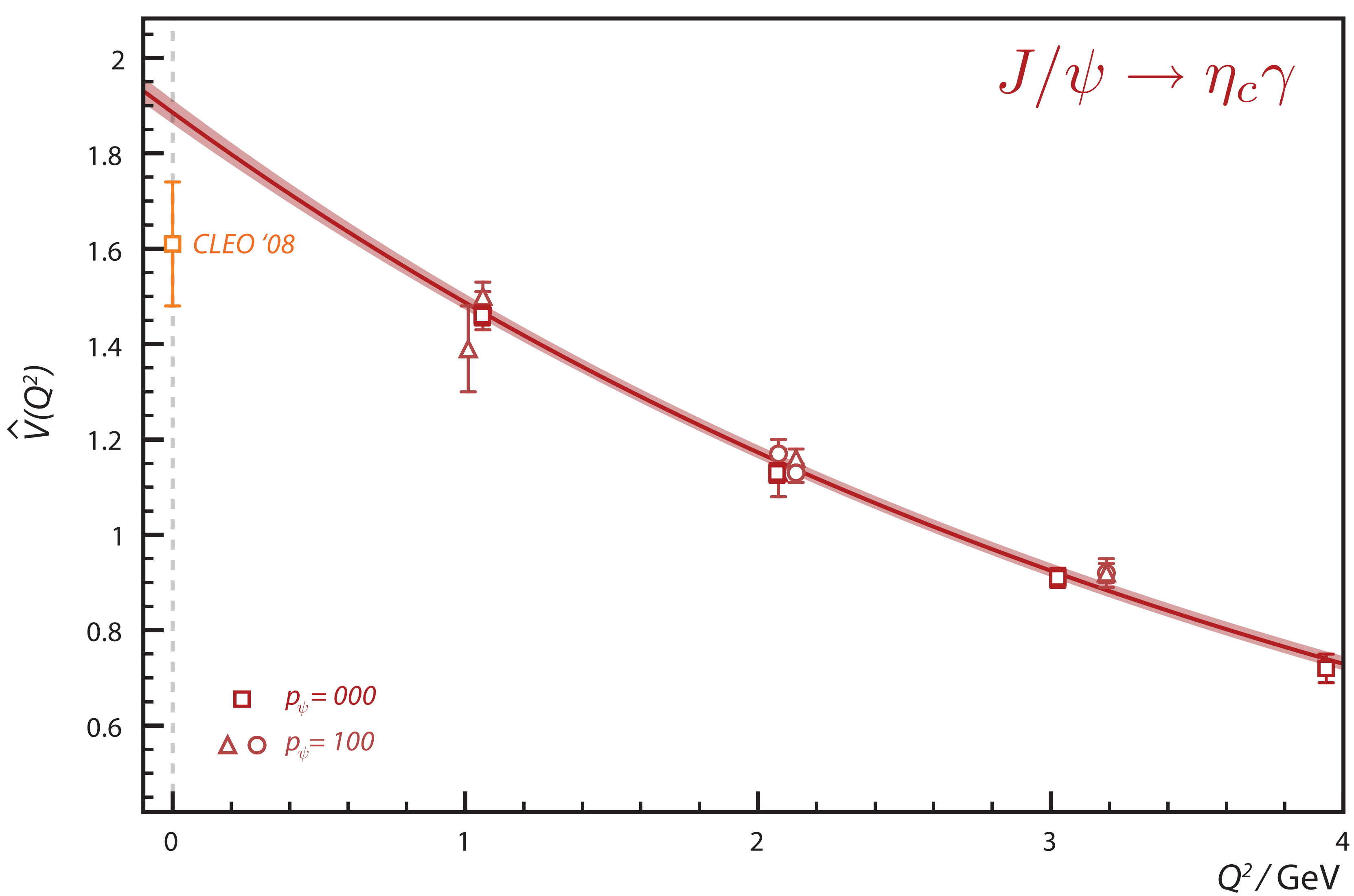

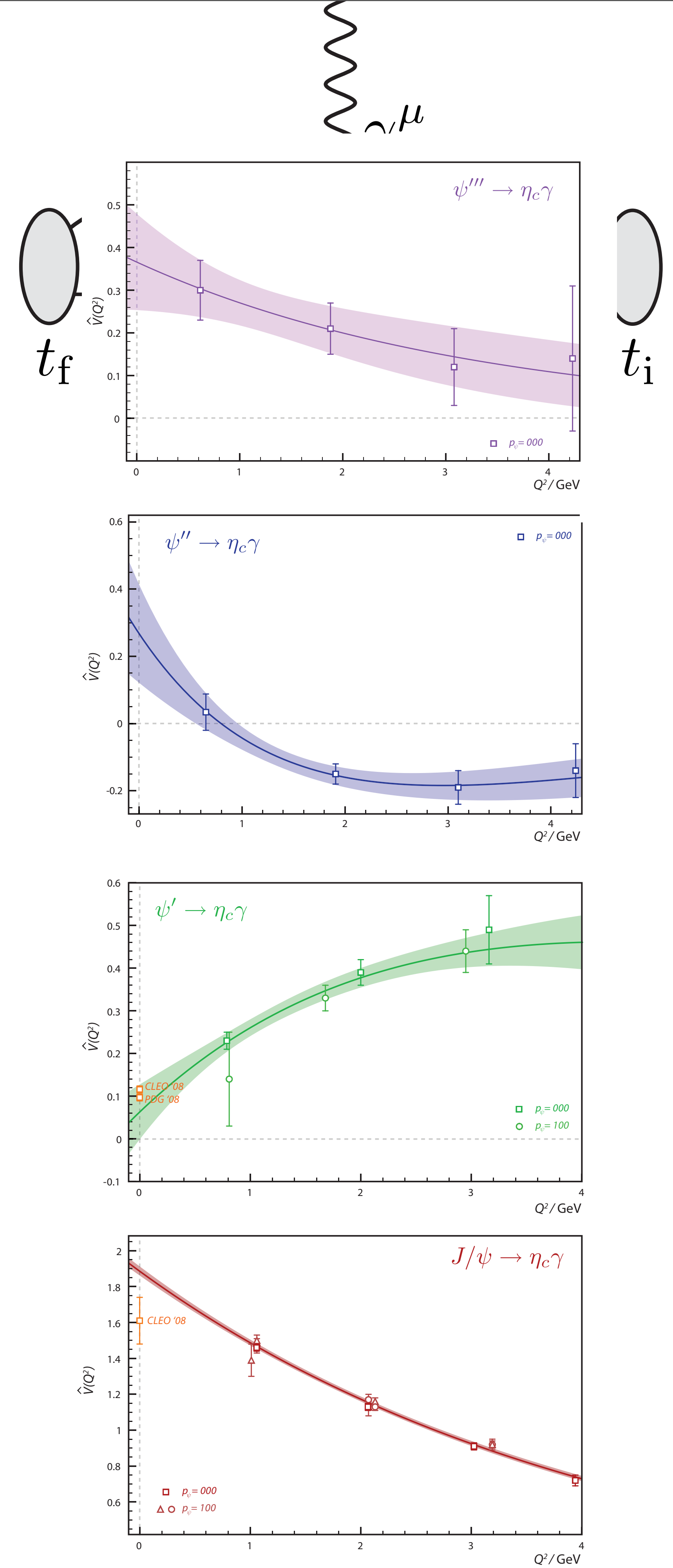

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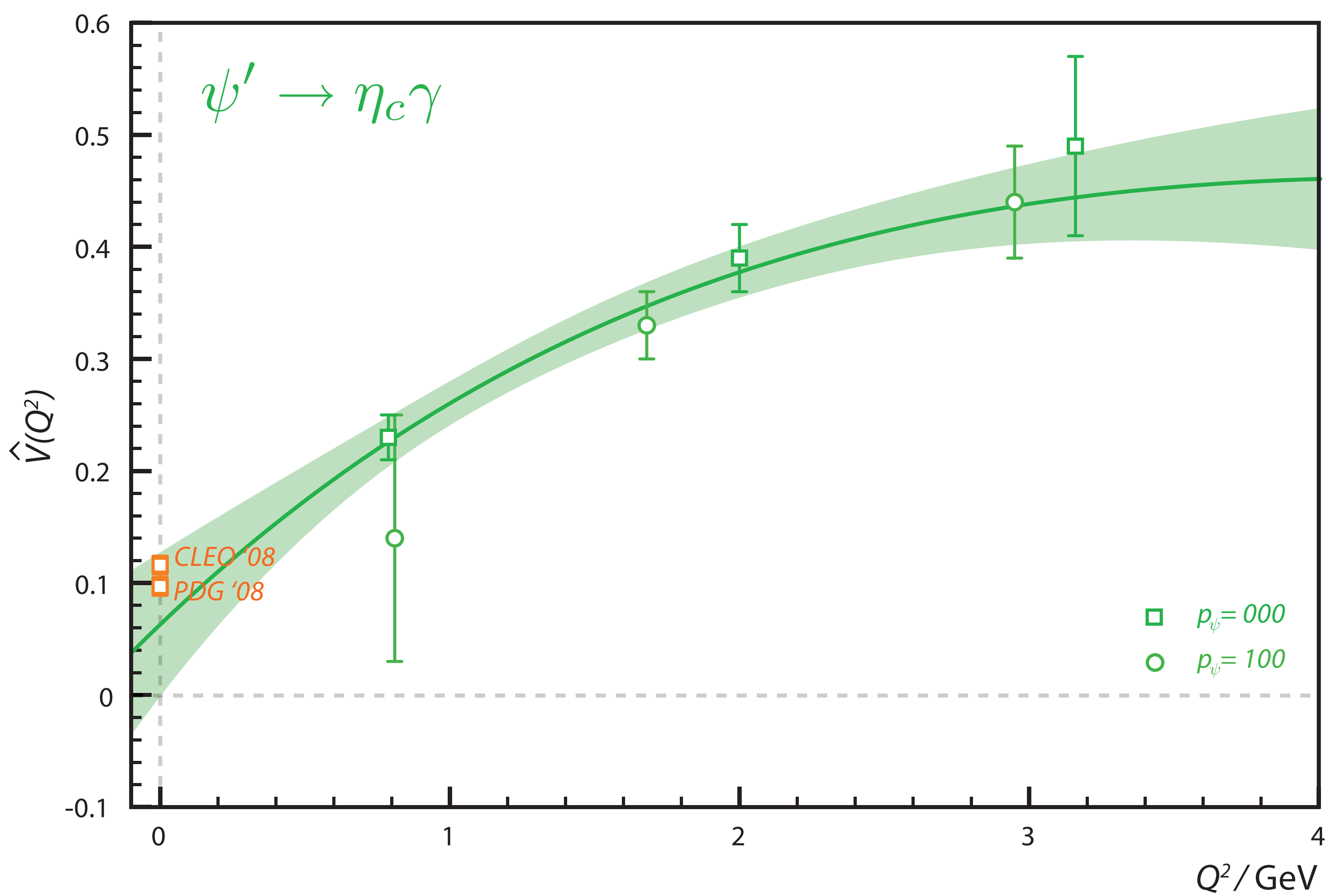

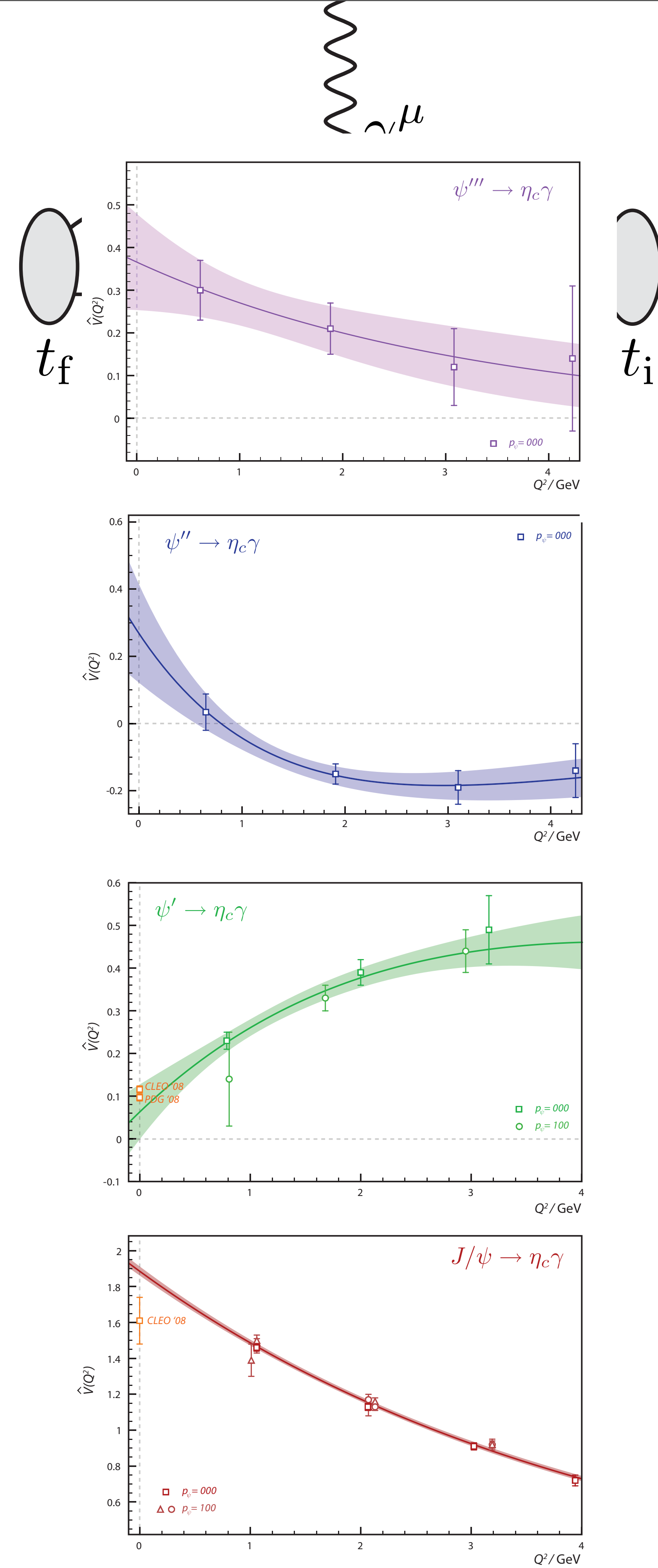

coupling to photons

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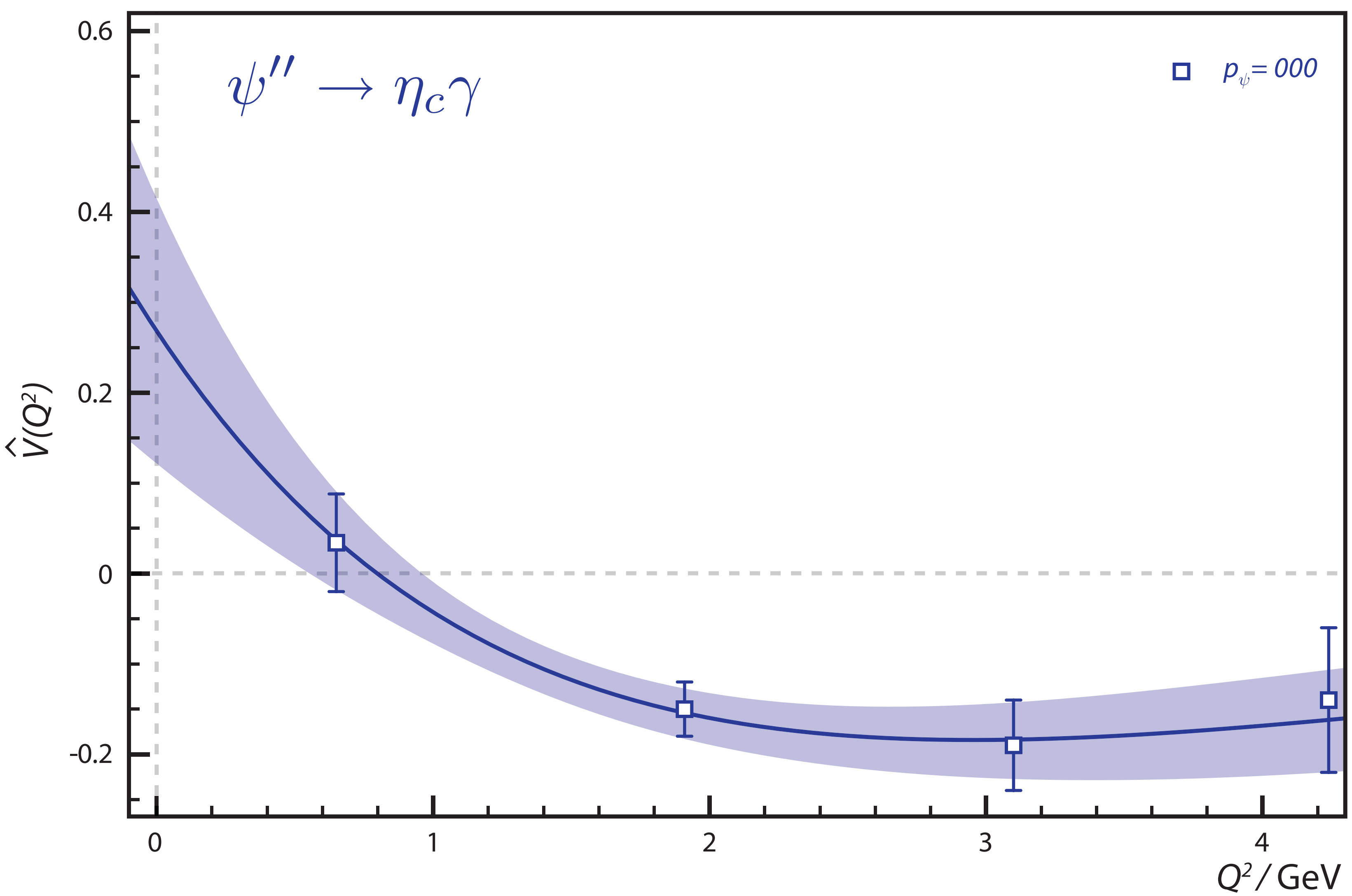

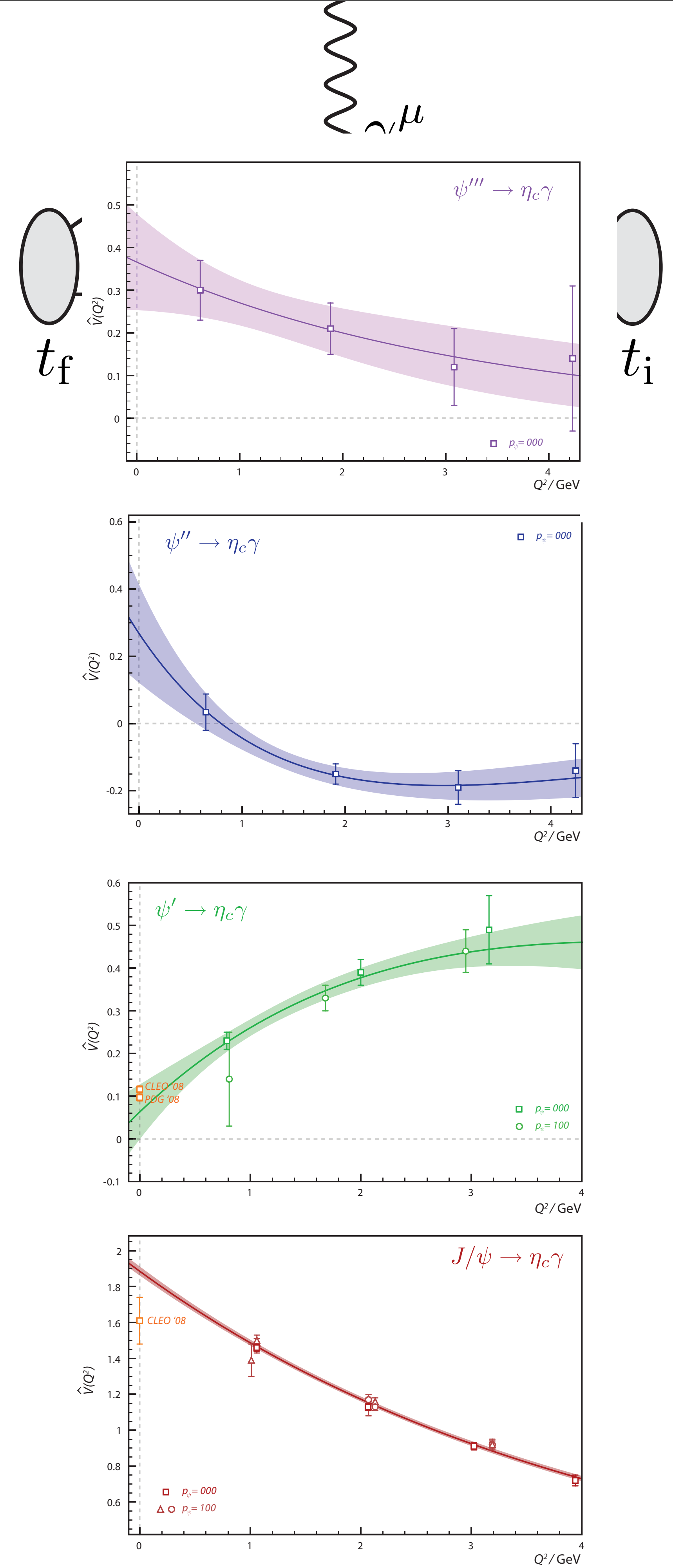

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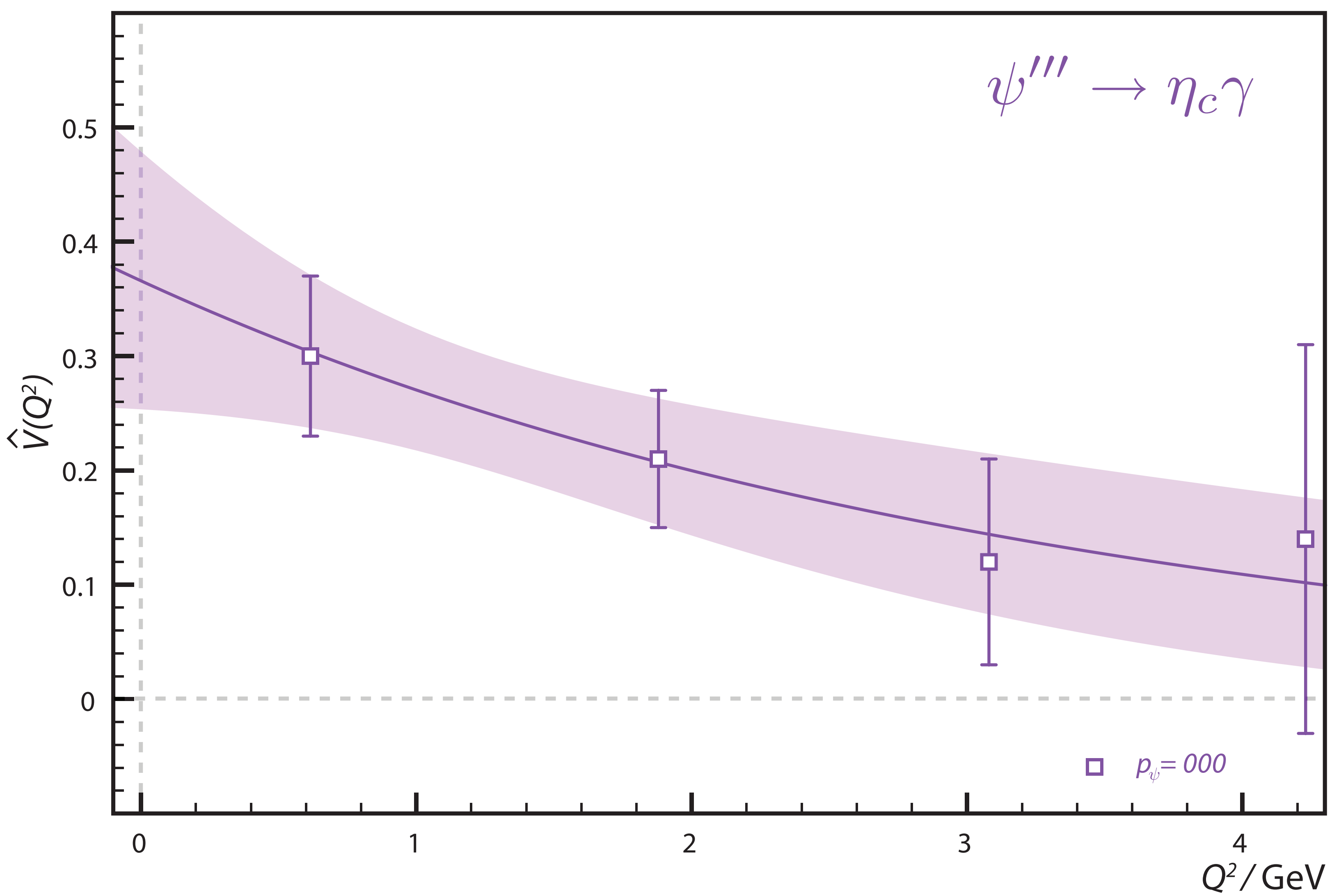
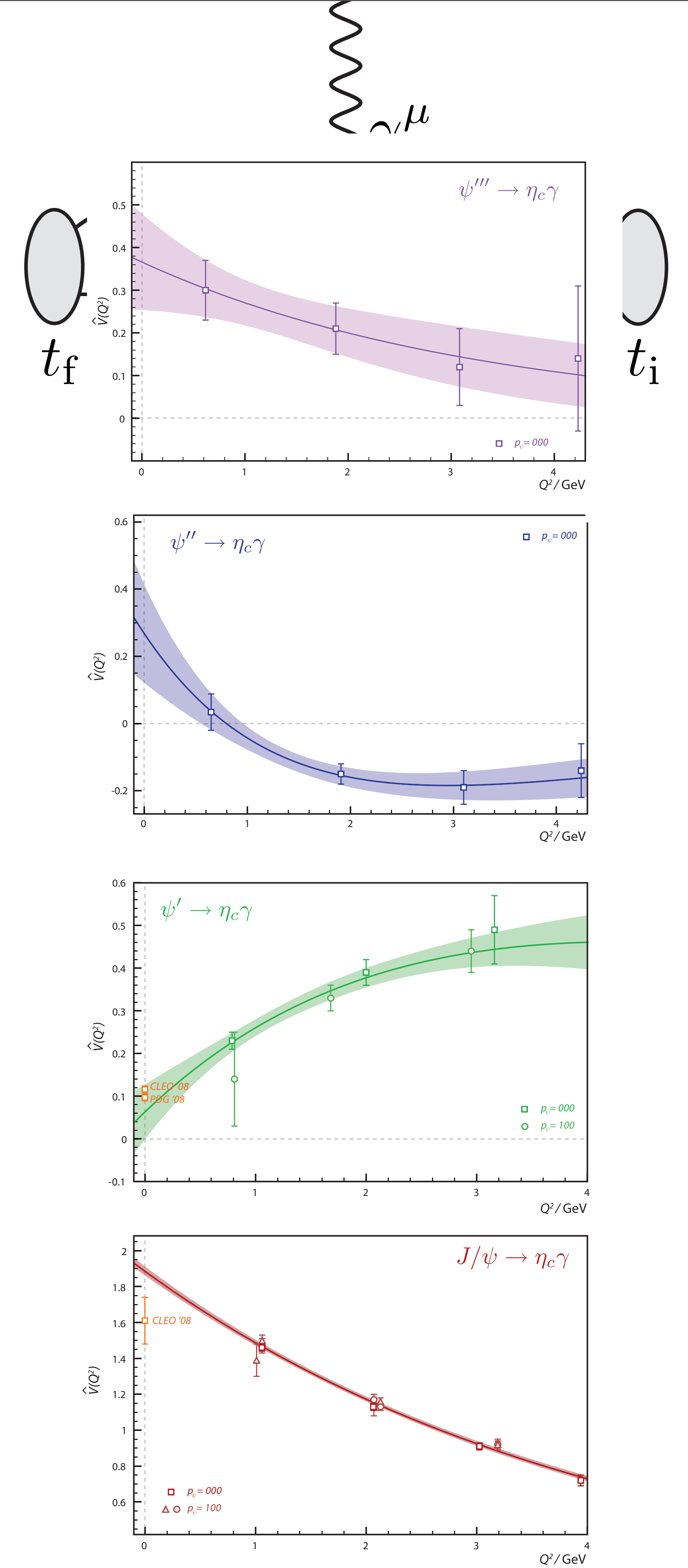
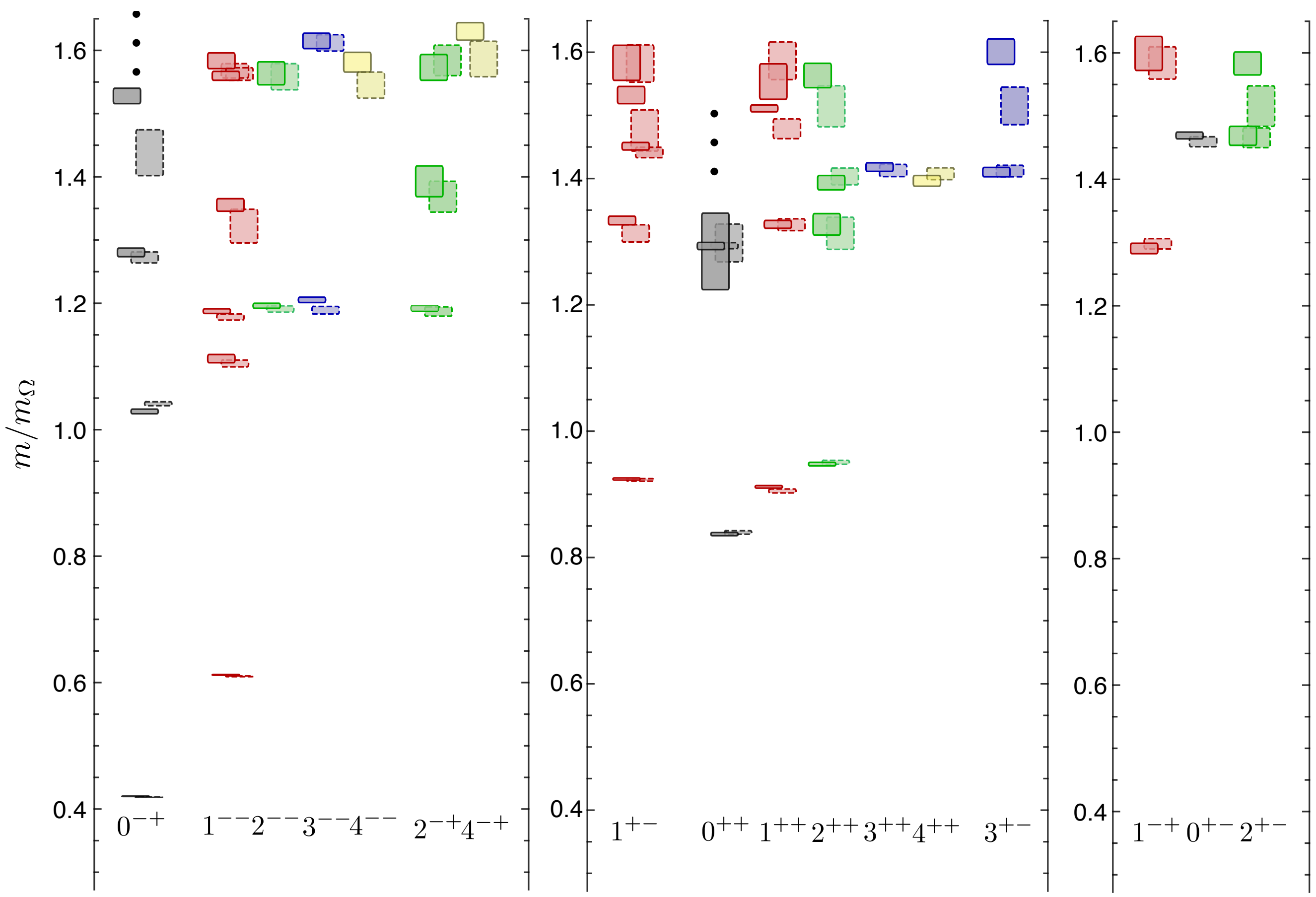




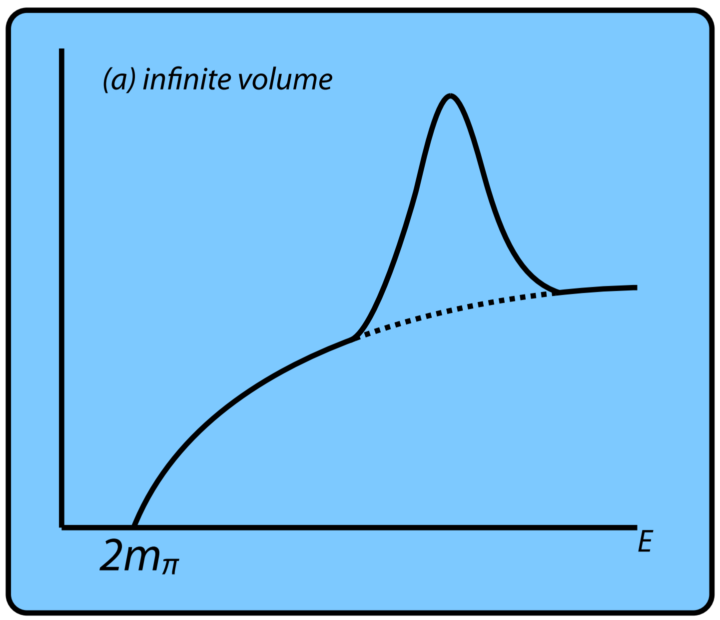
back to the spectrum

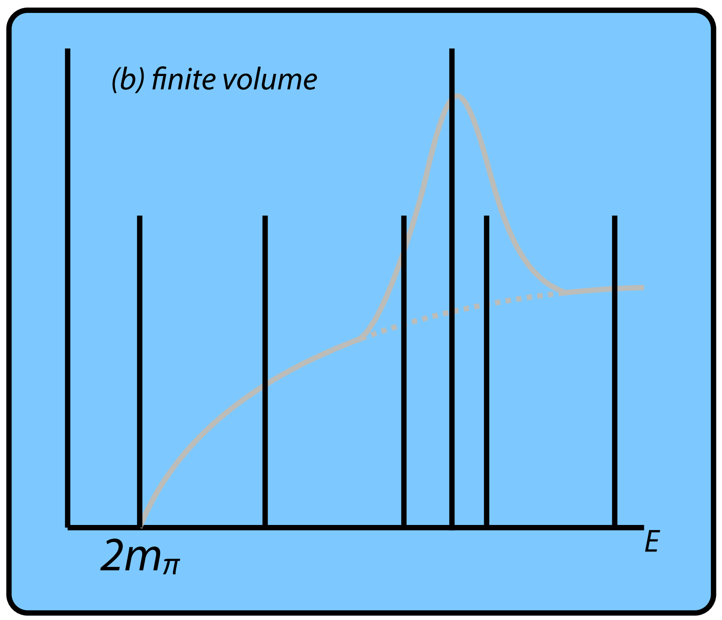


beautiful results - but we're not content

there are states missing that we know should be there

the 'continuum' of meson-meson scattering states !





e.g. a free particle $\varphi(x) = e^{ipx}$

periodic boundary conditions

$$\varphi(x+L) = \varphi(x)$$

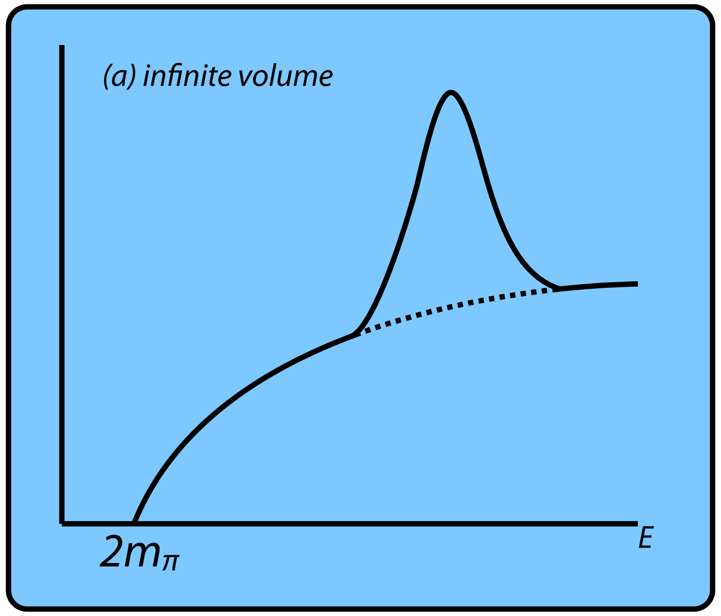
quantised momenta $p = n\frac{2\pi}{L}$

in ***infinite volume***, a continuous spectrum of ππ states

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$

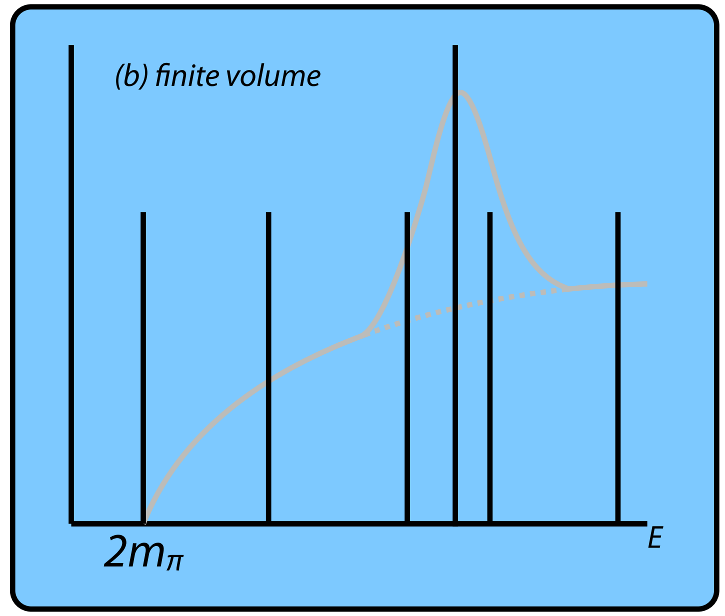
in ***finite volume***, a discrete spectrum of states

spectrum of finite volume field theory



in ***infinite volume***, a continuous spectrum of $\pi\pi$ states

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$



in ***finite volume***, a discrete spectrum of states

e.g. a free particle $\quad \varphi(x) = e^{ipx}$

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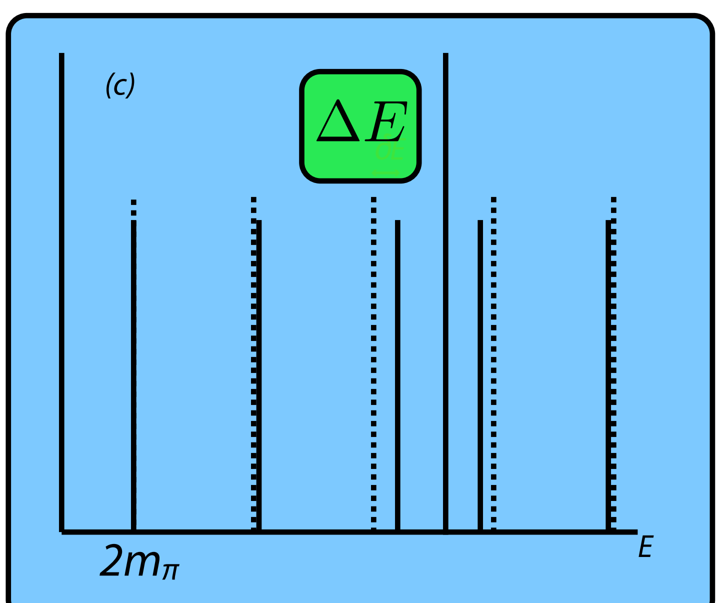


non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

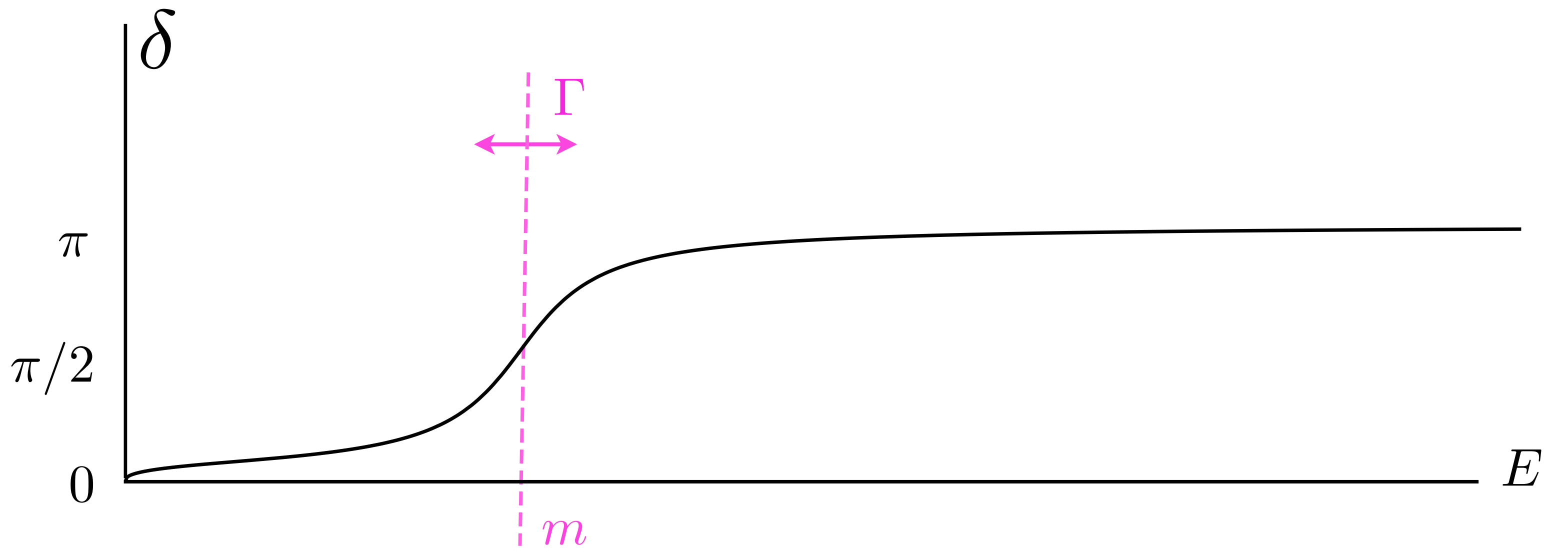
ΔE(L) ↔ δ(E) : Lüscher method

deviation from free energies depends upon the interaction and contains information about the scattering phase shift

reverse engineer

use known phase shift - anticipate spectrum

e.g. just a single elastic resonance



e.g.

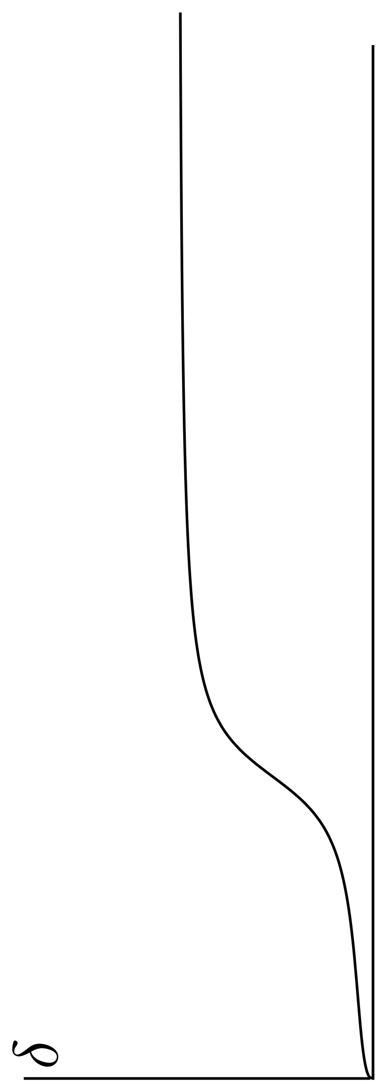
$$\pi\pi \longrightarrow \rho \longrightarrow \pi\pi$$

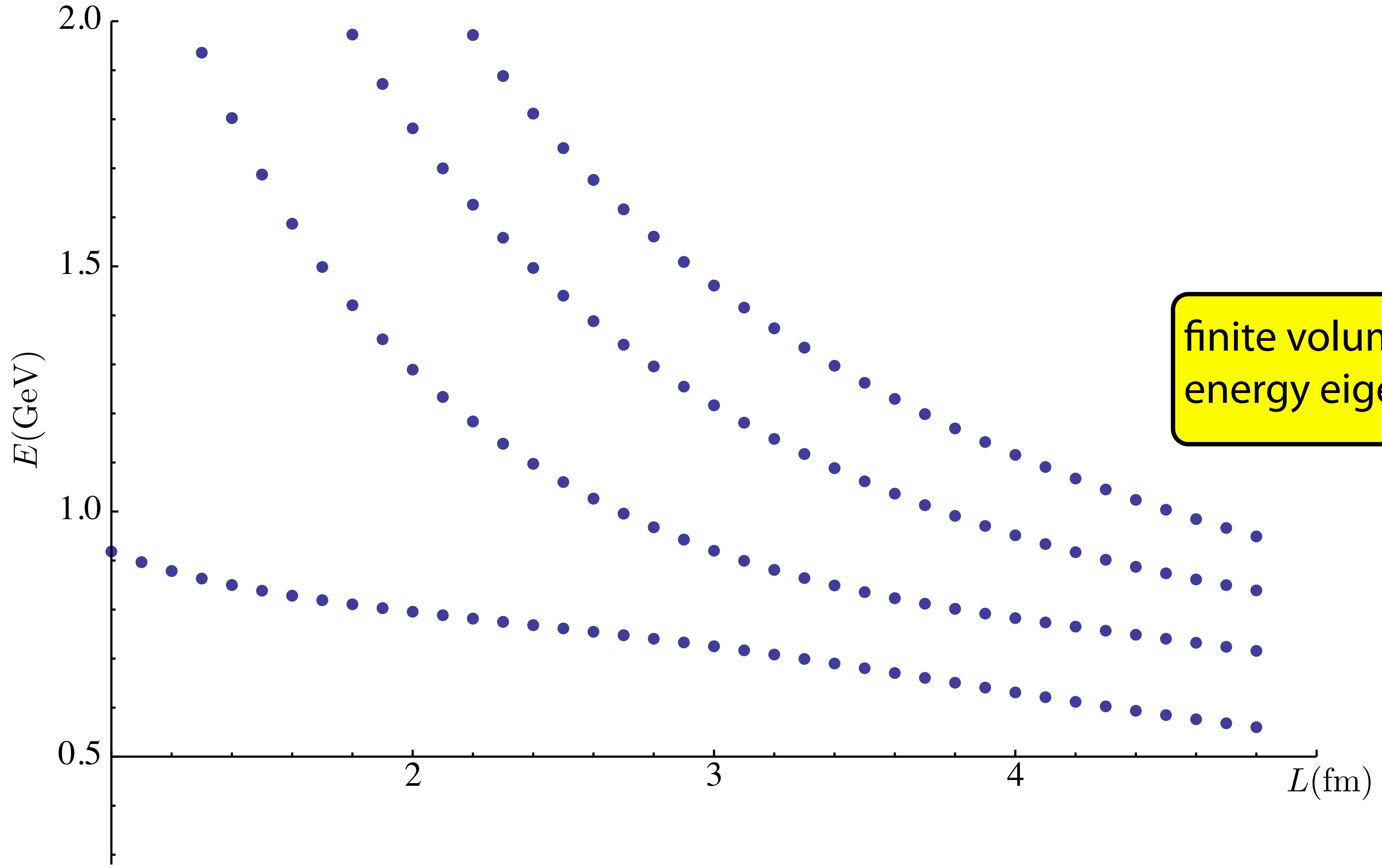
$$\pi N \rightarrow \Delta \rightarrow \pi N$$

Lüscher method
- essentially scattering in a periodic cubic box (length ***L***)
- finite volume energy levels ***E(δ,L)***

reverse engineer

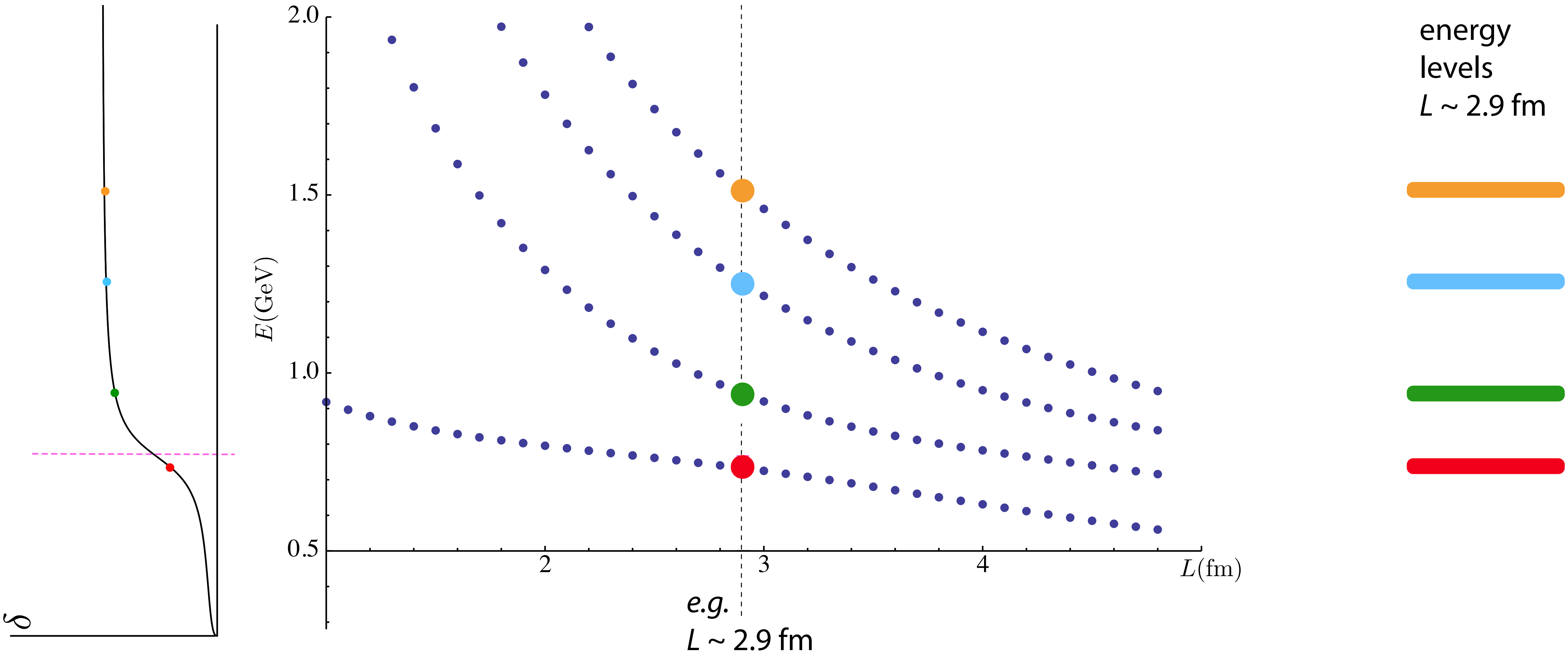
use known phase shift - anticipate spectrum





finite volume QCD energy eigenvalues

using the Lüscher method

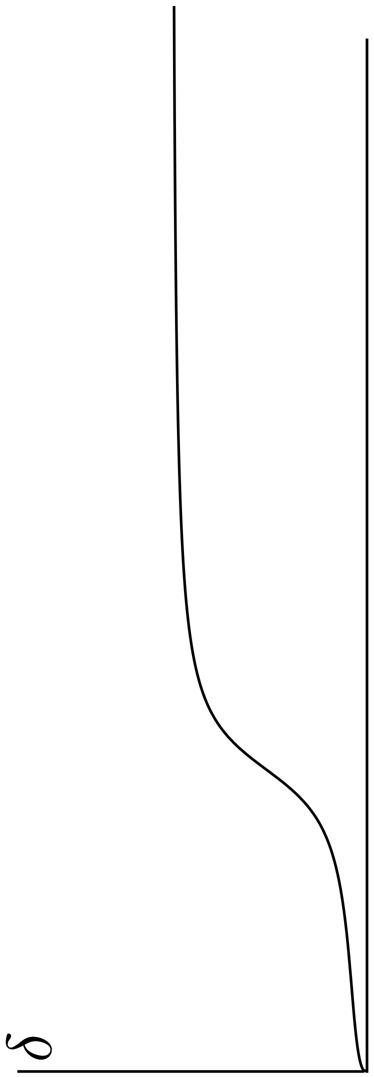


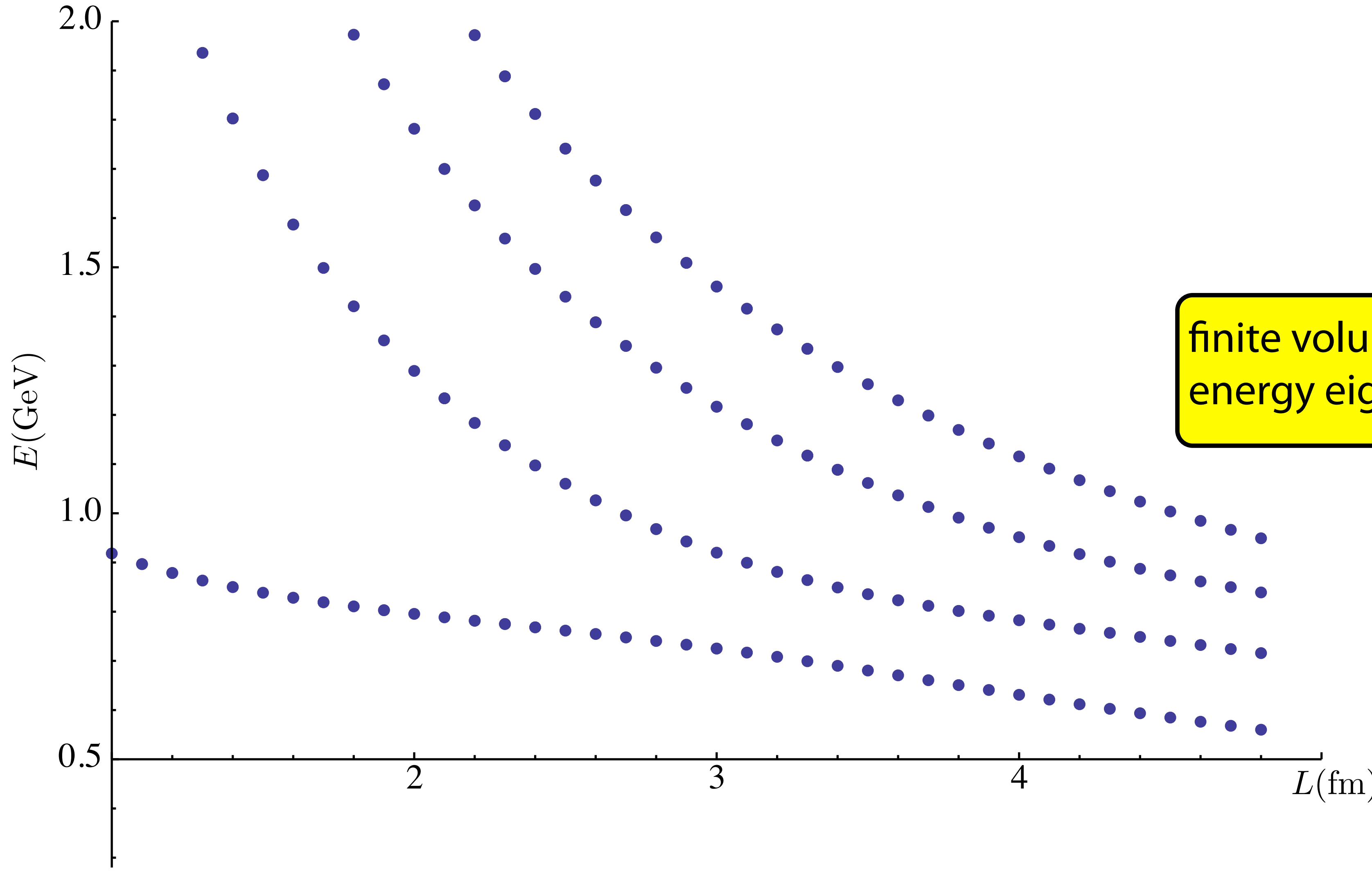
excited state spectrum at a single volume

discrete points on the phase shift curve

do more volumes, get more points

the interpretation

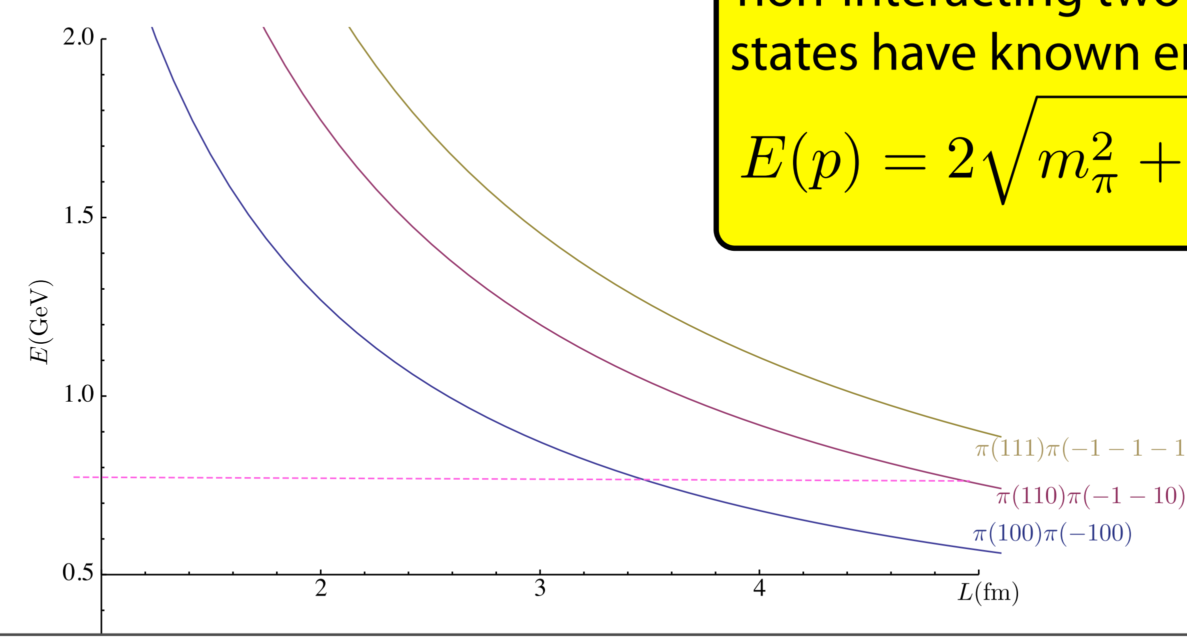




finite volume QCD energy eigenvalues

"non-interacting basis states"

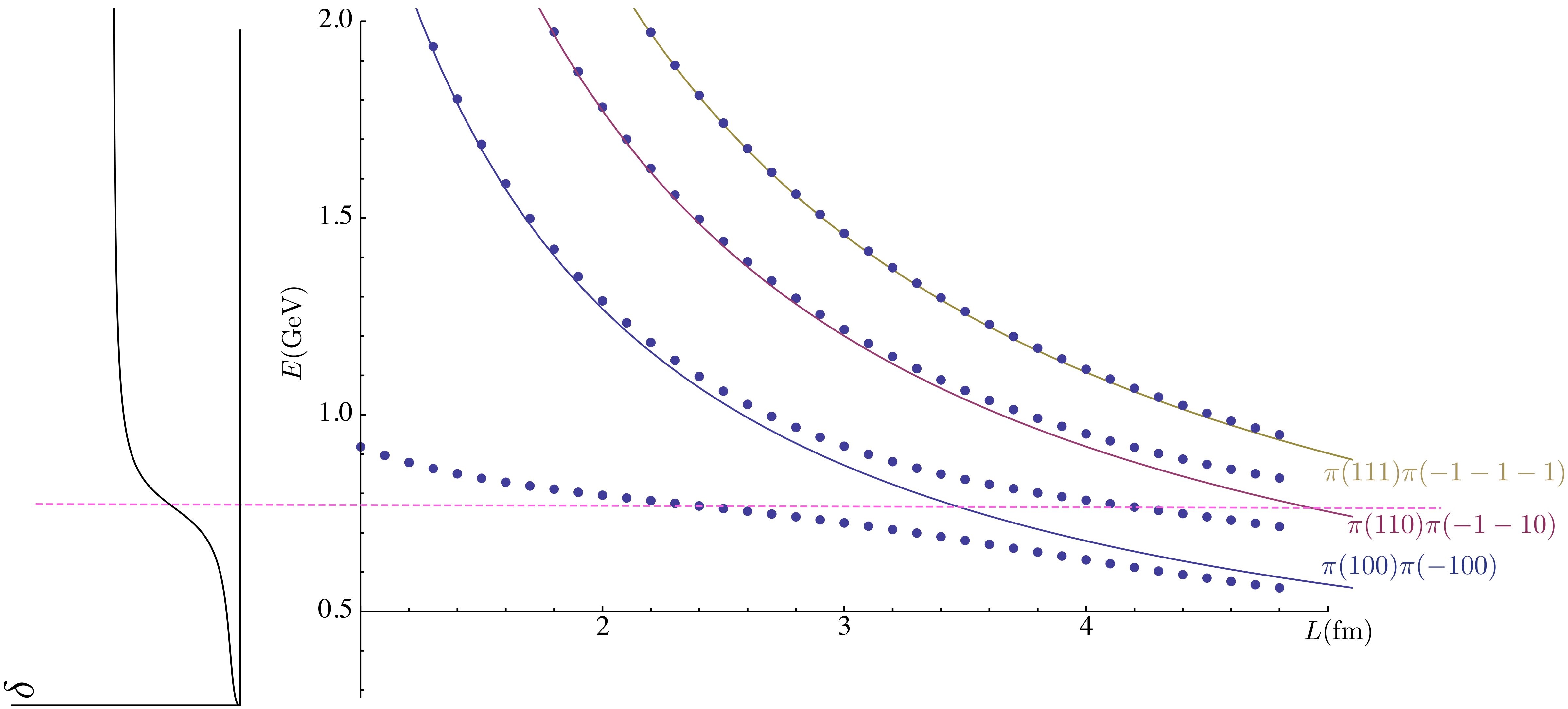
$$|q\bar{q}\rangle \qquad |\pi\pi_{100}\rangle \quad |\pi\pi_{110}\rangle \quad |\pi\pi_{111}\rangle$$



non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

the interpretation



"non-interacting basis states"

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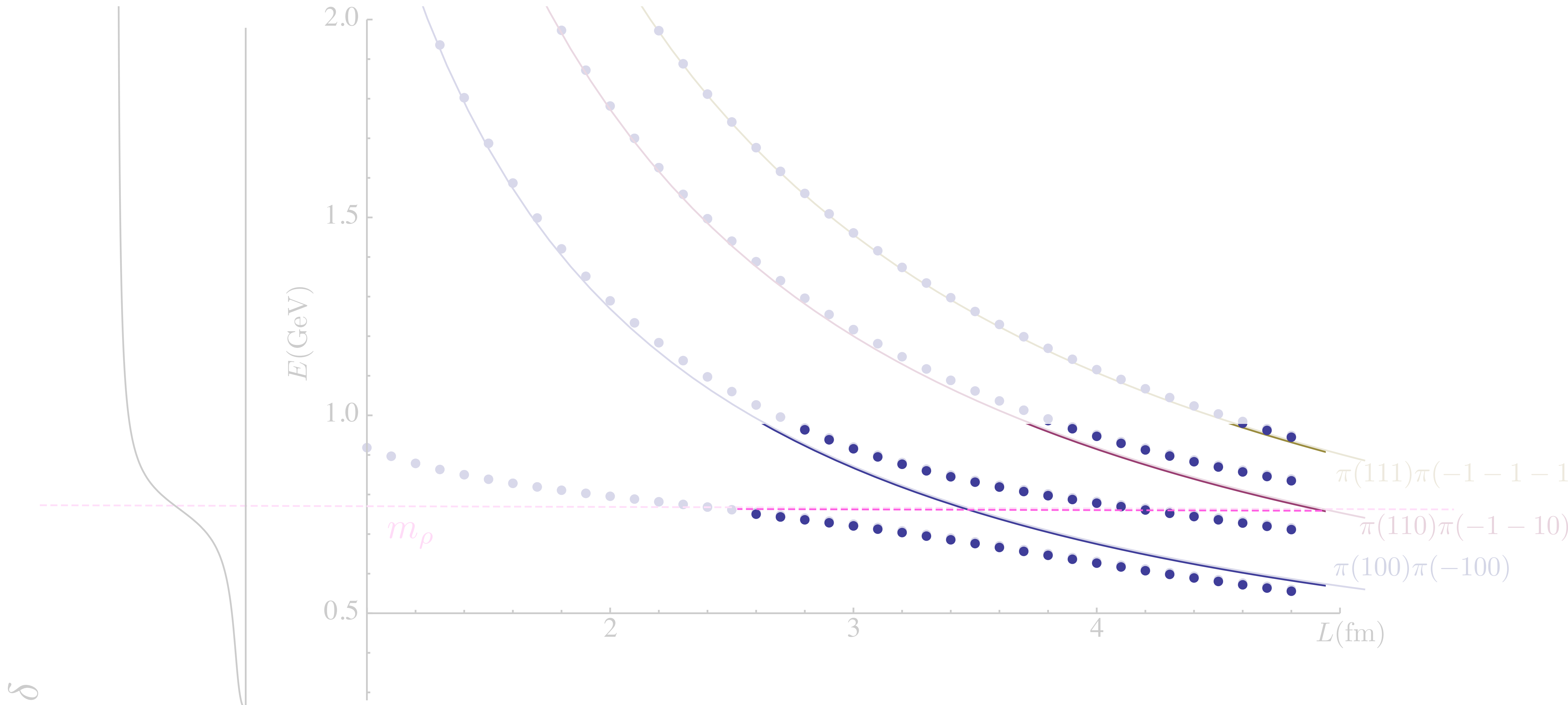
the interpretation

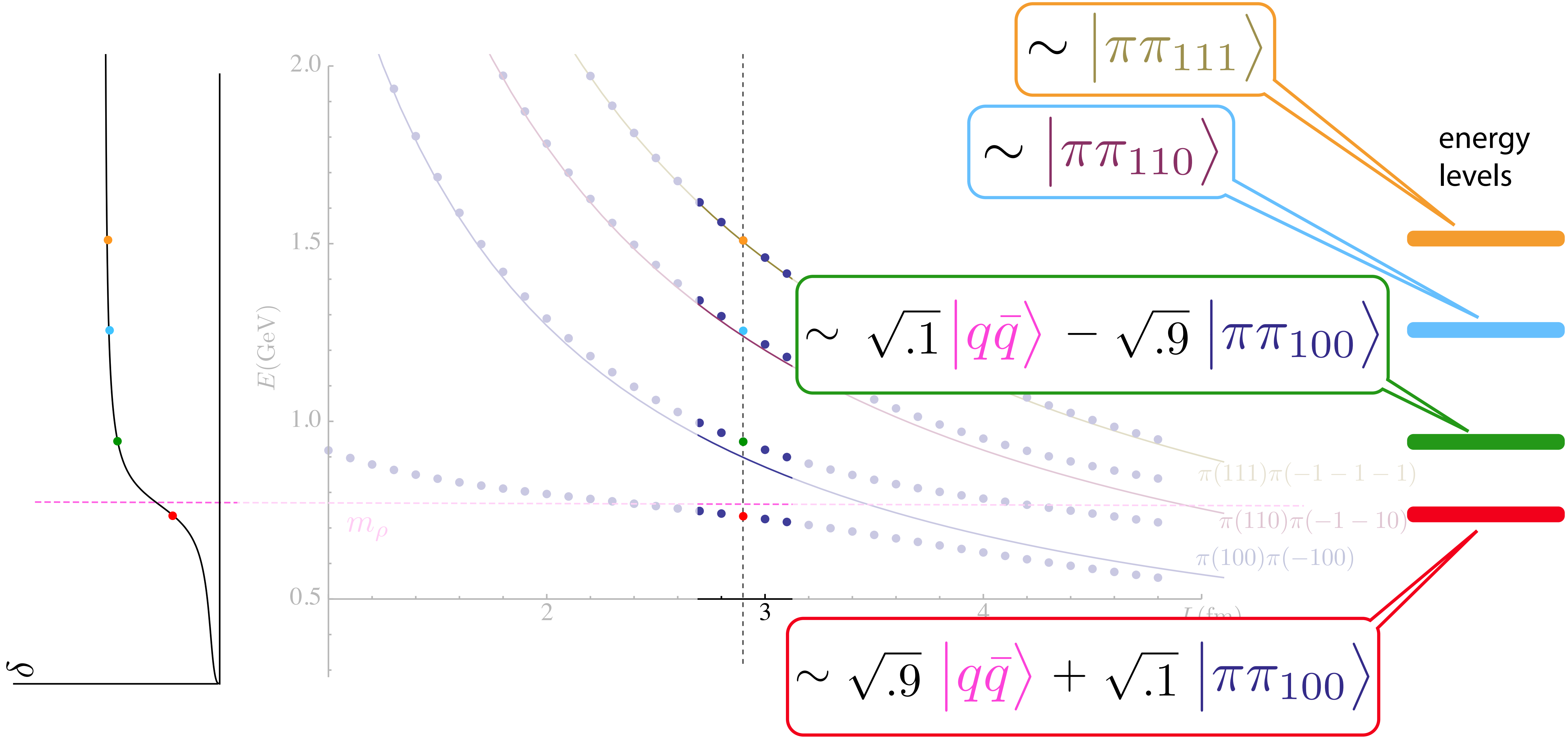


“non-interacting basis states”

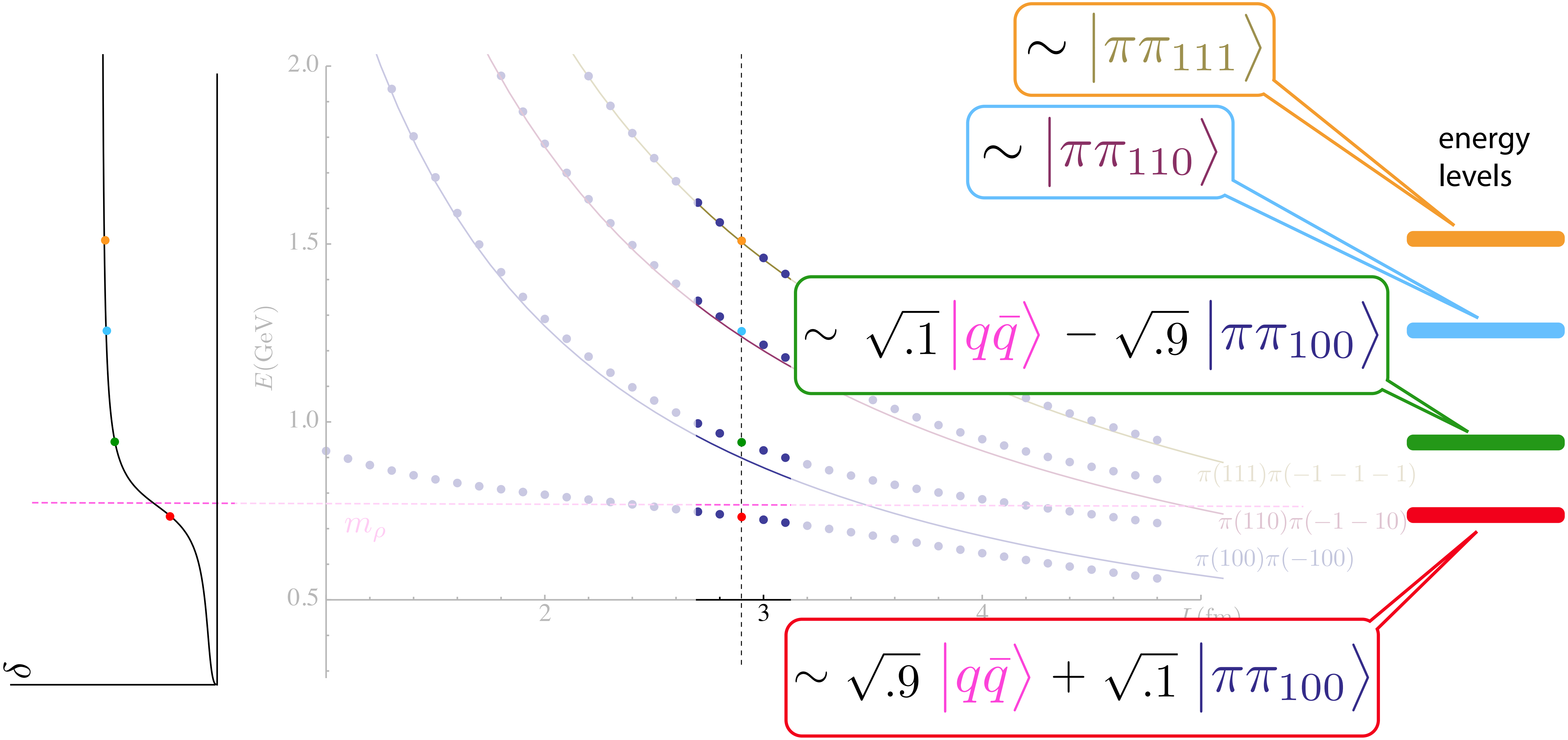
$$|q\bar{q}\rangle \qquad |\pi\pi_{100}\rangle \quad |\pi\pi_{110}\rangle \quad |\pi\pi_{111}\rangle$$

level repulsion - just like quantum mechanical pert. theory

the interpretation



the interpretation



... so just get on with it ...

there's a limitation in our current calculations

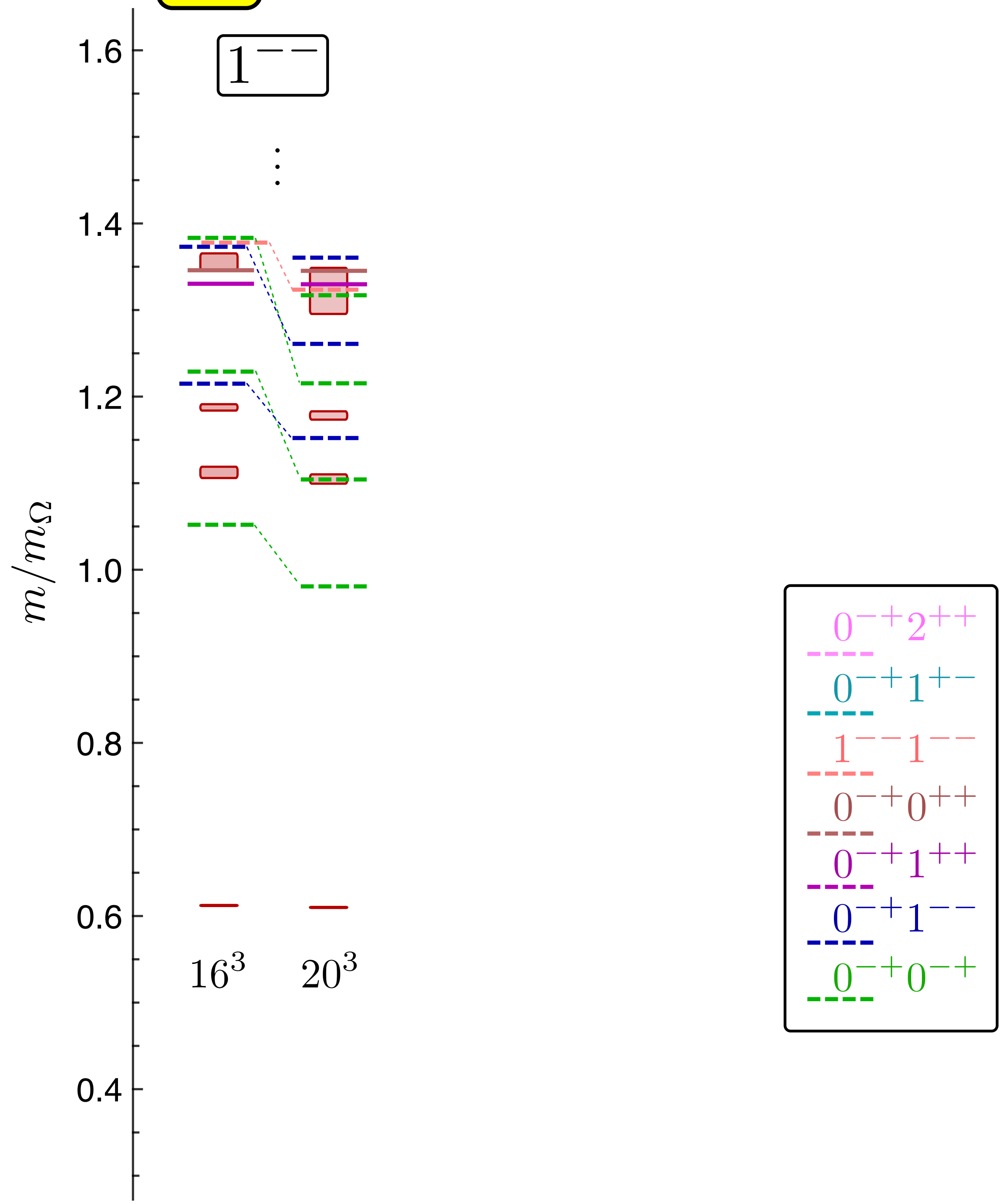
we don't see the meson-meson states in the spectrum

should we ? where will they be (roughly) ?

plot the *non-interacting* meson levels as a guide

$$\big|A(\vec{p})B(-\vec{p})\big\rangle \qquad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$$

e.g.



why aren't you seeing them ?

$$\langle 0 | \bar{\psi}_t \mathbf{\Gamma} \psi_t \; \bar{\psi}_0 \mathbf{\Gamma}' \psi_0 | 0 \rangle = \sum_n e^{-E_n t} \langle 0 | \bar{\psi}_0 \mathbf{\Gamma} \psi_0 | n \rangle \langle n | \bar{\psi}_0 \mathbf{\Gamma}' \psi_0 | 0 \rangle$$

all finite-volume QCD eigenstates ?

yes, but what if the operator can't make part of the eigenstate ?

$$\langle q\bar{q} \, | \bar{\psi}_0 \mathbf{\Gamma} \psi_0 | 0 \rangle \neq 0$$

$$\langle MM | \bar{\psi}_0 \mathbf{\Gamma} \psi_0 | 0 \rangle \sim \langle q\bar{q}q\bar{q} \, | \bar{\psi}_0 \mathbf{\Gamma} \psi_0 | 0 \rangle = 0$$

don't have orthogonal overlap onto multi-meson states

solution is to compute correlators including meson-meson like operators

i.e. $\bar{\psi}_0 \mathbf{\Gamma}_a \psi_0 \; \bar{\psi}_0 \mathbf{\Gamma}_b \psi_0$

$\pi\pi$ isospin=2

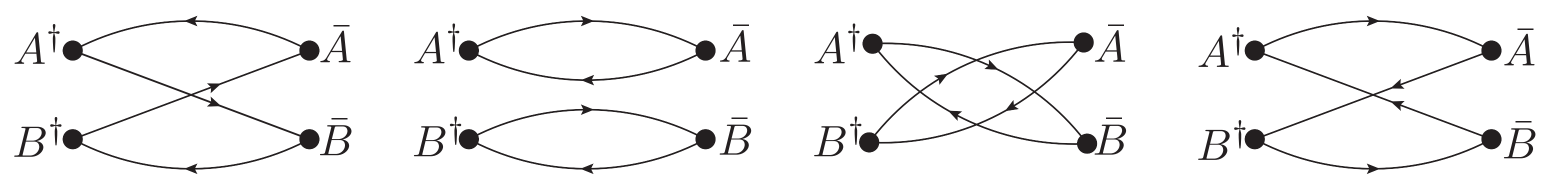
construct multi-meson operators, e.g.

$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}[\bar{\psi}\gamma_5\psi](\vec{x}) \cdot \sum_{\vec{y}} e^{-i(-\vec{p})\cdot\vec{y}}[\bar{\psi}\gamma_5\psi](\vec{y})$$

“looks” like a pair of pions with back-to-back momenta

basis of operators from different values of $|\vec{p}|$

few Wick contractions, but no annihilation



using the Lüscher method

excited state spectrum at a single volume

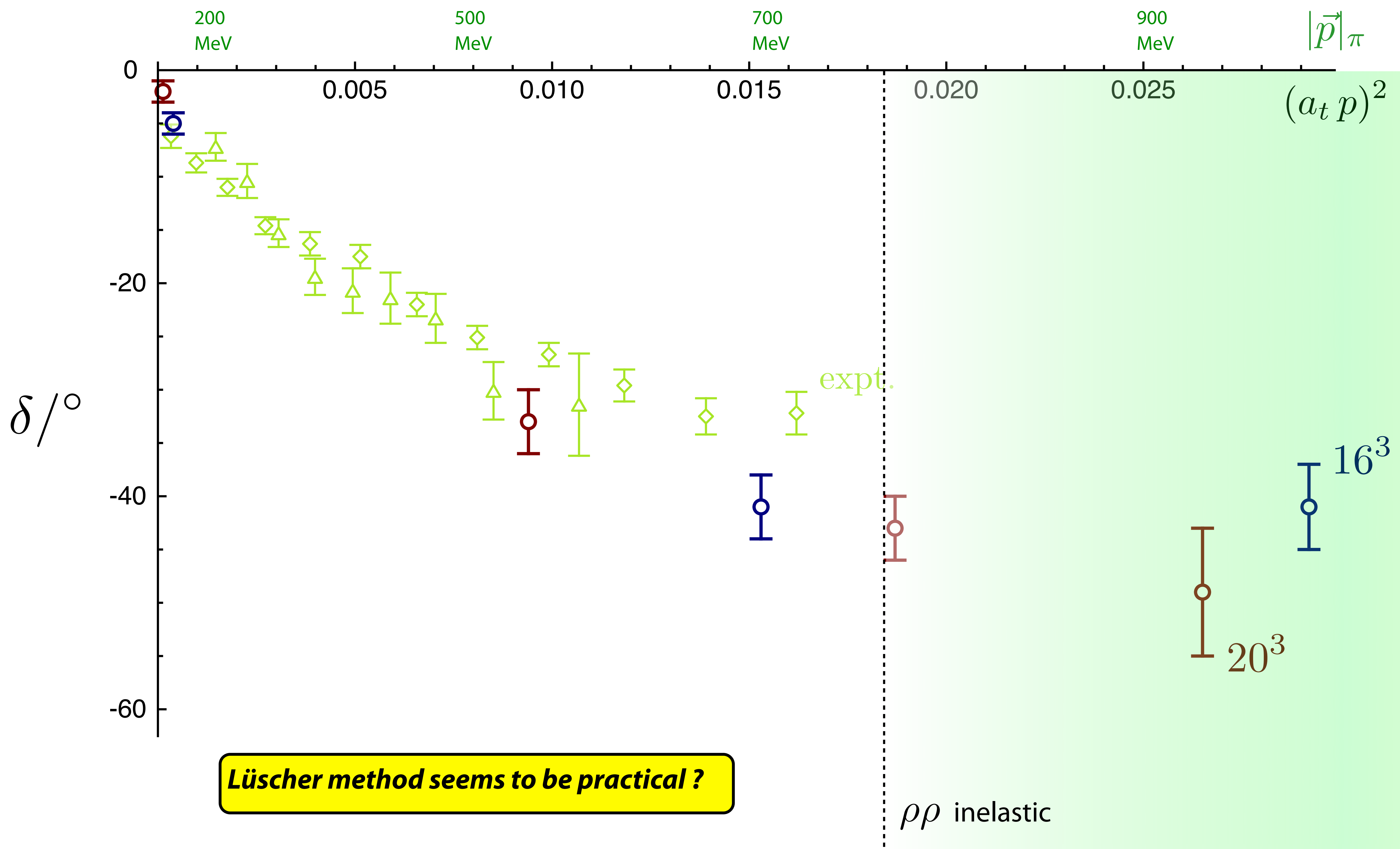
discrete points on the phase shift curve

do more volumes, get more points

computed ***16^3*** and ***20^3***

ππ isospin=2

$N_F = 2+1$ (u,d,s) $m_\pi \sim 400$ MeV



Lüscher method seems to be practical ?

summary

high performance computing necessary but not sufficient for progress

national supercomputing resources

USQCD managed resources

have a unique program of gauge field generation designed specifically for spectroscopy

have developed technology to extract multiple excited states

world leaders in this

need to do more to study resonances

include two-hadron operators

use Lüscher method

technology can be extended easily to study electromagnetic properties of hadrons

use the full set of non-perturbative information to ***"build a better model"***

variation with quark mass ?