

POLARIZED PDFs and HIGHER TWIST from NLO ANALYSIS of DIS and SIDIS: SOME CONTROVERSIAL ISSUES

Elliot Leader

Imperial College London

in collaboration with

A. V. Sidorov (Dubna) and D. B. Stamenov (Sofia)

CONTENTS

- 1) From experiment to $g_1(x, Q^2)$ in DIS—kinematics.
- 2) DIS theoretical expression for $g_1(x, Q^2)$.
- 3) Extension to SIDIS.
- 4) Data sample.
- 5) Results and comparison with DSSV.
- 6) Controversy about strange quark polarization
- 7) Controversy about Higher Twist.
- 8) Spin sum rule.

From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\Rightarrow}}{2 d\sigma_{unpold}} \quad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\uparrow} - d\sigma^{\rightarrow\downarrow}}{2 d\sigma_{unpold}}$$

$$(A_{\parallel}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$

From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\rightarrow}}{2 d\sigma_{unpold}} \quad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\uparrow} - d\sigma^{\rightarrow\downarrow}}{2 d\sigma_{unpold}}$$

$$(A_{\parallel}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$

If both A_{\parallel} and A_{\perp} measured: $\Rightarrow \frac{g_1}{F_1}$

From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\rightarrow}}{2 d\sigma_{unpold}} \quad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\uparrow} - d\sigma^{\rightarrow\downarrow}}{2 d\sigma_{unpold}}$$

$$(A_{\parallel}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$

If both A_{\parallel} and A_{\perp} measured: $\Rightarrow \frac{g_1}{F_1}$

If only A_{\parallel} measured:

$$\frac{A_{\parallel}}{D} = (1 + \gamma^2) \left[\frac{g_1}{F_1} \right] + (\eta - \gamma) A_2$$

$$\frac{A_{\parallel}}{D} \approx (1 + \gamma^2) \left[\frac{g_1}{F_1} \right] \quad \gamma^2 = \frac{4M^2 x^2}{Q^2}$$

NB γ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

NB γ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

It is ignored in the DSSV analysis

NB γ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

It is ignored in the DSSV analysis

Taking F_1 from experiment $\Rightarrow g_1(x, Q^2)_{exp}$

We utilize (in \overline{MS} scheme)

$$\begin{aligned} g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\ &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \end{aligned}$$

We utilize (in \overline{MS} scheme)

$$\begin{aligned}
 g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\
 &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2}
 \end{aligned}$$

$$\begin{aligned}
 g_1(x, Q^2)_{LT} &= \frac{1}{2} \sum_{flavors} e_q^2 \left\{ [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] \right. \\
 &\quad + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{ \Delta C_q(x/y) [\Delta q(y, Q^2) + \Delta \bar{q}(y, Q^2)] \\
 &\quad \left. + \Delta C_G(x/y) \Delta G(y, Q^2) \} \right\}
 \end{aligned}$$

We utilize (in \overline{MS} scheme)

$$\begin{aligned} g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\ &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \end{aligned}$$

$$\begin{aligned} g_1(x, Q^2)_{LT} &= \frac{1}{2} \sum_{flavors} e_q^2 \left\{ [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] \right. \\ &\quad + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{ \Delta C_q(x/y) [\Delta q(y, Q^2) + \Delta \bar{q}(y, Q^2)] \\ &\quad \left. + \Delta C_G(x/y) \Delta G(y, Q^2) \} \right\} \end{aligned}$$

Inclusive DIS determines ONLY the sum of quark and antiquark densities

Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE Q^2 and W^2 i.e.

$$1 \lesssim Q^2 \lesssim 4\text{GeV}^2 \quad 4 \lesssim W^2 \lesssim 10\text{GeV}^2$$

Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE Q^2 and W^2 i.e.

$$1 \lesssim Q^2 \lesssim 4\text{GeV}^2 \quad 4 \lesssim W^2 \lesssim 10\text{GeV}^2$$

We believe Higher Twist corrections are important.
 γ^2 term should not be neglected!

Extension to SIDIS

Aside from a kinematic factor, the SIDIS polarized cross-section, in NLO is

$$\begin{aligned}\Delta\sigma_p^h|_{NLO} &= \sum_i e_i^2 \Delta q_i \left[1 + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D_{q_i}^h \\ &+ \left(\sum_i e_i^2 \Delta q_i \right) \otimes \frac{\alpha_s}{2\pi} \Delta C_{qg} \otimes D_G^h \\ &+ \Delta G \otimes \frac{\alpha_s}{2\pi} \Delta C_{gq} \otimes \left(\sum_i e_i^2 D_{q_i}^h \right)\end{aligned}$$

This involves a double convolution and thus a double Mellin Transform.

The measured asymmetry is

$$A_{||}^h(x, Q^2)_{exp} = \frac{\Delta\sigma_p^h|_{exp}}{\sigma_p^h|_{exp}}$$

The measured asymmetry is

$$A_{||}^h(x, Q^2)_{exp} = \frac{\Delta\sigma_p^h|_{exp}}{\sigma_p^h|_{exp}}$$

TMC and HT corrections not known for SIDIS.....should be less important for kinematic range of present data. Thus use:

$$A_{||}^h(x, Q^2)_{exp} = \frac{\Delta\sigma_p^h|_{NLO}}{\sigma_p^h|_{exp}}$$

The measured asymmetry is

$$A_{||}^h(x, Q^2)_{exp} = \frac{\Delta\sigma_p^h|_{exp}}{\sigma_p^h|_{exp}}$$

TMC and HT corrections not known for SIDIS.....should be less important for kinematic range of present data. Thus use:

$$A_{||}^h(x, Q^2)_{exp} = \frac{\Delta\sigma_p^h|_{NLO}}{\sigma_p^h|_{exp}}$$

Use DSS Fragmentation Functions.....will use others as well

Note that DSS FFs are significantly different from others:

$D_g^{\pi^+} \gg$ Krezer (KRE) or Albino, Kniehl and Kramer (AKK) at large x .

$D_{s+\bar{s}}^{\pi^+} \gg$ AKK for $x \leq 0.7$

$D_{s+\bar{s}}^{K^+} \gg$ KRE, \ll AKK

$D_g^{K^+} \ll$ KRE and AKK

This needs study!

Parametrization

$$\Delta u + \Delta \bar{u} = A_U x^{\alpha_U} (1 - x)^{\beta_U} (1 + \epsilon_U \sqrt{x} + \gamma_U x)$$

$$\Delta \bar{u} = A_{\bar{u}} x^{\alpha_U} (1 - x)^{\beta} (1 + \gamma_{\bar{u}} x)$$

$$\Delta d + \Delta \bar{d} = A_D x^{\alpha_D} (1 - x)^{\beta_D} (1 + \gamma_D x)$$

$$\Delta \bar{d} = A_{\bar{d}} x^{\alpha_D} (1 - x)^{\beta}$$

$$\Delta s = \Delta \bar{s} = A_s x^{\alpha_s} (1 - x)^{\beta} (1 + \gamma_s x)$$

$$\Delta G = A_G x^{\alpha_G} (1 - x)^{\beta} (1 + \gamma_G x)$$

16 free parameters

The Data Sample

Inclusive DIS: 841 experimental points

Semi-inclusive DIS: 202 experimental points

The Data Sample

Inclusive DIS: 841 experimental points

Semi-inclusive DIS: 202 experimental points

Compared to DSSV, we use new COMPASS data on inclusive A_1 (proton) and on semi-inclusive deuterium asymmetries for π^\pm and K^\pm .

The Data Sample

Inclusive DIS: 841 experimental points

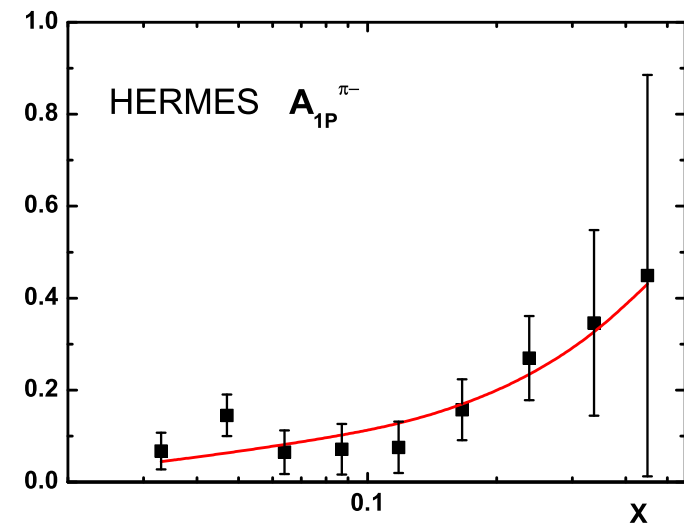
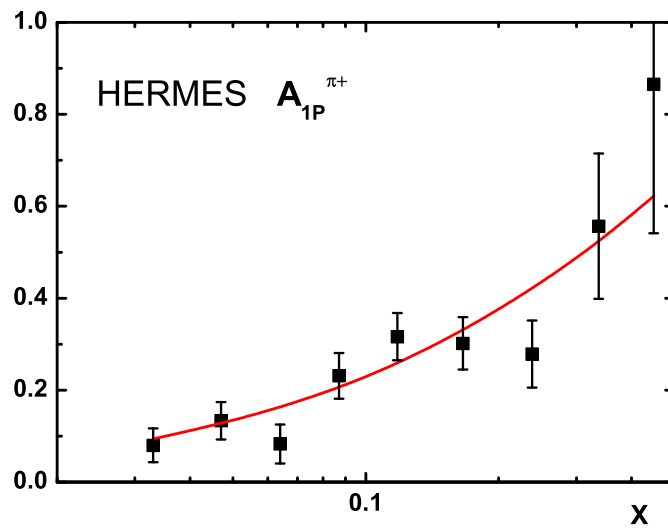
Semi-inclusive DIS: 202 experimental points

Compared to DSSV, we use new COMPASS data on inclusive A_1 (proton) and on semi-inclusive asymmetries on deuterium for π^\pm and K^\pm .

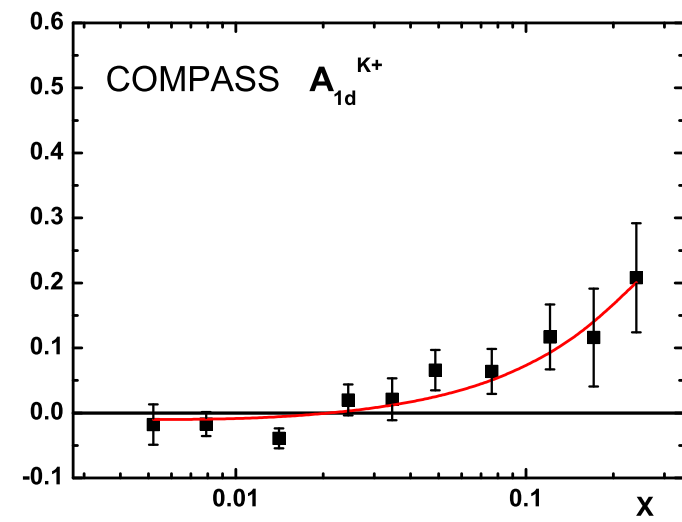
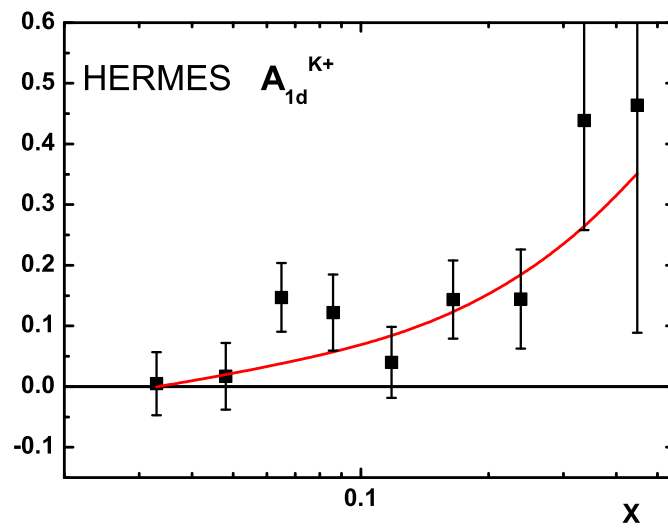
$$\text{DIS: } \chi^2_{NExpP} = 0.85 \quad \text{SIDIS: } \chi^2_{NExpP} = 0.90$$

$$\text{Overall } \chi^2_{DOF} = 0.88$$

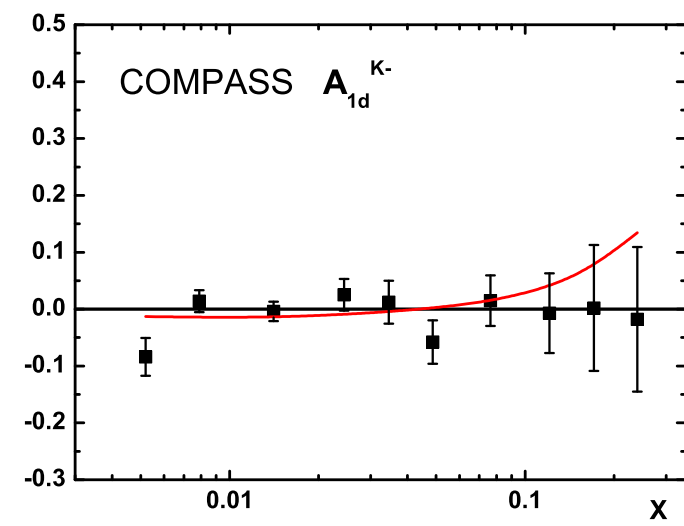
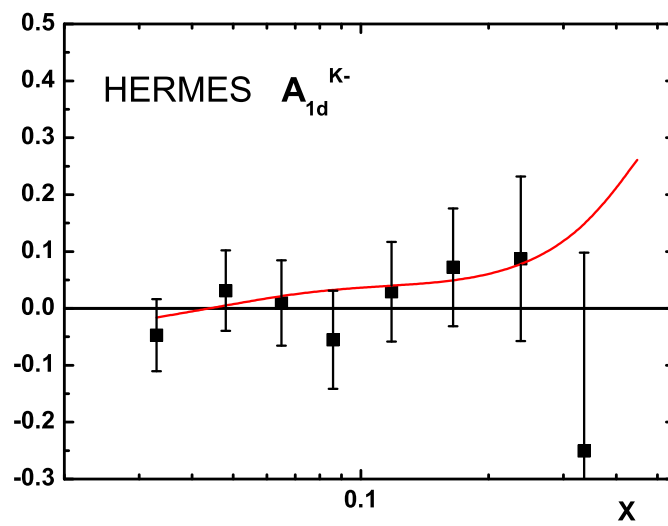
Fits to SIDIS data



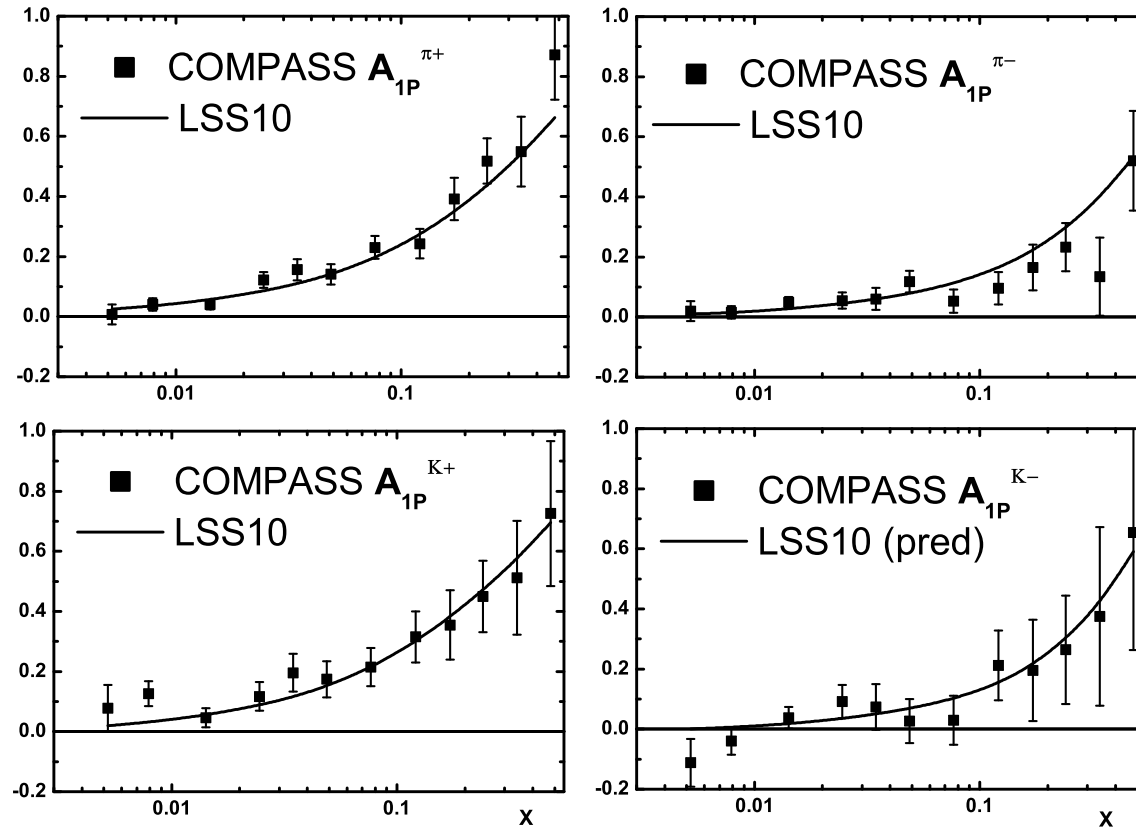
Fits to SIDIS data



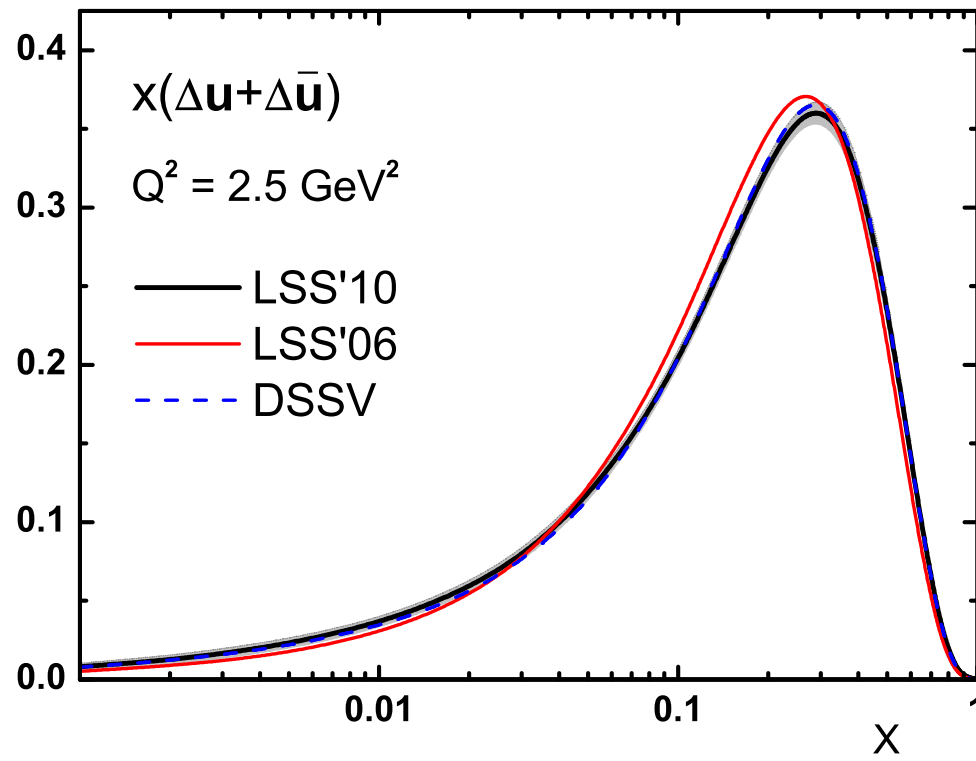
Fits to SIDIS data



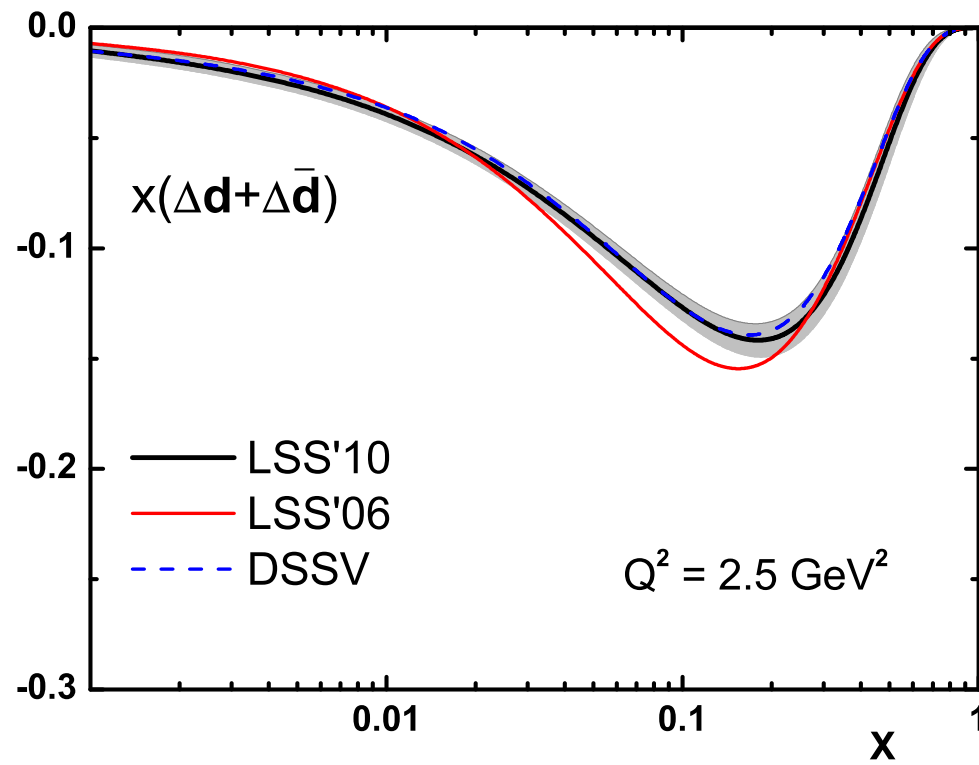
Predictions for COMPASS proton SIDIS data



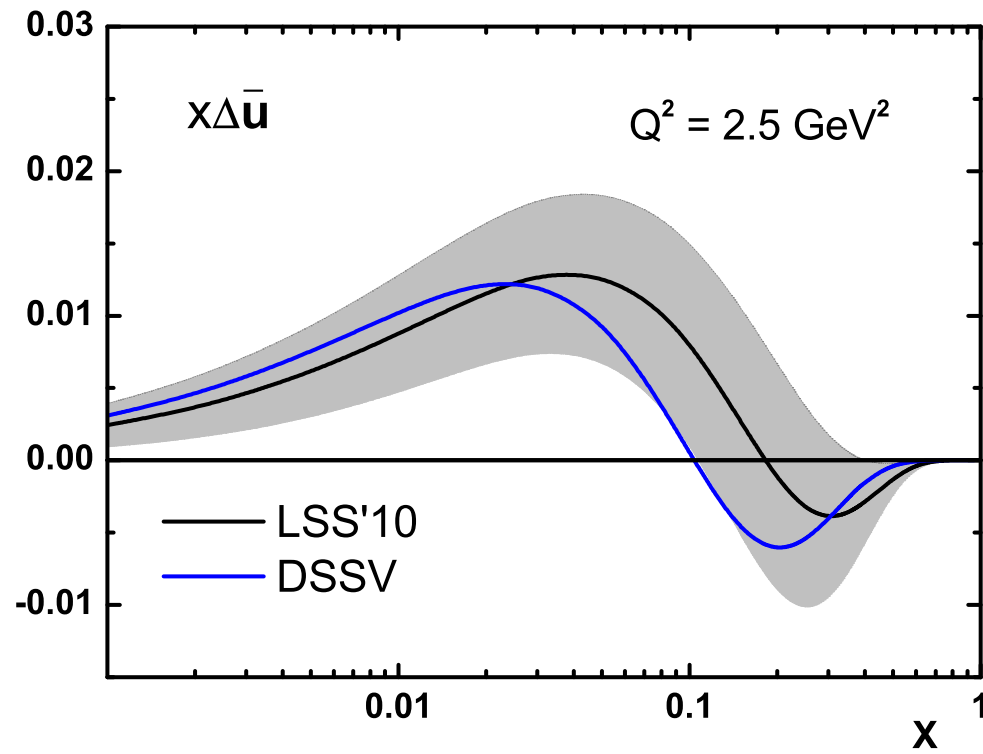
Results and comparison with DSSV: $\Delta u + \Delta \bar{u}$



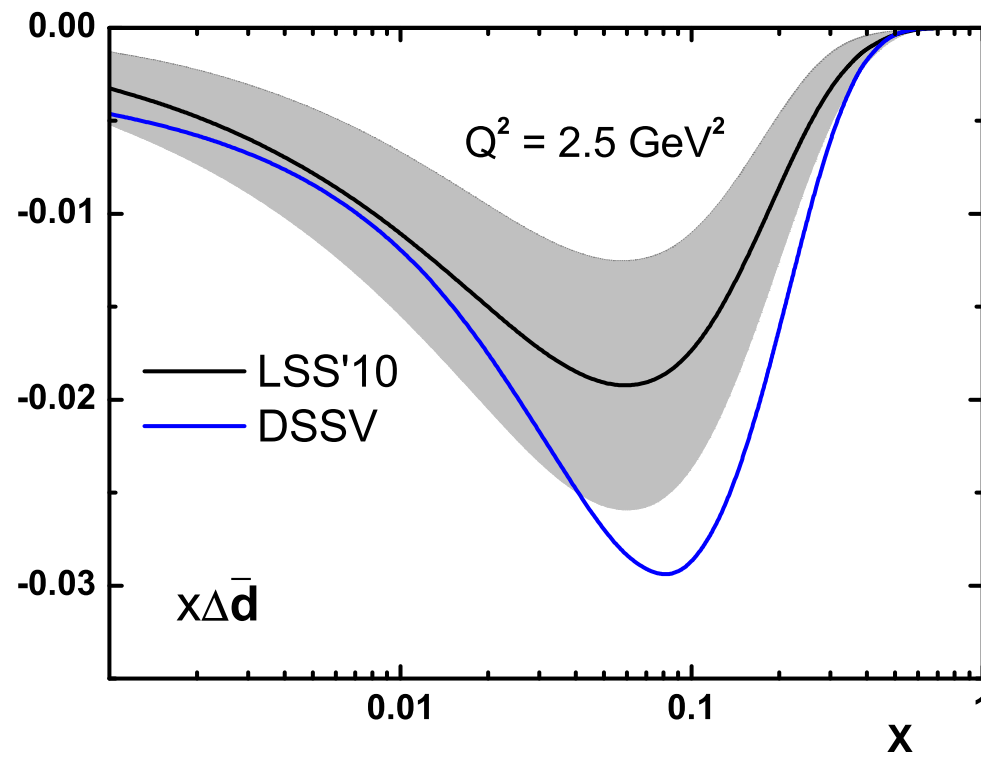
Results and comparison with DSSV: $\Delta d + \Delta \bar{d}$



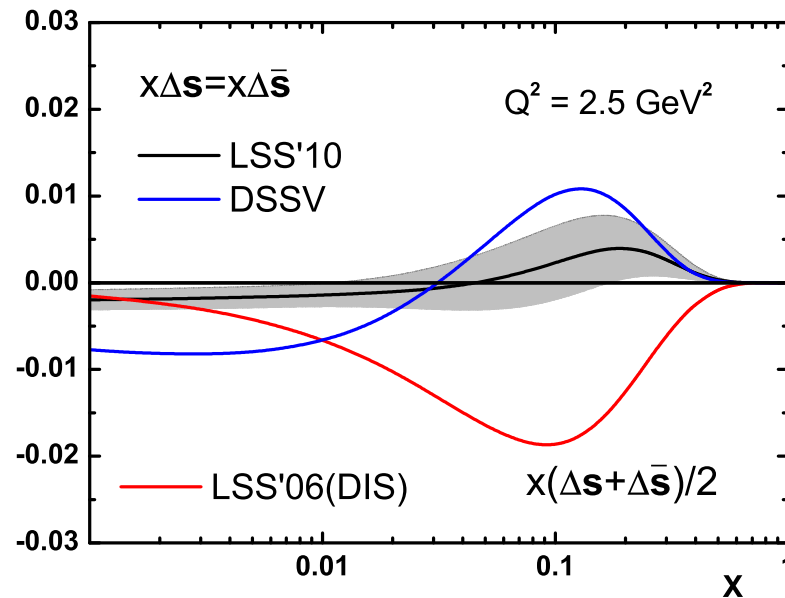
Results and comparison with DSSV: $\Delta\bar{u}$



Results and comparison with DSSV: $\Delta\bar{d}$



Results and comparison with DSSV: Strange quark

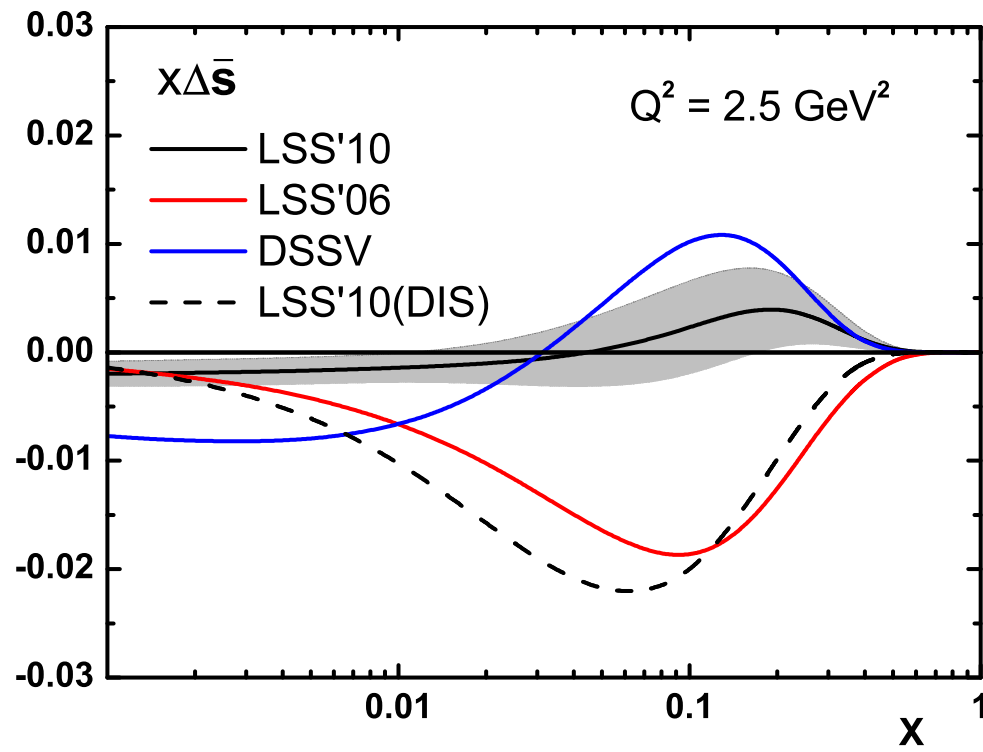


Note: DSSV use $\alpha_{\bar{s}} = \alpha_{\bar{d}}$ and find $= 0.16$

LSS find: $\alpha_{\bar{s}} = 0.05 \pm 0.02$ $\alpha_{\bar{d}} = 0.55 \pm 0.12$

Results and comparison with DSSV: Strange quark

Redo DIS including term $(1 + \gamma x)$ to permit sign change.



Δs is controversial

A red herring in papers on Polarized DIS

.....Inclusive polarized DIS gives **no** information about the *separate* sea quark densities.....

A red herring in papers on Polarized DIS

.....Inclusive polarized DIS gives **no** information about the *separate* sea quark densities.....

True!

But it does determine unambiguously $\Delta s(x) + \Delta \bar{s}(x)$

The controversy

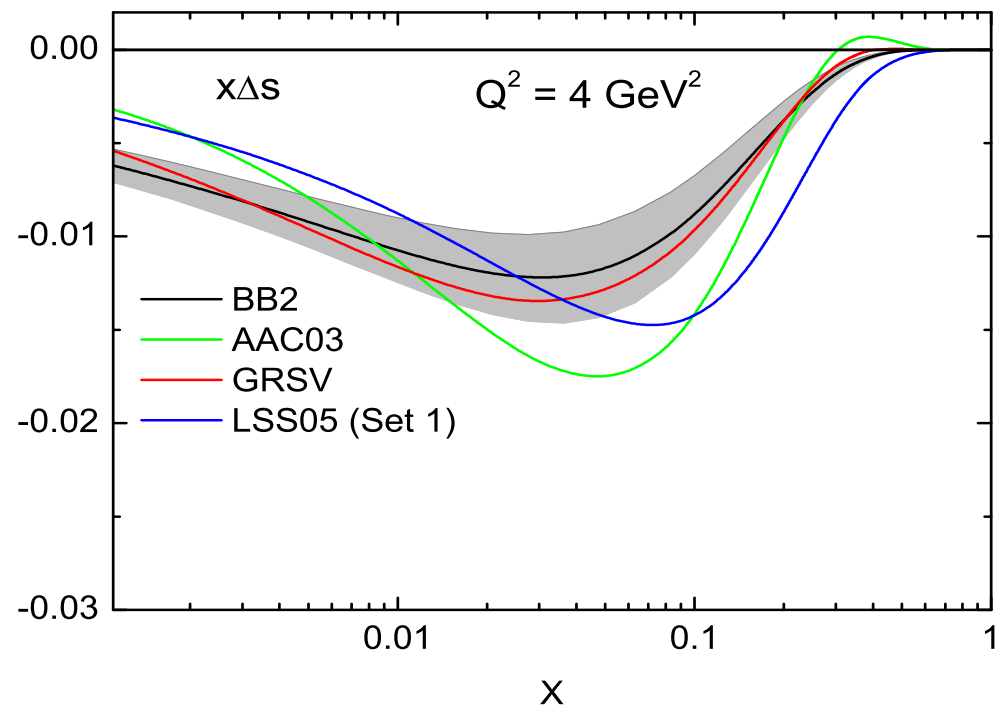
ALL inclusive DIS analyses give negative values for $\Delta s(x) + \Delta \bar{s}(x)$

The controversy

ALL inclusive DIS analyses give negative values for $\Delta s(x) + \Delta \bar{s}(x)$

ALL SIDIS, or combined DIS and SIDIS analyses, in LO and in NLO , give either positive or sign-changing results for $\Delta s(x) + \Delta \bar{s}(x)$.

The DIS situation



Constraint on positive values from SU(3) flavour

EL and D.B. Stamenov: PR D67 (2003) 037503

Define

$$\delta s(Q^2) \equiv \int_0^1 dx [\Delta s(x, Q^2) + \Delta \bar{s}(x, Q^2)]$$

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) = \frac{1}{6} \left[\frac{1}{2} a_3 + \frac{5}{6} a_8 + 2\delta s(Q^2) \right]$$

Constraint on positive values from SU(3) flavour

EL and D.B. Stamenov: PR D67 (2003) 037503

Define

$$\delta s(Q^2) \equiv \int_0^1 dx [\Delta s(x, Q^2) + \Delta \bar{s}(x, Q^2)]$$

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) = \frac{1}{6} \left[\frac{1}{2} a_3 + \frac{5}{6} a_8 + 2\delta s(Q^2) \right]$$

Rewrite as

$$a_8 = \frac{6}{5} \left[6\Gamma_1^p(Q^2) - \frac{1}{2} a_3 - 2\delta s(Q^2) \right]$$

Feed in

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004(stat) \pm 0.007(syst)$$

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003(stat) \pm 0.009(syst)$$

Feed in

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004(stat) \pm 0.007(syst)$$

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003(stat) \pm 0.009(syst)$$

Then, if $\delta_s \geq 0$ find

$$a_8 \leq 0.089 \pm 0.058 \quad a_8 \leq 0.197 \pm 0.068$$

Feed in

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004(stat) \pm 0.007(syst)$$

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003(stat) \pm 0.009(syst)$$

Then, if $\delta_s \geq 0$ find

$$a_8 \leq 0.089 \pm 0.058 \quad a_8 \leq 0.197 \pm 0.068$$

But $SU(3)_F$ seems good for hyperon decays viz. Fermilab KTeV $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$. Expect

$$a_8 = 0.585 \pm 0.025 \quad \text{i.e.} \quad 0.47 \leq a_8 \leq 0.70$$

Thus $\delta_s \geq 0$ implies huge breaking of $SU(3)_F$!

What's wrong?

1) Maybe we don't understand connection between DIS and SIDIS... a horrible thought which I will ignore.

What's wrong?

- 1) Maybe we don't understand connection between DIS and SIDIS... a horrible thought which I will ignore.
- 2) SIDIS involves fragmentation functions..... they are certainly poorly known....LSS will study effect of using other FFs.

What's wrong?

- 1) Maybe we don't understand connection between DIS and SIDIS... a horrible thought which I will ignore.
- 2) SIDIS involves fragmentation functions..... they are certainly poorly known....LSS will study effect of using other FFs.
- 3) Maybe $\Delta_s \neq \Delta_{\bar{s}}$

A little exercise taking $\Delta_s \neq \Delta_{\bar{s}}$

Define, at $x = 0.1$ and $Q^2 = 2.5$,

$$\begin{aligned}\Delta_{exact}^h &\equiv x\Delta_s D_s^h + x\Delta_{\bar{s}} D_{\bar{s}}^h \\ &= \frac{x}{2}[\Delta_s + \Delta_{\bar{s}}][D_s^h + D_{\bar{s}}^h] + \frac{x}{2}[\Delta_s - \Delta_{\bar{s}}][D_s^h - D_{\bar{s}}^h]\end{aligned}$$

where

$$D_q^h = \int_{0.2}^{0.85} dz D_q^h(z)$$

DSS: Pions

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^-}$$

$$\Delta_{exact}^{\pi} = x[\Delta_s + \Delta_{\bar{s}}][D_s^{\pi}] = 0.0008 \pm 0.0017 (\text{COMPASS})$$

Is this compatible with

$$\Delta_{exact}^{\pi} = x[\Delta_s + \Delta_{\bar{s}}]_{DIS}[D_s^{\pi}] = -0.0072 ?$$

Marginally

DSS: Kaons

$$D_s^{K^+} = D_{\bar{s}}^{K^-} \quad D_s^{K^-} = D_{\bar{s}}^{K^+}$$

$$\Delta_{COMPASS}^{K^+} = \Delta_{COMPASS}^{K^-} = 0.0013 \pm 0.0026$$

Is this compatible with

$$\Delta_{exact}^K = \frac{x}{2}[\Delta_s + \Delta_{\bar{s}}]_{DIS}[D_s^K + D_{\bar{s}}^K] + \frac{x}{2}[\Delta_s - \Delta_{\bar{s}}][D_s^K - D_{\bar{s}}^K] ?$$

DSS: Kaons

$$D_s^{K^+} = D_{\bar{s}}^{K^-} \quad D_s^{K^-} = D_{\bar{s}}^{K^+}$$

$$\Delta_{COMPASS}^{K^+} = \Delta_{COMPASS}^{K^-} = 0.0013 \pm 0.0026$$

Is this compatible with

$$\Delta_{exact}^K = \frac{x}{2}[\Delta_s + \Delta_{\bar{s}}]_{DIS}[D_s^K + D_{\bar{s}}^K] + \frac{x}{2}[\Delta_s - \Delta_{\bar{s}}][D_s^K - D_{\bar{s}}^K] ?$$

NO! Find you need

$$\begin{aligned} \frac{x}{2}[\Delta_s - \Delta_{\bar{s}}] &= -0.210 \pm 0.005 \text{ for } K^+ \\ &= 0.210 \pm 0.005 \text{ for } K^- \end{aligned}$$

Strange quark summary

There is a serious contradiction.

I guess it is caused by bad fragmentation functions

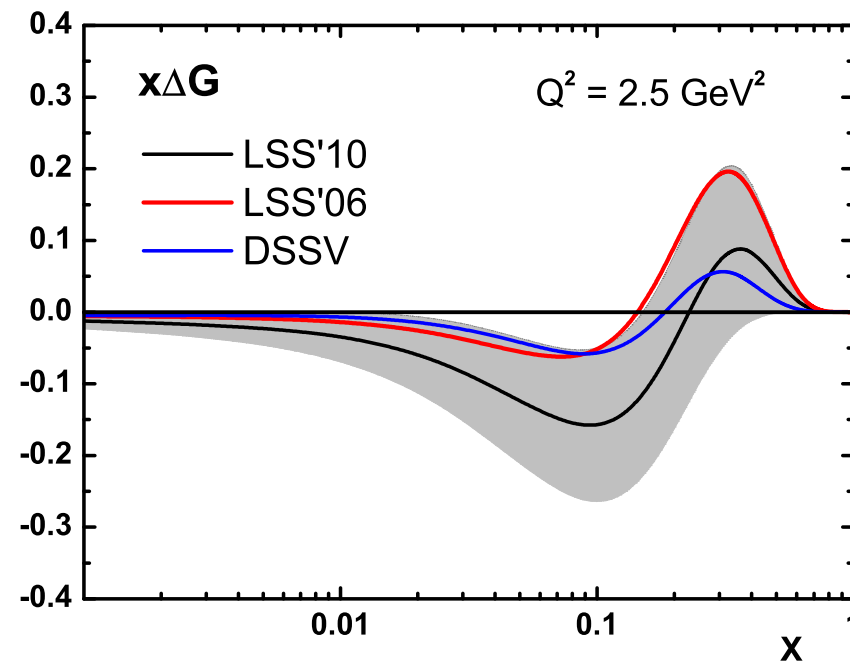
Strange quark summary

There is a serious contradiction.

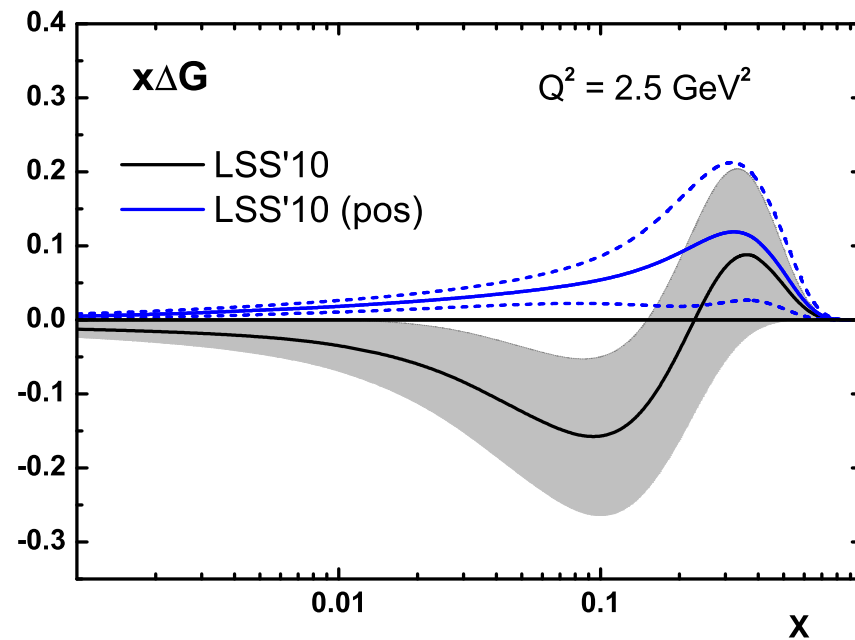
I guess it is caused by bad fragmentation functions

But it could be a signal of failure to understand the
connection between DIS and SIDIS

Results and comparison with DSSV: gluon



We also find an acceptable solution with positive ΔG



NB: has very little effect on $\Delta\bar{u}$, $\Delta\bar{d}$, $\Delta\bar{s}$.

Dashed lines: error bands —NB: Warning: error bands
do not reflect functional uncertainty!!!

The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

$$\begin{aligned} g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\ &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \end{aligned}$$

Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

The controversy about Higher Twist

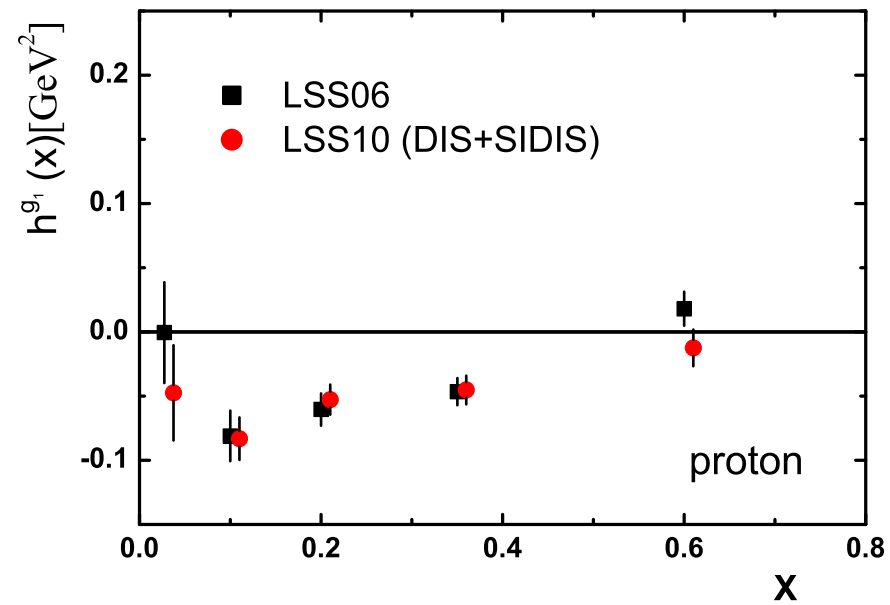
Following Operator Product Expansion (OPE), LSS use

$$\begin{aligned} g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\ &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \end{aligned}$$

Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

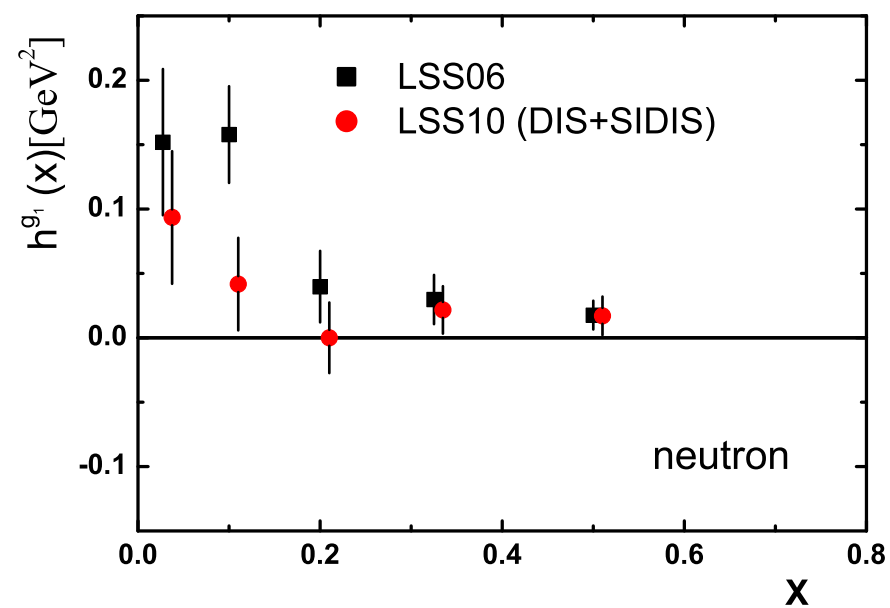
Possible slow scale i.e. Q^2 dependence in $h(x)$, the precise form of which is unknown, neglected compared to $1/Q^2$ variation.

We find significant HT contribution



Very important for CLAS data.

We find significant HT contribution



Blümlein and Böttcher (BB) (arXiv:1005.3113 v1)
disagree

They use

$$g_1(x, Q^2)_{exp} = g_1(x, Q^2)_{LT} \left[1 + \frac{C(x)}{Q^2} \right]$$

where any Q^2 dependence in $C(x)$ is neglected.

Blümlein and Böttcher (BB)(arXiv:1005.3113 v1)
disagree

They use

$$g_1(x, Q^2)_{exp} = g_1(x, Q^2)_{LT} \left[1 + \frac{C(x)}{Q^2} \right]$$

where any Q^2 dependence in $C(x)$ is neglected.

BB find no evidence for HT i.e. their $C(x)$ for protons and neutrons is compatible with zero.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2 , contradicting the use of $C(x)$ as Q^2 -independent.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2 , contradicting the use of $C(x)$ as Q^2 -independent.

If legitimate to neglect the Q^2 dependence in $C(x)$, then $h(x)$ must vary considerably with Q^2 .

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2 , contradicting the use of $C(x)$ as Q^2 -independent.

If legitimate to neglect the Q^2 dependence in $C(x)$, then $h(x)$ must vary considerably with Q^2 .

Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2 , contradicting the use of $C(x)$ as Q^2 -independent.

If legitimate to neglect the Q^2 dependence in $C(x)$, then $h(x)$ must vary considerably with Q^2 .

Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Since LSS formulation is closer in structure to the OPE we believe it to be the correct way to implement HT corrections.

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

This is incorrect. The correct relation should be

$$C_n(x) = \frac{1}{g_{1n}(x, Q^2)_{LT}} \left[\frac{2}{1 - 1.5\omega_D} g_{1d}(x, Q^2)_{LT} C_d(x) - g_{1p}(x, Q^2)_{LT} C_p(x) \right]$$

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

This is incorrect. The correct relation should be

$$C_n(x) = \frac{1}{g_{1n}(x, Q^2)_{LT}} \left[\frac{2}{1 - 1.5\omega_D} g_{1d}(x, Q^2)_{LT} C_d(x) - g_{1p}(x, Q^2)_{LT} C_p(x) \right]$$

Dangerous, since $g_{1n}(x, Q^2)_{LT}$ has a zero!

LSS Letter to BB—no response—so
(arXiv:1007.4781) “Comments on BB paper”

followed by Version 2 of BB, abandoning factorized
form for HT

“We prefer the additive case, since the twist-2 scaling
violations of $g_1(X, Q^2)$ do not influence $C_{p,d,n}(x)$.”

No reference to LSS

Claim no evidence for HT, but central values essentially
same as LSS. BB use only statistical errors, but, more
important, define error bars by $\Delta\chi^2 = 9.3$.

LSS method seems to agree with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx h^N(x) \quad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.018 \pm 0.008) GeV^2$$

LSS method seems to agree with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx h^N(x) \quad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.015 \pm 0.007) GeV^2$$

$$\bar{h}^p - \bar{h}^n = (-0.043 \pm 0.009) GeV^2$$

Seems to agree first moment analysis of $g_1^{(p-n)}$ of Duer et al. Also instanton model.

LSS method seems to agree with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx h^N(x) \quad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.015 \pm 0.007) GeV^2$$

$$\bar{h}^p - \bar{h}^n = (-0.043 \pm 0.009) GeV^2$$

Seems to agree first moment analysis of $g_1^{(p-n)}$ of Duer et al. Also instanton model.

$$\bar{h}^p + \bar{h}^n = (-0.013 \pm 0.009) GeV^2$$

$$|\bar{h}^p + \bar{h}^n| < |\bar{h}^p - \bar{h}^n|$$

Agrees with $1/N_C$ expansion.

The spin sum rule: $\overline{MS} : Q^2 = 4\text{GeV}^2$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}$$

Positive ΔG

$$\Delta G = 0.316 \pm 0.190 \quad \Delta\Sigma = 0.207 \pm 0.034$$

$$J_z = (0.42 \pm 0.19) + \text{OAM}$$

The spin sum rule: $\overline{MS} : Q^2 = 4\text{GeV}^2$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}$$

Positive ΔG

$$\Delta G = 0.316 \pm 0.190 \quad \Delta\Sigma = 0.207 \pm 0.034$$

$$J_z = (0.42 \pm 0.19) + \text{OAM}$$

Changing sign ΔG

$$\Delta G = -0.339 \pm 0.458 \quad \Delta\Sigma = 0.254 \pm 0.042$$

$$J_z = (-0.21 \pm 0.46) + \text{OAM}$$

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
- $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
- $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
- SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
- $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
- SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
- $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$: Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible?? **This is a serious discrepancy!**

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
- $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
- SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
- $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$: Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible?? **This is a serious discrepancy!**
- Higher Twist: LSS disagrees with BB, but seems to agree with moment studies

Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
- $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
- SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
- $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$: Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible?? **This is a serious discrepancy!**
- Higher Twist: LSS disagrees with BB, but seems to agree with moment studies
- ΔG still ambiguous. EIC, large Q^2 and small x could resolve.

SHORT BIBLIOGRAPHY

LSS2010: arXiv:1010.0574

DIS review: Kuhn, Chen, Leader, Prog. Part. Nucl. Phys., 63 (2009) 1

DSSV: Phys. Rev. Lett., 101 (2008) 072001

LSS: Phys. Rev. D75 (2007) 074027; Eur. Phys. J., 162 (2008) 19; Phys. Rev. D80 (2009) 054026

Deur et al: Phys. Rev. D78 (2008) 032001