Realistic Transverse Images of the Nucleon

Gerald A. Miller, U. of Washington

Theme- much form factor data exist, interpret form factor as determining transverse charge and magnetization densities, nucleon transverse densities known now to high precision, pion known fairly well

Outline-

1. How not to and how to analyze electromagnetic form factors- transverse density

2. Model independent proton, neutron transverse charge density

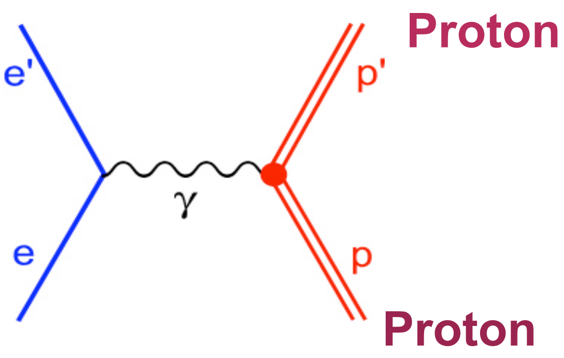
3. Pion time-like data and transverse charge density

Transverse Charge Densities.

Gerald A. Miller, arXiv:1002.0355 [nucl-th] ARNPS

Electron-nucleon scattering

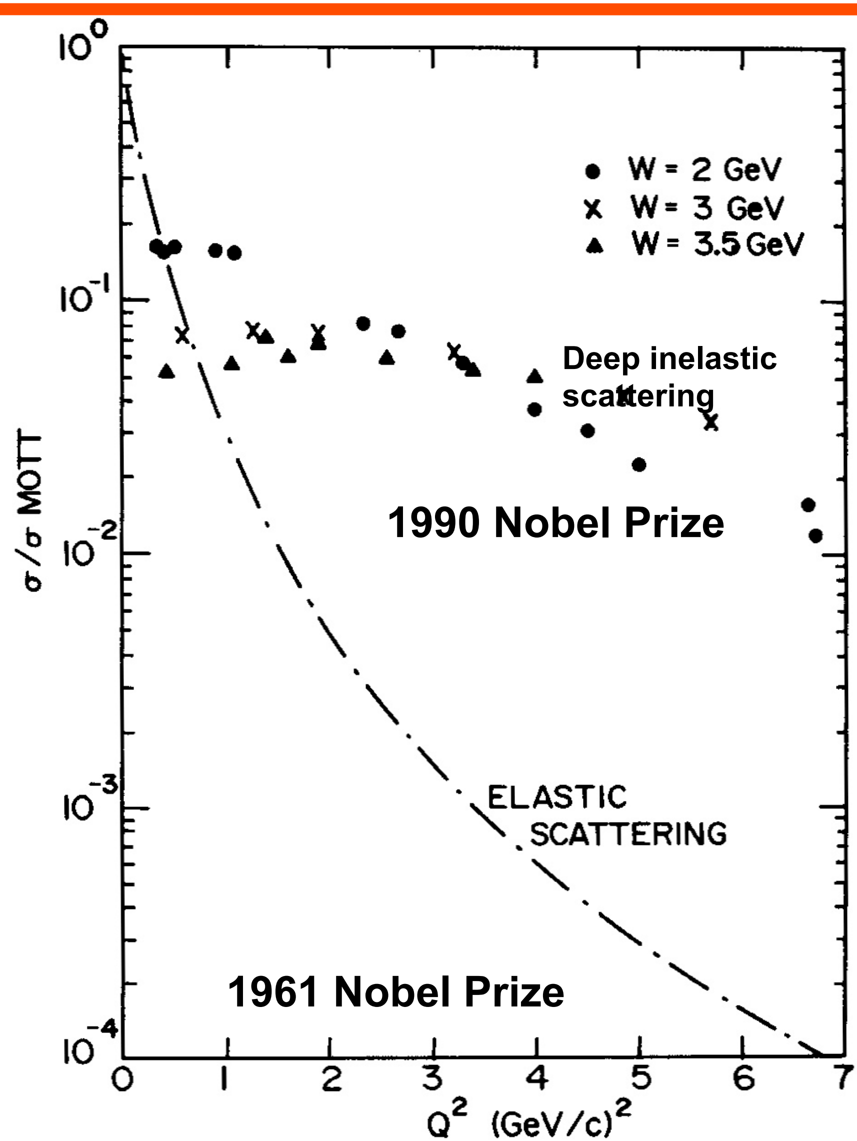
Elastic scattering



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(G_E^2 + \frac{\tau}{\epsilon} G_M^2\right) / (1+\tau)$$

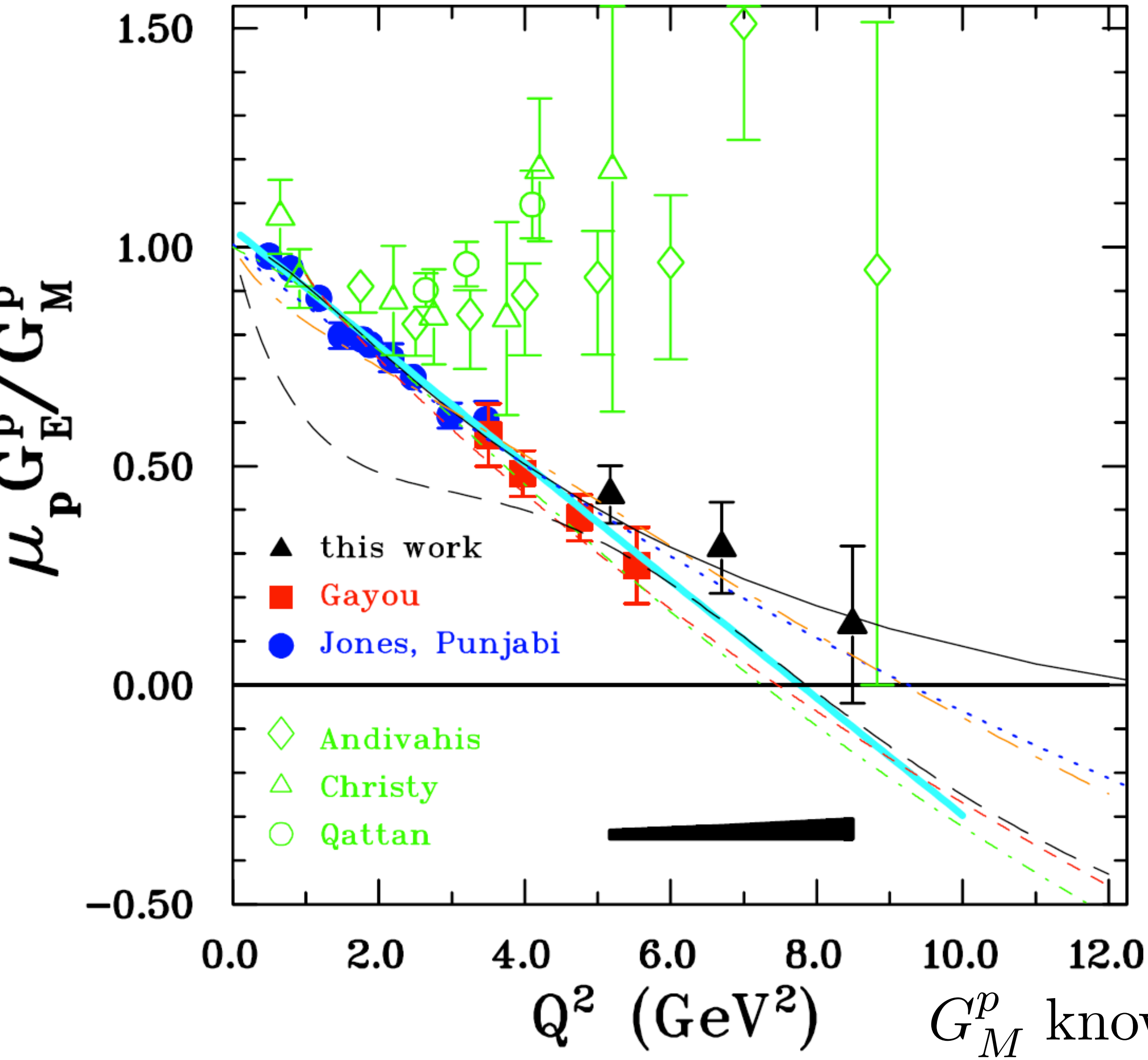
Cross section for scattering from a point-like object

Form factors for NON-pointlike nucleon



$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \; G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$

Proton

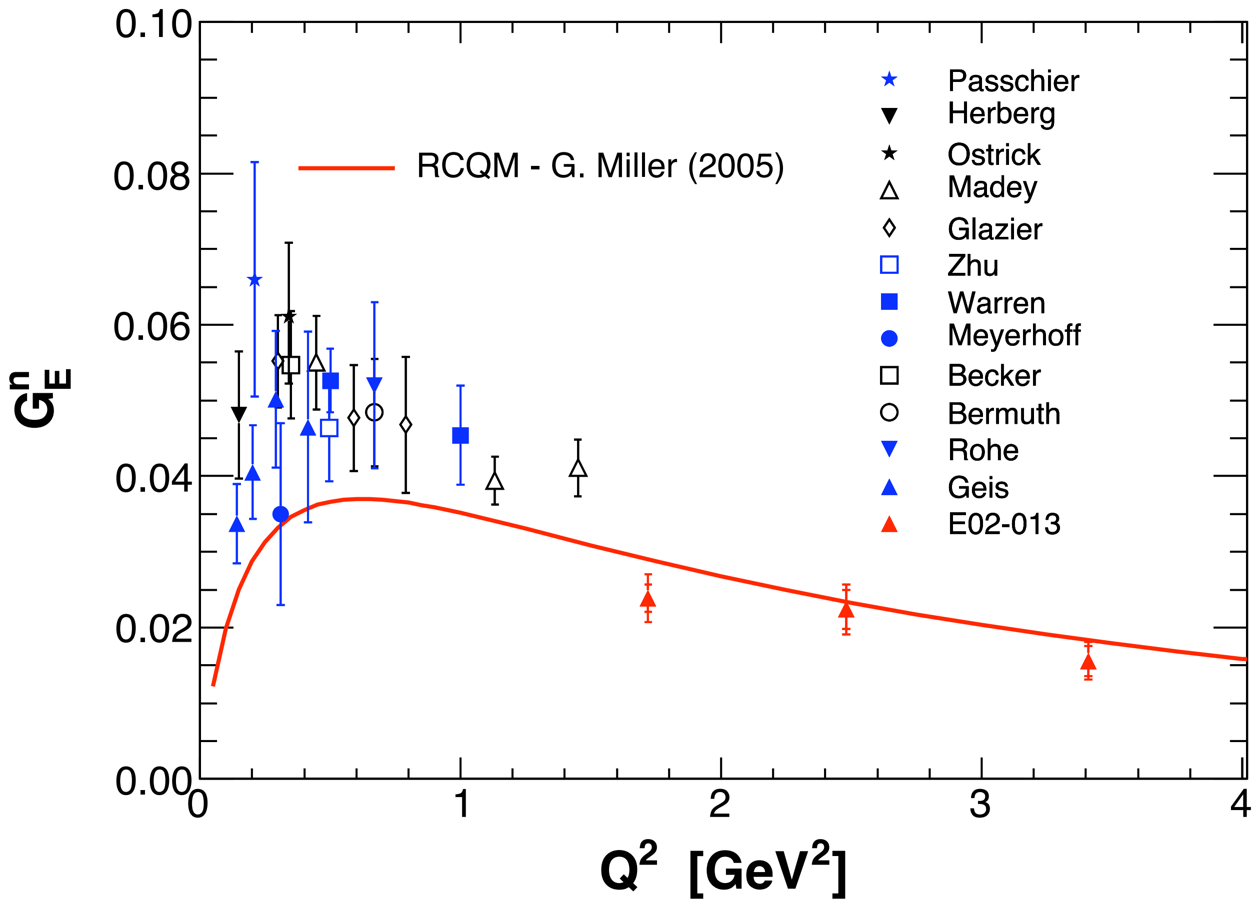


A.J.R. Puckett et al., Phys.Rev.Lett. 104:242301,2010.

G_M^p known $Q^2 \leq 31$ GeV^2

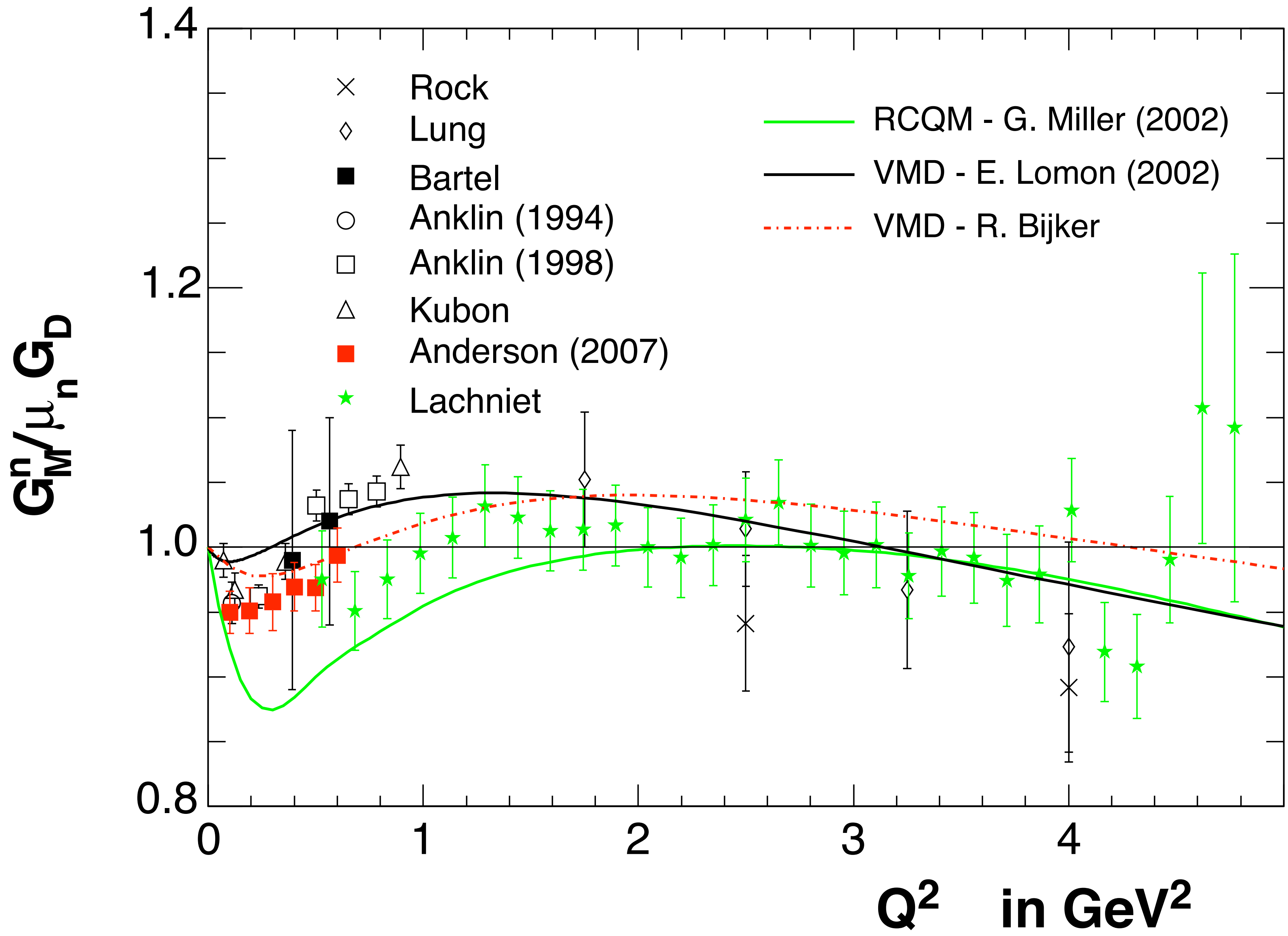
Electric Form Factor of the Neutron up to Q^2=3.4 GeV^2 using the Reaction $He^3(e,e'n)pp$.

S. Riordan *et al*. Aug 2010. e-Print: 1008.1738



Results for G_M^n

Jerry Gilfoyle



Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density **WRONG**

$$G_E(\vec{q}^{\,2}) = \int d^3r\rho(r)e^{i\vec{q}\cdot\vec{r}} \rightarrow \int d^3r\rho(r)(1-\vec{q}^{\,2}r^2/6+\cdots)$$

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

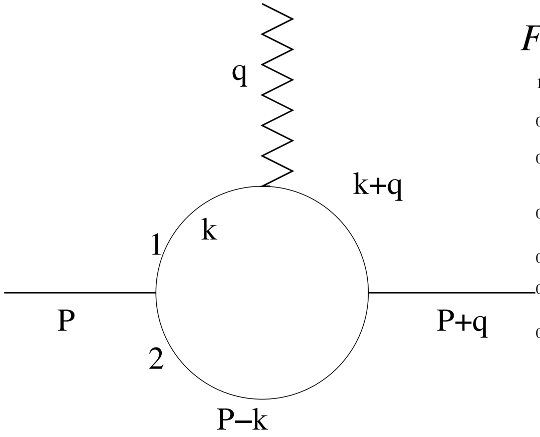
interpretation of Sachs FF is wrong

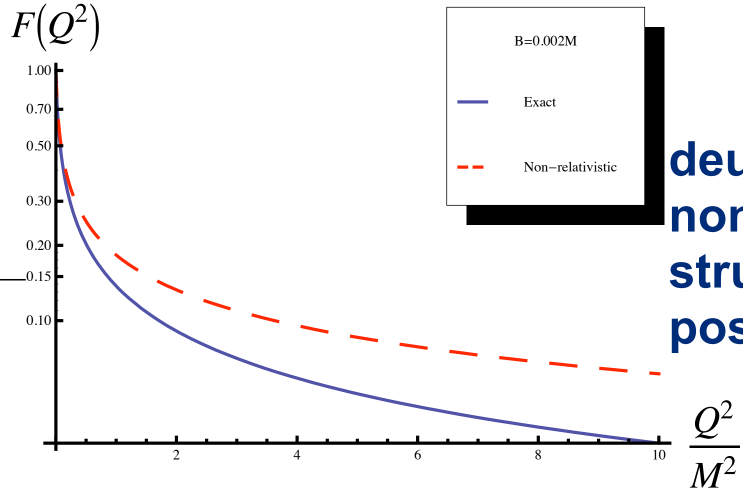
Final wave function is boosted from initial

Need relativistic treatment

Toy model GAM, Phys.Rev.C80:045210,2009.

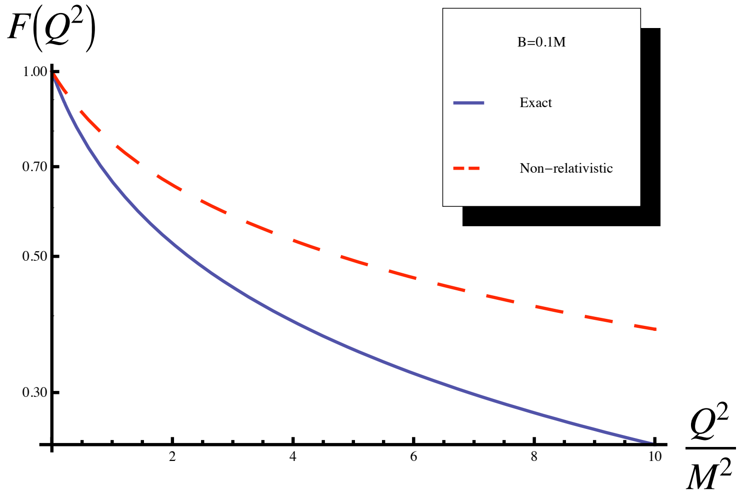
Scalar meson M, made of two scalar mesons, m
IF (M-2m)/M, small non-relativistic works





deuteron kinematics are non-relativistic: extract neutron structure function should be possible

(M-2m)/M=0.002



Relativity needed

(M-2m)/M=0.1

Exact *vs* non-relativistic Form factors for the case $m_1 = m_2 = m$.

Light front, Infinite momentum frame

$$\text{“}Time\text{”},\ x^+ = x^0 + x^3, \quad \text{“}Evolve\text{”},\ p^- = p^0 - p^3$$

$$\text{“}Space\text{”},\ x^- = x^0 - x^3, \quad \text{“}Momentum\text{”},\ p^+ (\text{Bjorken})$$

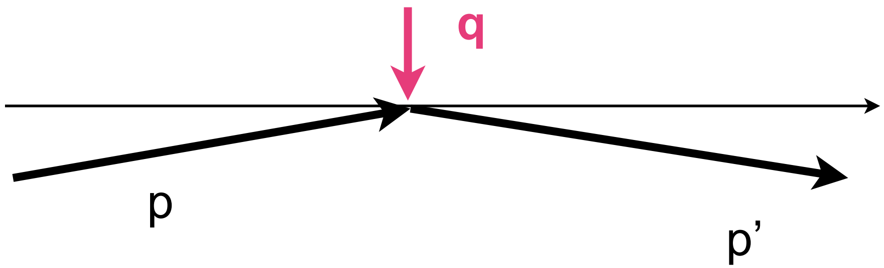
$$Transverse\ position,\ momentum \quad \mathbf{b}, \mathbf{p}$$

These variables are standard! GPDs, TMDs PDFs all use these

transverse boosts in kinematic subgroup

$$\mathbf{k} \rightarrow \mathbf{k} - k^+ \mathbf{v}$$

Space-like momentum transfer in the transverse direction



then density is 2 Dimensional Fourier Transform

Model independent transverse charge density

Charge Density operator in infinite momentum frame

Evaluate in state of **R=0,** superposition of **p** eigenstates

Definition of F_1

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

Soper '77

$$\text{Density is } u - \bar{u},\ d - \bar{d}$$

Impact parameter dependent GPD Burkardt

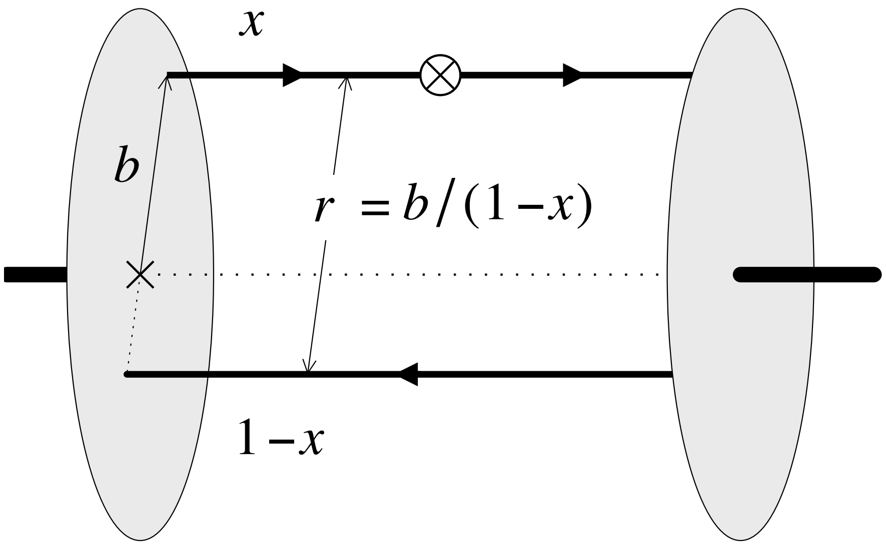
Probability that quark at b from CTM has long momentum fraction x: $\rho(x,b)$

$$\rho(b) = \int dx\,\rho(x,b)$$

Transverse density is integral over longitudinal position or momenta
example of Parseval's theorem

$$\mathbf{R} = 0 = \sum_i^N x_i \mathbf{b}_i$$

Quark of x=1, must have b=0



What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

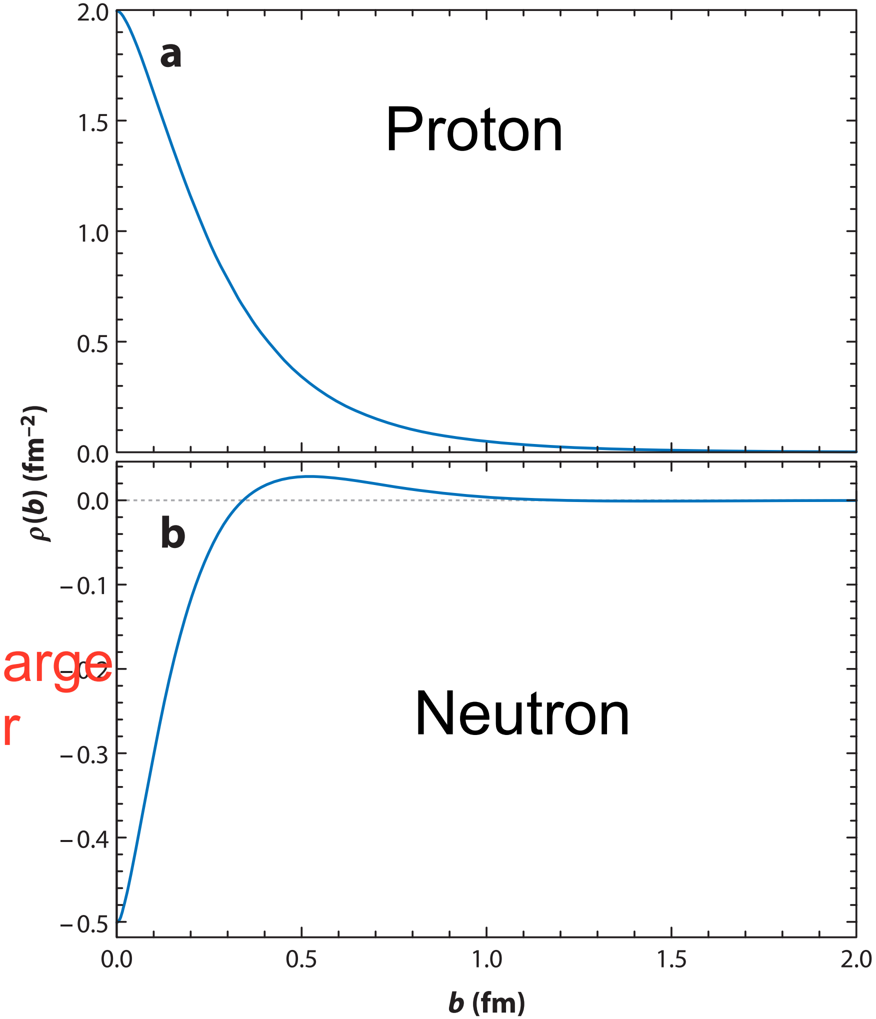
Fermi: n fluctuates to $p\pi^-$

p at center, pion floats to edge

One gluon exchange favors dud

Real question- how does form factor relate to charge density?

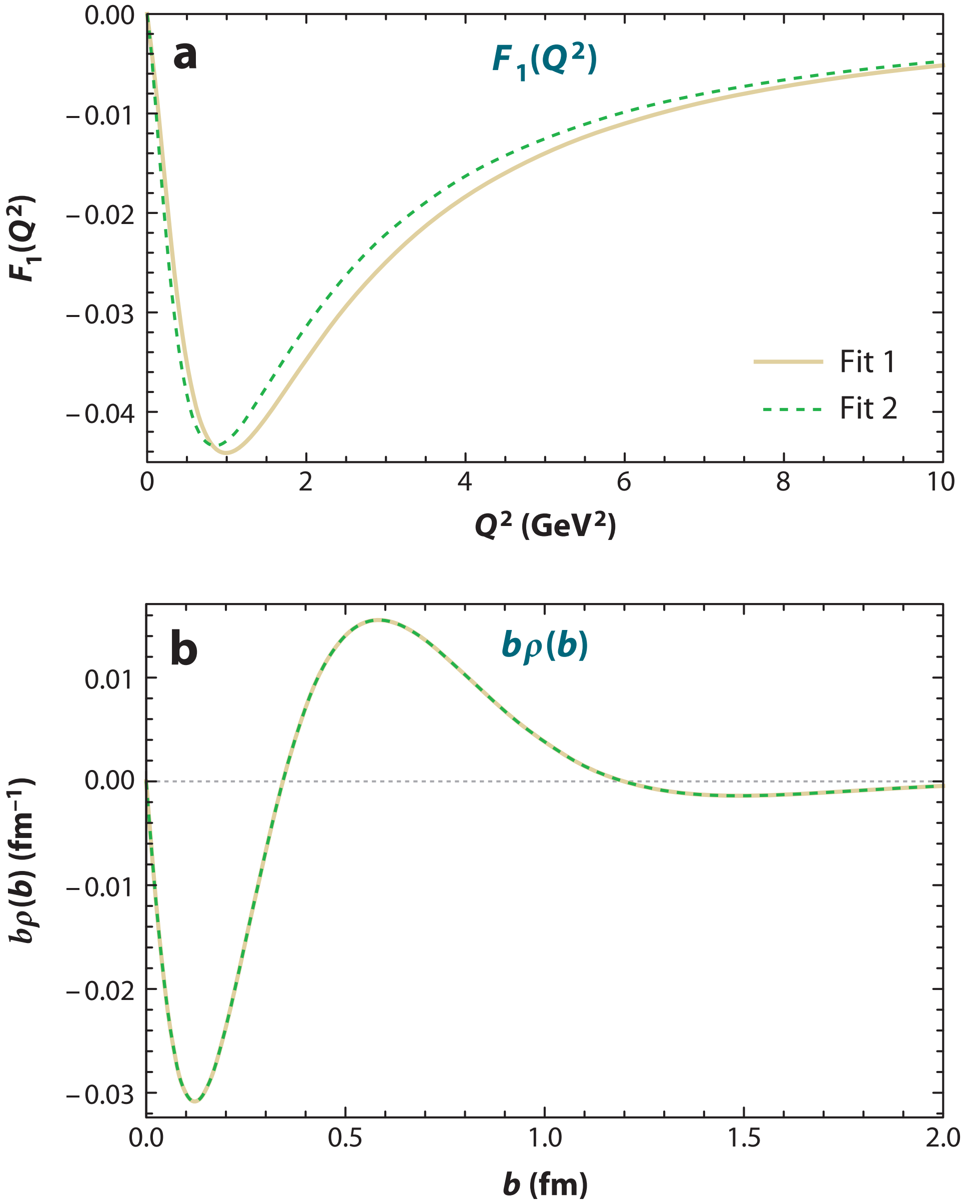
Transverse charge densities from parameterizations (Alberico)



Negative central charge density, negative for b<0.4 fm

Negative central density - GAM PRL '07

Neutron



F_1 is negative, so is central density

Negative at large b, pion cloud? see Strikman Weiss '10

arXiv:1004.3535

Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x,b)$
- Drell-Yan-West relation between high x DIS and high Q^2 elastic scattering
- High x related to low b, not uncertainty principle

$$\lim_{x\to 1} \nu W_2(x) = (1-x)^{2n-1} \leftrightarrow \lim_{Q^2\to\infty} F_1(Q^2) \sim \frac{1}{Q^{2n}}, n=2$$

- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or π^-

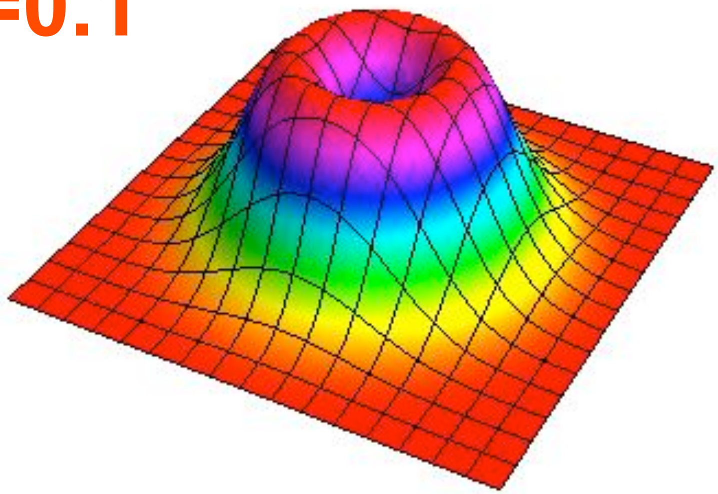
$$\text{Density is } u-\bar{u},\ d-\bar{d}$$
$$\pi^- \text{ is } \bar{u}d$$
$$\text{decreases } u \text{ contribution}$$
$$\text{enhances } d \text{ contribution}$$

Neutron interpretation ρ(x,b)

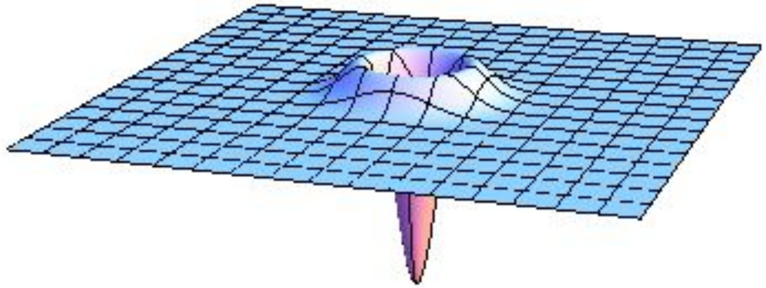
GAM, J. Arrington, PRC78,032201R '08

Using other people's models

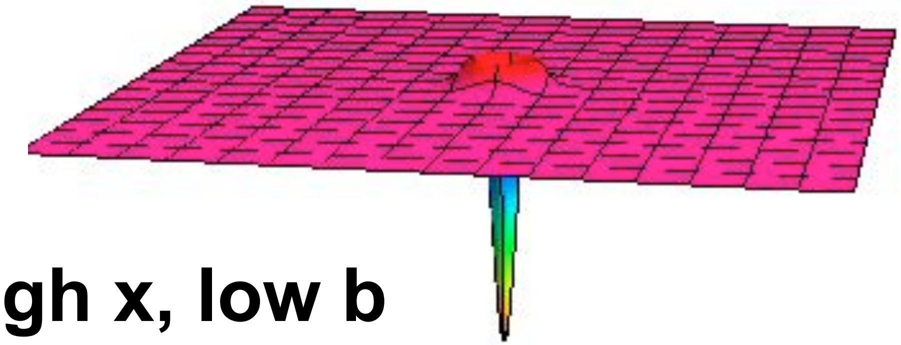
x=0.1



x=0.3

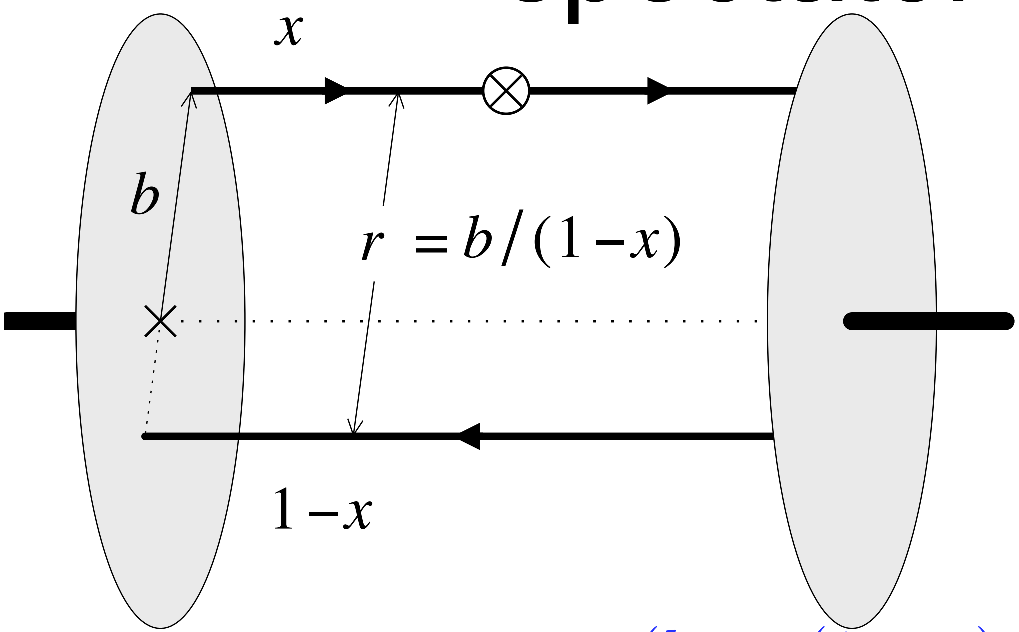


x=0.5

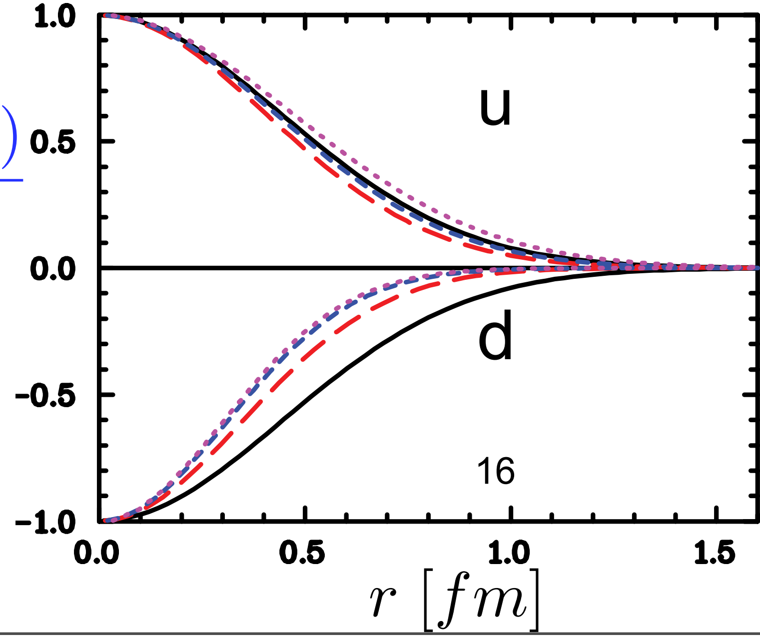


d or π^- dominates at high x, low b

Understand b: quark and spectator system



Several values of x, little variation



$$\frac{\rho(b = r(1-x), x)}{\rho(0, x)}$$

Model **dep**endent,
does **not** integrate to 0
Large x does not dominate the density

Transverse Nucleon anomalous magnetization density

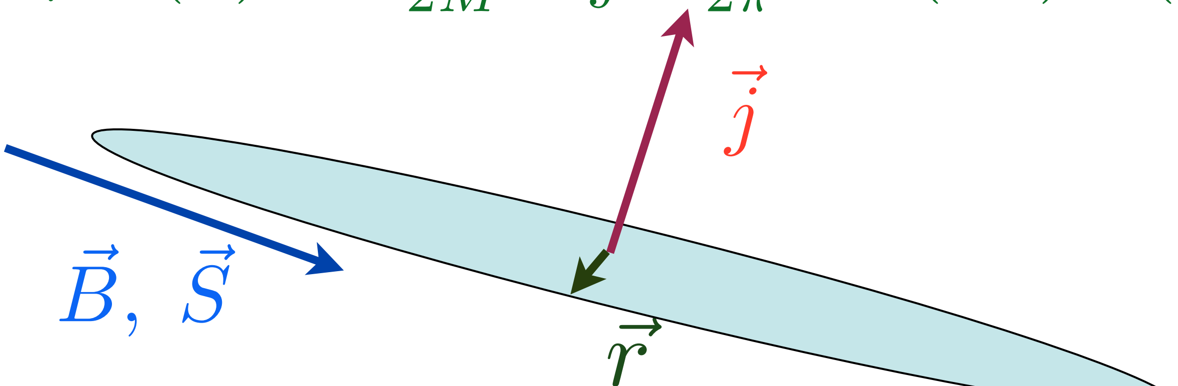
$$\vec{\mu} \cdot \vec{B} = \langle X | \int d^3 r \frac{1}{2} (\vec{r} \times \vec{j}) \cdot \vec{B} | X \rangle$$

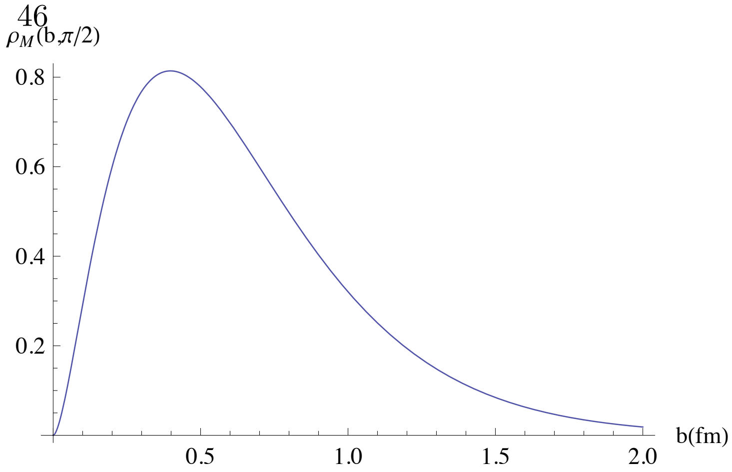
$\frac{1}{2} \vec{r} \times \vec{j}$ is magnetization density (OAM)   Spin included

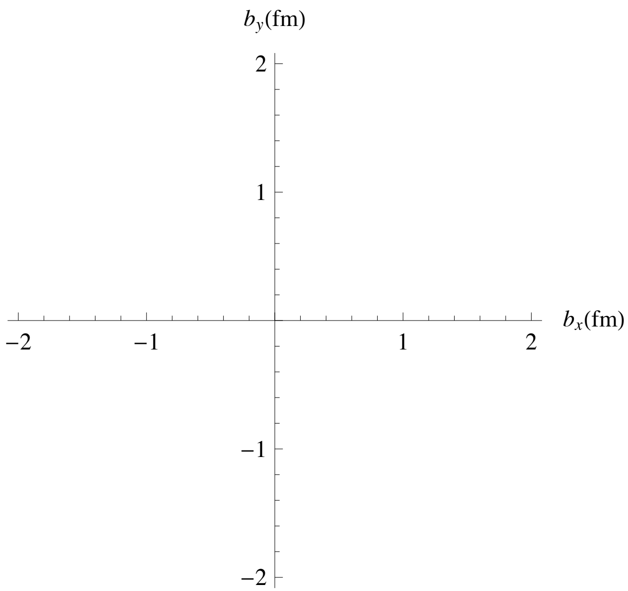
$\vec{B}$ in x-direction, $\vec{J}$ in z-direction

Magnetization density

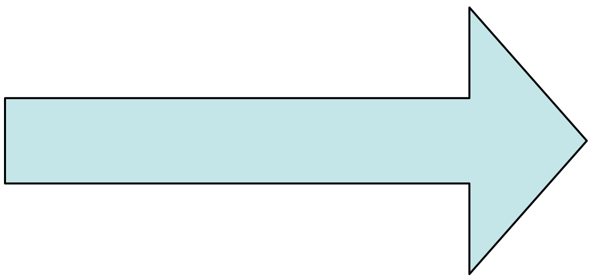
$$\rho_M(\mathbf{b}) = \frac{\sin^2 \phi}{2M} b \int \frac{Q^2 dQ}{2\pi} F_2(Q^2) J_1(Qb)$$







Direction of magnetic field



Realistic Transverse Images of the Proton Charge and Magnetic Densities

Siddharth Venkat, John Arrington, Gerald A. Miller, Xiaohui Zhan arXiv:1010.3629

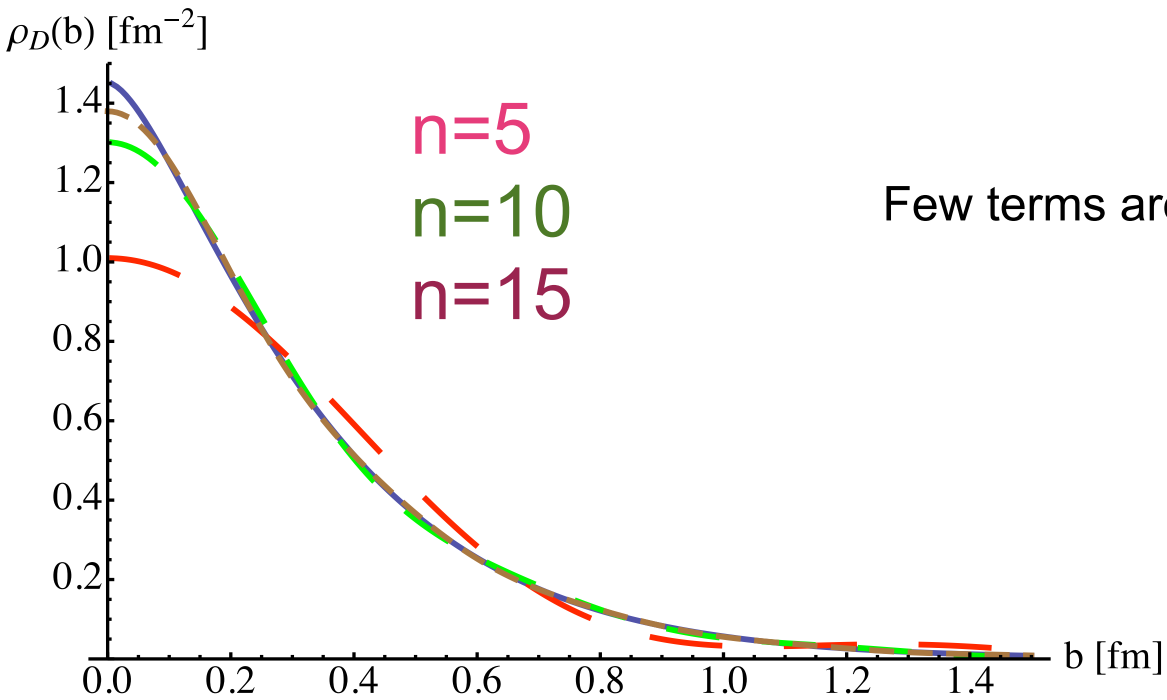
- Goals:
- Extract model-independent spatial information
- Deal with experimental uncertainties and
- Lack of information at higher Q^2
- Current interest- three dimensional structure of nucleon
- Technique should be extendable to other observables

The basic idea

$$\rho(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} J_1(X_n)^{-2} F(Q_n^2) J_0(X_n \frac{b}{R}),\ Q_n \equiv \frac{X_n}{R}.\ X_n \approx (n+3/4)\pi$$

Finite Radius Approximation FRA

- With R=3 fm, n=10, $Q_n^2 \approx 4\ \mathrm{GeV}^2$
- Dipole example
-



Few terms are necessary

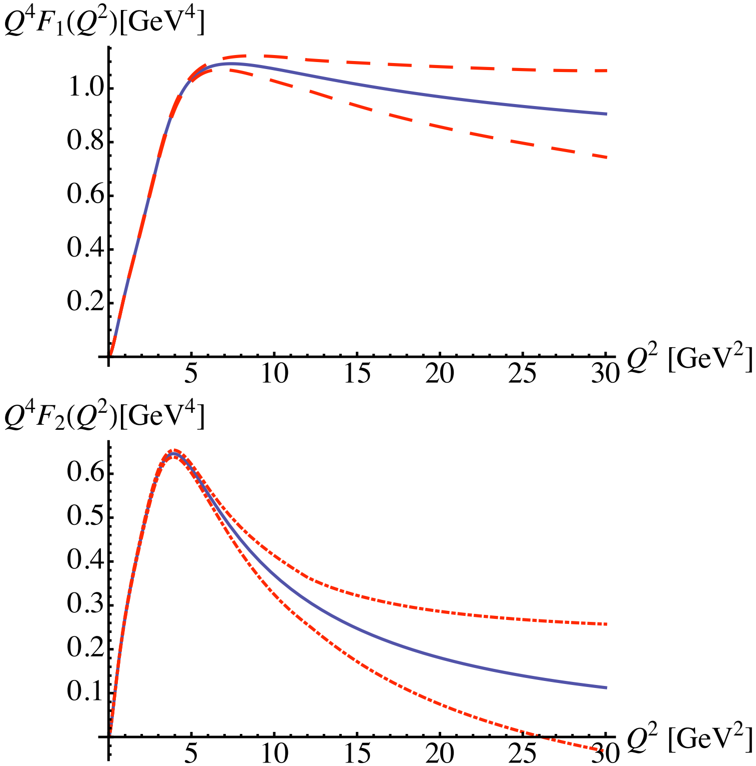
The proton data

- ep scattering up to 31 GeV^2
- $G_{E,M}$ extracted up to 10 GeV^2
- global analysis of world data
- two photon exchange: Blunden et al
- repeat Arrington et al analysis with Puckett data, evaluate analytic expression for $G_{E,M}$
- constrain slopes of $G_{E,M}$ to measured values
- uncorrelated uncertainties and normalization uncertainties included

$$(dF_1)^2 = (\frac{1}{1+\tau})^2(dG_E)^2 + (\frac{\tau}{1+\tau})^2(dG_M)^2$$

$$(dF_2)^2 = (\frac{1}{1+\tau})^2(dG_E)^2 + (\frac{1}{1+\tau})^2(dG_M)^2$$

The uncertainties

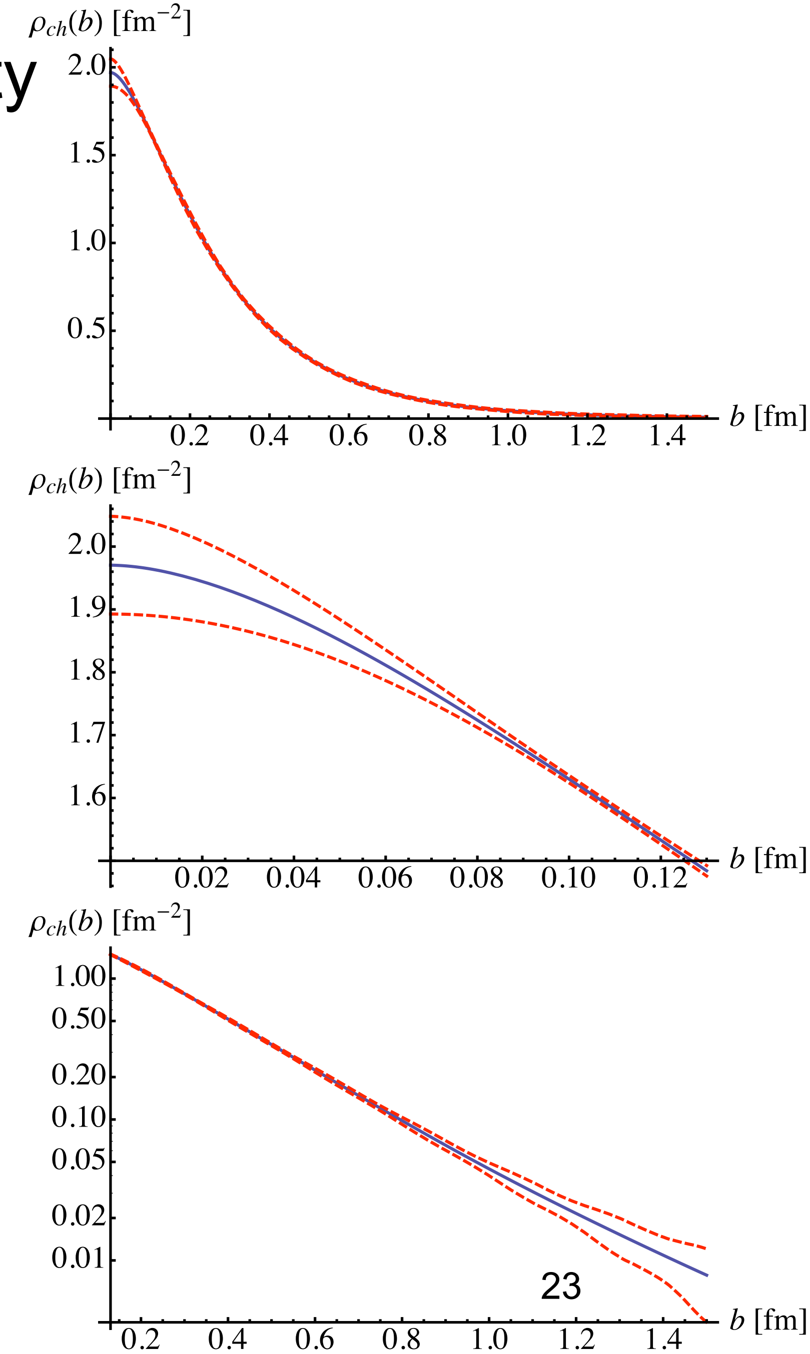


For $Q^2 < 30\ \mathrm{GeV}^2$,
use dF_1 in FRA to get $d\rho_{ch}(b)$

For Q^2 greater than 30 GeV^2,
use FRA and take $dF_1 = \pm |F_1(fit)|$

Proton transverse charge density

- Very well determined



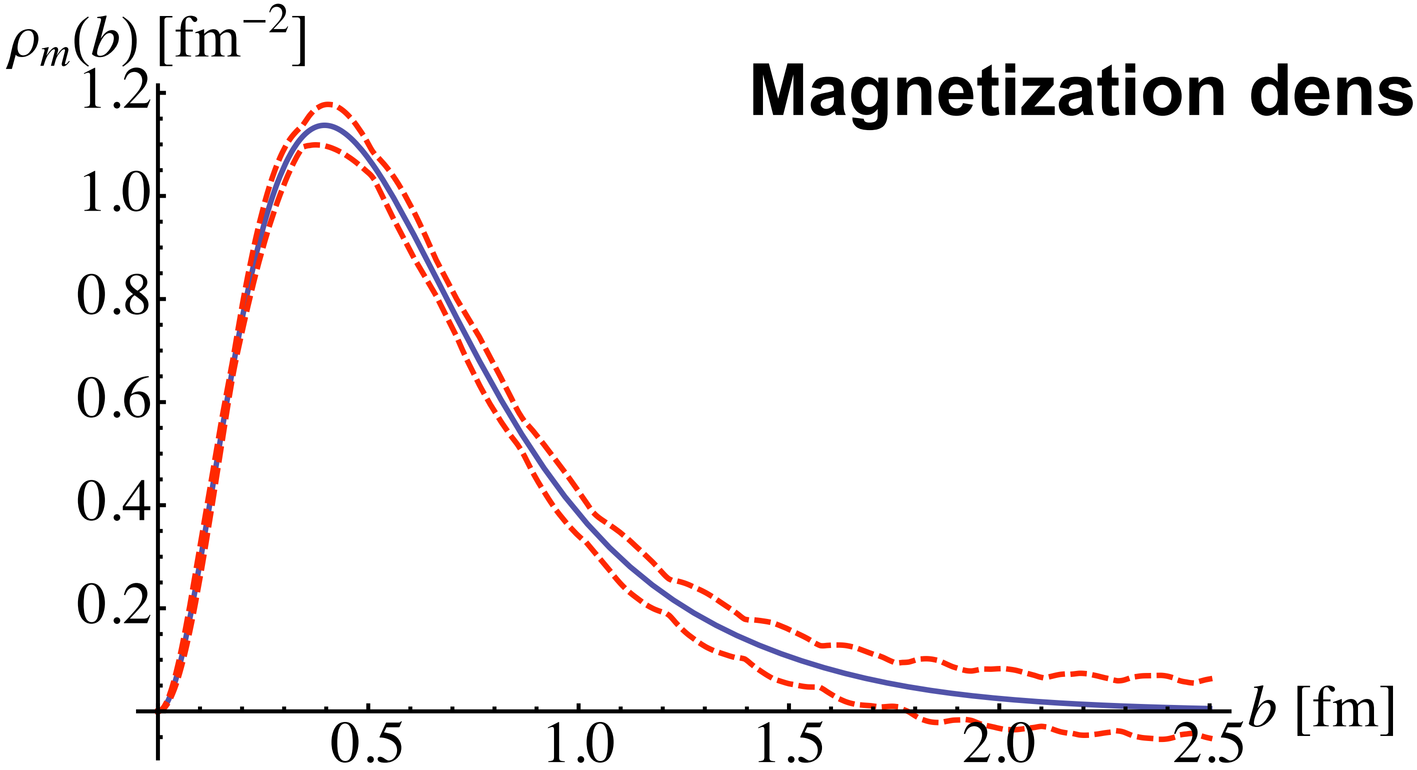
Other densities

$$\rho^{(\lambda)}(b) = \frac{1}{2\pi}\int QdQ\,J_\lambda(Qb)F^{(\lambda)}(Q^2).$$

$$\rho^{(\lambda)}(b) = \sum_{n=1}^{\infty} c_{n\lambda} J_\lambda(X_{\lambda,n}\frac{b}{R}), \quad c_{n,\lambda} \approx \widetilde{c}_{n,\lambda} = \frac{2}{R^2 J_{\lambda+1}(X_{\lambda,n})^2} F^{(\lambda)}(Q^2_{\lambda,n}),$$

$$Q_{\lambda,n} = \frac{X_{\lambda,n}}{R}$$

Magnetization density



Determination of F_π via Pion Electroproduction

At low Q^2<0.3 GeV^2, the π^+ form factor can be measured **exactly** using high energy π^+ scattering from atomic electrons.

⇒ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

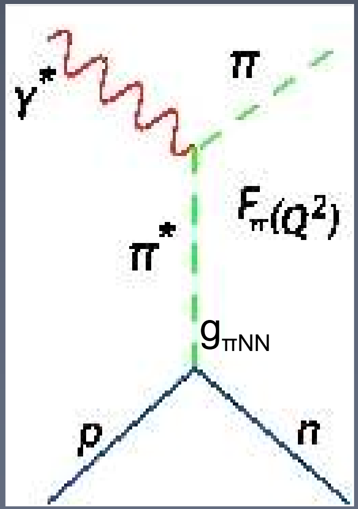
⇒ Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2, one must employ the $p(e,e'\pi^+)n$ reaction.

- t-channel process dominates σ_L at small $-t$.
- In the Born term model:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)$$



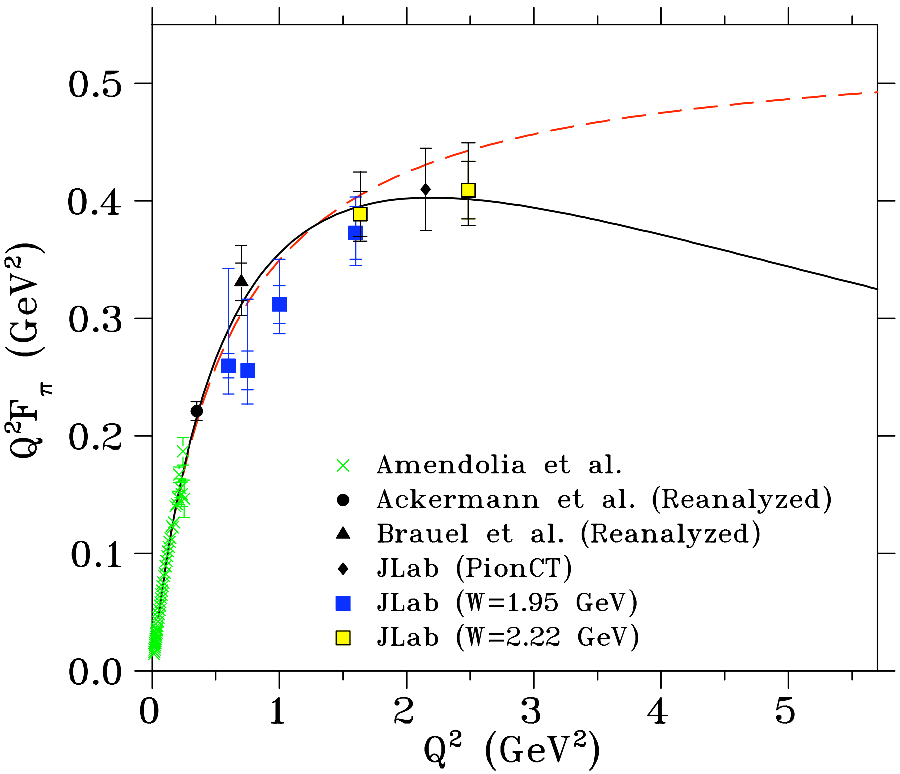
Pion Transverse Charge Density$_{\text{GAM}}$ **Phys.Rev.C79:055204**

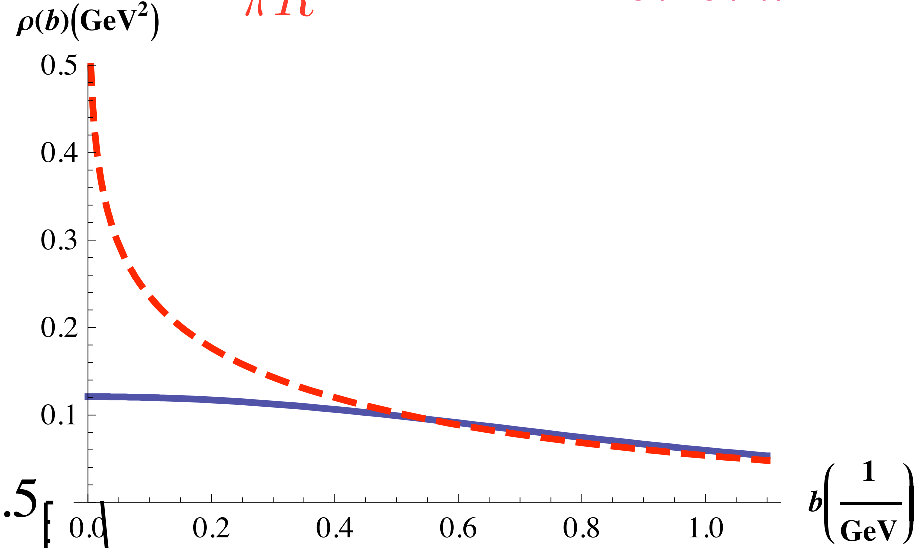
$$F_\pi(Q^2) = 1/(1 + R^2Q^2/6),$$

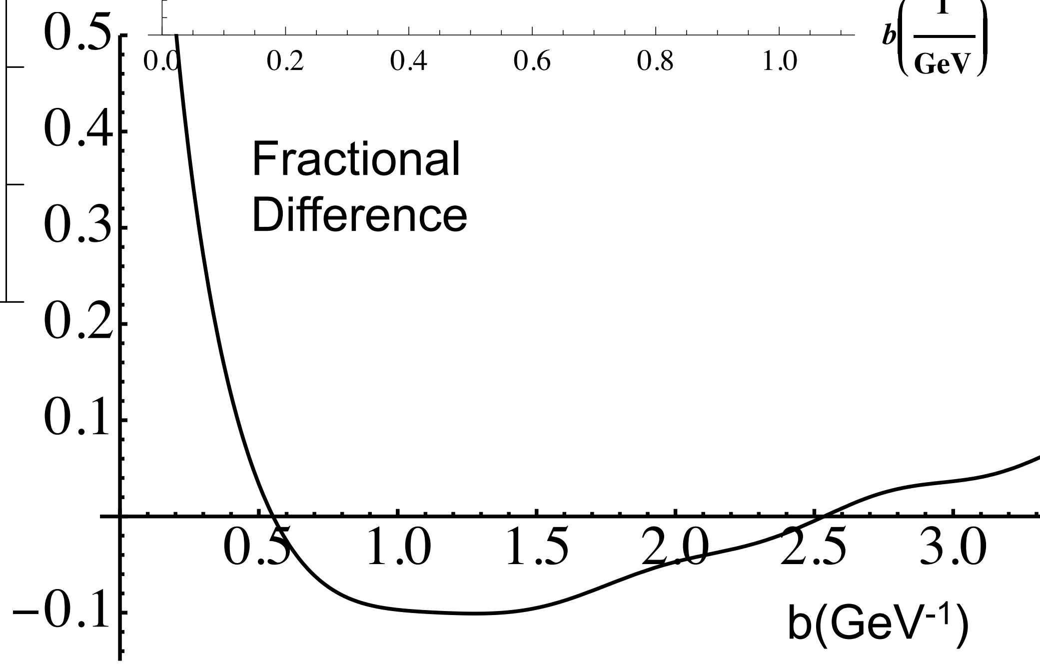
$$\rho_\pi(b) = \frac{3K_0(\frac{\sqrt{6}b}{R})}{\pi R^2}$$

Singular - varies as log (b)
small b, log(log(b)) in pQCD

Dashed- monopole fit, solid rel. cqm Huang



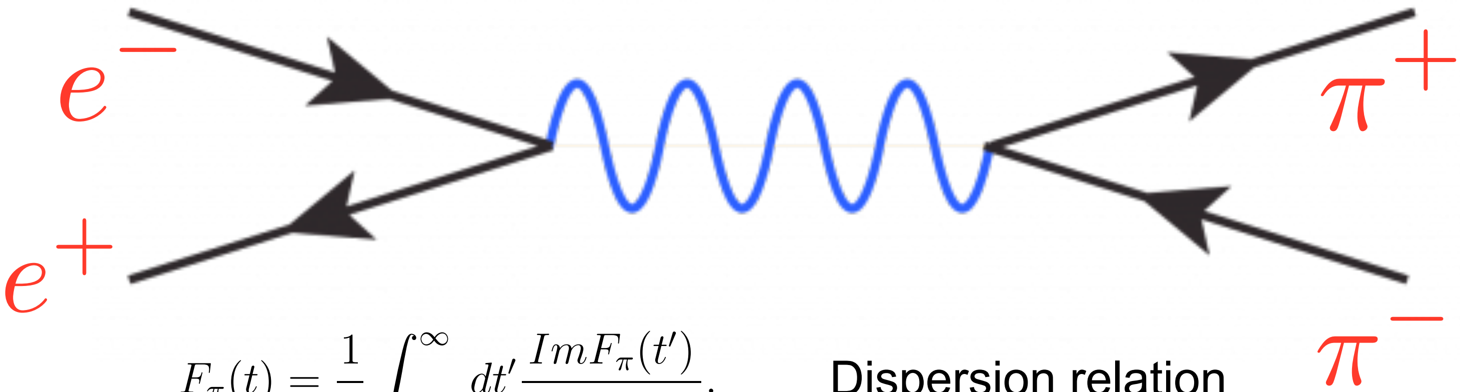




NT@UW-10-15

Pionic Transverse Density From Time-like and Space-Like Probes

Gerald A. Miller[1], Mark Strikman[2], Christian Weiss[3]



$$F_\pi(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{ImF_\pi(t')}{t' - t + i\epsilon}.$$

Dispersion relation

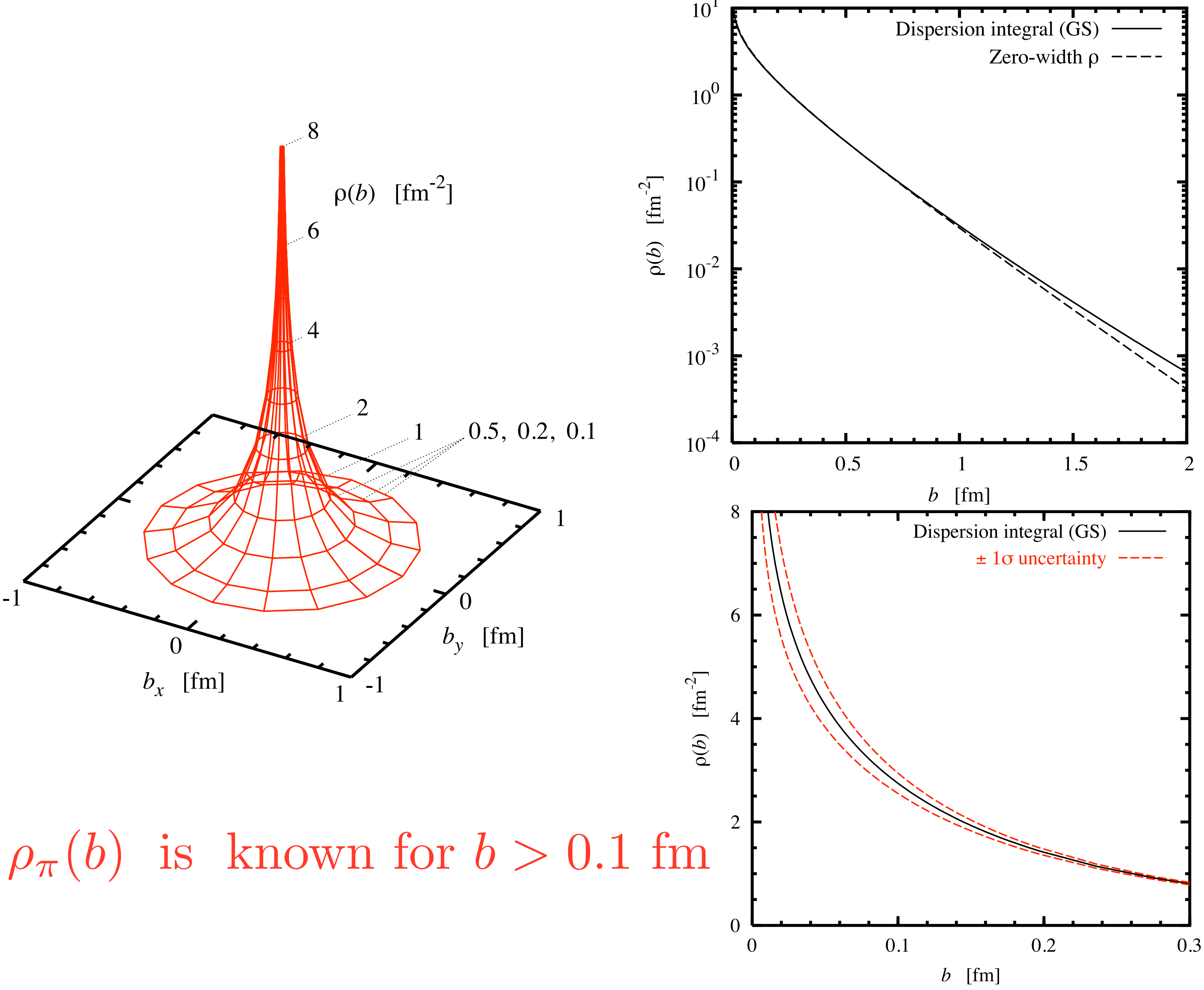
Use this expression in equation for transverse density.

$$\rho(b) = \frac{1}{2\pi}\int_{4m_\pi^2}^{\infty} dt K_0(\sqrt{t}b)\frac{ImF_\pi(t)}{\pi}.$$

Low t' dominates except for very small values of b

Model needed: C. Bruch et al E. J Phys.C39, 41: Vector Meson Dominance Gouranis Sakurai

Pion Transverse Charge Density



$\rho_\pi(b)$ is known for $b > 0.1$ fm

Summary

- Much data exist, Jlab12 will improve data set
- Charge density is not a 3 dimensional Fourier transform of G_E
- Interpret form factor as determining transverse charge and magnetization densities
- Nucleon transverse densities known now to high precision
- New Finite Radius Approximation FRA technique can be used for other spatial variables
- Pion transverse density known well

Spares follow

Generalized transverse densities-

$$\mathcal{O}_q^\Gamma(px,\mathbf{b}) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger(0,\mathbf{b})\Gamma q_+(x^-,\mathbf{b})$$

$$\rho^\Gamma(b) = \int dx \sum_q e_q \langle p^+, \mathbf{R}=\mathbf{0}, \lambda | \mathcal{O}_q^\Gamma(p^+x,\mathbf{b}) | p^+, \mathbf{R}=\mathbf{0}, \lambda \rangle$$

$\int dx$ sets $x^- = 0$, get $q_+^\dagger(0,\mathbf{b})\Gamma q_+(0,\mathbf{b})$ **Density!**

$\Gamma = \frac{1}{2}(1+\mathbf{n}\cdot\ \boldsymbol{\gamma}\gamma^5)$ gives spin-dependent density

Local operators calculable on lattice M. Göckeler et al PRL98,222001 $\widetilde{A}''_{T10} \sim \mathrm{sdd}$ spin-dependent density

Schierholtz, 2009 -this quantity is not zero, proton is not round

Observing shape of proton

- Transverse coordinate space density is a GPD, observe on lattice
- Transverse momentum space density is a TMD, can be observed in

$$e, \uparrow p \rightarrow e' \pi \; X$$

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

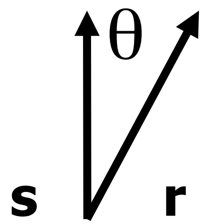
$$\langle \mathbf{r}_p|\psi_{1,1/2s}\rangle = R(r_p)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}_p|s\rangle$$

$$\rho(r) = \langle\psi_{1,1/2s}|\delta(\mathbf{r}-\mathbf{r}_p)|\psi_{1,1/2s}\rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r},\mathbf{n}) = \langle\psi_{1,1/2s}|\delta(\mathbf{r}-\mathbf{r}_p)\frac{(1+\boldsymbol{\sigma}\cdot\mathbf{n})}{2}|\psi_{1,1/2s}\rangle$$

$$= \frac{R^2(r)}{2}\langle s|\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}(1+\boldsymbol{\sigma}\cdot\mathbf{n})\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}|s\rangle$$



$\mathbf{n}\parallel\hat{\mathbf{s}}:$ $\quad\rho(\mathbf{r},\mathbf{n}=\hat{\mathbf{s}}) = R^2(r)\cos^2\theta$

$\mathbf{n}\parallel-\hat{\mathbf{s}}:$ $\quad\rho(\mathbf{r},\mathbf{n}=-\hat{\mathbf{s}}) = R^2(r)\sin^2\theta$

non-spherical shape depends on spin direction

Summary

- **Much data exist, Jlab12 will improve data set**
- **Interpret form factor as determining transverse charge and magnetization densities**
- **Nucleon transverse densities known now to high precision,**
- **Pion known fairly well**
-

Relativistic formalism- kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

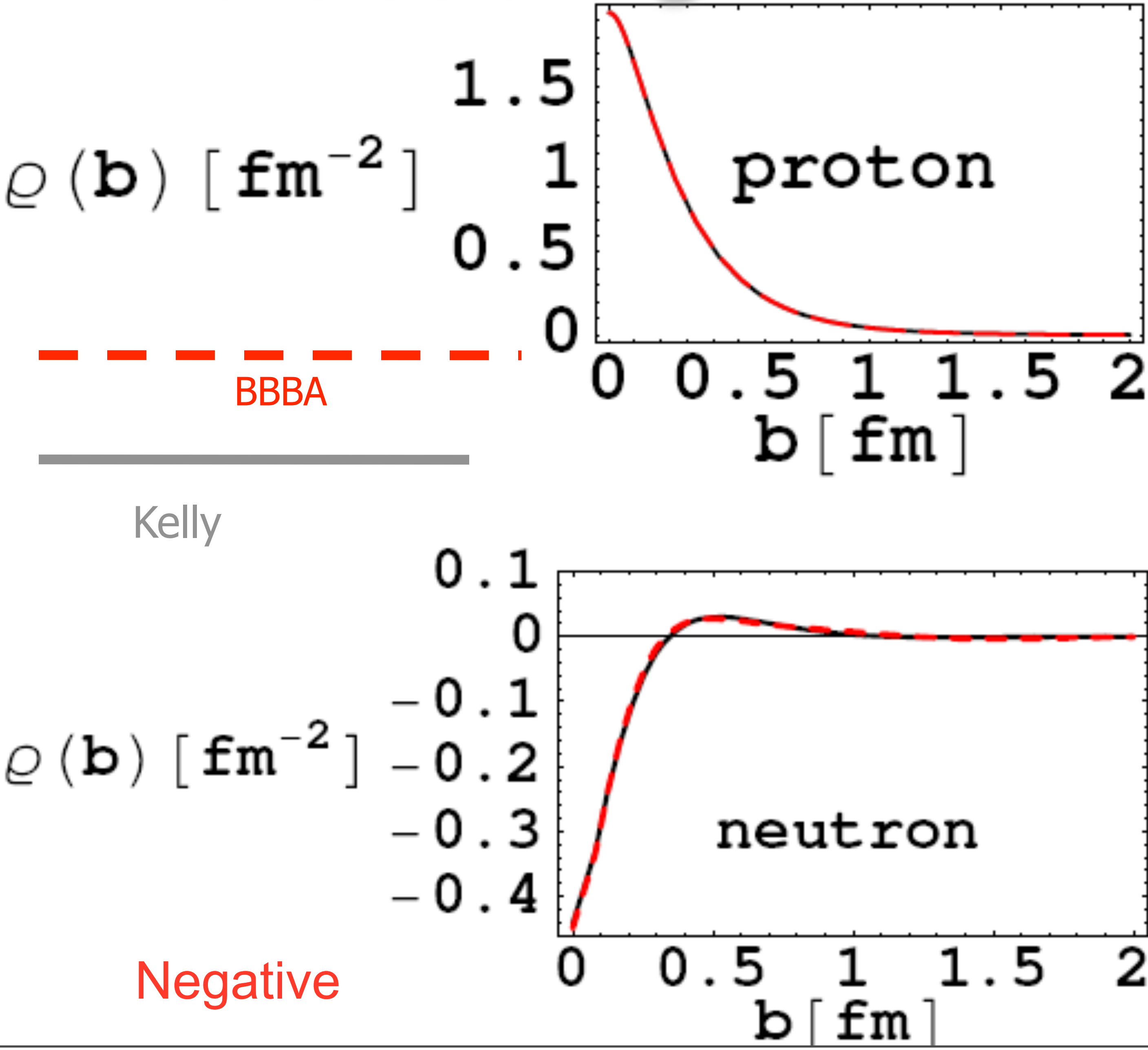
$$k^+ \rightarrow k^+, \;\; \mathbf{k} \rightarrow \mathbf{k} - k^+\mathbf{v}$$

k^- such that k^2 not changed

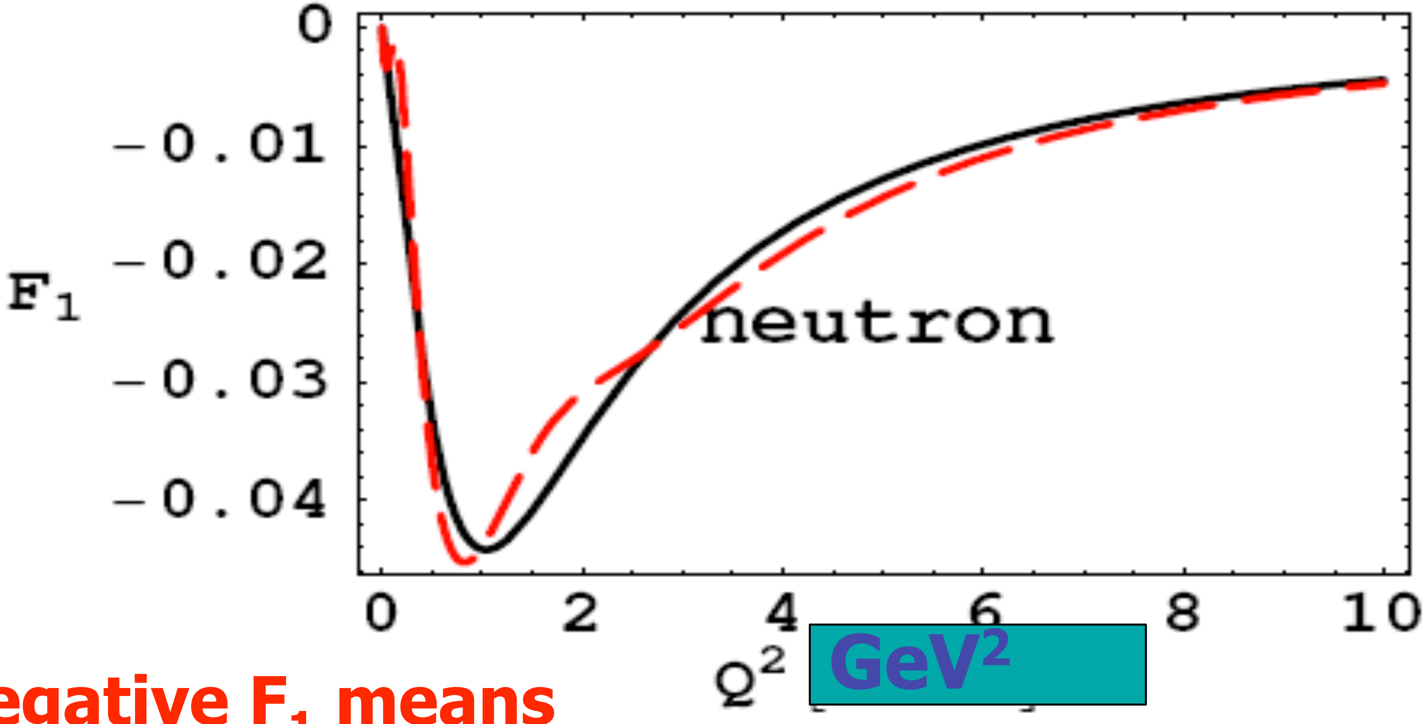
Just like non-relativistic with k^+ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform, also

$$q^+ = q^0 + q^3 = 0, -q^2 = Q^2 = \mathbf{q}^2$$

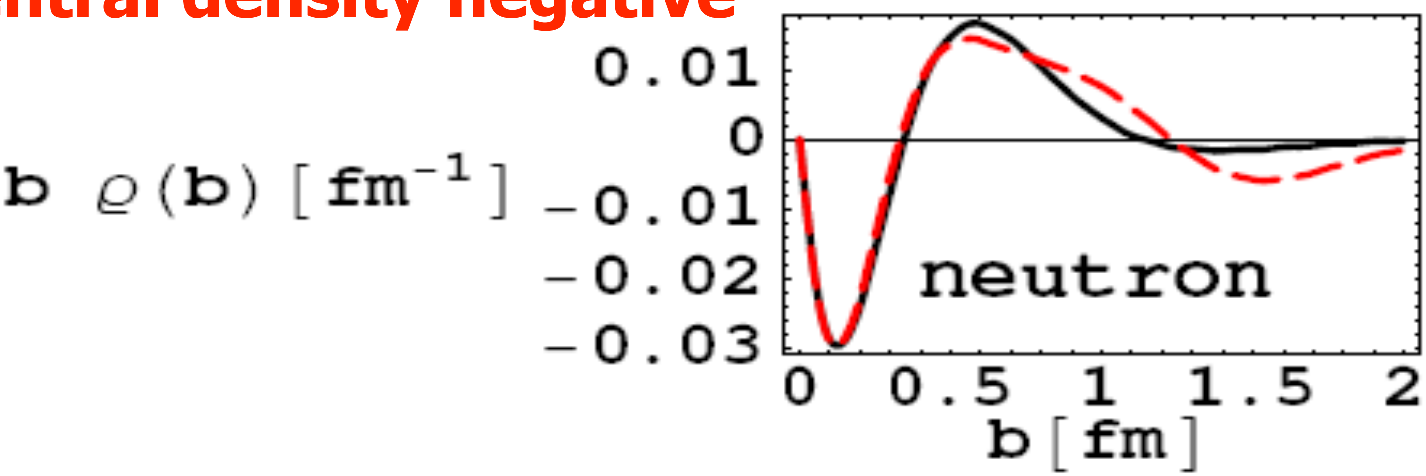
Transverse charge densities



Negative



Negative F_1 means central density negative

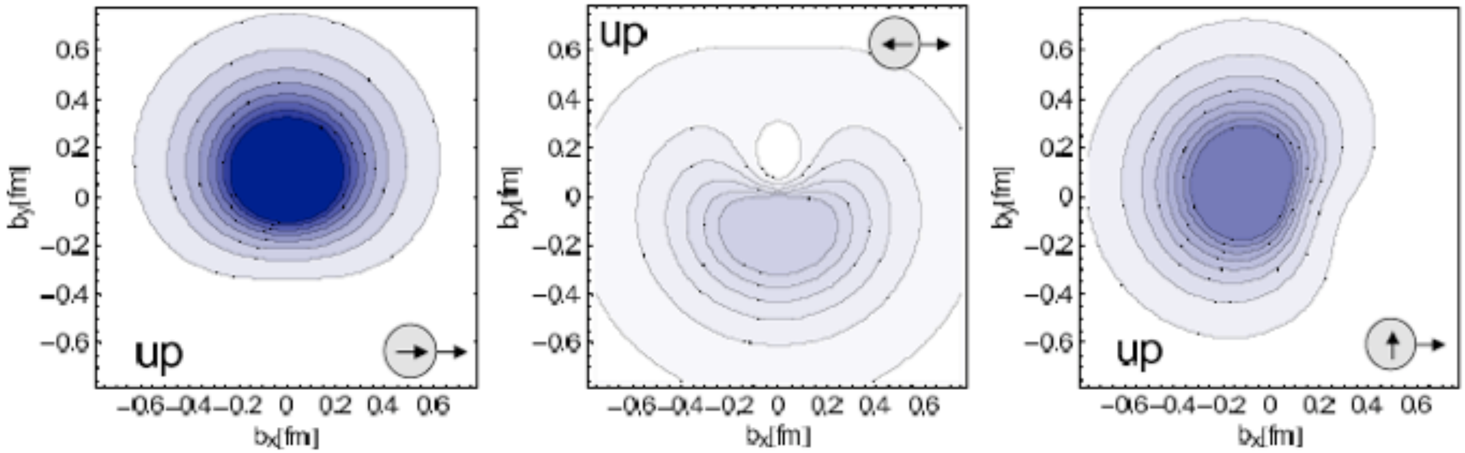


Return of the cloudy bag model

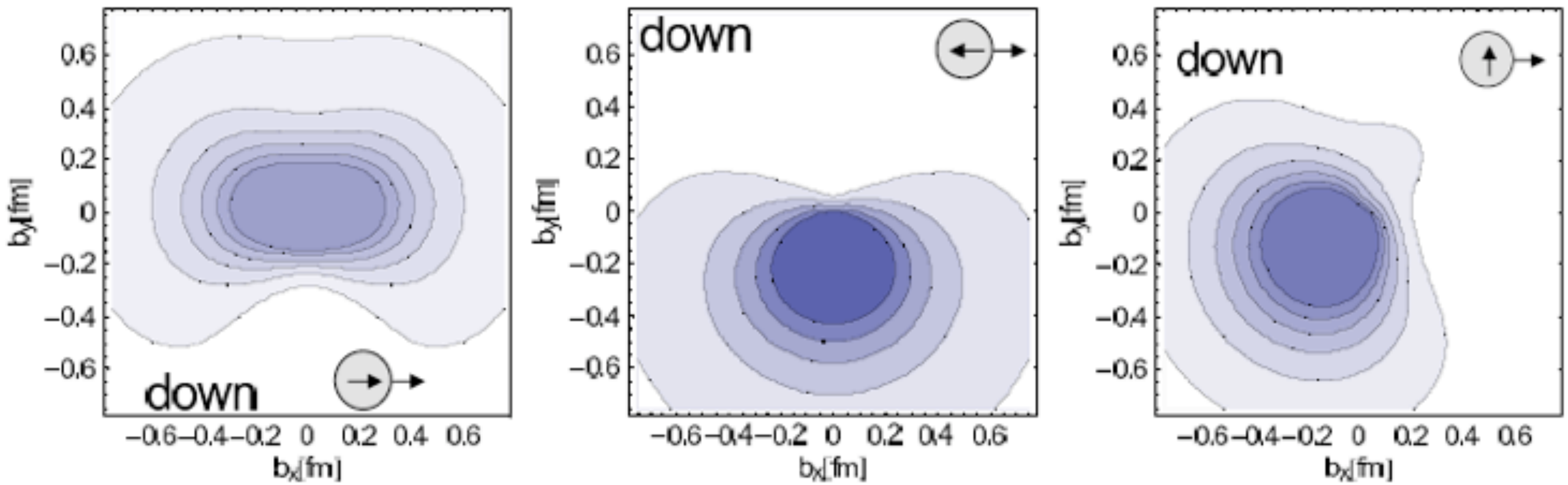
- In a model nucleon:bare nucleon + pion cloud - parameters adjusted to give negative definite F_1, pion at center causes negative central transverse charge density
- Boosting the matrix element of J^0 to the infinite momentum frame changes G_E to F_1

Rinehimer and Miller
PRC80,015201, 025206

Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...



This is not zero!



Transverse Momentum Distributions - momentum space density

In a state of fixed momentum

$\Phi_q^\Gamma(x, \mathbf{K})$ give probability of quark of given 3-momentum

$h_{1T}^\perp$ gives momentum-space spin-dependent density

measurable experimentally

hard to calculate on lattice because - gauge link

Relation or not between GPD and TMD

$$\mathrm{GPD}:$$

$$\langle P', S'| \int \frac{dx^-}{4\pi} \bar{q}(-\frac{x^-}{2}, \mathbf{0})\gamma^+ q(\frac{x^-}{2}, \mathbf{0}) e^{ix\bar{p}^+ x^-} |P, S\rangle_{\, x^+ = 0}$$

$$= \frac{1}{2\bar{p}^+} \bar{u}(P', S') \left(\gamma^+ H_q(\xi, t) + i \frac{\sigma^{+\nu}\Delta_\nu}{2M} E_q(x, \xi, t) \right) u(P, S)$$

$$\mathrm{TMD}:$$

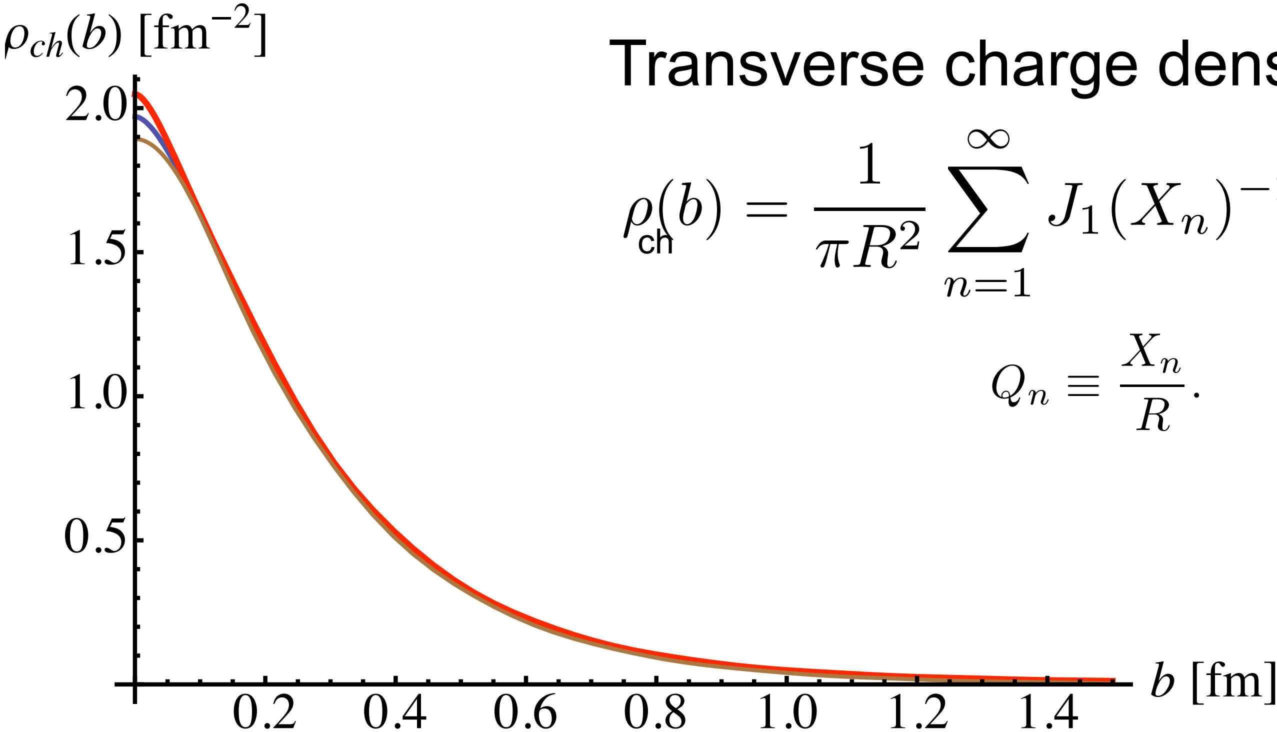
$$\Phi_q^\Gamma(x = \frac{k^+}{P^+}, \mathbf{k}) = \langle P, S| \int \frac{d\zeta^- d^2\zeta}{2(2\pi)^3} e^{ik\cdot\zeta} \bar{q}(0)\Gamma q(\zeta)|P, S\rangle_{\zeta^+=0}$$

GPD: nucleons have different momenta, but FT local in coordinate space if integrate over x

TMD: nucleons have same momenta, operator is local in momentum space

How well are these known now?

- Analyze effect of experimental errors and errors due to finite range of Q^2



Transverse charge density

$$\rho_{ch}(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} J_1(X_n)^{-2} F_1(Q_n^2) J_0(X_n \frac{b}{R}),$$

$$Q_n \equiv \frac{X_n}{R}.$$

Venkat, Arrington, Miller, Zhan **arXiv:1010.3629**

Both can be obtained Wigner distribution operator

$$W_q^\Gamma(\zeta^-, \boldsymbol{\zeta}, k^+, \mathbf{k})$$

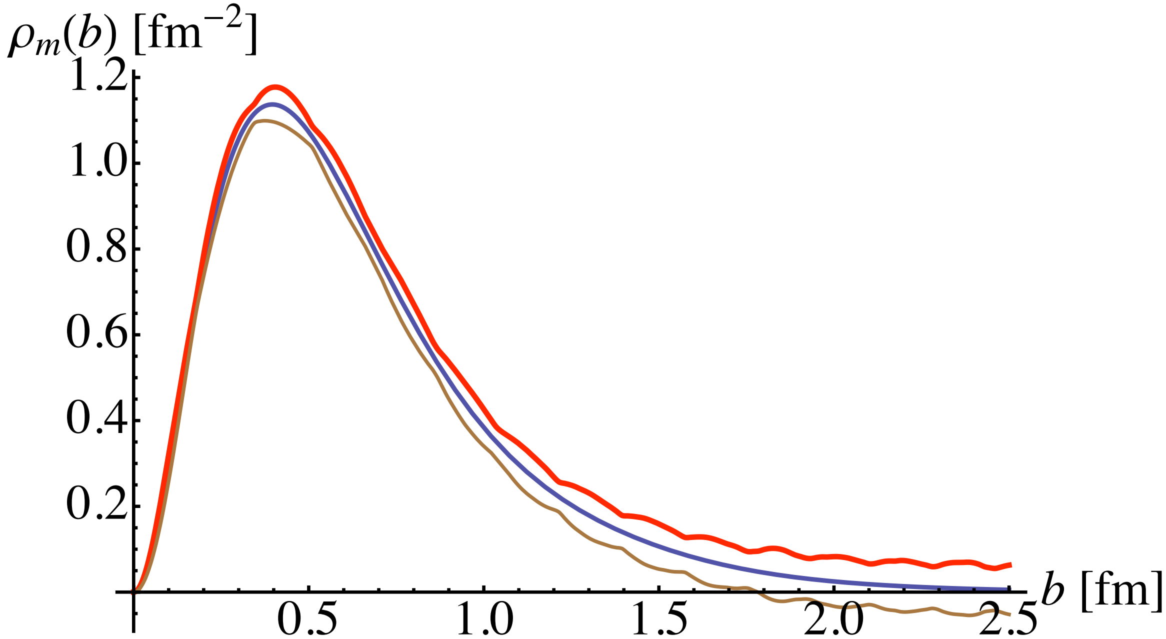
$$= \frac{1}{4\pi}\int d\eta^- d^2\eta e^{ik\cdot\eta}\bar{q}(\zeta^- - \frac{\eta^-}{2}, \boldsymbol{\zeta} - \frac{\boldsymbol{\eta}}{2})\Gamma q(\zeta^- + \frac{\eta^-}{2}, \boldsymbol{\zeta} + \frac{\boldsymbol{\eta}}{2})$$

$$H_q(x,\xi,t) = \langle P', S'| \int \frac{d^2\mathbf{k}}{(2\pi)^2} W_q^{\gamma^+}(\zeta^- = 0, \zeta = 0, k^+, \mathbf{k})|P, S\rangle$$

$$\Phi_q^\Gamma(x,\mathbf{k}) = \langle P, S| \int \frac{d\zeta^-}{(2\pi)^2} W_q^\Gamma(\zeta^-, \zeta, k^+, \mathbf{k})|P, S\rangle$$

Anomalous magnetization density

$$\rho_m^{FRA} = \frac{1}{\pi R_2^2} \sum_{n=1}^{\infty} J_2^{-2}(X_{1,n}) b Q_{1,n} F_2(Q_{1,n}^2) J_1(Q_{1,n} b), \; Q_{1,n} \equiv \frac{X_{1,n}}{R_2}$$



Summary

- **Form factors, GPDs, TMDs, understood from unified light-front formulation**
- **Neutron central transverse density is negative-consistent with Cloudy Bag Model**
- **Proton is not round- lattice QCD spin-dependent-density is not zero**
- **Experiment can whether or not proton is round by measuring** $h_{1T}^{\perp}$

Summary

- **Form factors, GPDs, TMDs, understood from unified light-front formulation**
- **Neutron central transverse density is negative- consistent with Cloudy Bag Model**
- **Proton is not round- lattice QCD spin-dependent- density is not zero**
- **Experiment can whether or not proton is round by measuring** $h_{1T}^{\perp}$

The Proton

Cloudy Bag Model~1980

PHYSICAL REVIEW D VOLUME 24, NUMBER 1 1 JULY 1981

Cloudy bag model of the nucleon

A. W. Thomas and S. Théberge
TRIUMF, University of British Columbia, Vancouver, British Columbia, Canada V6T2A3

Gerald A. Miller
Institute for Nuclear Theory and Physics Department, FM-15, University of Washington, Seattle, Washington 98195

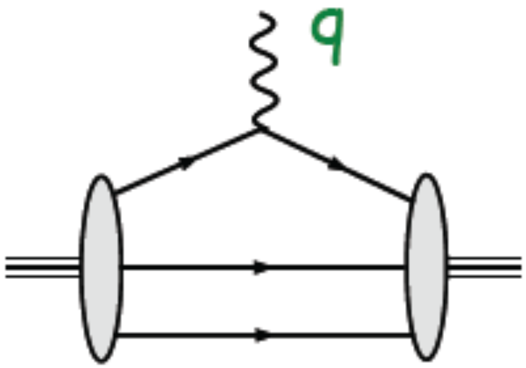
(Received 28 January 1981)

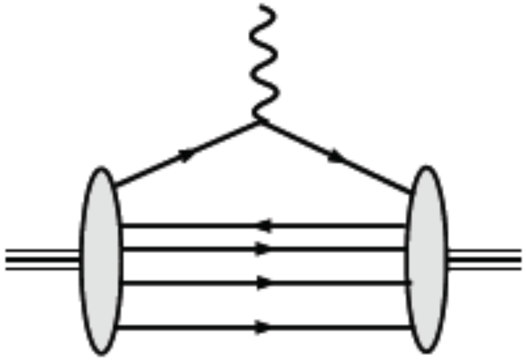
A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

Many successful predictions

One feature- pion penetrates to the bag interior

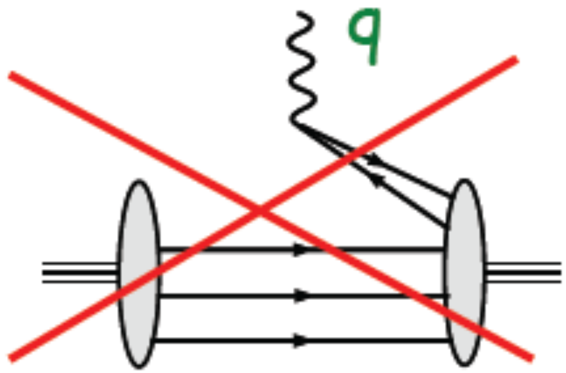
interpretation of FF as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

Absent in a Drell-Yan Frame

$$q^+ = q^0 + q^3 = 0$$

From Marc Vanderhaeghen