Connections between Neutron-Rich Matter and Neutron Stars

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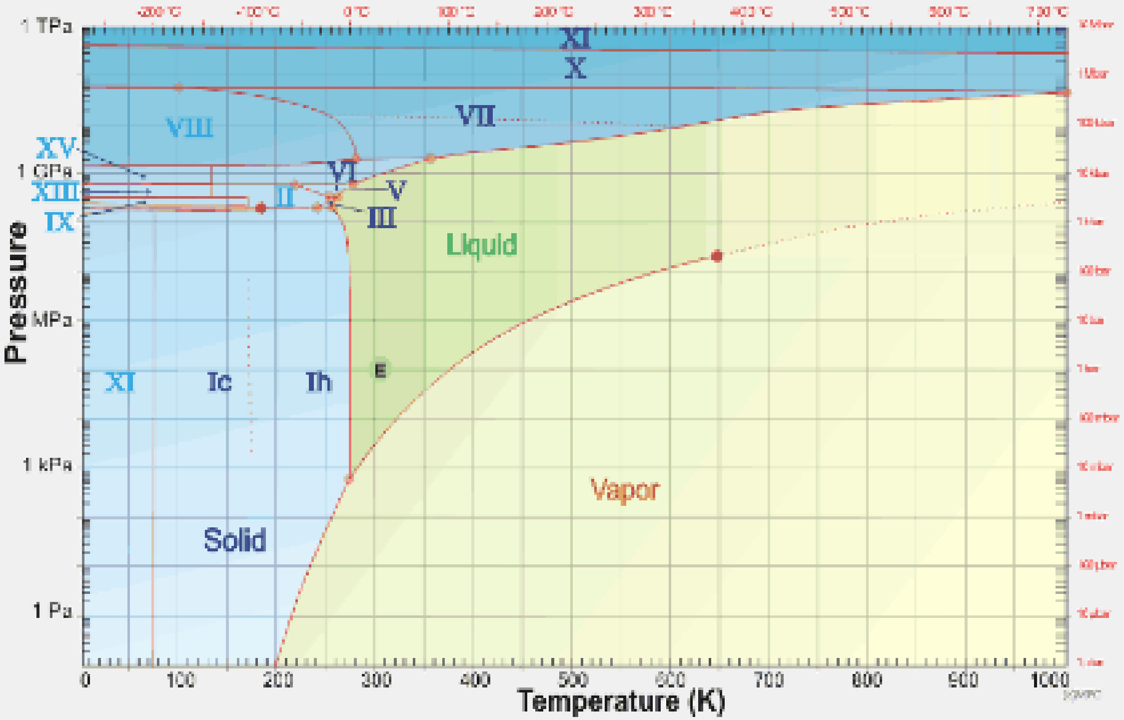
Based on work with E. F. Brown, J. M. Lattimer, and M. Prakash

Outline

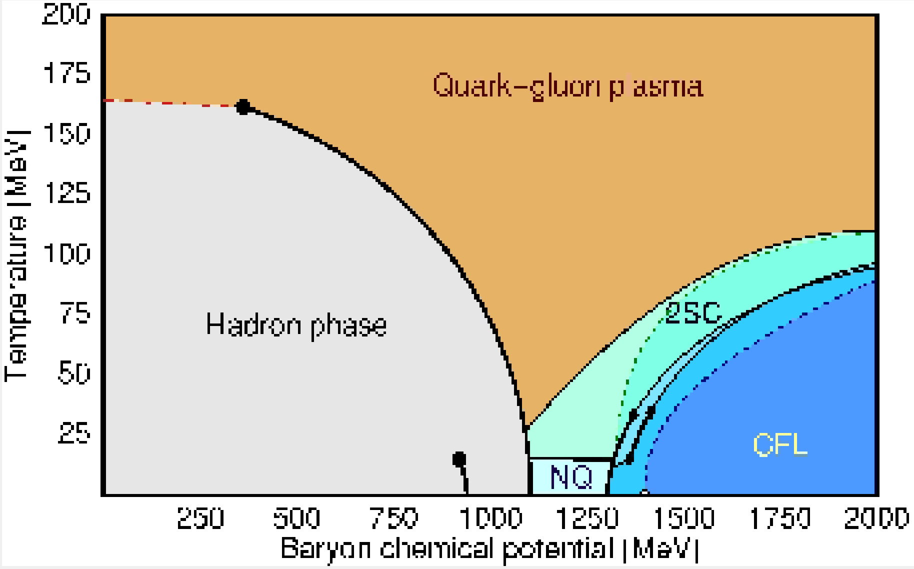
- Neutron stars and the symmetry energy
- Pressure, the EOS, and the neutron skin thickness of lead
- Neutron star radii
- Bayesian inference
- Fastest sound in the universe
- Neutron star cooling and direct Urca
- Quartic terms, short-range correlations, and superfluidity

The QCD phase diagram

- QCD: The theory which describes the interactions of nucleons and quarks



http://www.lsbu.ac.uk/water/phase.html

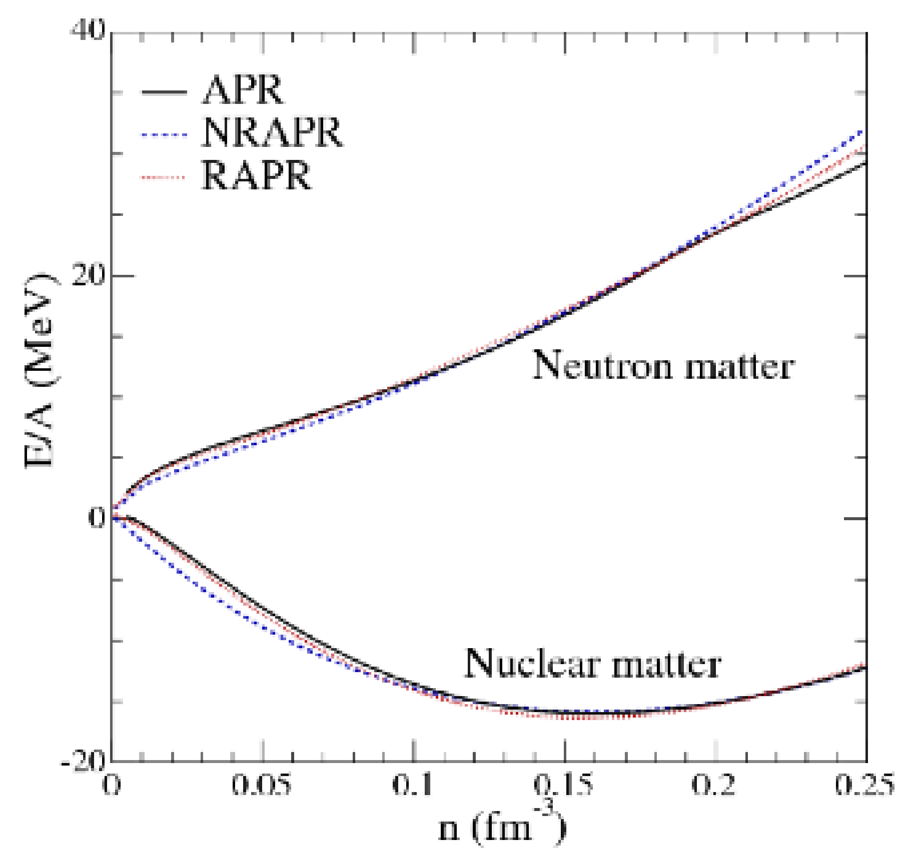


Rüster et al. (2006)

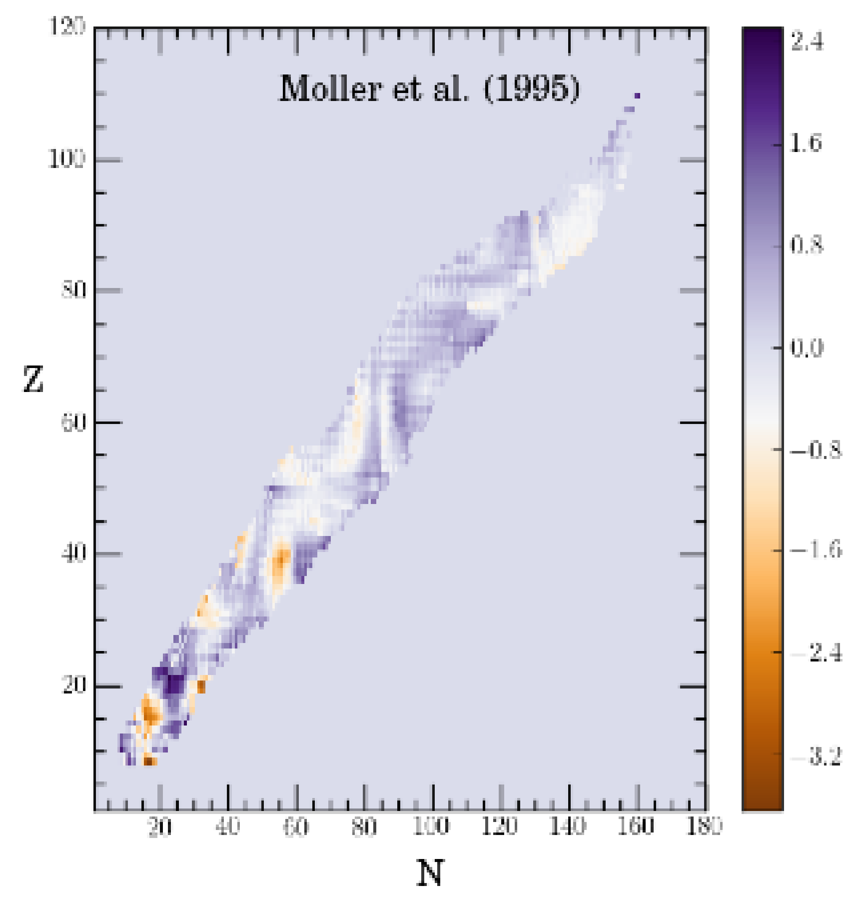
- Heavy-ion collisions and lattice QCD sensitive primarily to high T, low μ regions

- Electromagnetic and gravitational wave observations of neutron star-related phenomena are the **best** probe of cold, dense (and non-perturbative) QCD.

The Nuclear Symmetry Energy



Steiner et al. (2005)

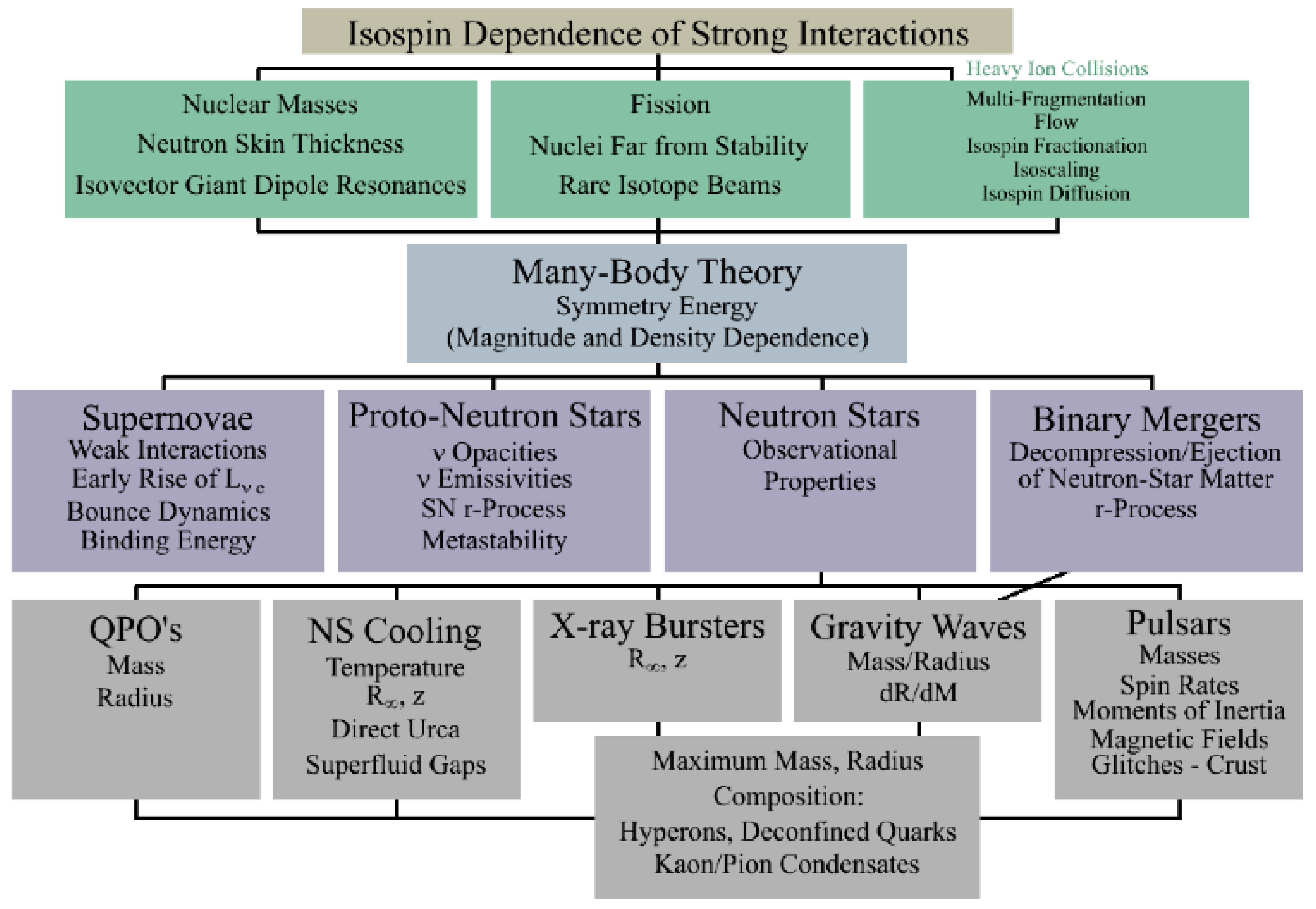


Comparison of Moller et al. masses to experiment

- $S(n_B) \equiv E_{\mathrm{neut}}(n_B) - E_{\mathrm{nuc}}(n_B)$
- S is the value at the nuclear saturation density $S = S(n_0)$
- L is the derivative, $L = 3n_0 S'(n_0)$
- $P(n_0, x = x_\beta) \approx n_0^2 S'(n_0)$

  Lattimer and Prakash (2007)

Bridging Nuclear and Astrophysics



Isospin Dependence of Strong Interactions

Nuclear Masses
Neutron Skin Thickness
Isovector Giant Dipole Resonances

Fission
Nuclei Far from Stability
Rare Isotope Beams

Heavy Ion Collisions
Multi-Fragmentation
Flow
Isospin Fractionation
Isoscaling
Isospin Diffusion

Many-Body Theory
Symmetry Energy
(Magnitude and Density Dependence)

Supernovae
Weak Interactions
Early Rise of $L_{\nu e}$
Bounce Dynamics
Binding Energy

Proto-Neutron Stars
ν Opacities
ν Emissivities
SN r-Process
Metastability

Neutron Stars
Observational
Properties

Binary Mergers
Decompression/Ejection
of Neutron-Star Matter
r-Process

QPO's
Mass
Radius

NS Cooling
Temperature
R_∞, z
Direct Urca
Superfluid Gaps

X-ray Bursters
R_∞, z

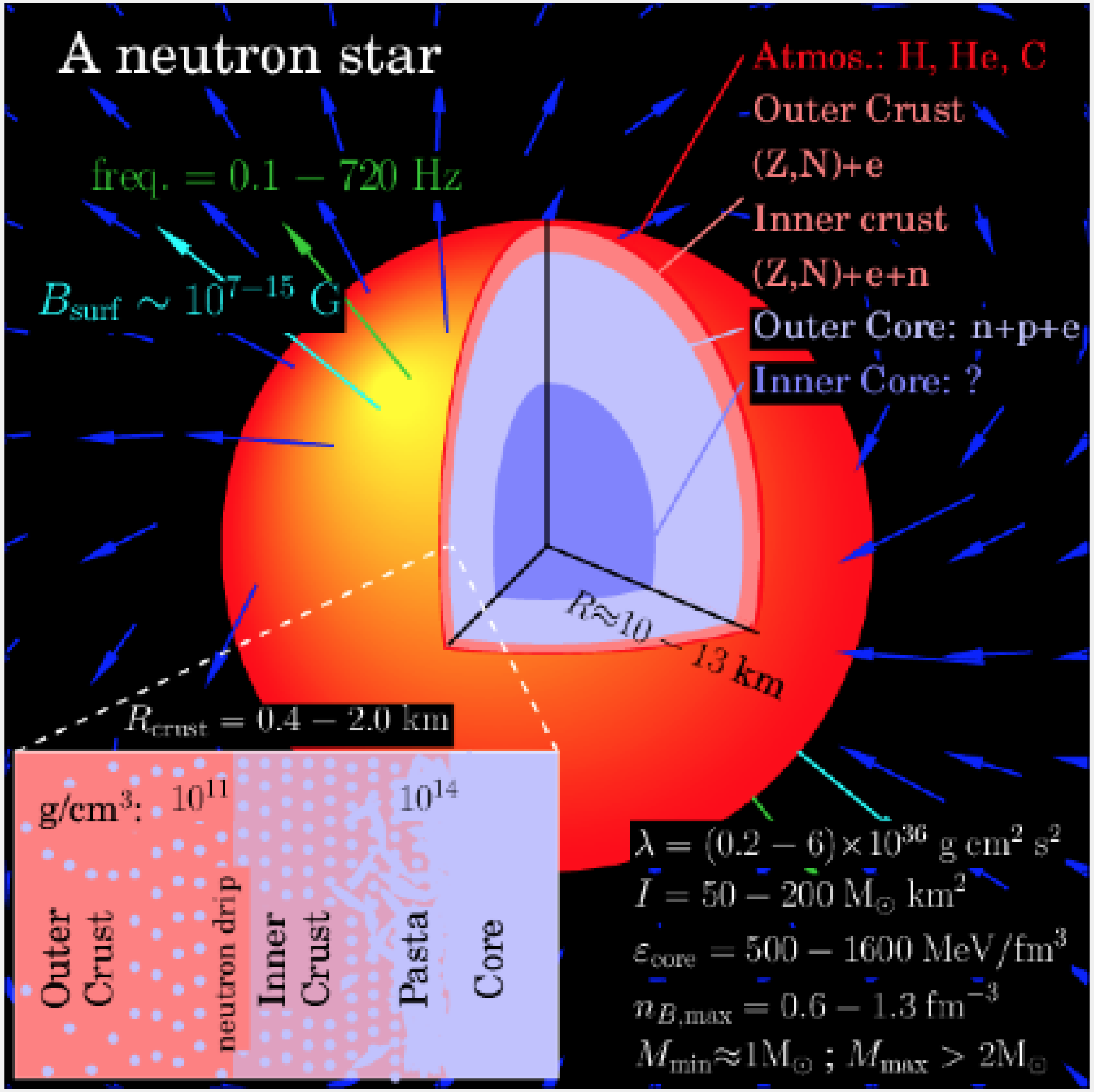
Gravity Waves
Mass/Radius
dR/dM

Pulsars
Masses
Spin Rates
Moments of Inertia
Magnetic Fields
Glitches - Crust

Maximum Mass, Radius
Composition:
Hyperons, Deconfined Quarks
Kaon/Pion Condensates

Steiner, et al. (2005)

Neutron Star Composition



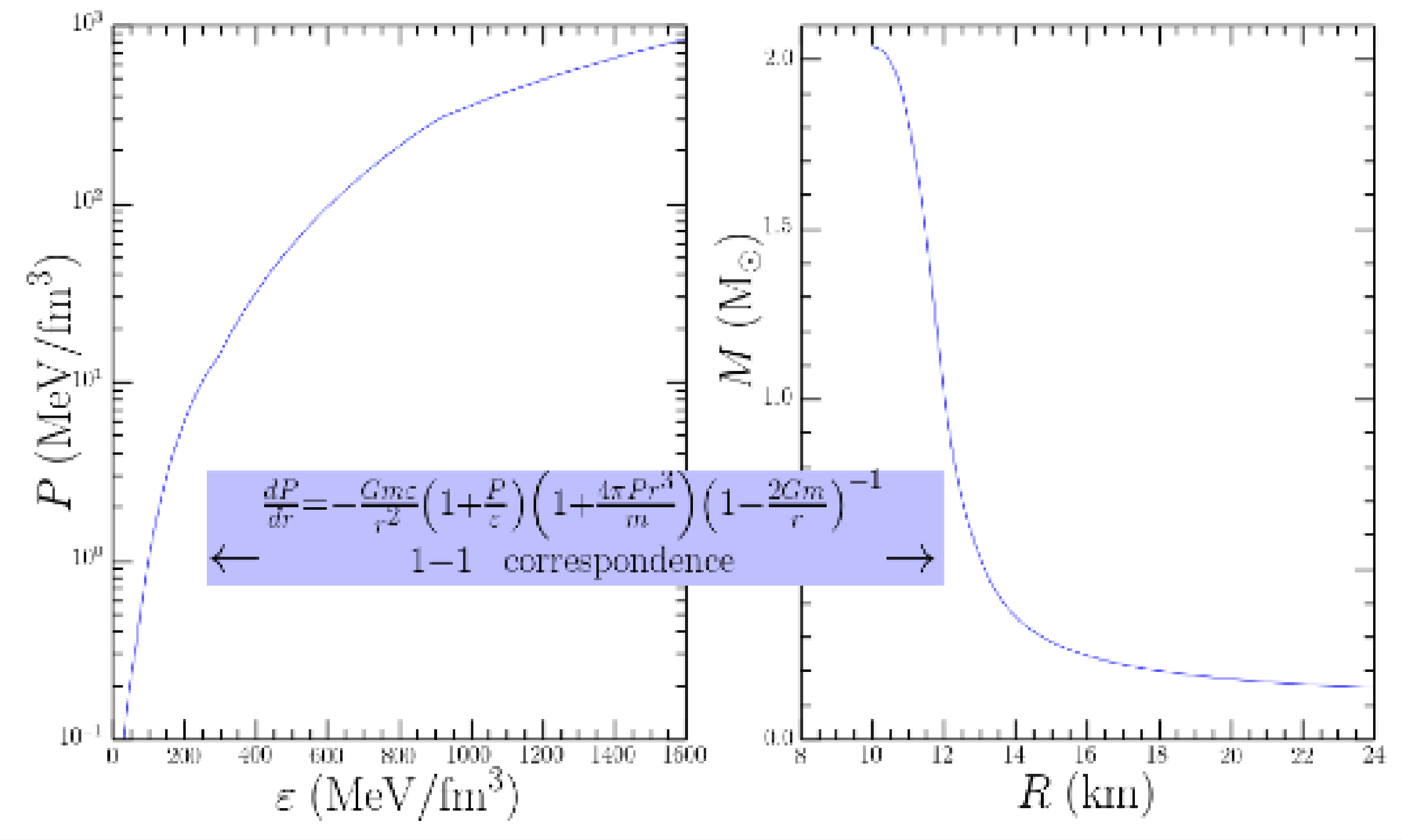
Inspired by D. Page; open source (python)

- Outer crust: of neutron-rich nuclei $\mu_n = \mu_p + \mu_e$
- Inner crust: neutron-rich nuclei embedded in a sea of quasi-free superfluid neutrons
- Outer core: fluid of neutrons, protons, and electrons
- Inner core: hyperons? Bose condensates? deconfined quark matter?

What are the correct degrees of freedom for the effective field theory which describes dense matter?

Neutron Star Masses and Radii and the EOS

- Neutron stars (to better than 10%) all lie on one universal mass-radius curve
  (Largest correction is rotation)

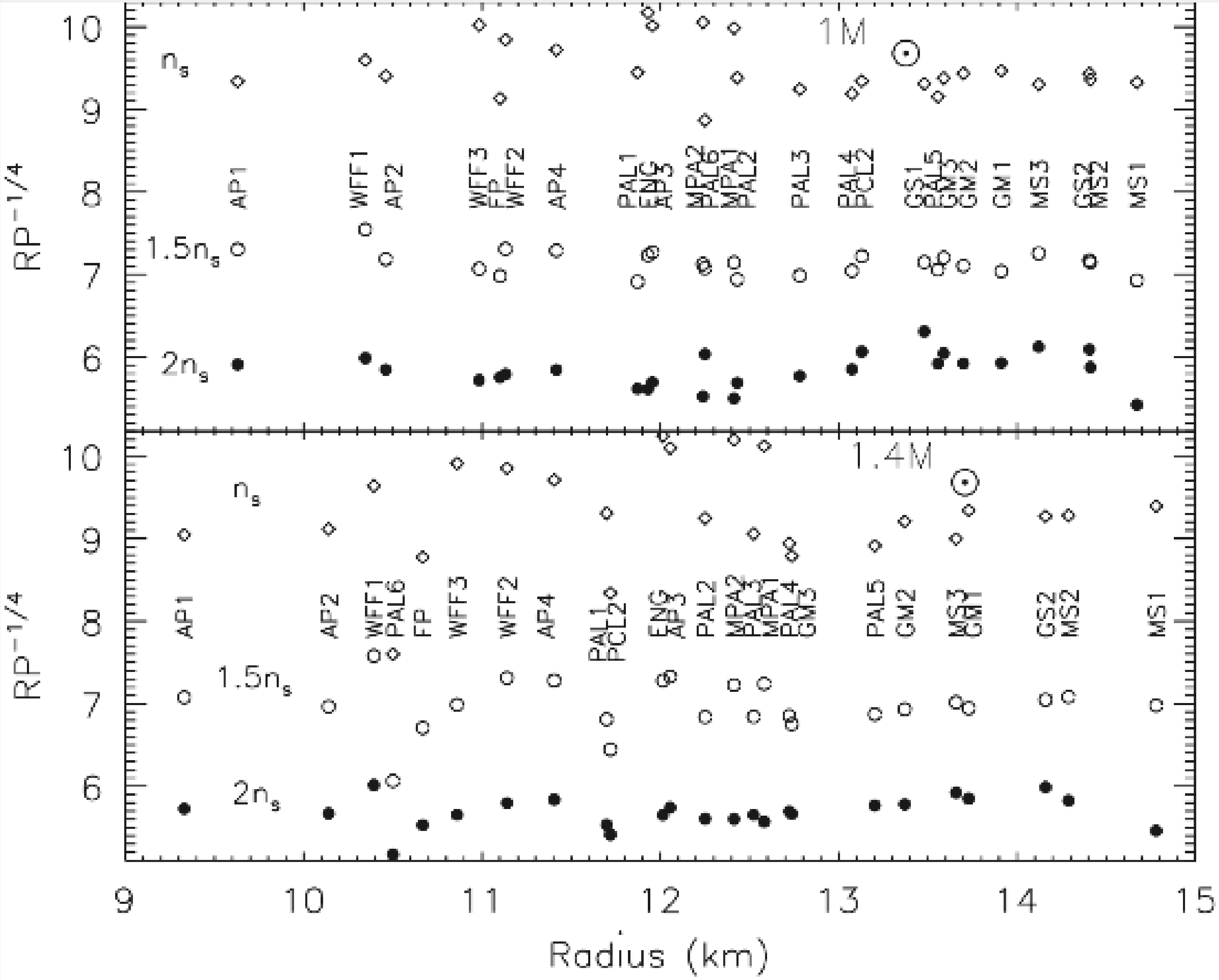


$$\frac{dP}{dr}=-\frac{Gm\varepsilon}{r^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi P r^3}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

← 1–1 correspondence →

- As of 2007, neutron star radii ranged from 8-16 km
  Lattimer and Prakash (2007)
- Two 2 $M_\odot$ neutron stars
  (important for later)

Correlation Between Pressure and Radius



Lattimer and Prakash (2001)

- Radius is particularly sensitive to the pressure of neutron-star matter
- Caveat emptor: this is still model dependent

Correlations with the Skin Thickness of Lead

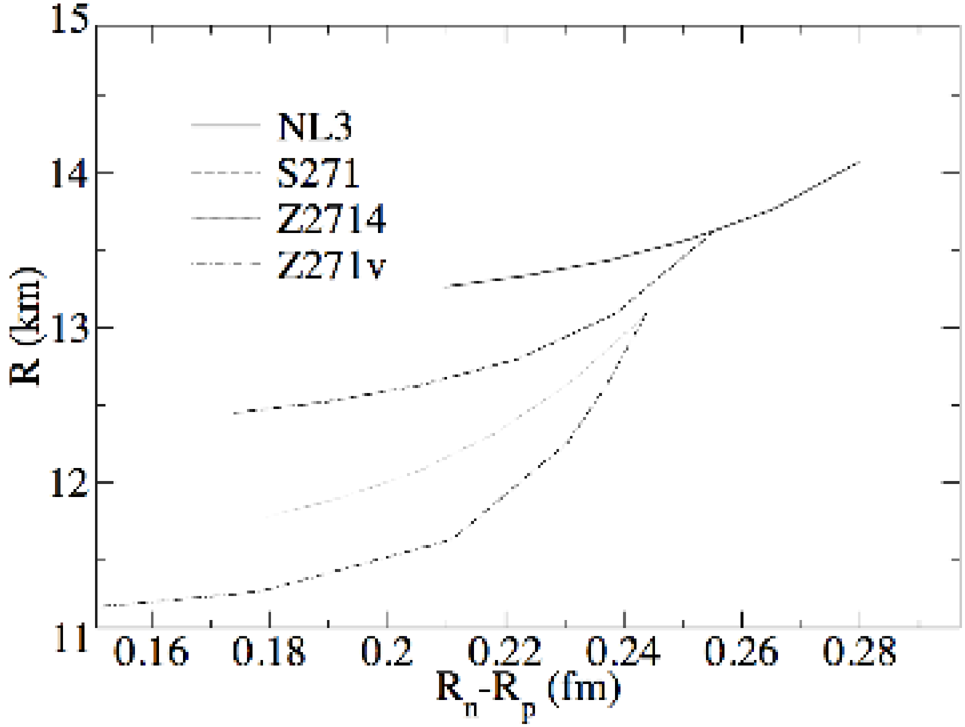
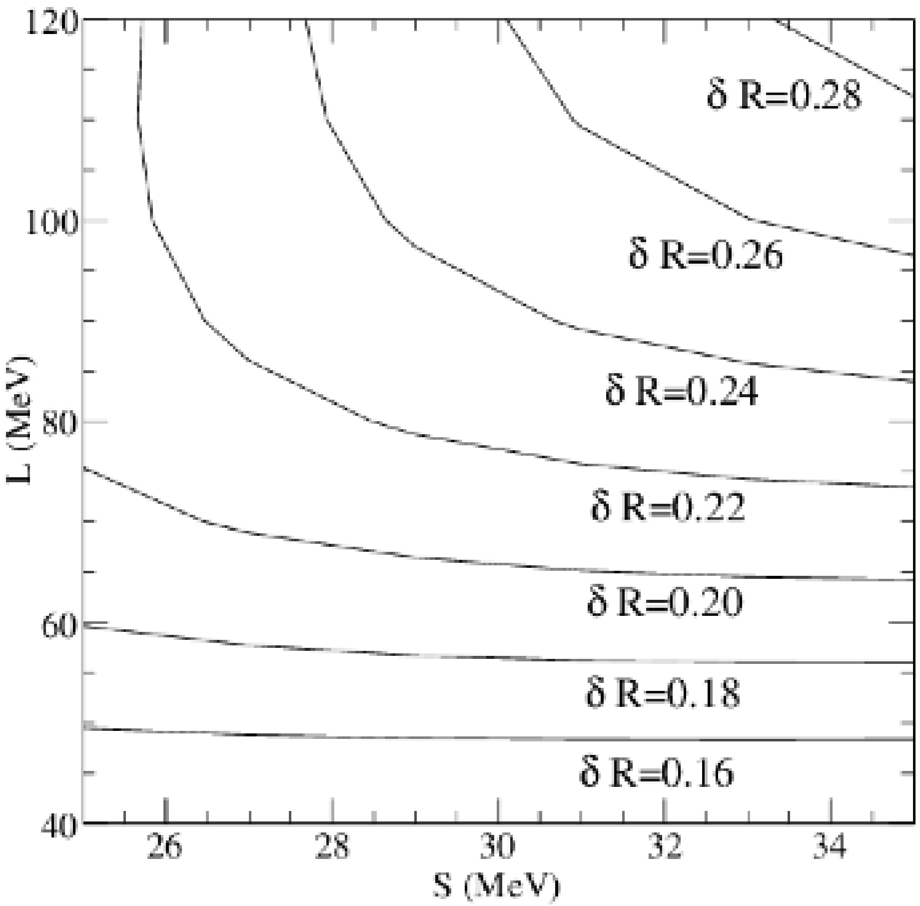


FIG. 3. Radius of a 1.4 solar mass neutron star versus neutron-minus-proton radius in ^{208}Pb for the four parameter sets described in the text.

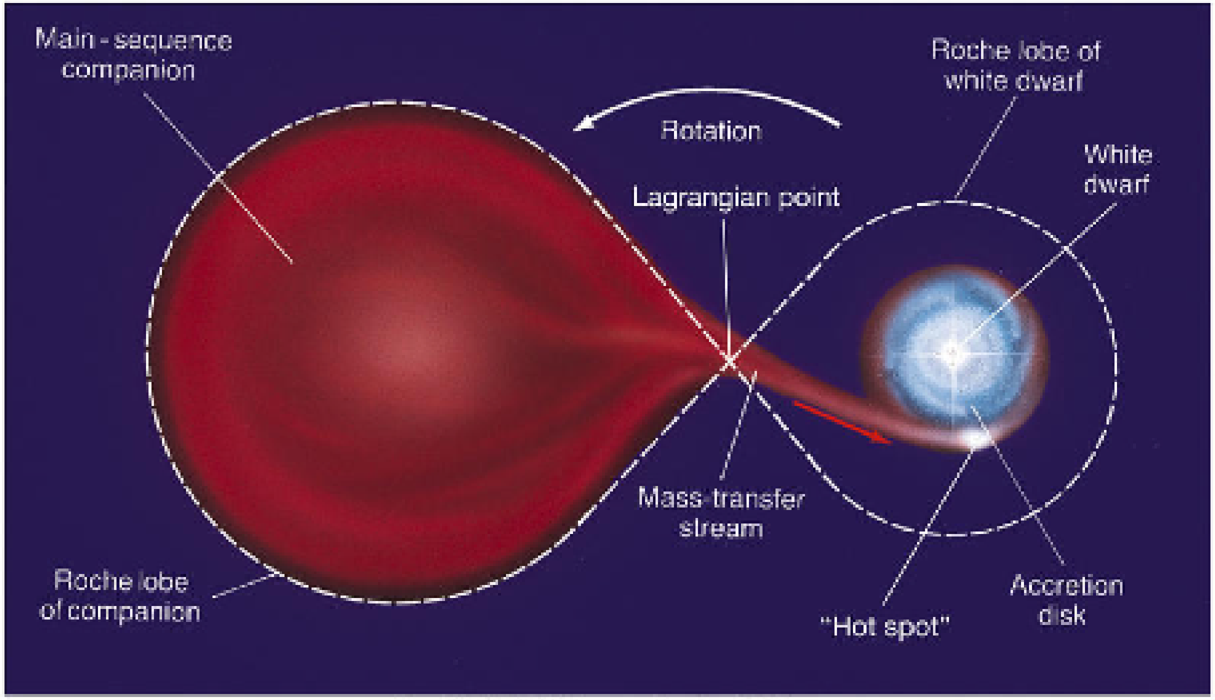
Horowitz and Piekarewicz (2001), but taken from the arXiv v1.



Steiner, unpublished work based on a droplet model

- Skin thickness correlated with neutron star radii
- Skin thickness correlated with L
- These lines are fuzzier in reality – model dependence

Low-mass X-ray binaries



- H and He accreted is unstable
- Accretion is unstable and sporadic
- X-ray burst, burns H and He to heavier elements

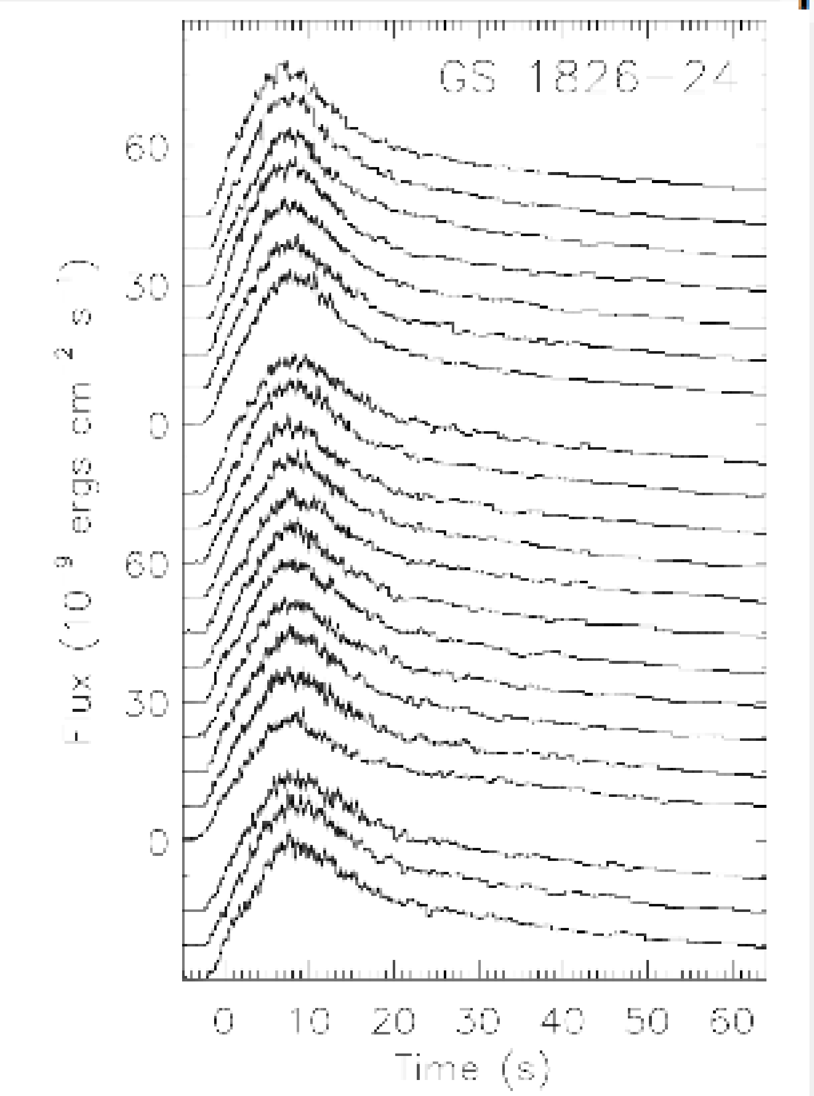


FIG. 1.— Profiles of 20 X-ray bursts from GS 1826−24 observed by *RXTE* between 1997–2002, plotted with varying vertical offsets for clarity. The upper group of 7 bursts were observed in 1997–98, the middle group of 10 bursts in 2000, while the lower group of 3 were observed in 2002. The bursts from each epoch have been time-aligned by cross-correlating the first 8 seconds of the burst. Error bars indicate the 1σ uncertainties.

X-ray bursts from GS 1826-24 from Galloway et al. (2004)

Radius Measurements in qLMXBs

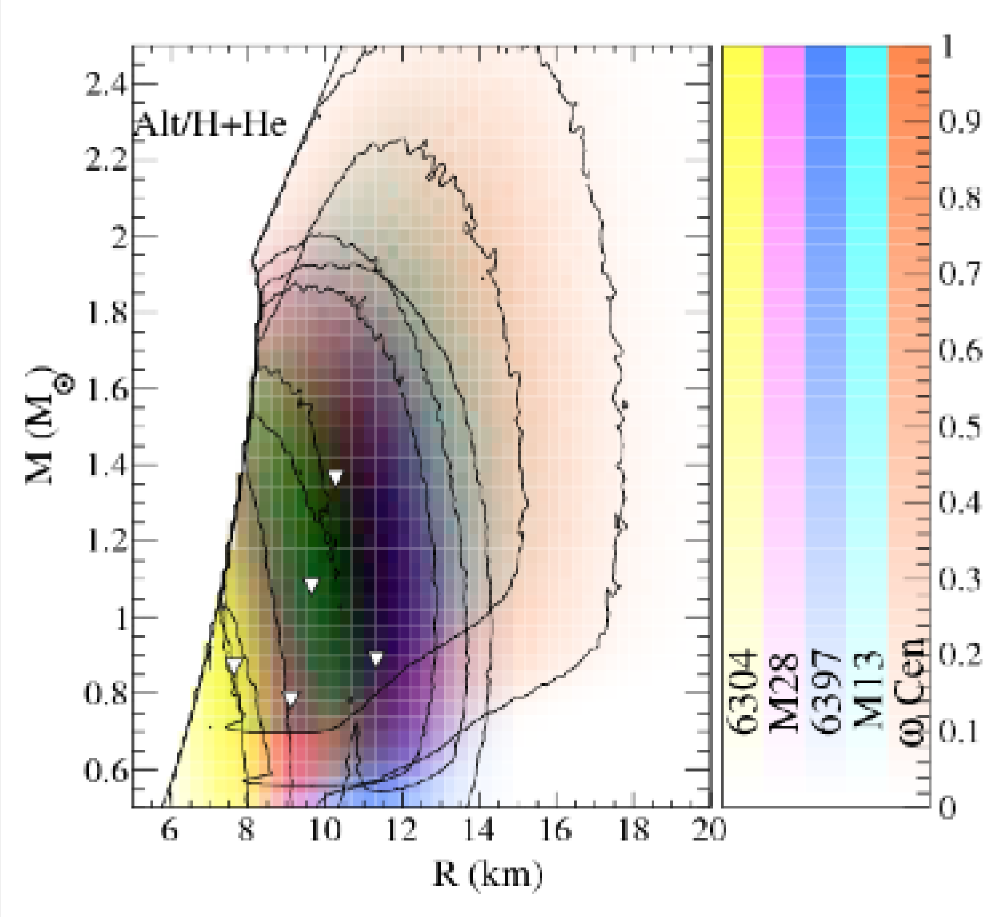
Quiescent LMXBs

- Blackbody-like spectrum of X-rays

$$F \propto T_{\rm eff}^4 \left(\frac{R_\infty}{D} \right)^2$$

i.e. Rutledge et al. (1999)

- Measure flux of photons and their energy distribution
- Know distance if in a globular cluster
- Neutron star atmosphere calculation needed to determine $T/T_{\rm eff}$



Lattimer and Steiner (2013) - Probability distributions for five neutron stars, colors added together

- Between 8 and 12 radius data points all together

Dominated by Systematic Uncertainties

- Composition of neutron star atmosphere
- Absorption of X-rays by interstellar matter
    - Connection to galatic chemical evolution: X-ray absorption impacted by model of solar system abundances
- Distance measurement difficulties
- X-ray detector calibration
- Model-dependent extraction of M-R curve from data

- Current astrophysical constraints on the neutron skin in lead are limited by uncontrollable systematics

Role of Bayesian Inference

- Underconstrained problem!

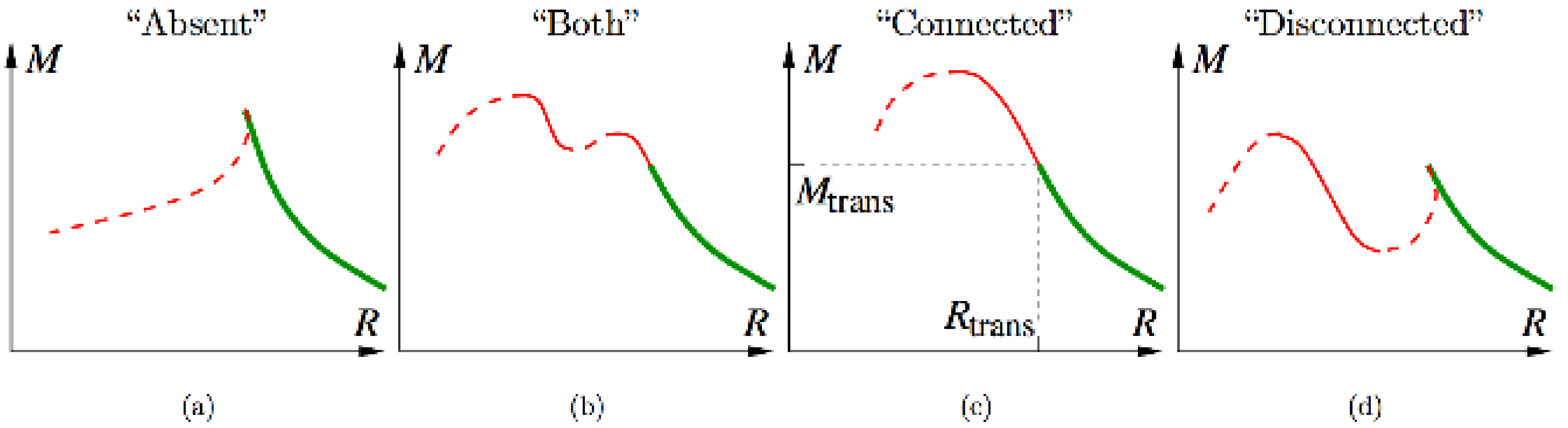


FIG. 2: Four possible topologies of the mass-radius relation for hybrid stars. The thick (green) line is the hadronic branch. Thin solid (red) lines are stable hybrid stars; thin dashed (red) lines are unstable hybrid stars. In (a) the hybrid branch is absent. In (c) there is a connected branch. In (d) there is a disconnected branch. In (b) there are both types of branch. In realistic neutron star $M(R)$ curves, the cusp that occurs in cases (a) and (d) is much smaller and harder to see [13, 14]

Alford et al. (2013)

- Bayes theorem: $P[\mathcal{M}_i|D] = \dfrac{P[D|\mathcal{M}_i]P[\mathcal{M}_i]}{\sum_j P[D|\mathcal{M}_j]P[\mathcal{M}_j]}$
- Determine parameters through marginalization, i.e.

$$P(\mathcal{M}_i^0) = \int \delta(\mathcal{M}_i - \mathcal{M}_i^0) P[D|\mathcal{M}_i]P[\mathcal{M}_i]\, d\mathcal{M}$$

- Choice of prior distribution introduces additional model dependence

Extrapolation: The Tail Wags the Dog

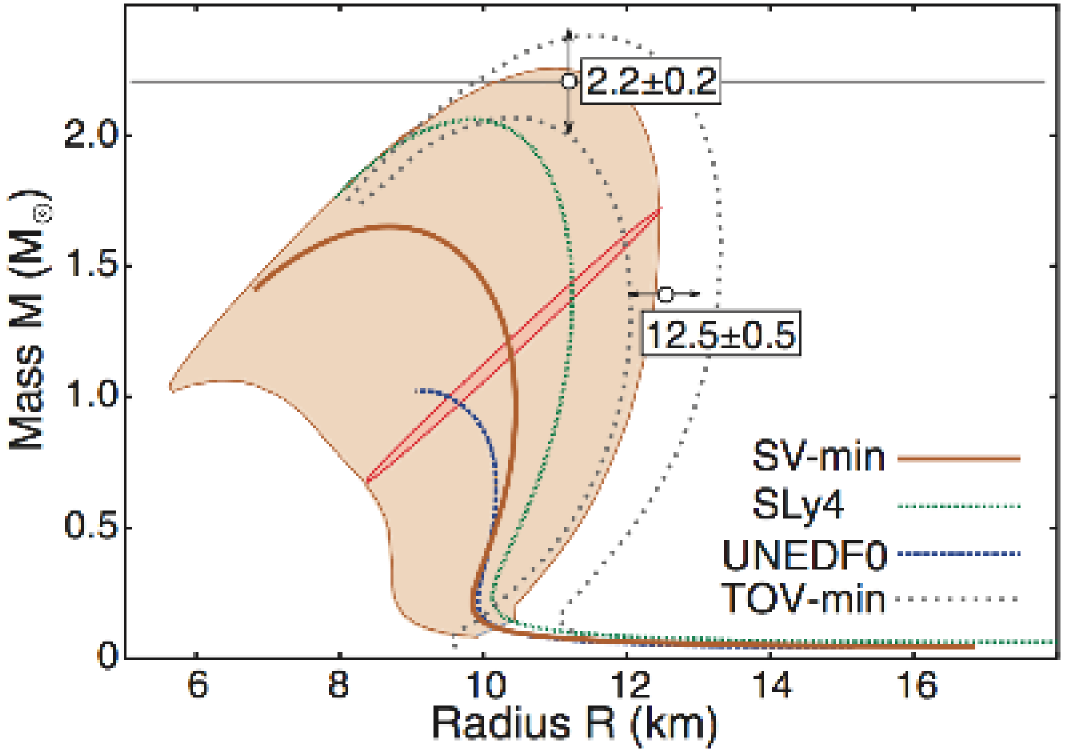
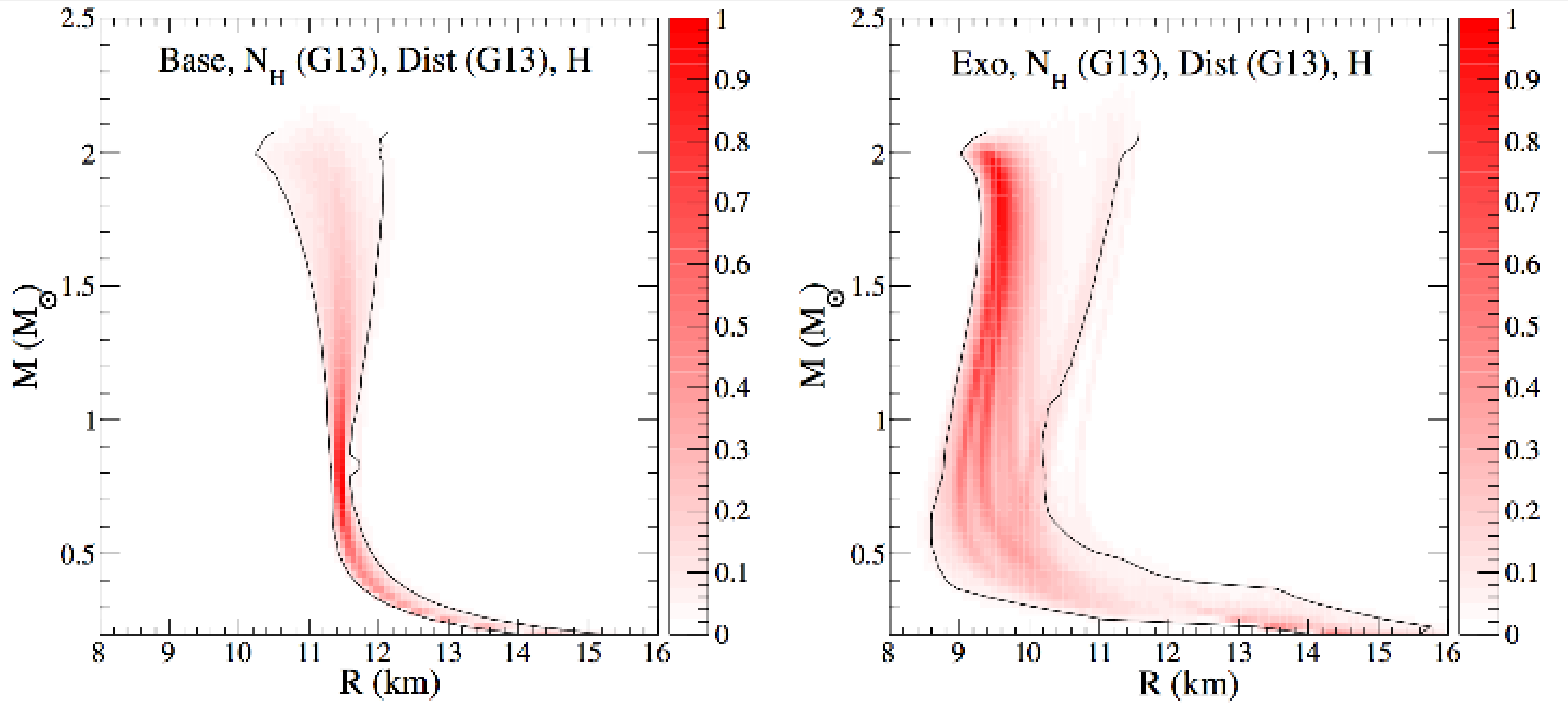


FIG. 2. (Color online) Mass-radius relation of SLy4 [1], UNEDF0 [22] and SV-min [24]. The uncertainty band for SV-min is shown. This band is estimated by calculating the covariance ellipsoid for the mass M and the radius R at each point of the SV-min curve as indicated by the ellipsoid. Also depicted (dotted lines) are uncertainty limits for TOV-min.

Erler et al. (2013)

- Many works extrapolate models of the nucleon-nucleon interaction up to very large densities
- Bayesian inference can alleviate this problem
- Arrange prior distribution to properly reweigh the dog and its tail
- Important issue when fitting disparate data sets to one model
- Related to "overfitting"

Presence of Phase Transitions Above Saturation



Lattimer and Steiner (2014)

- Choice of prior distribution has a significant affect on radius, even after fixing data
- Phase transitions tend to spoil relationship between radius and L
- Demonstration of the danger of extrapolations to high density

Implications for L and radii

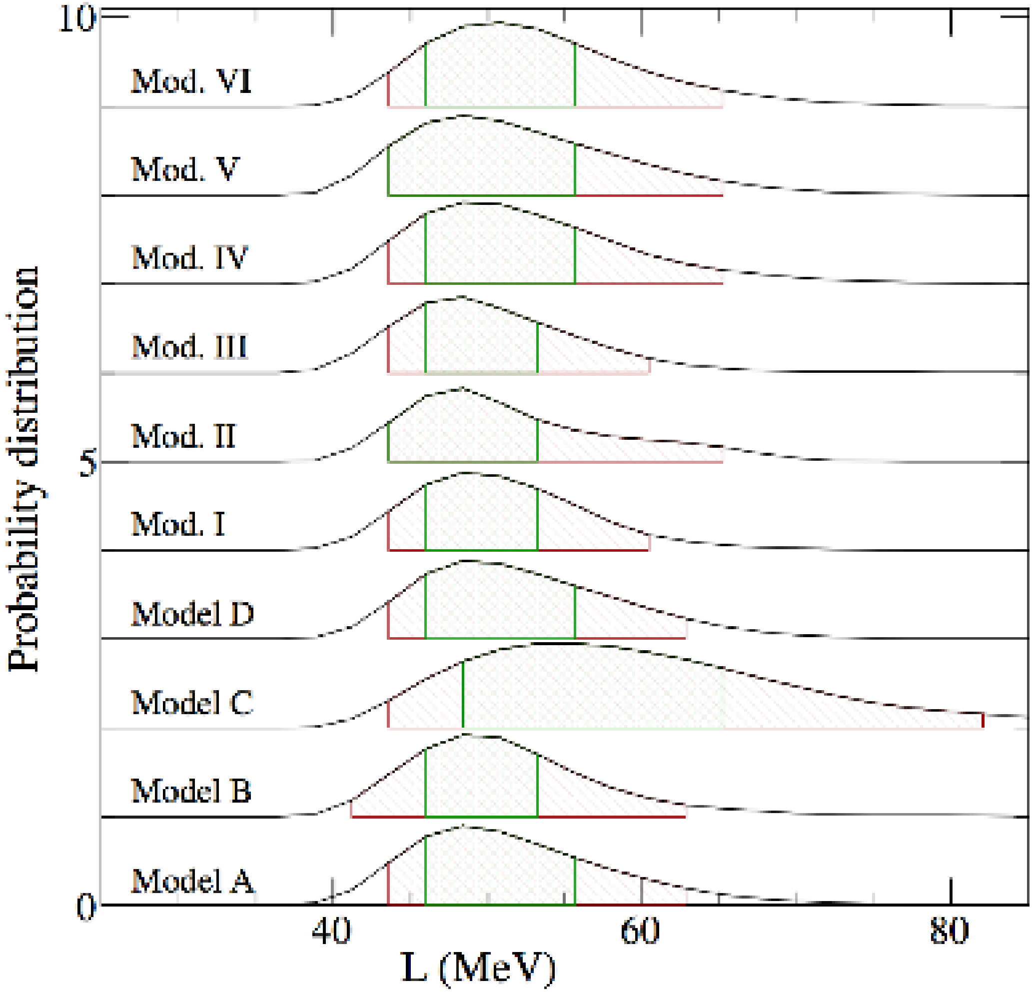


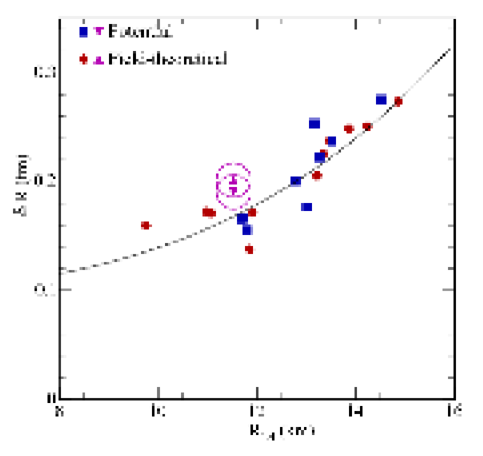
Figure 4. Limits on the density derivative of the symmetry energy, *L*. The single-hatched (red) regions show the 95% confidence limits and the double-hatched (green) regions show the 68% confidence limits.

Steiner et al. (2013)

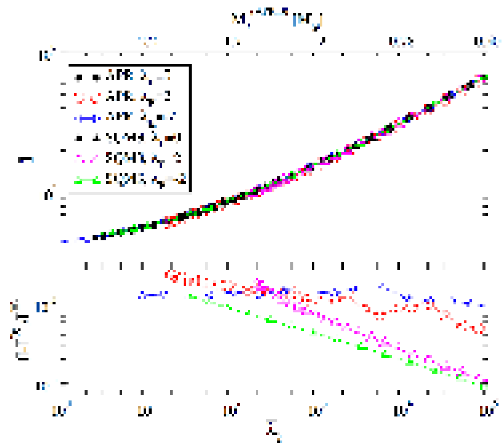
- Trying several different prior distributions
- Model C prefers strong phase transitions
- Less certain constraints on the skin thickness of lead
- Radius of a 1.4 $M_\odot$ neutron star is about 10.5 - 13 km

Representative Models

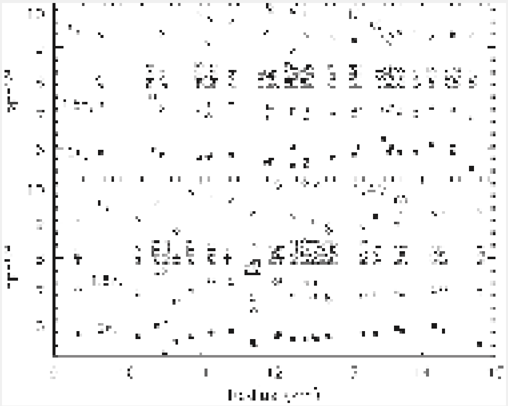
- It has been common to use a small set of models to represent a large parameter space



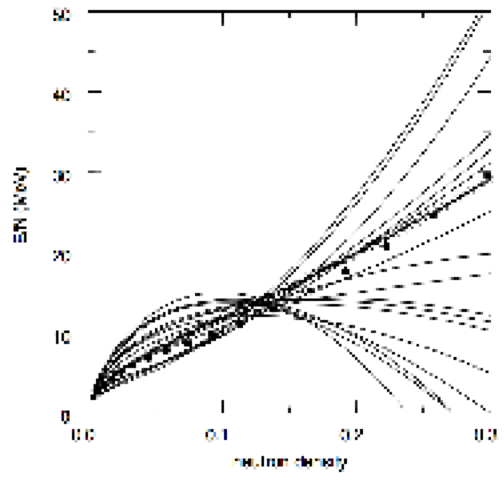
Steiner et al. (2005)



Yagi and Yunes (2015)



Lattimer and Prakash (2001)



Brown (2000)

- This is like doing a many-dimensional Monte Carlo integral with only a handful of haphazard points

- New computational power has allowed for more complete explorations of parameter space

Speed of sound

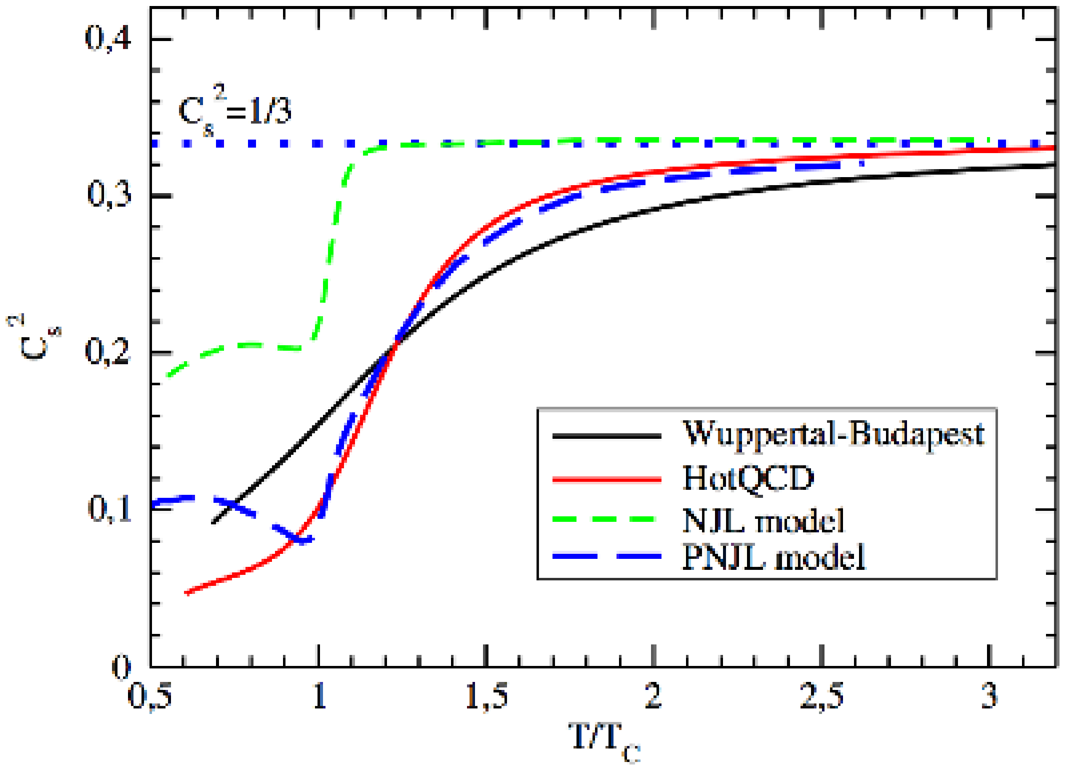


FIG. 4 (color online). Speed of sound squared as a function of T/T_C. The red solid curve corresponds to the HotQCD data, the black solid one to the Wuppertal-Budapest data, the short-dashed one to the NJL model, and the long-dashed one to the Polyakov loop extended NJL model fit to the HotQCD lattice data.

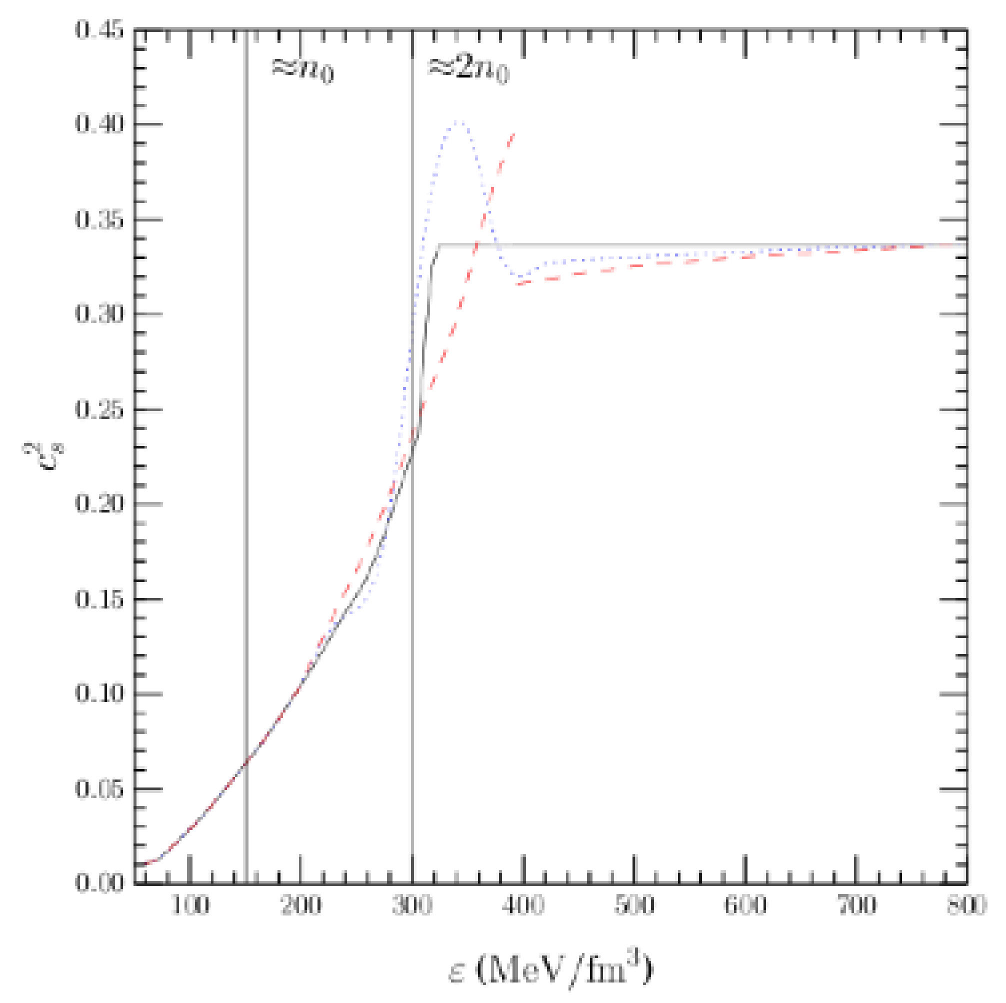
Plumari et al. (2011)

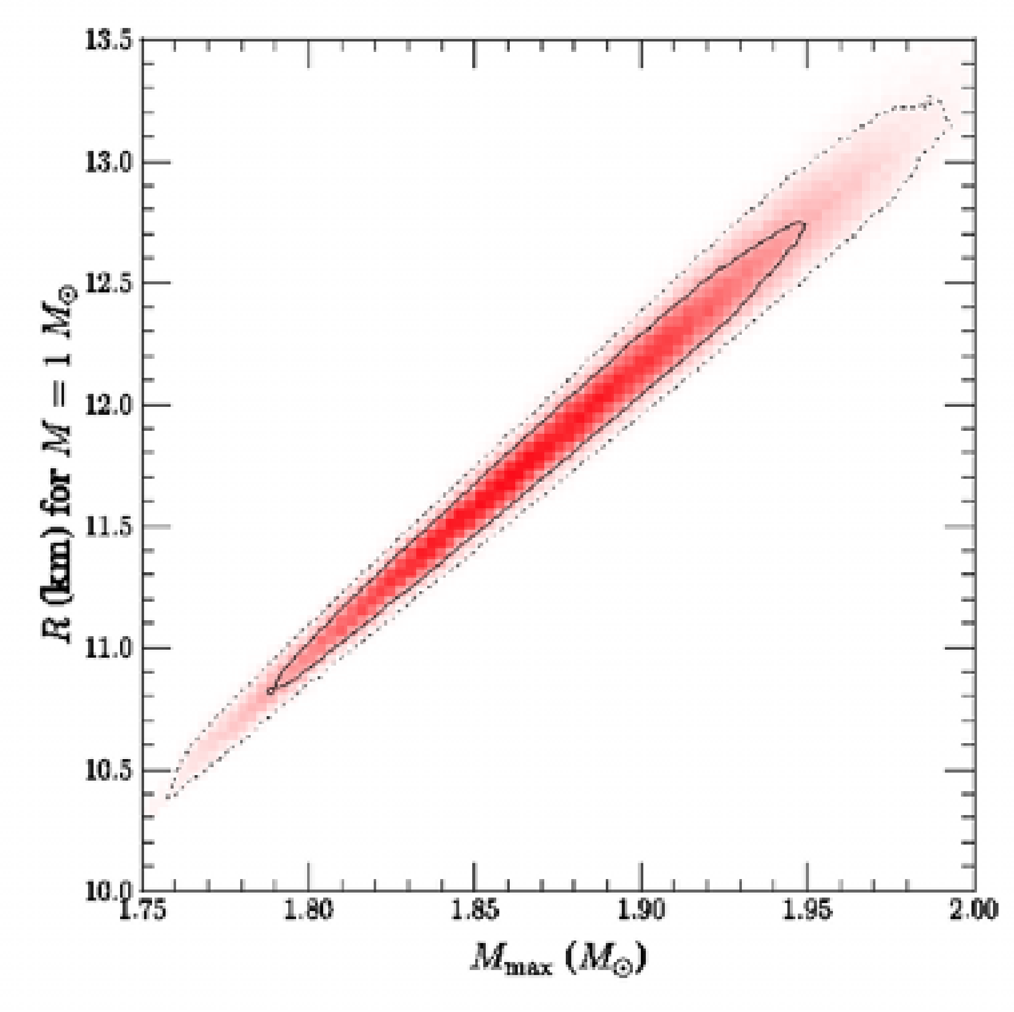
- The speed of sound at zero density and finite temperature: $c_s^2 \rightarrow 1/3$ as $T \rightarrow \infty$
- What happens at high density and zero temperature?
- Perturbation theory suggests c_s^2 increases to 1/3 from below
  Kurkela et al. (2010)
- $c_s^2 \approx 1/12$ in neutron matter at the saturation density
- Is $c_s^2 > 1/3$ anywhere in the universe?

A Hypothesis

- Remember $c_s^2 = dP/d\varepsilon$
- Ok, let's assume that $c_s^2 < 1/3$
- This limits the pressure, thus also limits the maximum mass
- Can we produce a $2\ \mathrm{M}_\odot$ neutron star?

Assume $c_s^2 < 1/3$ everywhere





Bedaque and Steiner (2015)

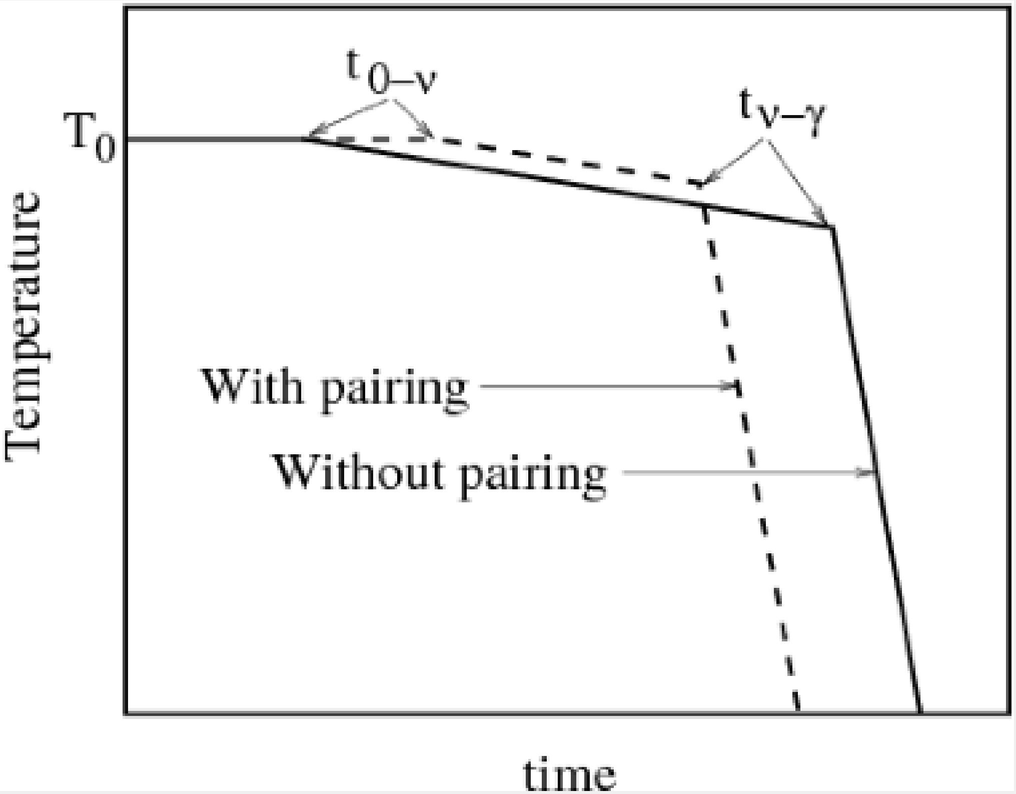
- Make the speed of sound as large as possible (black curve)
- Failure! c_s^2 must be non-trivial at high densities. Why?
- May imply a phase transition at high-density, or the introduction of some new length scale

Thermal Emission from Isolated Neutron Stars

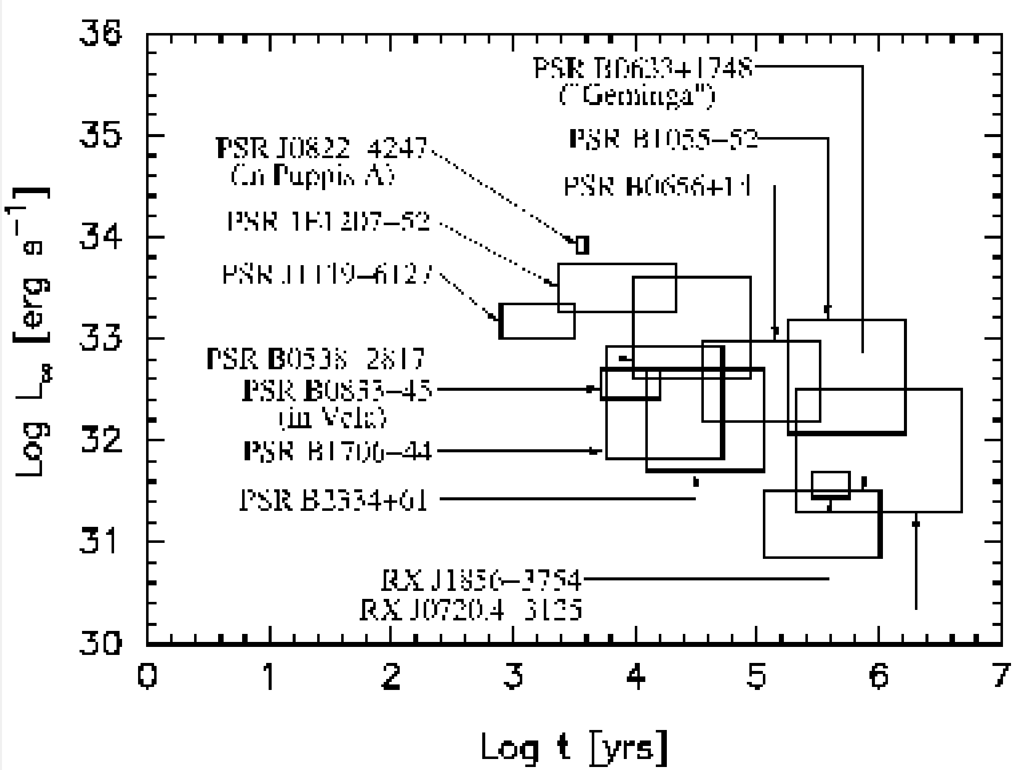
- No distance measurement required
- Requires a model of the NS atmosphere to associate the observed spectrum with a luminosity or temperature

$$C_V \frac{dT}{dt} = L_\nu + L_\gamma\,, \quad L_\gamma \sim T^{2+4\alpha}\,, \quad L_\nu \sim T^8 \text{ (Modified Urca)}\,, \quad C_V \sim CT$$

- Age assumed from spin-down age or associated with a supernova remnant



Page, et al (2004)



Page, et al (2009)

Neutron Star Cooling Processes

- Direct Urca: $n \to p + e + \bar{\nu}, \quad Q \sim T^6$
- Only possible if enough protons are around

$$2s \equiv p_{Fn} + p_{Fp} + p_{Fe}$$

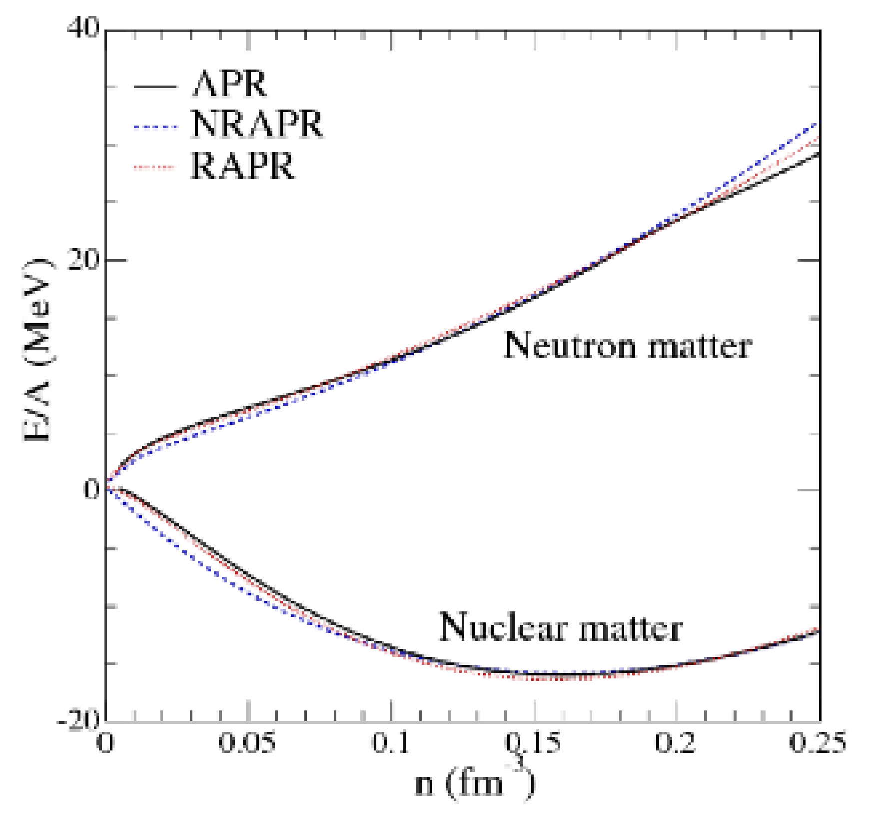
- Condition for direct Urca is:

$$s(s - p_{Fn})(s - p_{Fp})(s - p_{Fe}) \geq 0$$

Lattimer et al. (1991), this expression from Steiner (2006)

- $4(1 - 2n_p/n)S(n) = \mu_e$
- Also blocked by superfluidity because it breaks pairs
- Modified Urca cooling: $n + n \to n + p + e + \bar{\nu}, \quad Q \sim T^8$
- Also quark and hyperon direct Urca processes

The Nuclear Symmetry Energy, Revisited



Steiner et al. (2005)

- Define $\alpha \equiv (n_n - n_p)/n$

$$S(n) = \frac{1}{2n}\left(\frac{\partial(nE/A)}{\partial\alpha}\right)_{\alpha=0}$$

- $\tilde{S}(n) = (E/A)_{\mathrm{neut}}(n) - (E/A)_{\mathrm{nuc}}(n)$
- If E/A is quadratic in α, $S(n) = \tilde{S}(n)$
- $S \equiv S(n_0)$
- L is the derivative, $L \equiv 3n_0 S'(n_0)$

- $4S(n) = \partial_\alpha \left[\mu_n(n,\alpha) - \mu_p(n,\alpha)\right]$
- $\tilde{S}(n)$ probed in neutron matter,
  $S(n)$ probed in nuclei, e.g. isovector giant dipole

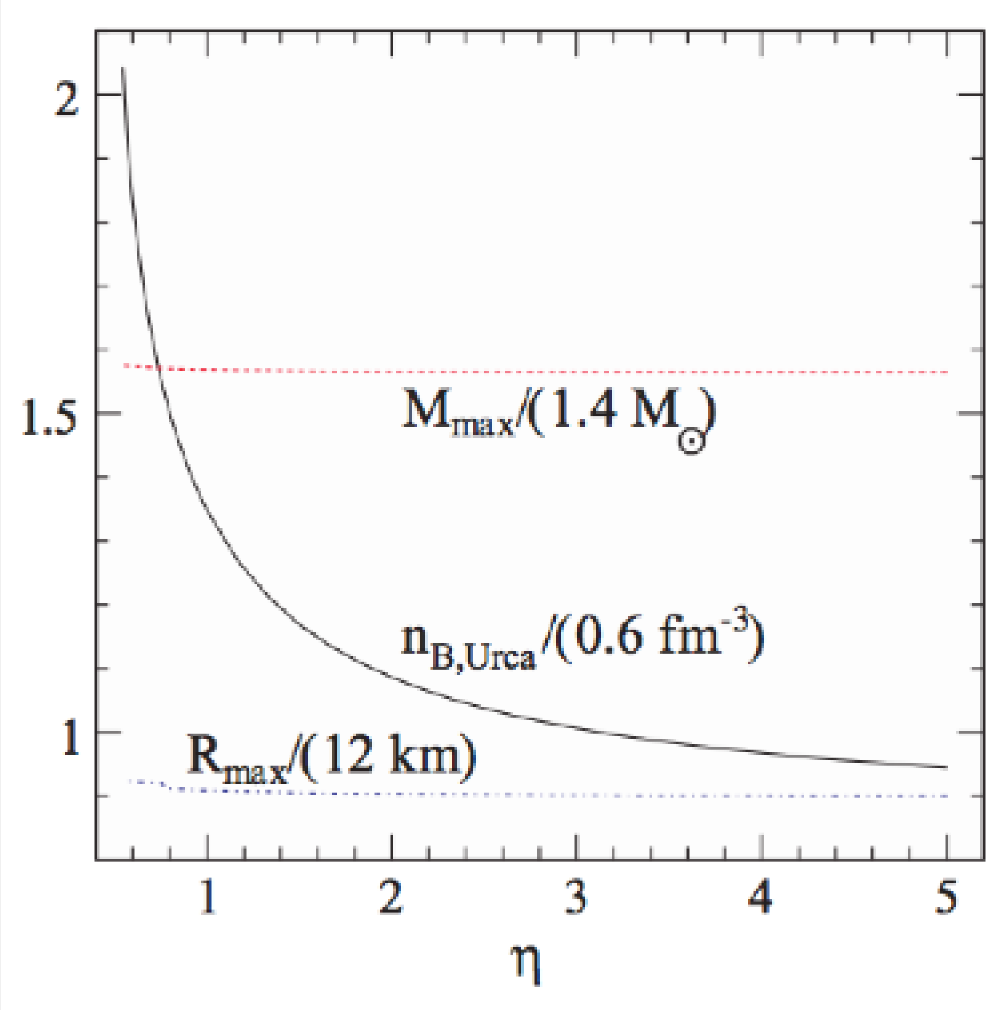
Quartic Terms and the Direct Urca Process

$$(E/A)(n,\alpha) = (E/A)_{\rm nuc}(n,\alpha) + \alpha^2 S(n) + \alpha^4 Q(n)$$

- Below saturation, quartic terms are likely "small", above saturation densities, they may be important

$$\eta(n) \equiv \frac{4S(n) + 5Q(n)}{4S(n) + Q(n)}$$

- $3/7 < \eta(n) < 5$
- Complicates connection between symmetry energy and direct Urca
- Superfluidity also very important, and depends on L



Steiner (2006)

More Complications from Short-Range Correlations

- Most calculations of the direct Urca process have assumed a Fermi-like distribution function for the nucleons

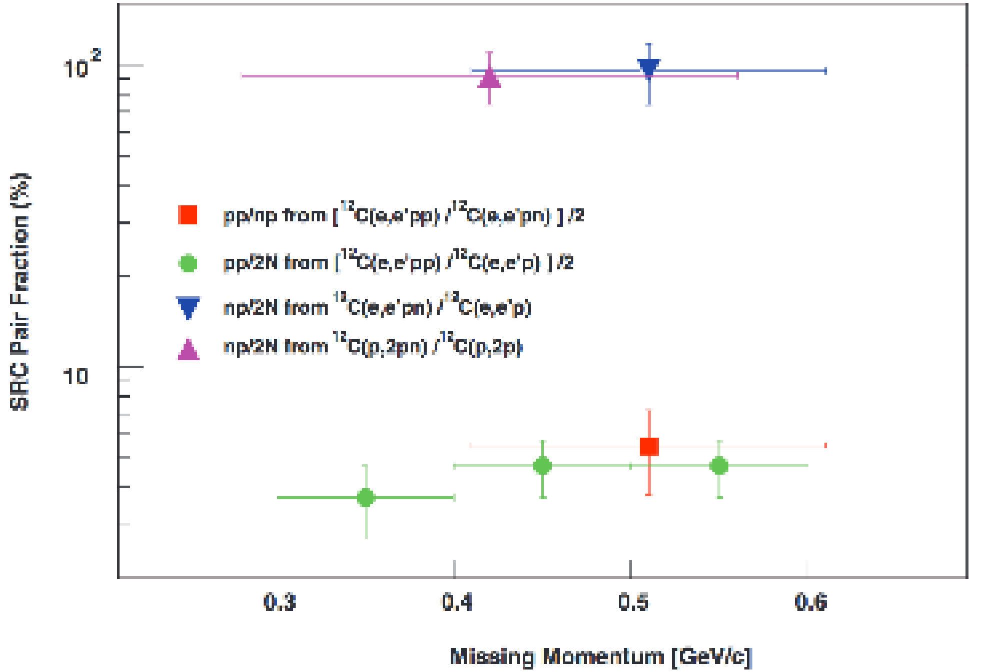


Fig. 2. The fractions of correlated pair combinations in carbon as obtained from the (e,e'pp) and (e,e'pn) reactions, as well as from previous (p,2pn) data. The results and references are listed in table 51.

Subedi et al. (2008)

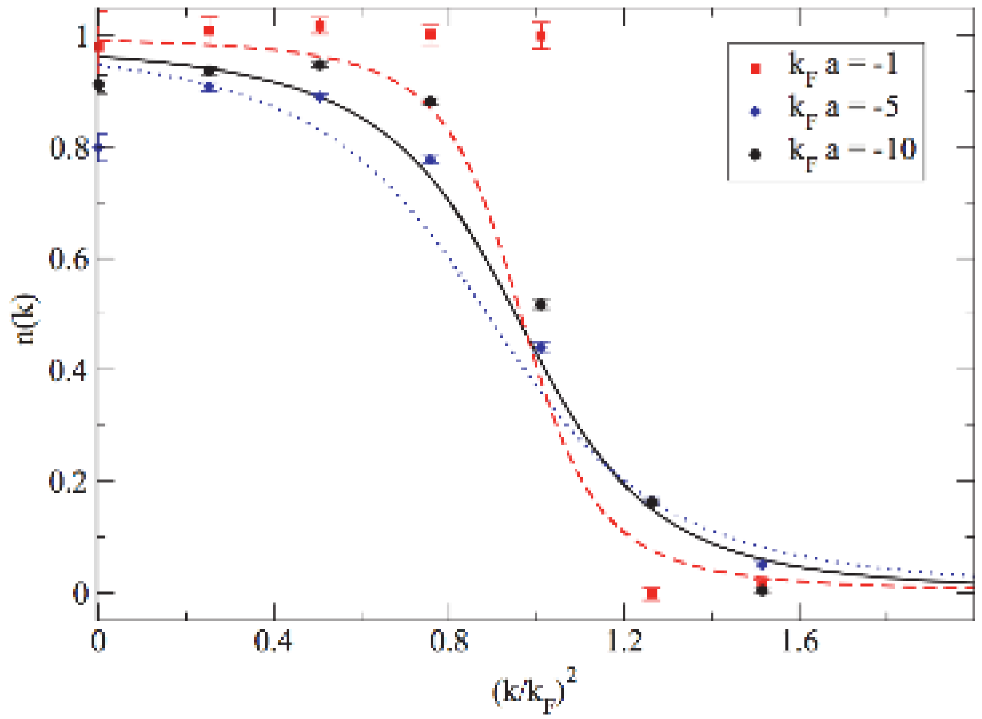


FIG. 8. (Color online) The neutron-matter momentum distribution in QMC versus $(k/k_F)^2$ at $k_F a = -1$ (squares), $k_F a = -5$ (diamonds), and $k_F a = -10$ (circles). Also shown are the continuum BCS results at $k_F a = -1$ (dashed line), $k_F a = -5$ (dotted line), and $k_F a = -10$ (solid line).

Gezerlis and Carlson (2010)

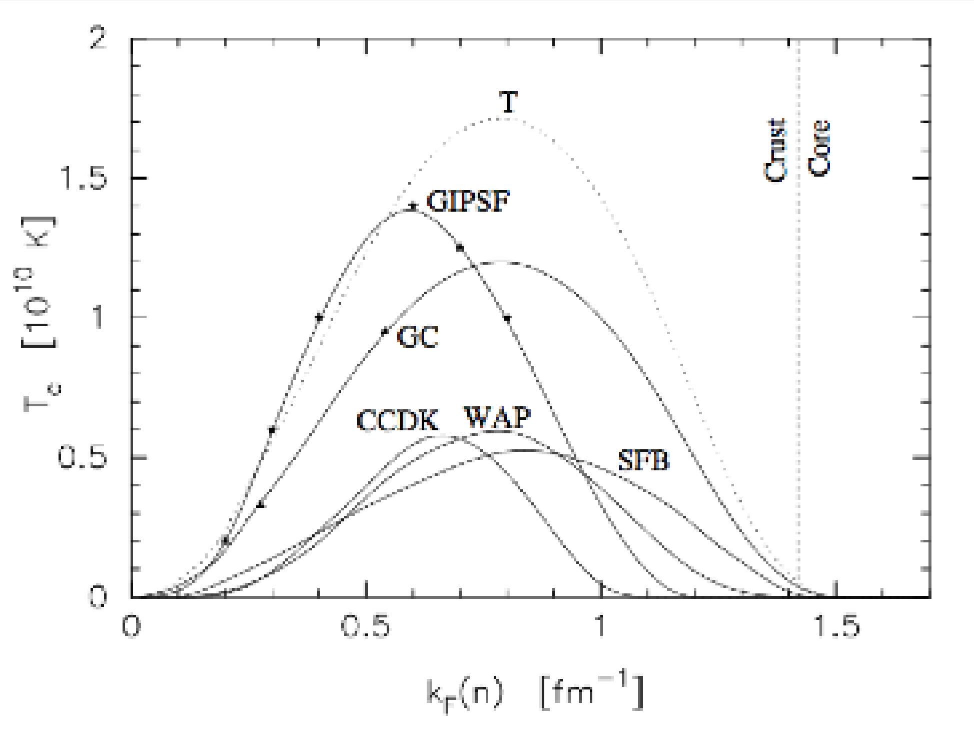
- Short-range correlations modify this picture
- E.g. may suppress superfluidity
  Frankfurt and Strikman, 0806.0997

Summary

- Nuclear physics provides an important input for describing astronomical phenomena
- Astronomical observations are helping us understand QCD and the nucleon-nucleon interaction
- Neutron star radii are most likely between 10 and 13 km
- L is most likely between 40 and 80 MeV
- We need more data, particularly data which is not dominated by systematic uncertainties
- Neutron star cooling may help us determine the composition and transport properties of dense nucleonic matter

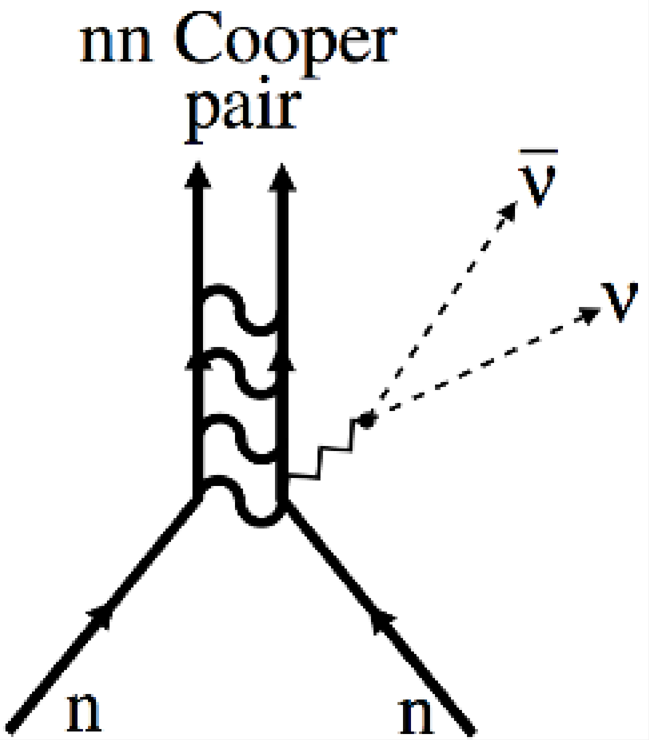
Neutron Star Superfluidity

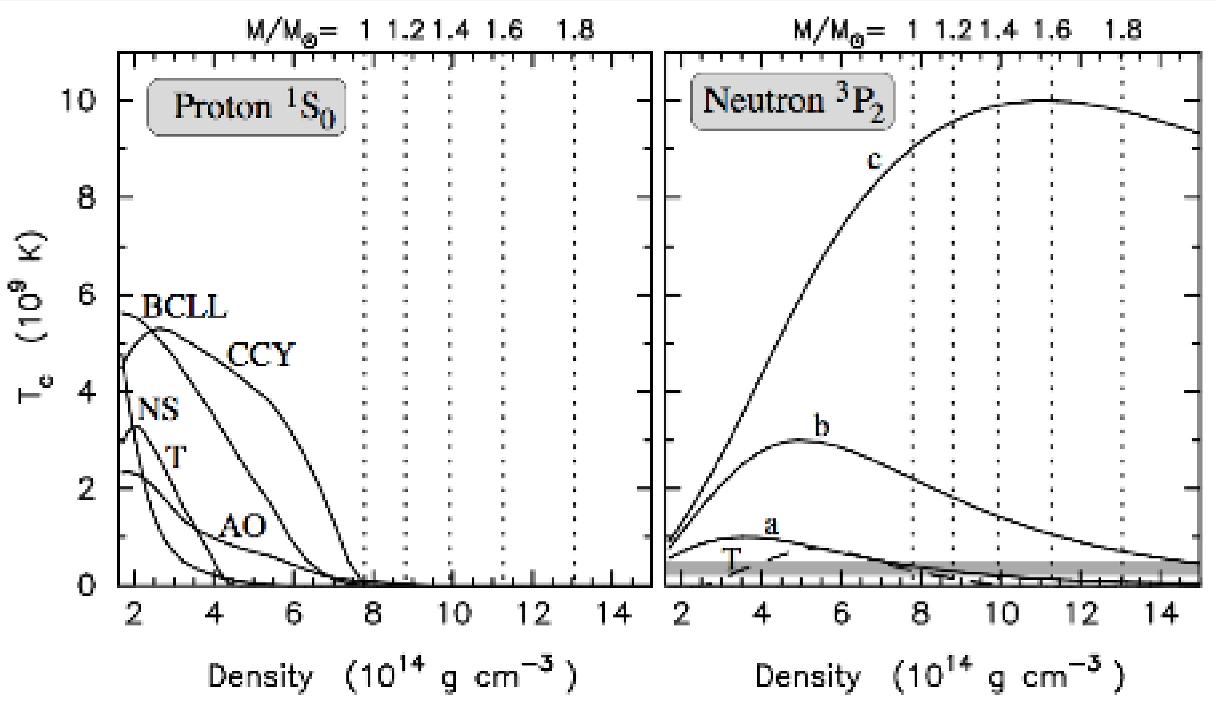
(See our review at 1302.6626)



- $^1S_0$ gap increases with increasing density, but drops off at higher densities because of n-n repulsion
- Superfluidity can block cooling processes
- ...but it opens up new ways of cooling

Steiner and Reddy (2009)

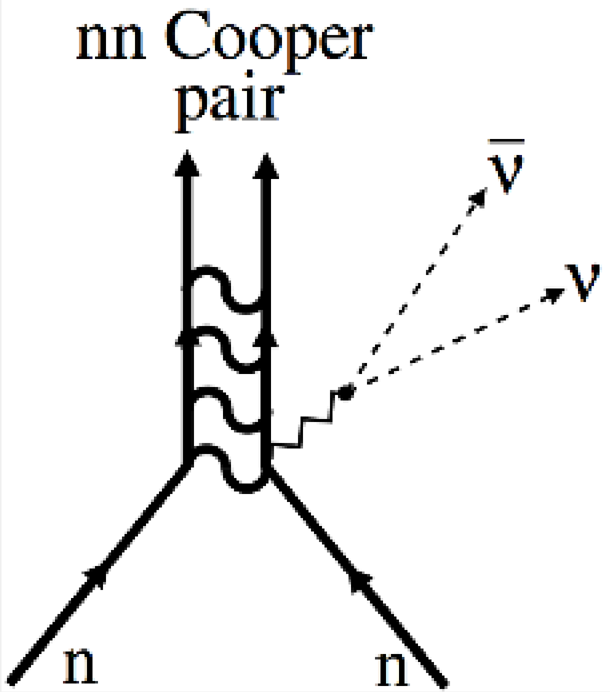


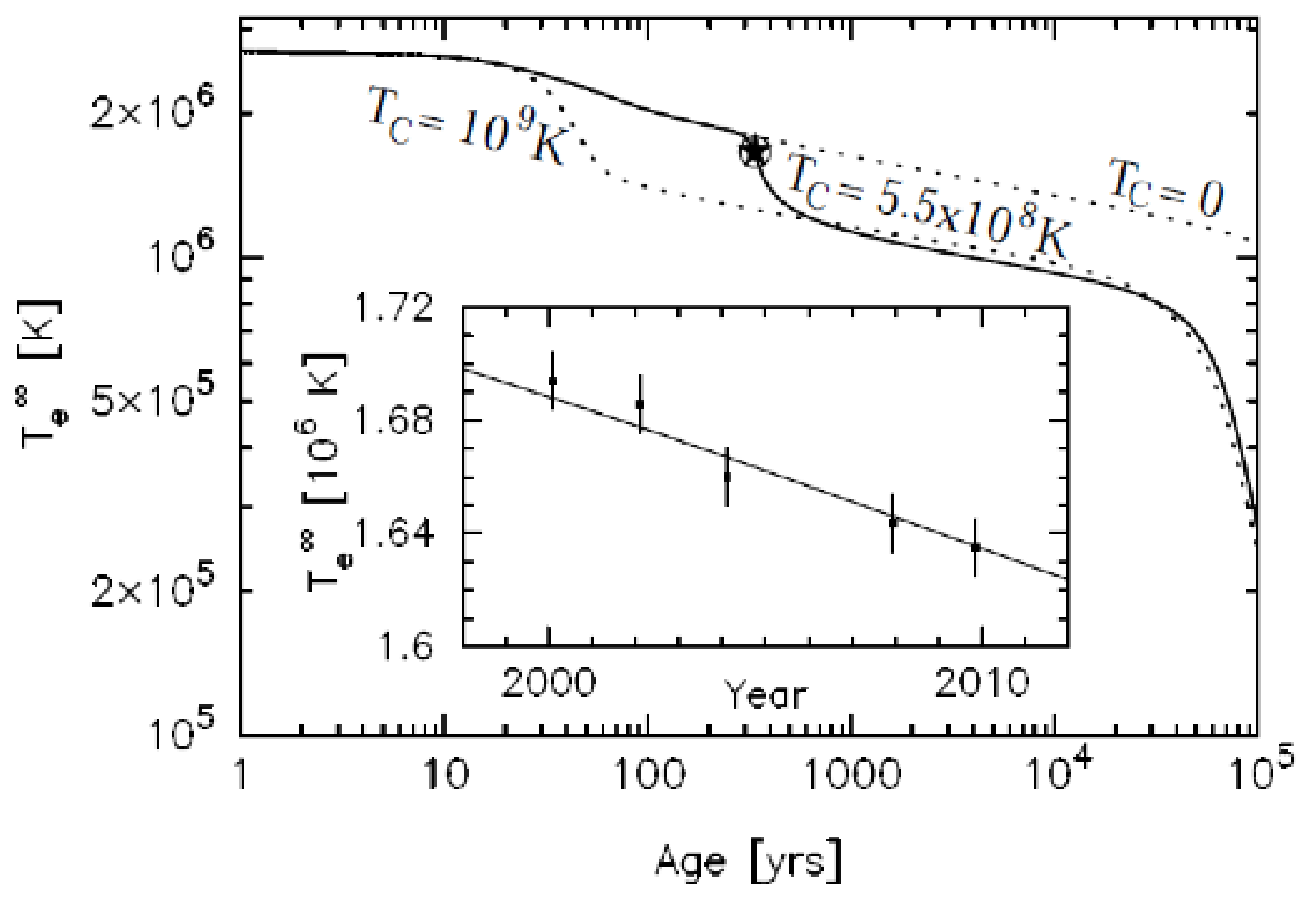


Detecting Neutron Star Superfluidity

- The large slope is only well reproduced by the neutron triplet superfluid transition and associated emissivity

- Cas A requires a very particular triplet gap $\Delta(T = 0) \propto T_C$

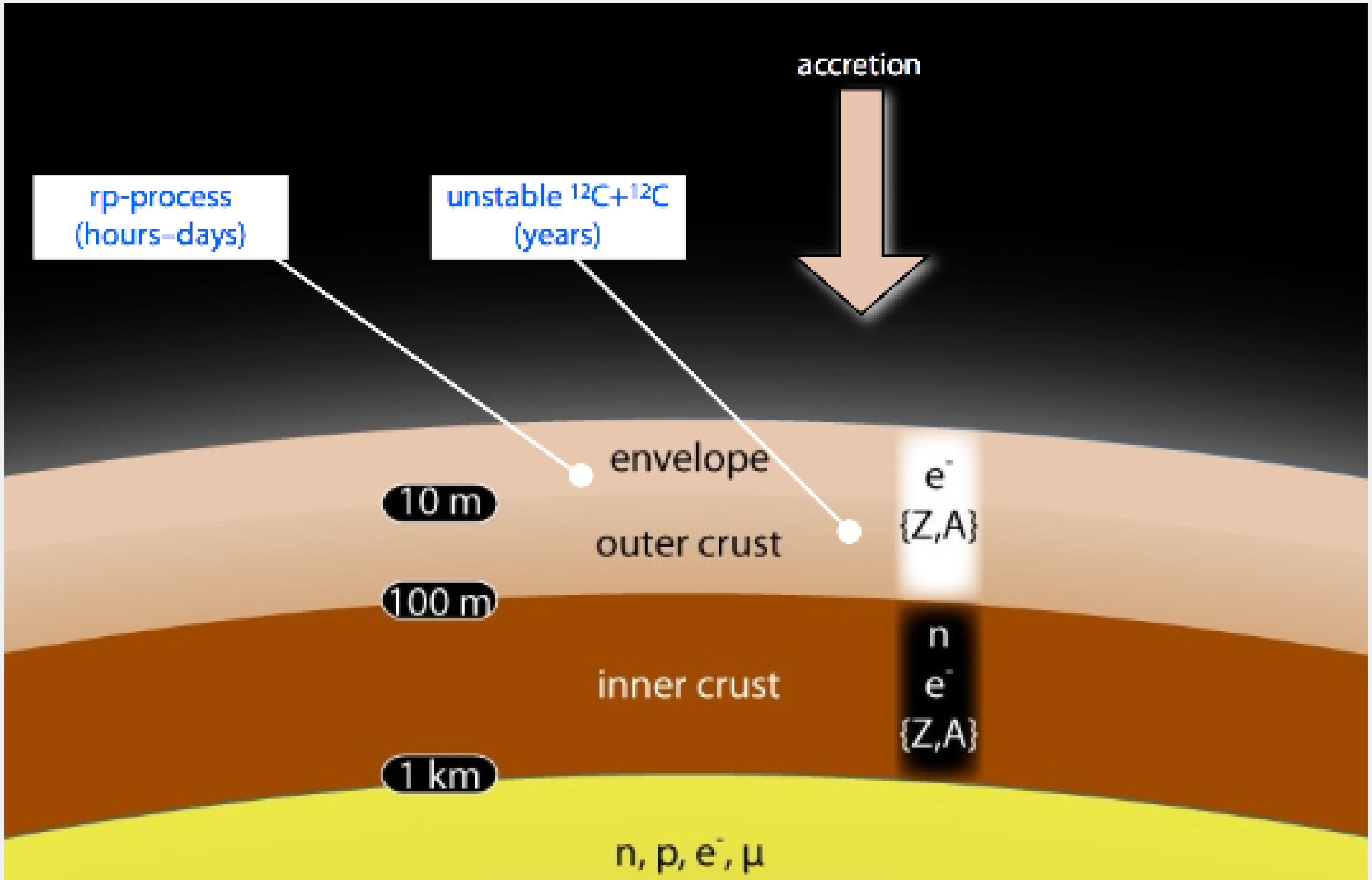




- If you form a Cooper pair, you gain energy

Page, et al. (2011)

Accreted Neutron Star Crusts

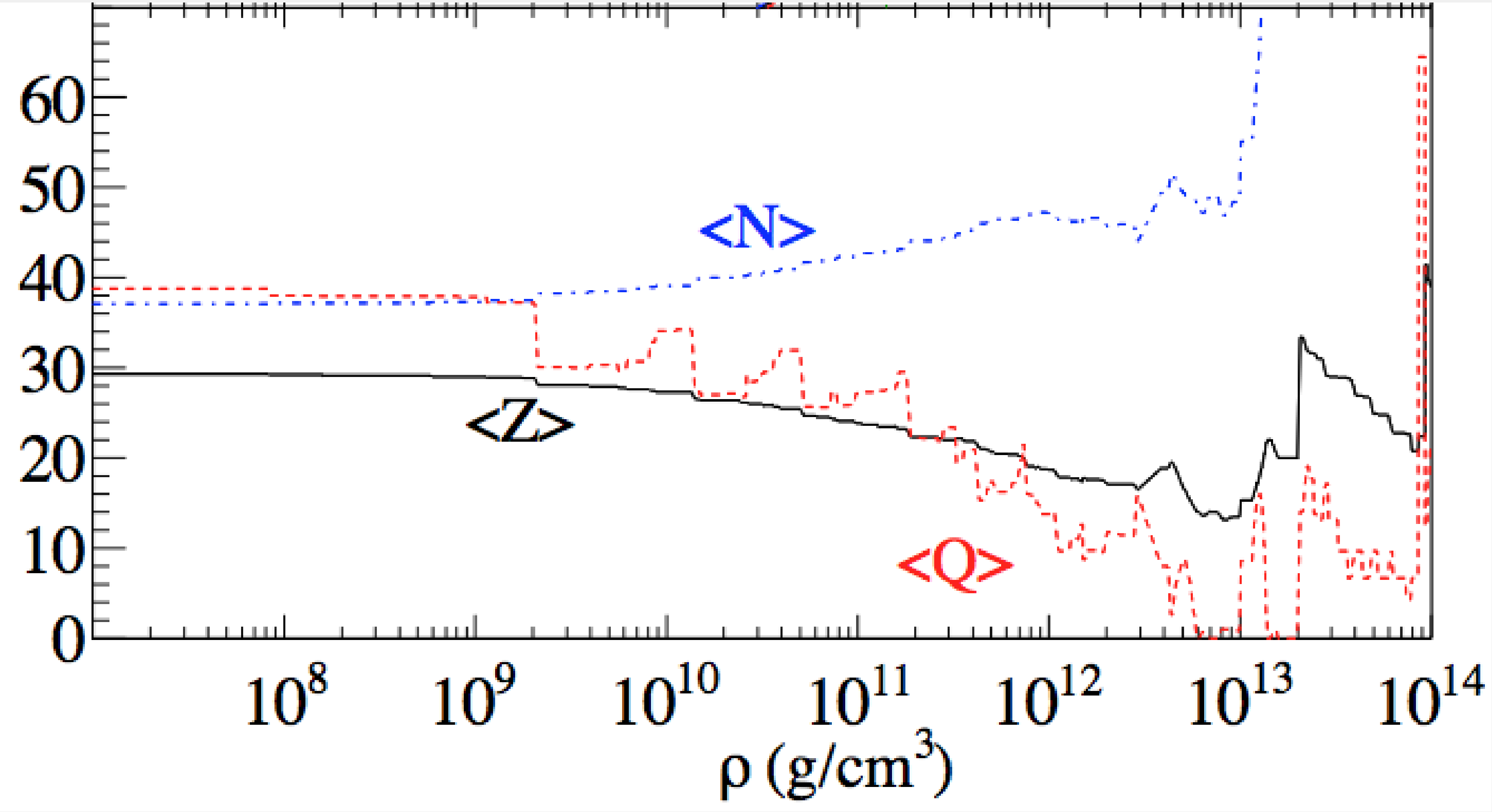


Adapted from E. Brown

- X-ray burst ashes undergo electron captures, neutron emissions, and pycnonuclear fusions
- Heat is released: "deep crustal heating"

A Multicomponent Model of the Deep Crust

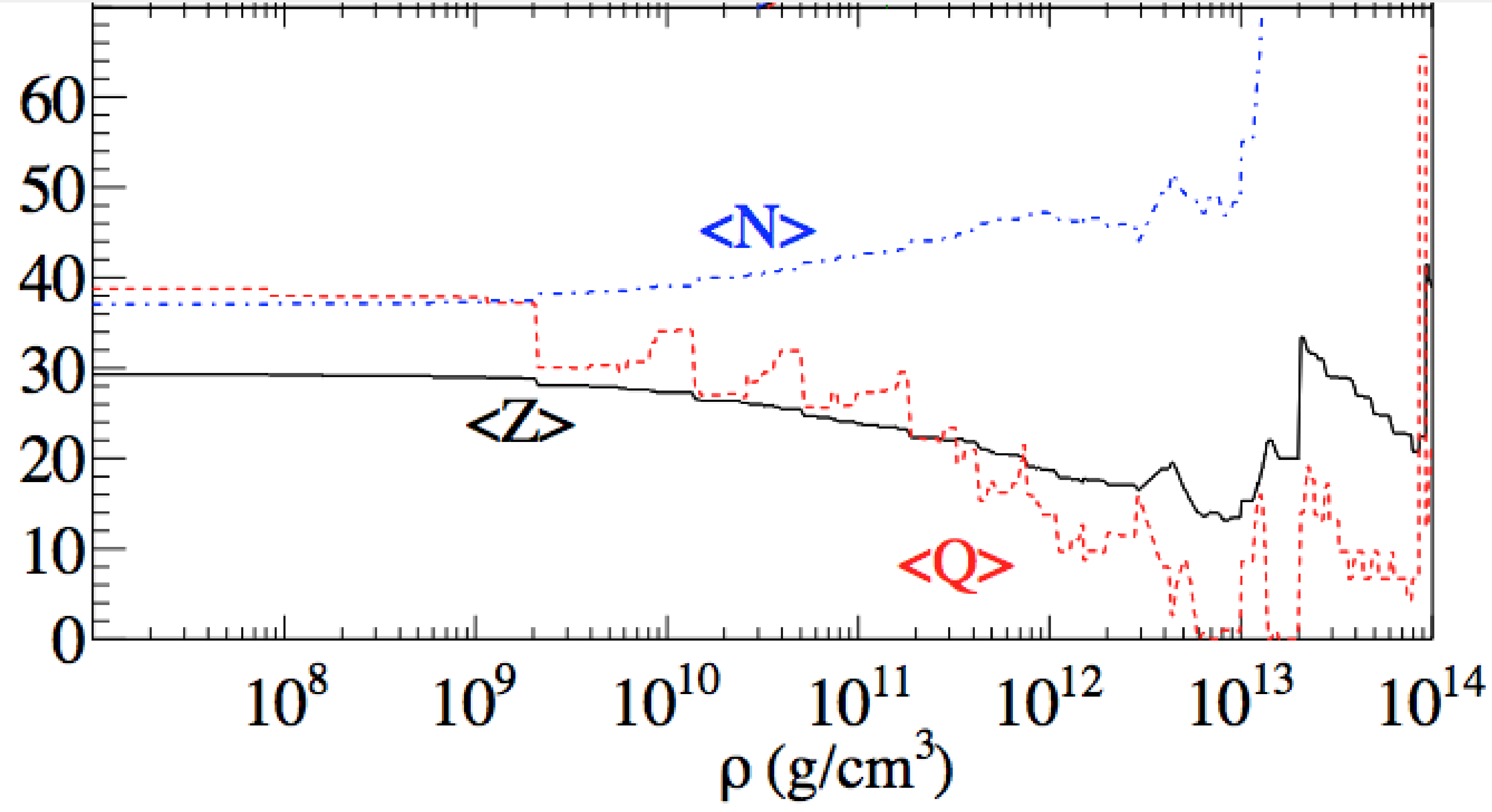
- Multi-component simulation of the compression of X-ray burst ashes
- Reaction rates depend critically on the properties of very neutron-rich nuclei
- Most of the heat is released in the very outer layers of the inner crust



Steiner (2012)

Symmetry Energy and Deep Crustal Heating

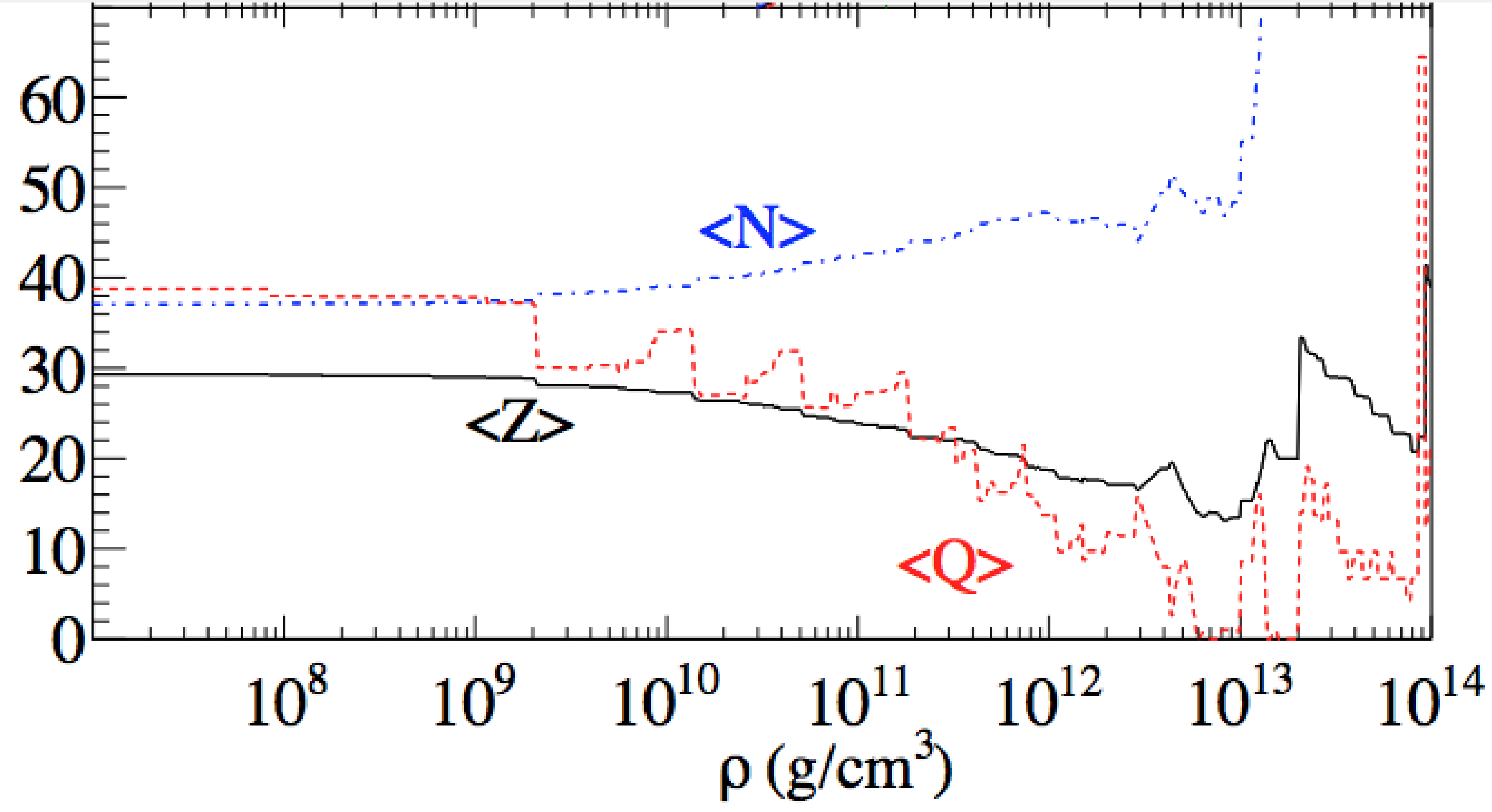
- Vary masses based on nucleon-nucleon interactions with different symmetry energies
- Skyrme models: SLy4 (L=45 MeV) and Gs (L=90 MeV)
- Find that SLy4 gives 2.4 MeV per nucleon while Gs gives 4.8 MeV per nucleon



Steiner (2012)

Neutron-rich Nuclei and Deep Crustal Heating

- Important cycles which generate energy involving neutron-rich nuclei
- Two ^{40}Mg nuclei
- 6 electron captures and 18 neutron emissions ⇒ Two ^{22}C nuclei
- Fuse to one ^{44}Mg, emit another 4 neutrons



Steiner (2012)