Investigating proton form-factors with initial-state radiation

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Physics Seminar @ JLab
The Radius Puzzle

\[ R_p = ? \]
Two ways of climbing the mountain

\[ r_p = ? \]
Extraction of FF via Rosenbluth Separation.

- Best estimate for radius:

\[
r_E^2 = -6\hbar^2 \frac{d}{dQ^2} G_E(Q^2) \bigg|_{Q^2=0}
\]
Proton’s charge form-factor

- Radius from Bernauer’s measurements: \( r = (0.879 \pm 0.008) \text{ fm} \)
There is small difference in energy between energy levels $2S_{1/2}$ and $2P_{1/2}$ due to QED vacuum fluctuations.
Lamb shift in Hydrogen

- Change in level energy (approximately):

\[
\Delta E_{Lamb}^{nl} \propto |\psi_{nl}(0)|^2
\]

\[
E(nS) \approx -\frac{R_\infty}{n^2} + \frac{\Delta E_{Lamb}^{1S}}{n^3}
\]

\[
\Delta E_{Lamb}^{1S} \approx \left(8.172 + 1.56 \ r_p^2\right) MHz
\]

- Significant effect in S-states and only tiny change in P-states.

- The center of the hydrogen atom is not empty. **Proton is here!**

- Different **n-dependence** of the two terms allows the determination of \(R_\infty\) and \(r_p\) from at least two different measurements.
Spectroscopic measurements

- Direct (RF) and indirect (laser) spectroscopy measurements:

Radius from spectroscopic measurements: \( r = (0.8758 \pm 0.0077) \text{ fm} \)
Due to larger mass muon much closer to the nucleus, resulting in a more pronounced Lamb shift effect.

- The largest signal is given by the $2S_{1/2}^F=1$ and $2P_{3/2}^F=2$ transition.

- The QED calculation predict:

\[
\Delta E = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV}
\]

- Finite size of the proton contributes 1.8% of the energy difference.
The mean position of the peak:

\[ f_{2S-2P} = 49881.88(76) \text{ GHz} \]

\[ \Delta E = 206.2949(32) \text{ meV} \]

The resulting radius:

\[ r_p = 0.84184(36)(56) \text{ fm} \]
The ever changing radius!

- The 6σ discrepancy in the $r_p$ measurements.
Why is the puzzle so important?

- Knowledge of basic properties of the nucleon.
- The radius is strongly correlated to the Rydberg constant.
- Problems in nuclear scattering data?
- Bringing different interpretations of nuclear scattering data to an agreement.
- Do we understand QED?
Proton’s charge form-factor

- Data available only for $Q^2 > 0.004 \text{ (GeV/c)}^2$.
- Need to avoid extrapolations to zero!
Relating to Lamb shift measurements

\[ Q_{\text{vac}}^2 \geq 4m_e^2 \sim 10^{-6}\,\text{GeV}^2 \]

\[ Q_{\text{vertex}}^2 \geq \frac{\hbar^2}{R_e^2} \sim 10^{-9}\,\text{GeV}^2 \]

Realistically accessible \( Q^2 > 10^{-4}\,\text{GeV}^2 \).

- Region of \( Q^2 < 0.004\,\text{(GeV)}^2 \) is extremely hard to reach.
- Kinematic range is **limited by available experimental apparatus**.
- **Novel techniques are needed** to explore extremely low \( Q^2 \) regime.
Dominated by coherent sum of two Bethe-Heitler diagrams.

By comparing data to simulation ISR information can be reached.

Measured $\delta \sigma$ linearly proportional to the $\delta G_E$ between data and model.
Based on standard A1 framework.

Detailed description of apparatus.

Exact calculation of the leading order diagrams:

The NL-order **virtual and real** corrections included via effective corrections to the cross-section.
Going beyond peaking approximation

- Traditional peaking approximations insufficient for such experiment.

- **Secondary objective:** Measurements at higher $Q^2$ for validating the radiative corrections in a region, where FFs are well known.

  Important for experiments, e.g. VCS, which require high-precision knowledge of the radiative corrections.

\[ \sigma_a(\Delta E) = \sigma_{Elastic} e^{\delta(\Delta E)} \]
The ISR experiment

- Full experiment done in August 2013. Four weeks of data taking.

**Electron Beam:**
- Energy: 195, 330, 495 MeV
- Current: 10nA – 1μA
- Rastered beam

**Spectrometer A:**
- Luminosity monitor (const. setting)
- Momentum: 180, 305, 386 MeV/c
- Angles: 50°, 60°

**Spectrometer B:**
- Data taking
- Angle: 15.3°
- Momentum:
  - 48 - 194 MeV/c (35 setups)
  - 156 - 326 MeV/c (12 setups)
  - 289 - 486 MeV/c (9 setups)

**Spectrometer C:**
- Not used

**Luminosity monitors:**
- pA-meter
- Förster probe
- SEM

**Beam control module:**
- Communicates with MAMI and ensures very stable beam.
- BPM and pA-meter measurements performed automatically every 3min.
Kinematic settings

- Overlapping settings to control systematic uncertainty.
Target Frame contributions #1

- Presence of target frame results in the deficiency of the elastic events.
Target Frame contributions #2

- ... and in the abundance of bogus events in radiative tail of the elastic peak.
Spec. B encompasses a long entrance flange.

Events rescattered from the snout cover the whole vertex acceptance.
Results

- Existing apparatus limited reach of ISR experiment to $E' \sim 130$ MeV.

- Elastic points excluded.

- Simulation performed with Bernauer parameterization of form factors.

- A percent agreement between the data and simulation demonstrates that the radiative corrections are well understood!
Assuming flawless description of radiative corrections, form factors can be extracted from the data.

First measurement of $G_E^p$ at $0.001 \text{ GeV}^2 \leq Q^2 \leq 0.004 \text{ GeV}^2$
ISR Proton radius

- $G_E^p$ modeled with the polynomial fit.

$$G_E^p(Q^2) = n \left( 1 - \frac{r_E^2}{6} Q^2 + \frac{a}{120} Q^4 - \frac{b}{5040} Q^6 \right)$$

Terms $(a,b)$ known from previous analyses [Distler et al.]

- The obtained radius:

$$r_E = \left( 0.810 \pm 0.035_{\text{stat.}} \pm 0.074_{\text{syst.}} \pm 0.003_{\text{mod.}} \right) \text{fm}$$
Lever arm is important

- Determining radius analogous to measuring elasticity of a rod!
- Measuring deviations $x$ with fixed precision $\Delta x$.
- Measuring further away from pivot is relatively more precise.

\[ \frac{\Delta x}{x} \ll 1 \]

- Not knowing the exact behavior of a rod near the pivot.
Problem of small lever arm

- Measuring near the pivot point gives us insufficient lever arm!

\[ \frac{\Delta x}{x} \approx 1 \]

- Insufficient precision to extract the elasticity (radius).
- No precise information on the absolute position of the origin.
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- The fit (with statistical errors only) reports the reduced $\chi^2$ of 3.2.

Result is dominated by systematic effects!
Uncertainties

Total systematic uncertainty of cross-section $\leq 1.0\%$
Hypersonic jet target

- Target developed for MAGIX, but could be used also in A1.
- No metal frame near the vertex.
- No target walls.
- Width of the jet 2mm (point-like target)

by Črt Harej
Expected uncertainties with JetISR

- **NNLO Corrections ($\leq 0.45\%$)**
- **Detectors (0.2\%)**
- **Luminosity (0.17\%)**
- **Statistical (0.2\%)**

Total systematic uncertainty of cross-section $\leq 0.5\%$

- Uncertainty of NNLO theoretical corrections will be reduced to 0.2\% and total uncertainty to 0.3\%.
Hypersonic jet target

- Target developed for MAGIX, but could be used also in A1.

- No metal frame near the vertex.
- No target walls.
- Width of the jet 2mm (point-like target)

- Density of $10^{-4}$ g/cm$^3$ at 15 bar.
- Luminosity of $10^{34}$/cm$^2$s can be achieved at MAMI.

- Experiment approved by PAC 2016
Summary

- A pilot experiment has been performed at MAMI to measure $G_{E_p}$ at very low $Q^2$.

- A new technique for FF determination based on ISR has been successfully validated.

- Reach of the first ISR experiment limited by unforeseen backgrounds.

- The jet target opens possibility for reaching the ultimate goal of measuring form factors at $10^{-4}$ GeV$^2$. 
Thank you!