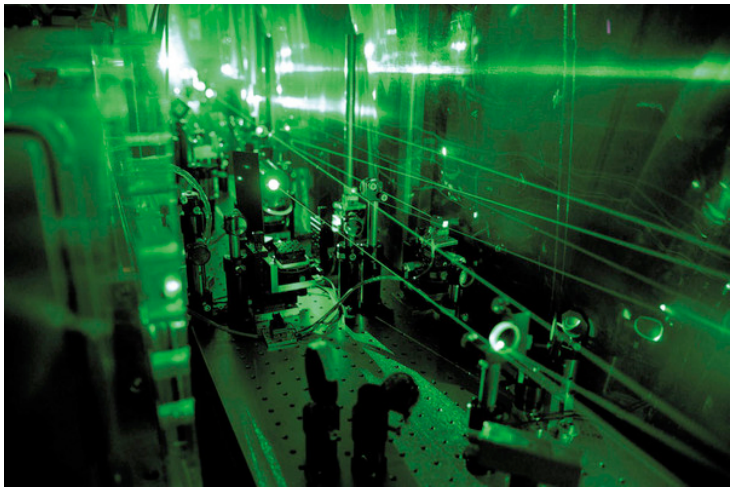
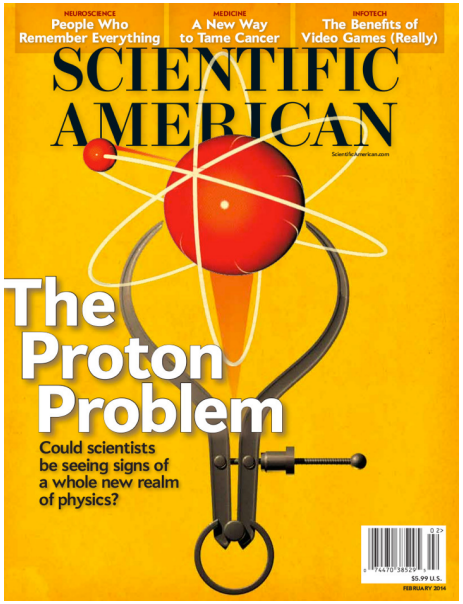
Leptophobic Scalar Boson and Muonic Puzzles

Gerald A. Miller, University of Washington

Pohl et al Nature 466, 213 (8 July 2010)





Feb. 2014

Pohl, Gilman, Miller, Pachucki (ARNPS63, 2013)

$$r_p^2 \equiv -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

muon H r_p =0.84184 (67) fm

electron H r_p =0.8768 (69)fm

electron-p scattering r_p =0.875 (10)fm

PRad at JLab- lower Q^2

4 % Difference

4 % in radius: why care?

- Can't be calculated to that accuracy
- 1/2 cm in radius of a basketball

4 % in radius: why care?

- Can't be calculated to that accuracy
- 1/2 cm in radius of a basketball

Is the muon-proton interaction the same as the electron-proton interaction? - many possible ramifications

Summary/Outline

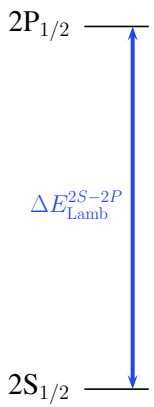
- If all of the experiments, and their analyses, are correct a lepton universality is violated- a new scalar boson can explain the puzzles
- Need to check that doesn't violate existing constraints- it doesn't s
- Direct detection is needed.

PRL **117,** 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

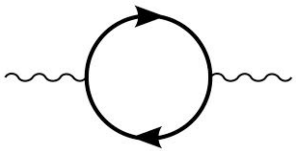
Yu-Sheng Liu,[*] David McKeen,[†] and Gerald A. Miller[‡]

muonic hydrogen experiment



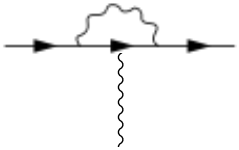
The Lamb shift is the splitting of the degenerate $2S_{1/2}$ and $2P_{1/2}$ eigenstates

Dominant in μH



vacuum polarization 205 of 206 meV

Dominant in eH



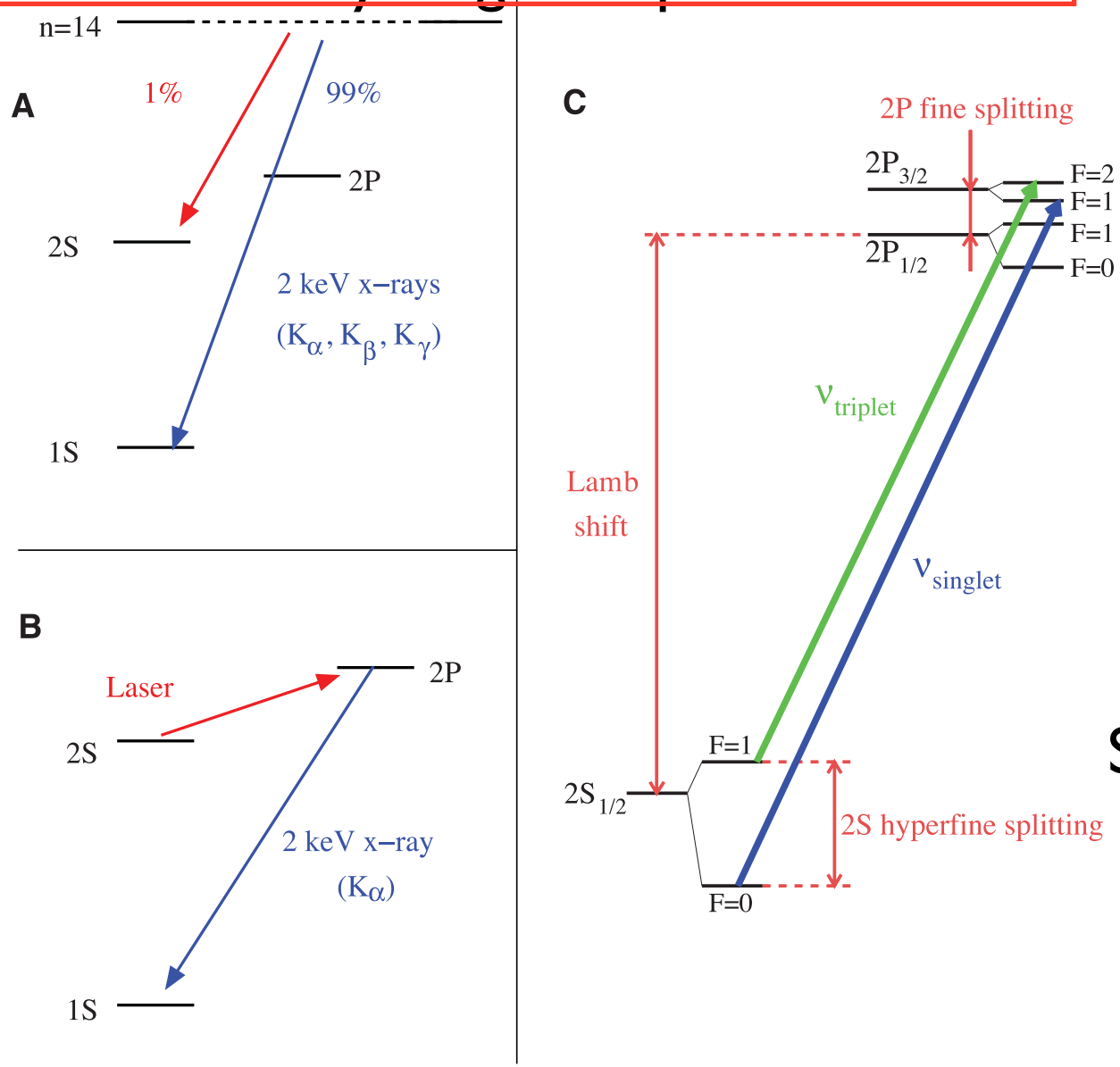
electron self-energy

Proton radius in Lamb shift

$$\Delta E = \langle \Psi_S | V_C - V_C^{\rm pt} | \Psi_S \rangle = \frac{2}{3}\pi\alpha |\Psi_S(0)|^2 (-6G'_E(0))$$

Muon/electron mass ratio 205! 8 million times larger for muon

Muonic Hydrogen Experiment



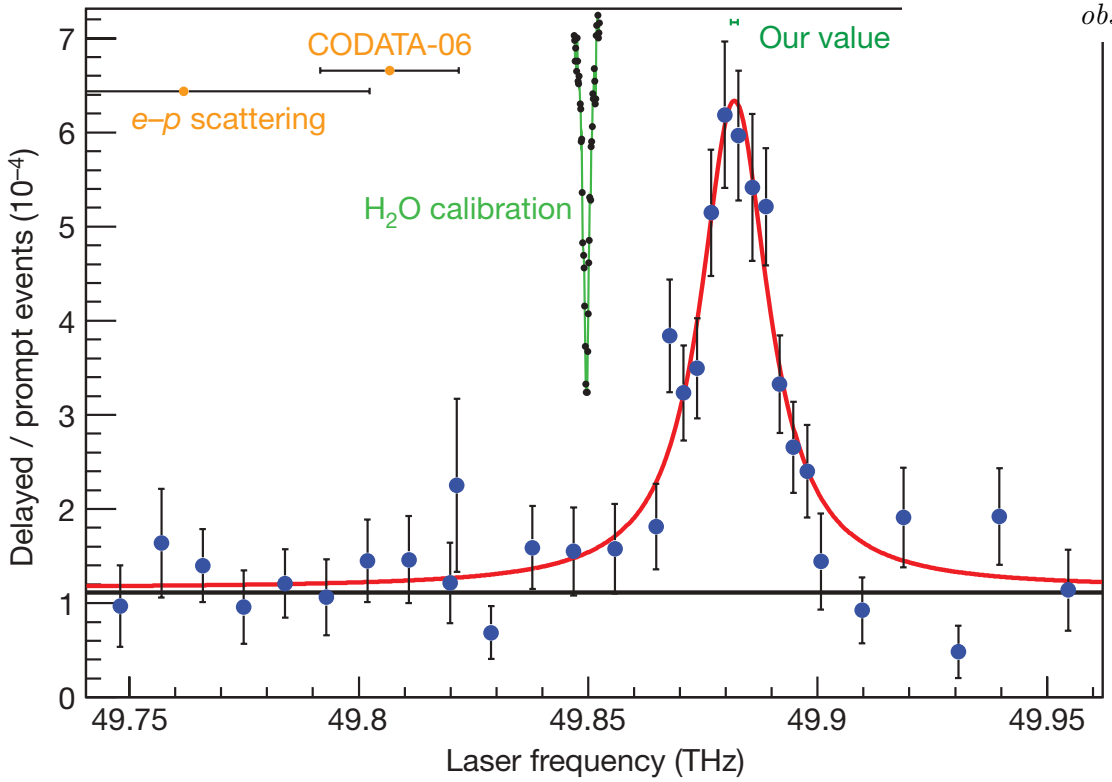
From 2013 Science paper

Fig. 1. (**A**) Formation of μp in highly excited states and subsequent cascade with emission of "prompt" $K_{\alpha,\beta,\gamma}$. (**B**) Laser excitation of the 2S-2P transition with subsequent decay to the ground state with K_α emission. (**C**) 2S and 2P energy levels. The measured transitions ν_s and ν_t are indicated together with the Lamb shift, 2S-HFS, and 2P-fine and hyperfine splitting.

The experiment:
results disagree with previous measurements & world average

2010 Rock Solid!

"The 1S-2S transition in H has been measured to 34 Hz, that is, 1.4×10^{-14} relative accuracy. Only an error of about 1,700 times the quoted experimental uncertainty could account for our observed discrepancy."

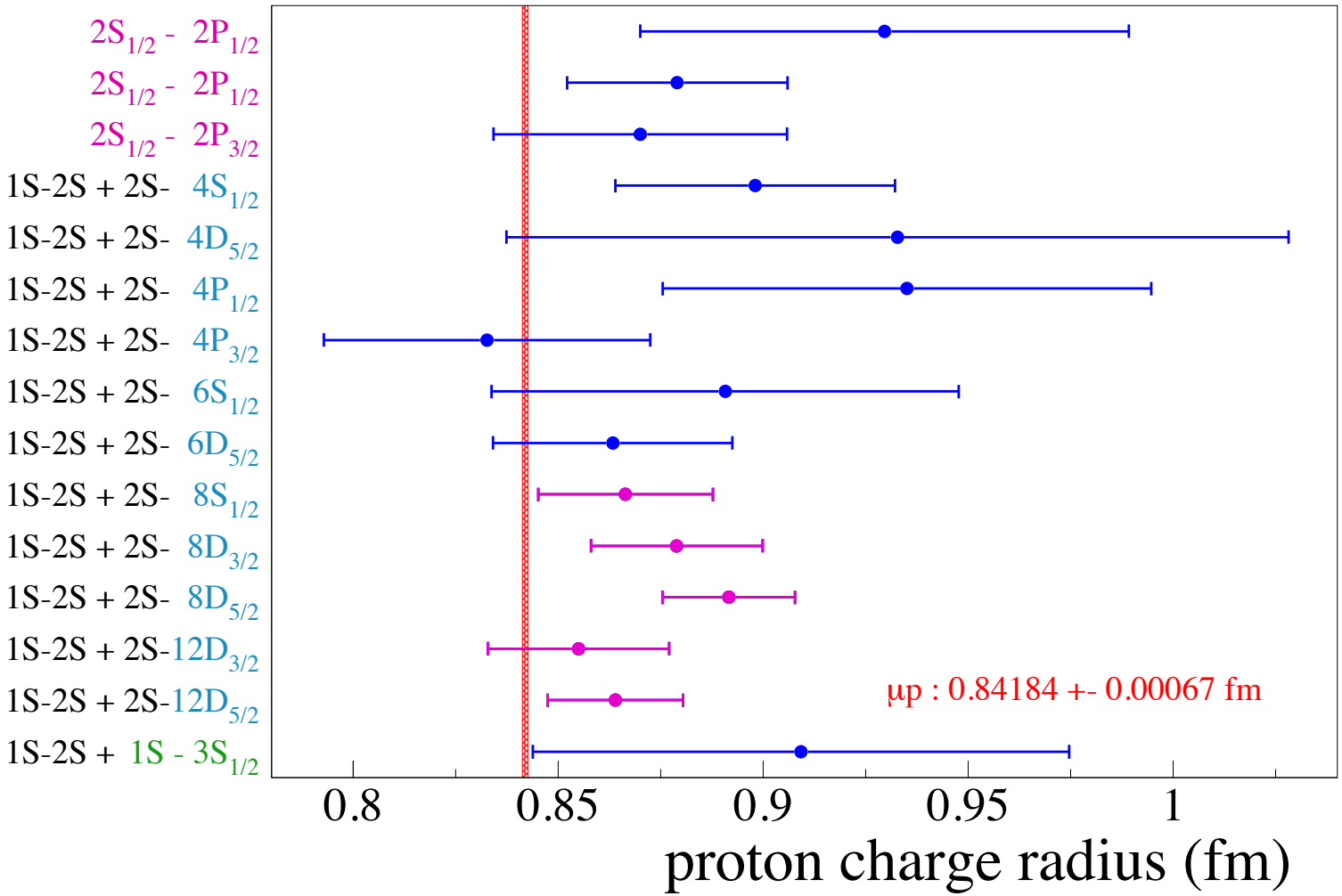


7 standard deviation difference in r_p- or value of Rydberg constant has to be shifted- (12 figures) or new physics!

Electronic Hydrogen -Pohl

- Need two levels to get Rydberg and Lamb shift-have ~ 20 available

$$E(nS) \approx \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$



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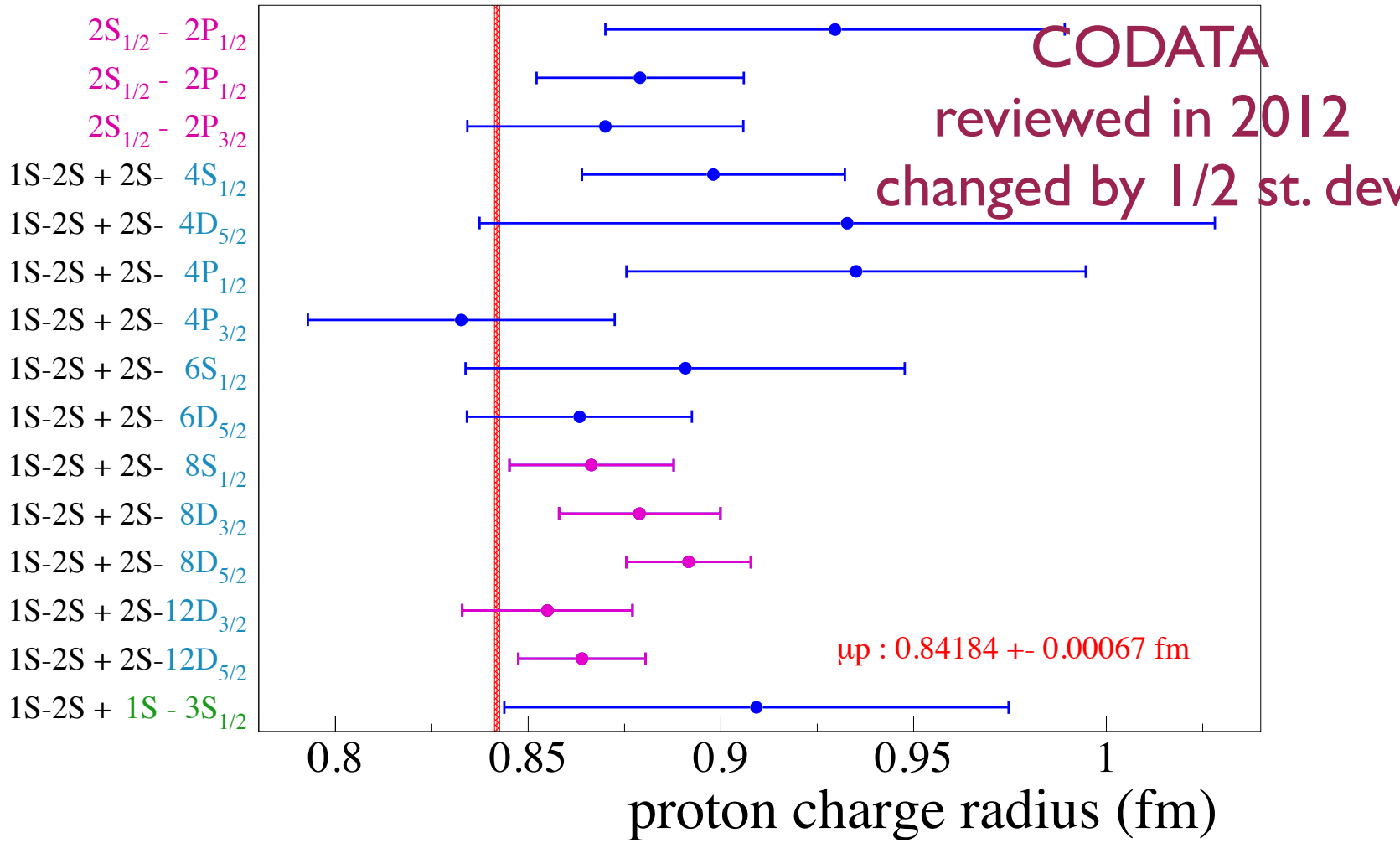

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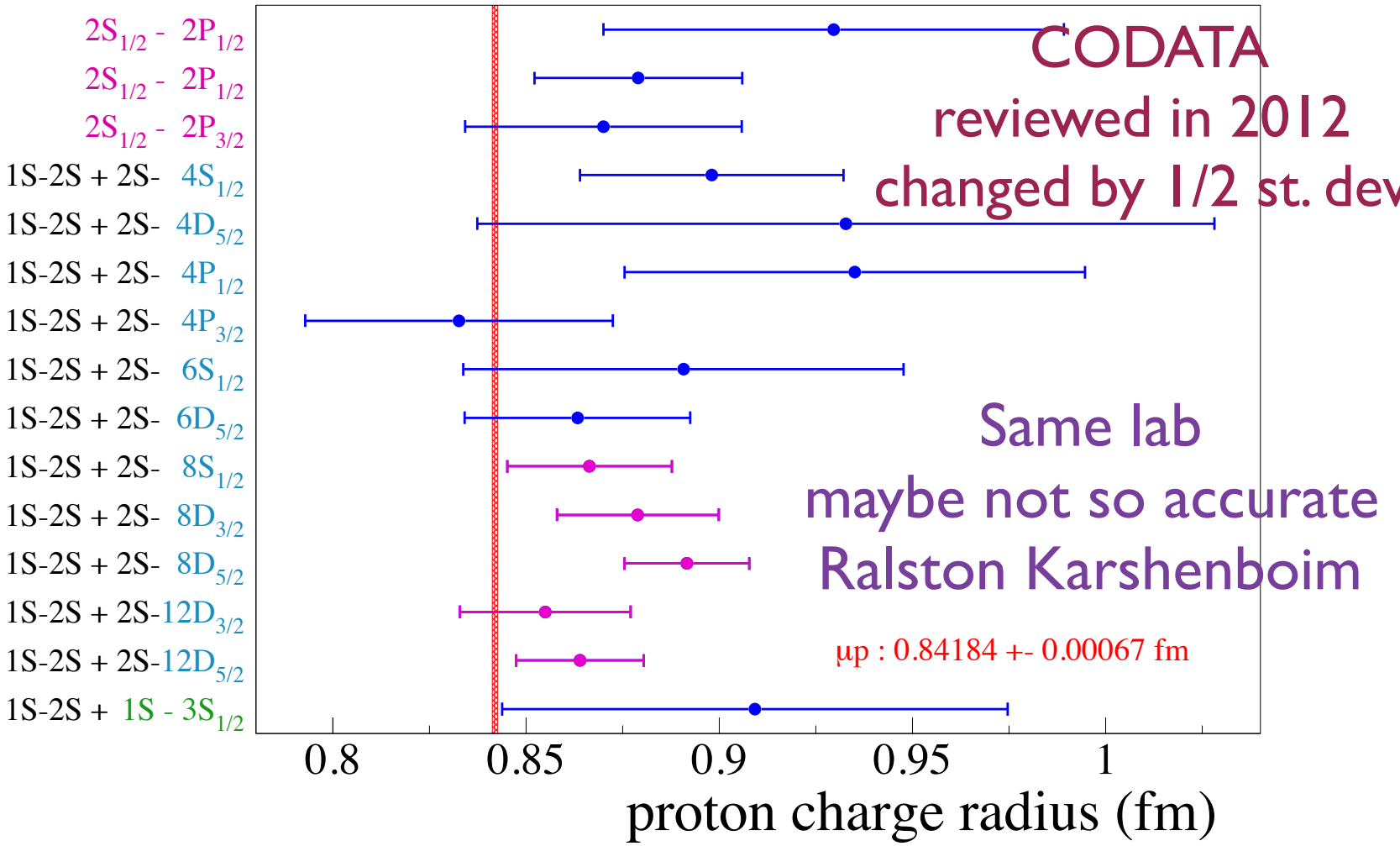
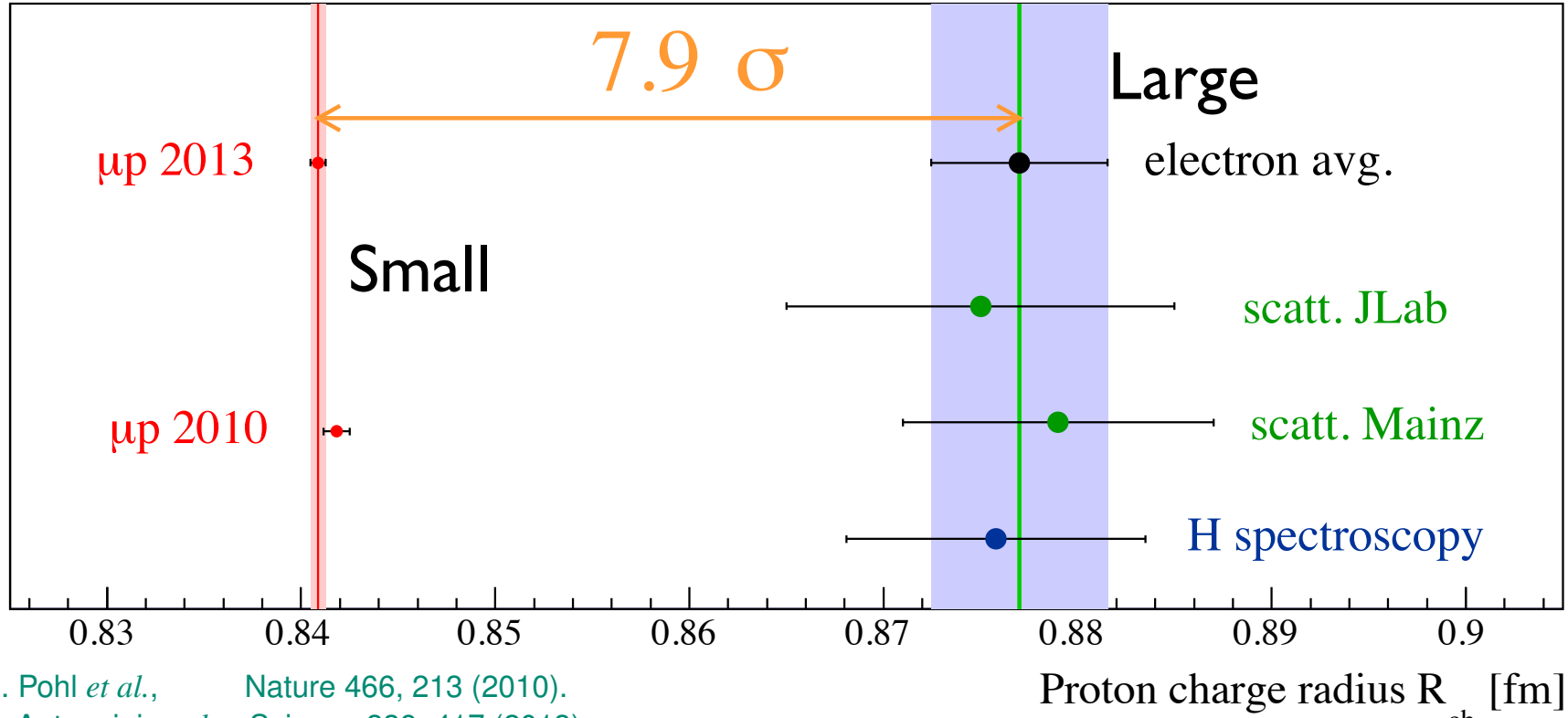


The proton radius puzzle In a picture

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm



R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini *et al.*, Science 339, 417 (2013).

What energy difference corresponds to 4% in radius?

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV
computed

Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state needed

QED theory ?

- Pohl et al table 32 terms!
- Most important -HFS- measured Jan '13
- QED theory not responsible-

electron

muon

A new effect on mu-H energy shift must vary at least as fast as lepton mass to the fourth power, if short-ranged

An effect on electron, but not muon free of this constraint

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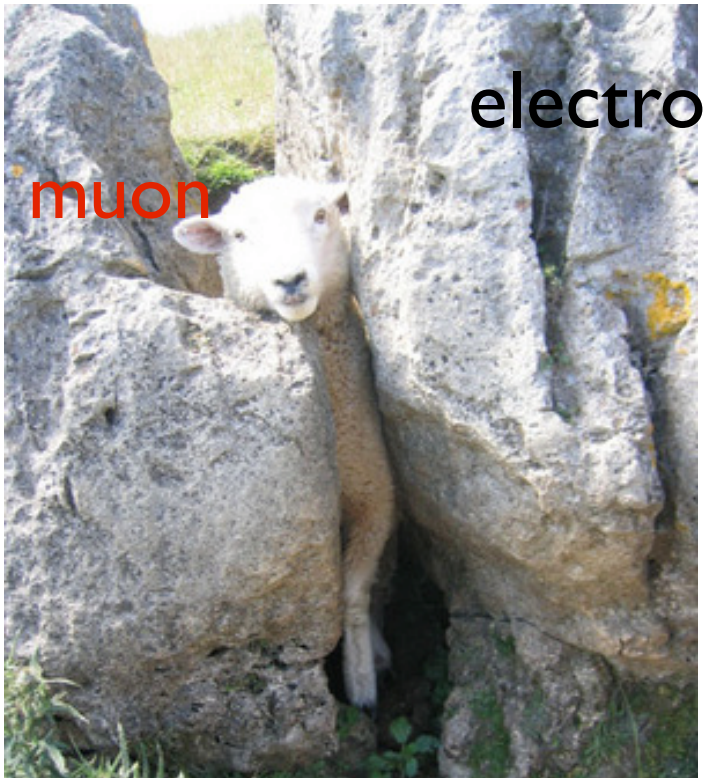

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Deuteron

Electron (D-H) isotope shift (2S-1S) 2 photon spectroscopy PRL 104, 233001
$r_d^2 - r_p^2 = 3.82007(65)$
$\mu - D$ Lamb shift $r_d = 2.12562(78)$ fm Science 353 (2016) 669
CODATA (2010)$r_d = 2.1424(26)$ fm - mainly electron scattering
Use $r_p = 0.84087$ in $r_d^2 - r_p^2 = 3.82007(65)$ gives $r_d = 2.12769$ fm
μD and Electron (D-H) isotope shift are consistent$\rightarrow$ redo eD scattering?

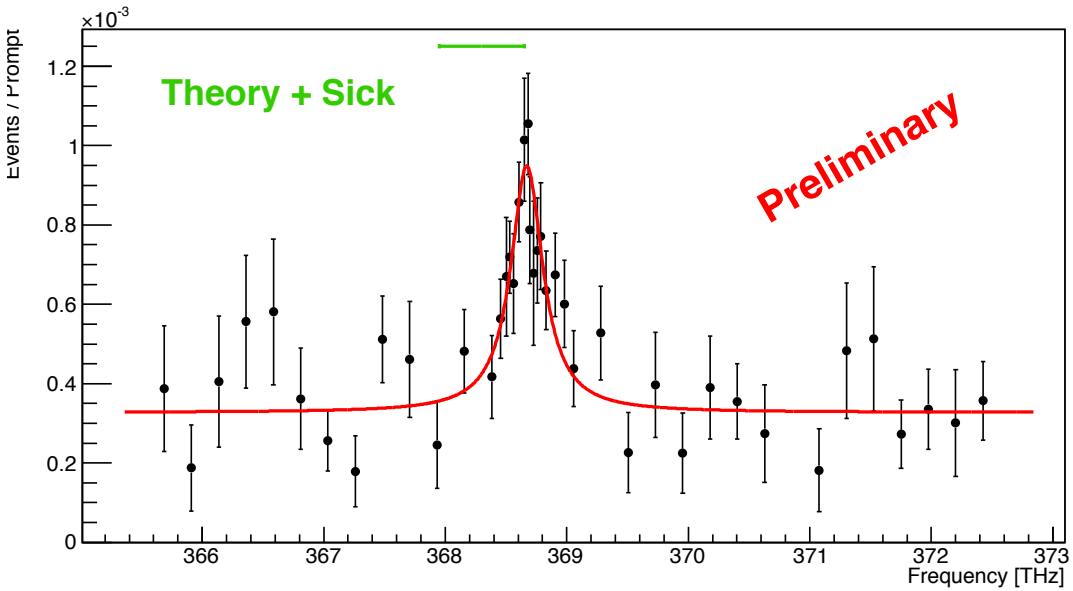
Using the CODATA deuteron radius corresponds to

$$\delta E_L^{\mu \mathrm{D}} = -0.438(59) \text{ meV compared with } 0.307 \text{ in proton}$$

Puzzle is worse!

Secret results!

^{4}He



- The transition has been found at the expected position i.e., whithin the uncert. given by r_{He} from $e-$He scattering.
- New physics model of Pospelov excluded
- Zavattini value from old $\mu\,\mathrm{He}^+$ experiment excluded

Need to summarize all 2S-2P contributions

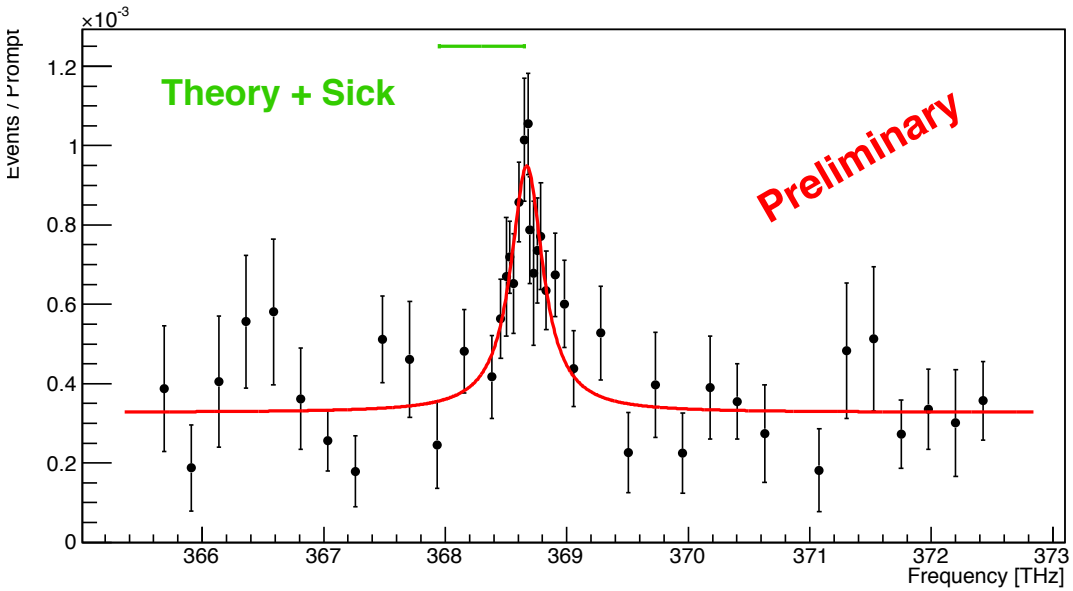
^{4}He nuclear charge radius

$1.681(4)$ fm	$u_r = 2 \times 10^{-3}$	[Sick]
$1.677(1)$ fm	(VERY preliminary)	[$\mu\,\mathrm{He}^+$]

No new effect in ^{4}He within 1 st. dev.

Secret results!

^{4}He



^{3}He expected soon

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Several new electron spectroscopy experiments

- Independent measurement of Rydberg constant. This would change only extracted r_p nothing else
- 2S-6S UK, 2S-4P Germany, 1S-3S France
- 2S-2P classic, Canada
- Highly charged single electron ions NIST

2S-4P has reported preliminary results- small radius not yet published

Possible resolutions

- ~~QED bound-state calculations not accurate- very unlikely- this includes recoil effects~~
- Electron H spectroscopy not so accurate
- Strong interaction effect in two photon exchange diagram
  - soft proton
- ~~More e^+e^- pairs than $\mu^+\mu^-$ pairs in the proton~~
- Muon interacts differently than electron!-new particles, ~~gravity, non-commutative geo~~metry

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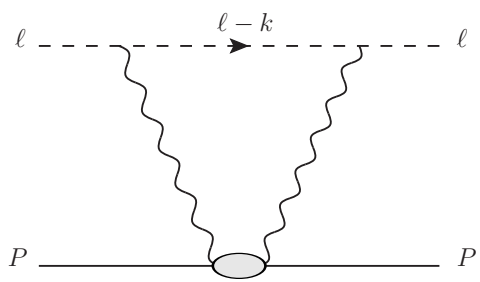
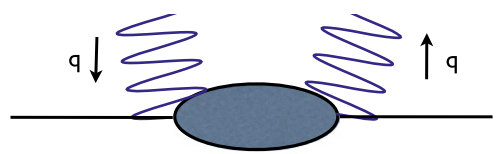
Our idea



energy shift proportional to lepton mass^4

$T^{\mu\nu} =$



$$= -(g^{\mu\nu} - \cdots)T_1 + (P^\mu - \cdots)(P^\mu - \cdots)T_2$$

Dispersion relation: $Im[T_1] \propto W_1$ measured

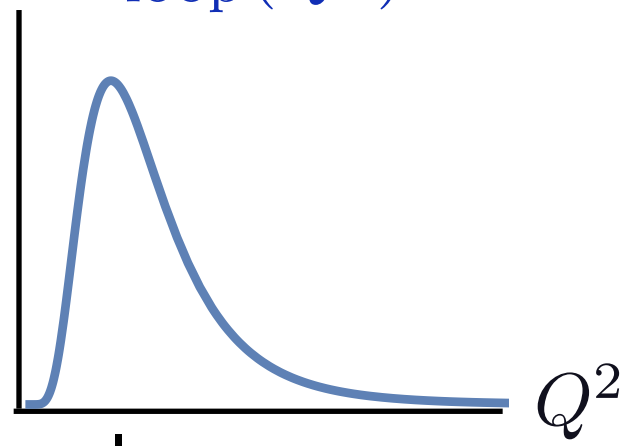
Large virtual photon energy ν, $W_1 \sim \nu$ integral over energy diverges

Subtraction function needed: $\bar{T}_1(0, Q^2)$ zero energy

Hill & Paz- big uncertainty in dispersion approach

Almost unknown $\bar{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} \sim \frac{\alpha^2}{m}\Psi_S^2(0)\int^\infty \frac{dQ^2}{Q^2}\ldots F_{\rm loop}(Q^2)$$



number of

Infinite $F_{\rm loop}(Q^2)$ give 0.31 meV satisfy all constraints

Recast in EFT- parameters seem natural

Soft proton

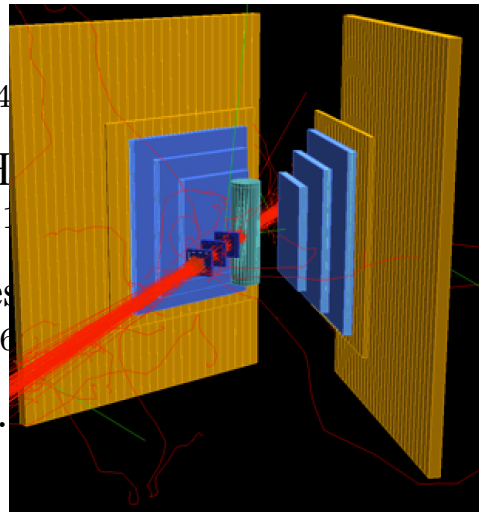
e^+/e^- and μ^+/μ^- scattering on proton

So what? MUSE expt

A Proposal for the Paul Scherrer Institute πM1 beam line

Studying the Proton "Radius" Puzzle with μp Elastic Scattering

J. Arrington,[1] F. Benmokhtar,[2] E. Brash,[2] K. Deiters,[3] C. Djalali,[4]
Fuchey,[6] S. Gilad,[7] R. Gilman (Contact person),[5] R. Gothe,[4] D. H
Ilieva,[4] M. Kohl,[9] G. Kumbartzki,[5] J. Lichtenstadt,[10] N. Liyanage,
Z.-E. Meziani,[6] K. Myers,[5] C. Perdrisat,[13] E. Piasetzsky (Spokes
Punjabi,[14] R. Ransome,[5] D. Reggiani,[3] A. Richter,[15] G. Ron,[16]
E. Schulte,[6] S. Strauch,[4] V. Sulkosky,[7] A.S. Tadapelli,[5] and L.



determining the proton radius through muon scattering, with simultaneous electron scattering measurements.

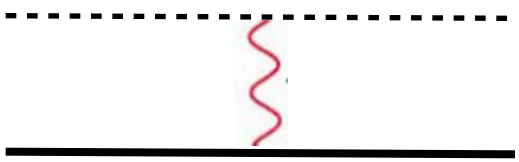
PSI proposal R-12-01.1

2 photon exchange idea is testable

http://www.physics.rutgers.edu/~rgilman/elasticmup/

muon scattering

$$\mathcal{M} = \mathcal{M}^{(1)} + \mathcal{M}^{(2)}$$



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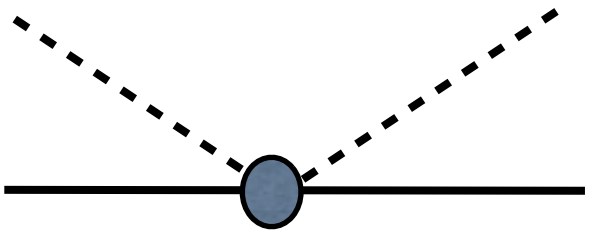

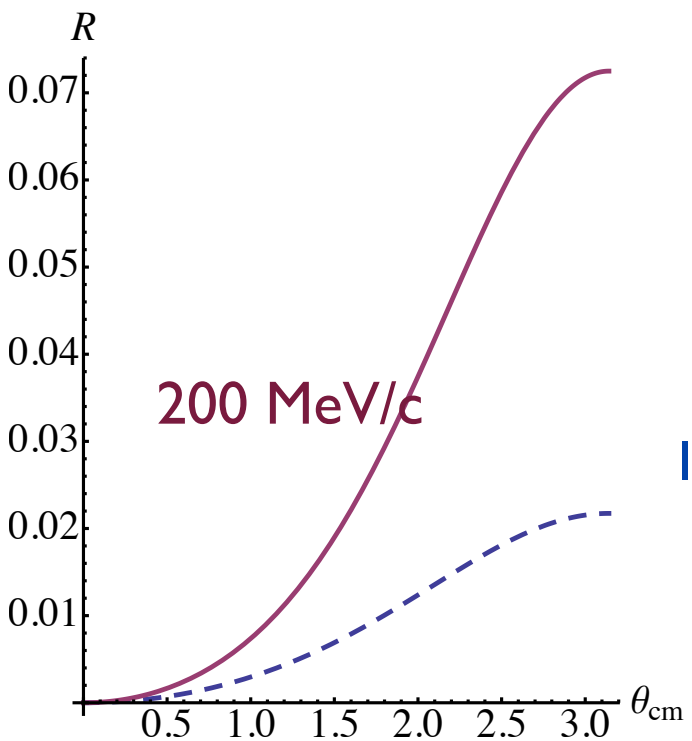

$$R = 2\frac{\mathrm{Re}[(\mathcal{M}^{(1)})^* \mathcal{M}^{(2)}]}{|\mathcal{M}^{(1)}]|^2}$$

~5 % effect should be seen

~10 % for ratio +/-

Radians

Soft proton idea

- explains muon Lamb shift
- no change to electron Lamb shift
- no hyperfine interaction
- can adjust neutron term so Deuteron is OK
- easily testable in muon-proton scattering
- easily testable in heavier muonic atoms

Nuclear dependence of short-ranged mu-p effects

- Energy shift is proportional to square of muon wave function at the origin

GAM 1501.01036

- Suppose you have effect that gives energy shifts Ep (on proton) E_n (on neutron)

$$E_A = \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p+NE_n)\left(1-\mathcal{O}(\frac{R_A^2}{a_\mu^2})\right) \approx \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p+NE_n),$$

Nuclear shift

Square of wave fun

Counting

Size of nucleus

My model: ~0.3 meV (1+0.3)(8)(2) =-6.3 meV about 6 st. dev off **RIP any short range idea**

Two photon effect is not zero

- Doesn't explain proton radius puzzle but
- largest source of uncertainty in the analysis of muonic-atom experiments
- MUSE experiment!

Possible resolutions

- ~~QED bound-state calculations not accurate- very unlikely- this includes recoil effects~~
- Electron experiments not so accurate
- ~~Strong interaction effect in two photon exchange diagram-my work- soft proton~~
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Another muon opportunity-anomalous moment

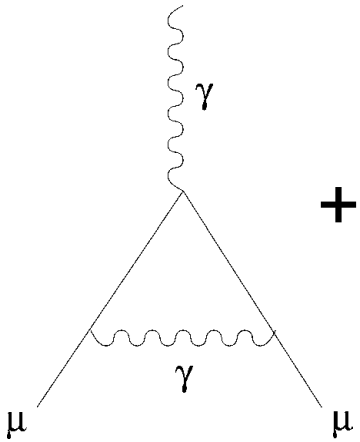

Figure 1 The first-order QED correction to g-2 of the muon.

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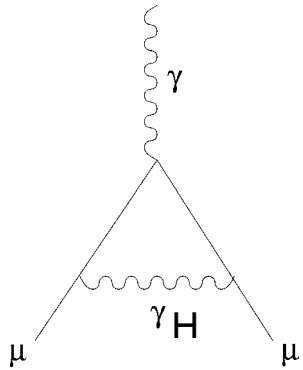

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3.6 st. dev anomaly now
fix add heavy photon
interacts preferentially with
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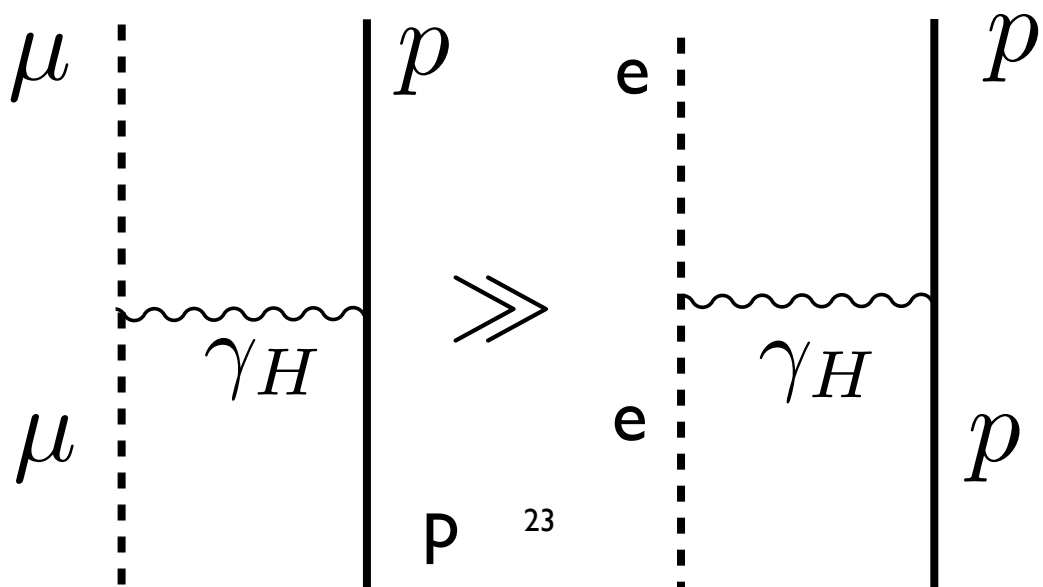
Muon data is g-2 - BNL exp't,
Hertzog- Lab now...

Lamb shift

Maybe dark matter, energy particles show up in muon physics!



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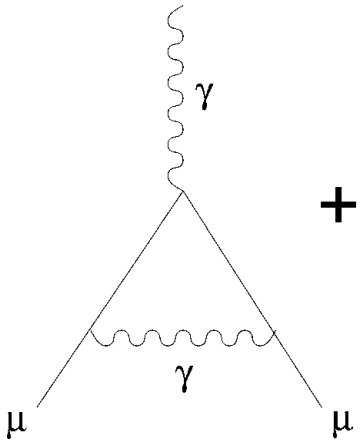

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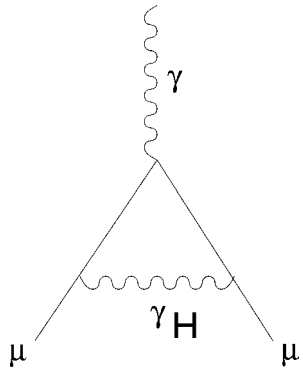

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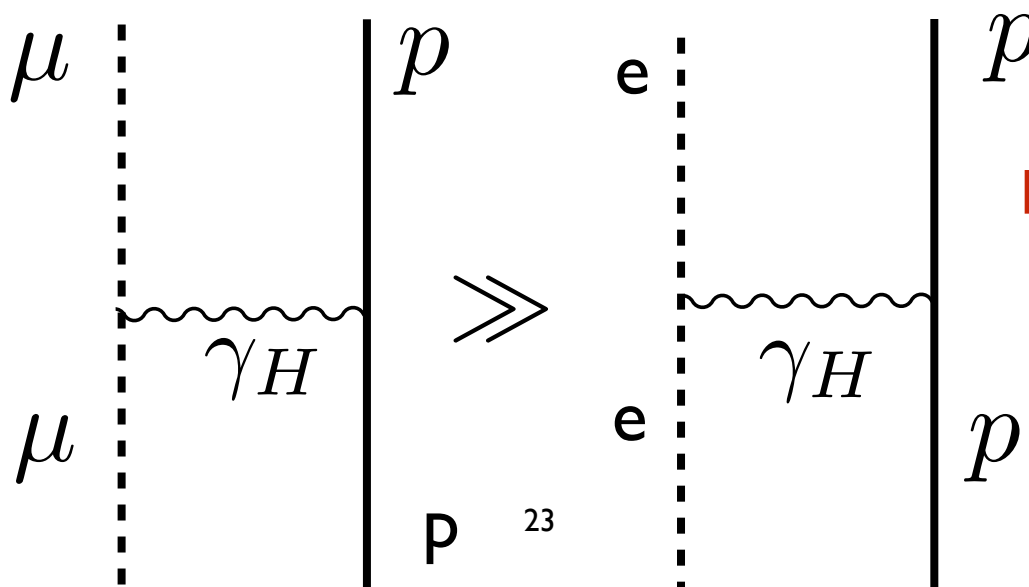

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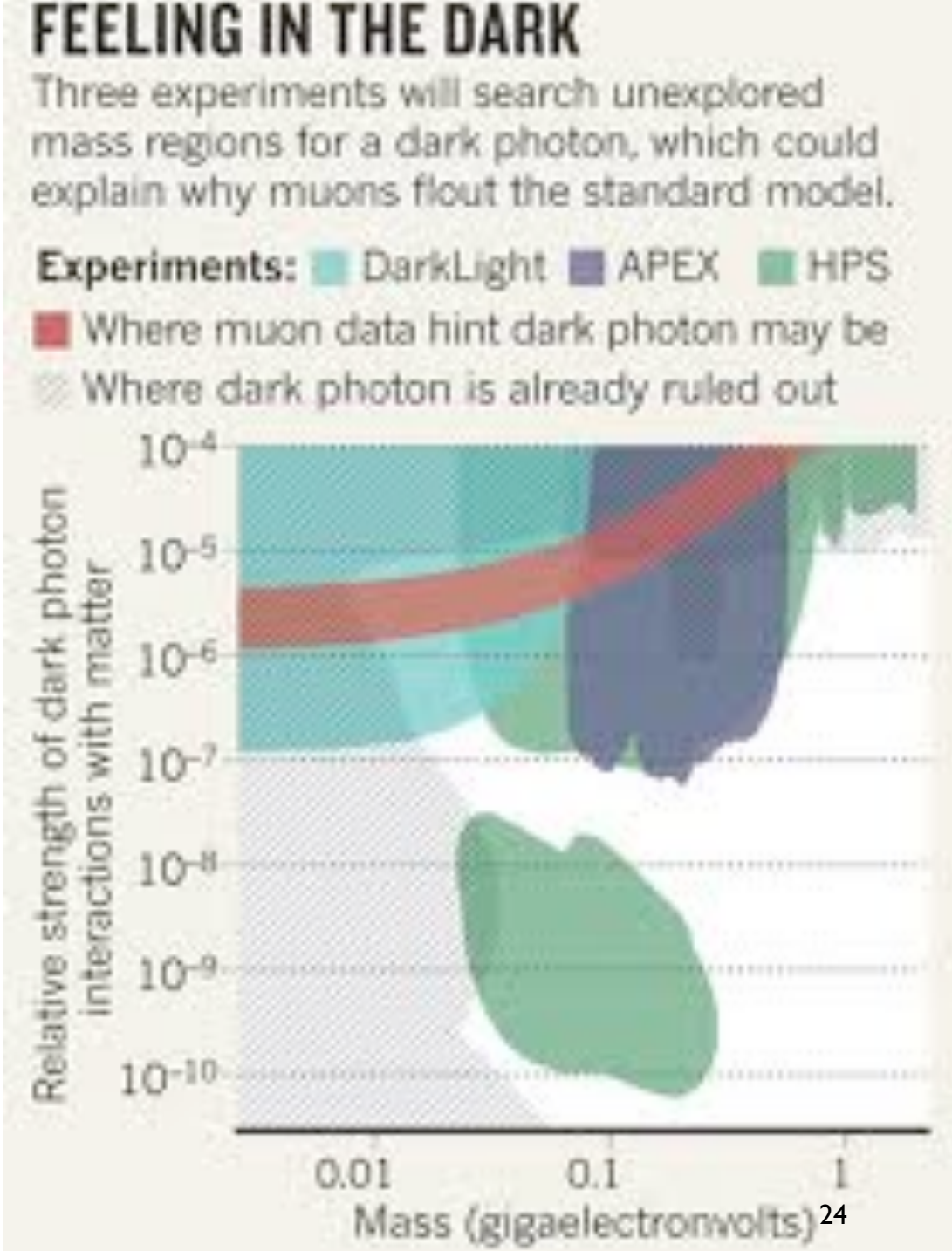
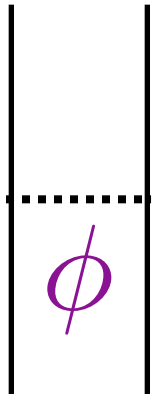
R Essig '15 APS



Scalar exchange



Looking for new scalars is not new
Low mass Higgs searches

$$p(1.88\,\mathrm{MeV}) +^{19} F \rightarrow \alpha +^{16} O^*(6.05)$$
$$^{16}O^*(6.05) \rightarrow^{16} O(GS) + \phi$$

Kohler et al PRL 33, 1628 (1974)

Freedman et al. PRL 52, 240 (1984)

$$p +^3 H \rightarrow^4 He(20.1) \rightarrow 4He(GS) + \phi$$

No Scalars found, but assumed coupling constants were much larger than what we will use

Lepton-universality violating one boson exchange

- Tucker-Smith & Yavin PRD83, 101702 new particle scalar or vector coupling
- Brax & Burrage scalar particles PRD 83, 035020 &'14
- Batell, McKeen & Pospelov PRL 107, 011803 new gauge boson kinetically mixing with $F^{\mu\nu}$ plus scalar for muon mag. mom. 1401.6154 W decays enhanced
- Carlson Rislow PRD 86, 035013 fine tune scalar pseudoscalar or polar and axial vector couplings
- Barger et al PRL106,153001 - new particles ruled out but assumes universal coupling
- Kaon decays provide constraints

New scalar bosons must

ϕ

- give μ-p Lamb shift
- almost no hyperfine in μ proton
- small effect for D, almost no effect ^{4}He
- consistent with g-2 of μ and electron
- many other constraints
- be found

PRL **117,** 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

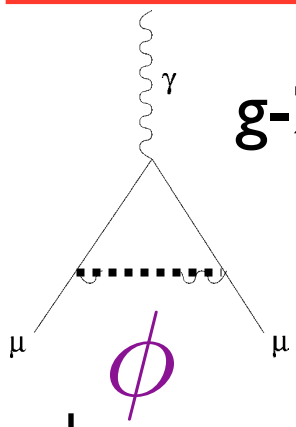
Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,[*] David McKeen,[†] and Gerald A. Miller[‡]

Our approach

- Exchange of ϕ, accounts proton radius and $(g-2)_\mu$
- $\mathcal{L} \supset e\epsilon_f \phi \bar{f} f$, $\epsilon_f = g_f/e$ and e is the electric charge of the proton.
- TuckerSmith, Izaguirre et al. made assumptions on coupling constants
- We make no assumptions re signs, magnitudes of coupling constants.
- The Lamb shift in muonic hydrogen fixes $\epsilon_\mu \epsilon_p > 0$ each > 0
- ϵ_e and ϵ_n are allowed to have either sign.

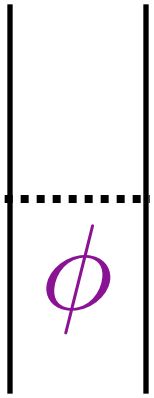
First constraints



g-2

$$\Delta a_\ell = \frac{\alpha \epsilon_\ell^2}{2\pi} \int_0^1 dz \frac{(1-z)^2(1+z)}{(1-z)^2 + (m_\phi/m_\ell)^2 z}.$$

$\Delta a_\mu = 287(80) \times 10^{-11}$, $\Delta a_e = 1.5 \times 10^{-12}$ From ^{87}Rb



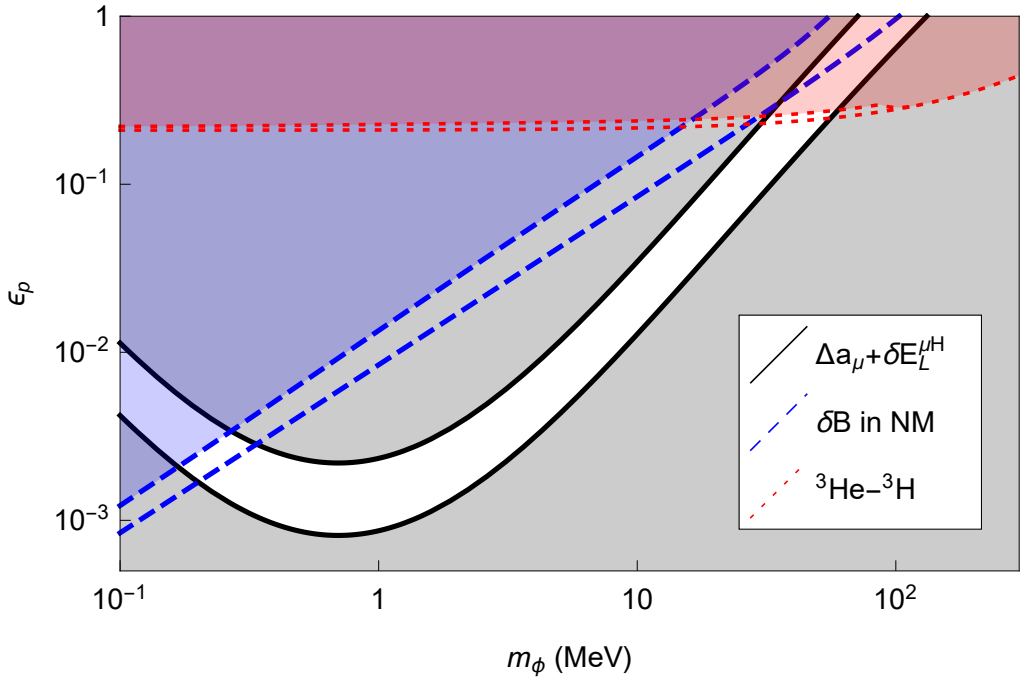
$$V(r) = -\epsilon_{f_1}\epsilon_{f_2}\alpha \frac{e^{-m_\phi r}}{r}.$$

$$|\epsilon_e| \ll \epsilon_\mu$$

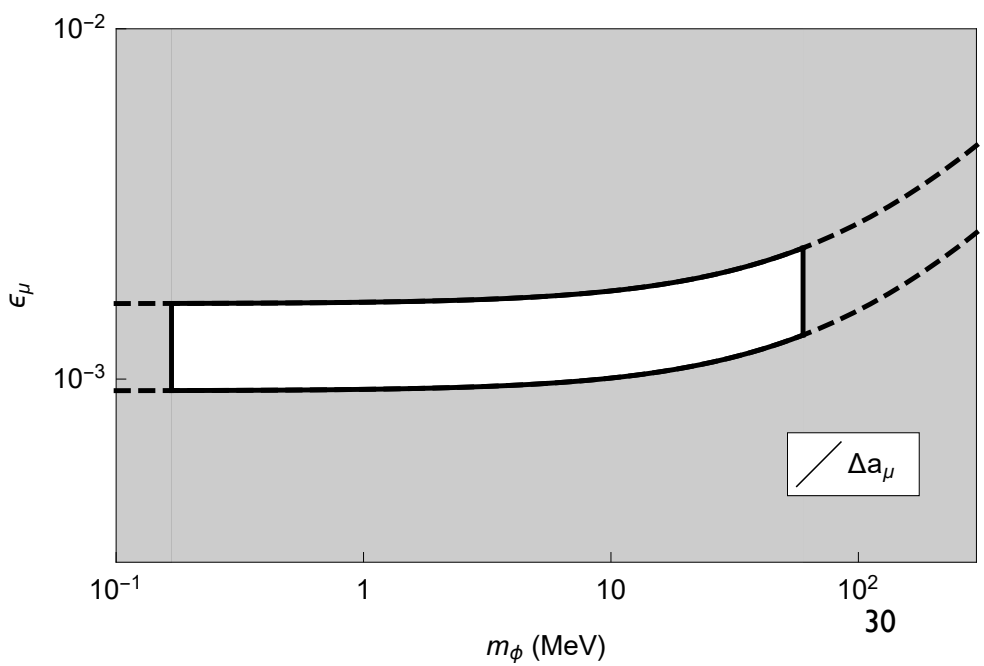
Bohr radius

$$\delta E_L^{\ell \mathrm{N}} = -\frac{\alpha}{2a_{\ell\mathrm{N}}} \epsilon_\ell [Z\epsilon_p + (A-Z)\epsilon_n] \frac{(a_{\ell\mathrm{N}} m_\phi)^2}{(1+(a_{\ell\mathrm{N}} m_\phi))^4}$$

$$\delta E_L^{\mu \mathrm{H}} = -0.307(56)\ \mathrm{meV}$$



Unshaded allowed by muon g-2 and muon-p Lamb shift



Unshaded allowed by muon g-2

Nuclear Physics constraints

ϵ_n / ϵ_p

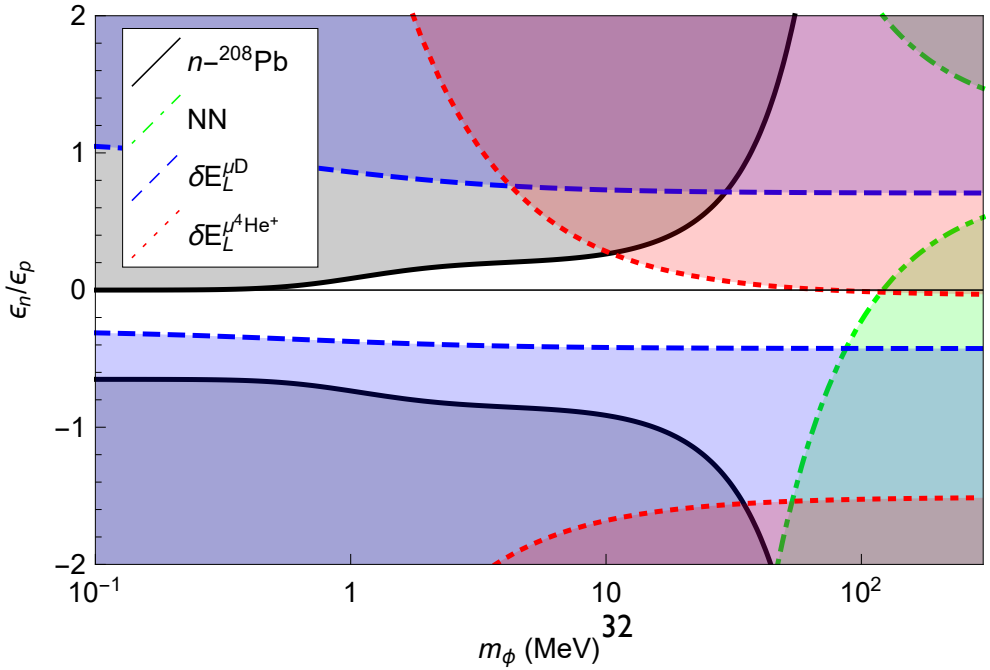
- Low energy scattering of neutrons on ^{208}Pb using ϕ-nucleon coupling g_N.
  $\frac{g_N^2}{e^2} \to \frac{A-Z}{A}\epsilon_n^2 + \frac{Z}{A}\epsilon_p\epsilon_n$ cancellation evades previous limits
- NN charge-independence breaking scattering length
  $\Delta a = (a_{pp} + a_{nn})/2 - a_{np}$, measured: 5.64(60) fm, theory: 5.6(5)
  Scalar boson exchange: $\Delta a_\phi \propto \int_0^\infty \Delta V \bar{u} u_{np} dr \leq 1.6$ fm (2 S.D.)
- Change in binding energy/A infinite nuclear matter: less than 1 MeV
- binding energy B(^3He) $- B(^3H)$= 763.76 keV due to Coulomb (693 keV) + strong force charge symmetry breaking (68 keV) ϕ exchange $<$ 30 keV

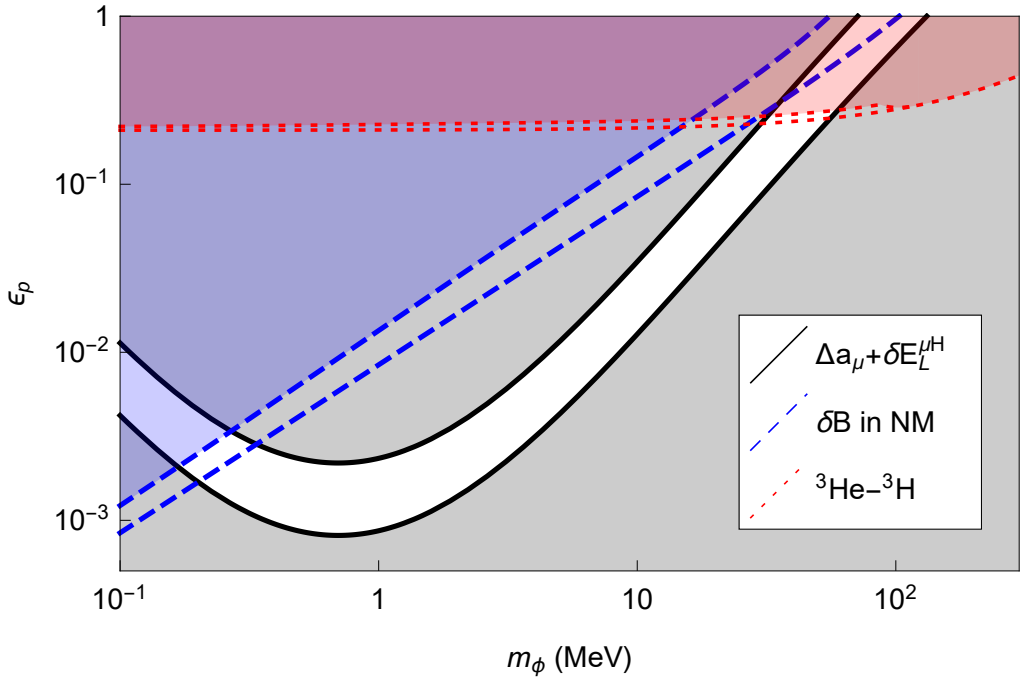
Muonic D, ^{3,4}He

$\delta E_L^{\mu\mathrm{D}} = -0.438(59)$ meV compared with 0.307 in proton

There is a (smaller) effect in the neutron as well

^{3,4}He data are coming, providing additional constraints
μ^4He, preliminary radii muonic Lamb shift, + elastic electron scatterin
$\rightarrow \delta E_L^{\mu^4\mathrm{He}} \approx -1.4(1.5)$ meV





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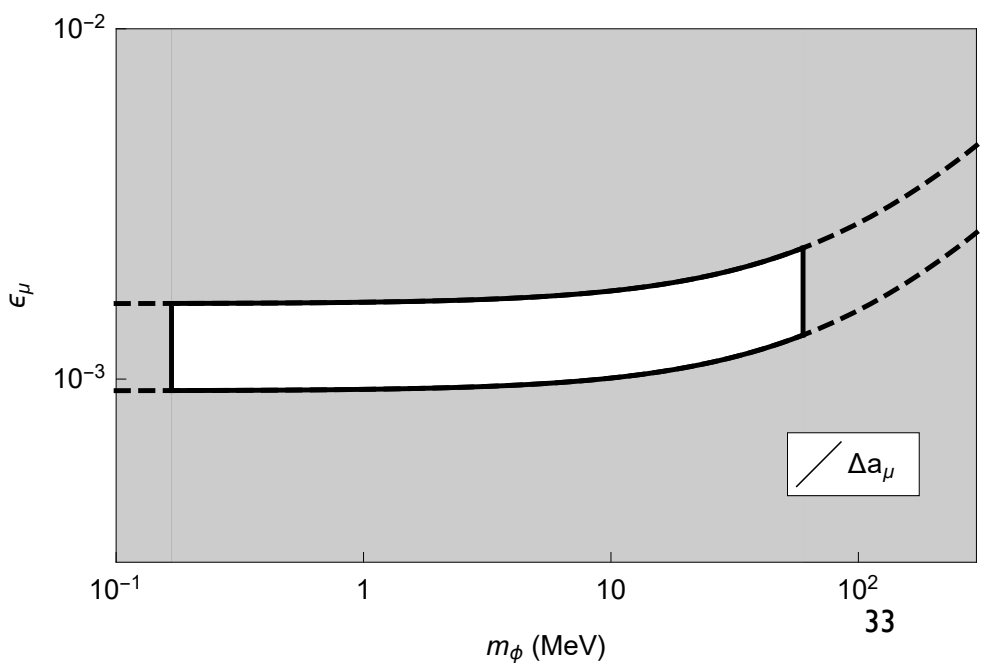


Unshaded allowed by muon g-2

Coupling to electron ϵ_e

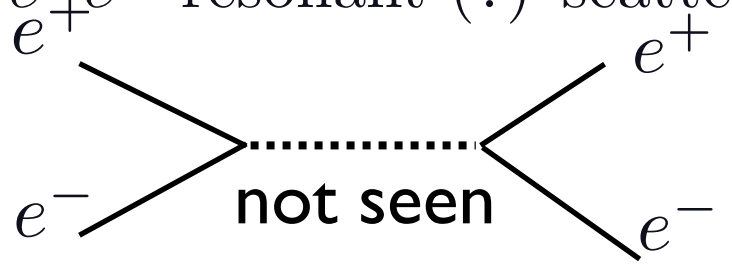
Anomalous magnetic moment

$\Delta a_\mu = 287(80) \times 10^{-11}$, $\Delta a_e = 1.5 \times 10^{-12}$ From ^{87}Rb

Hydrogen atom Lamb shift

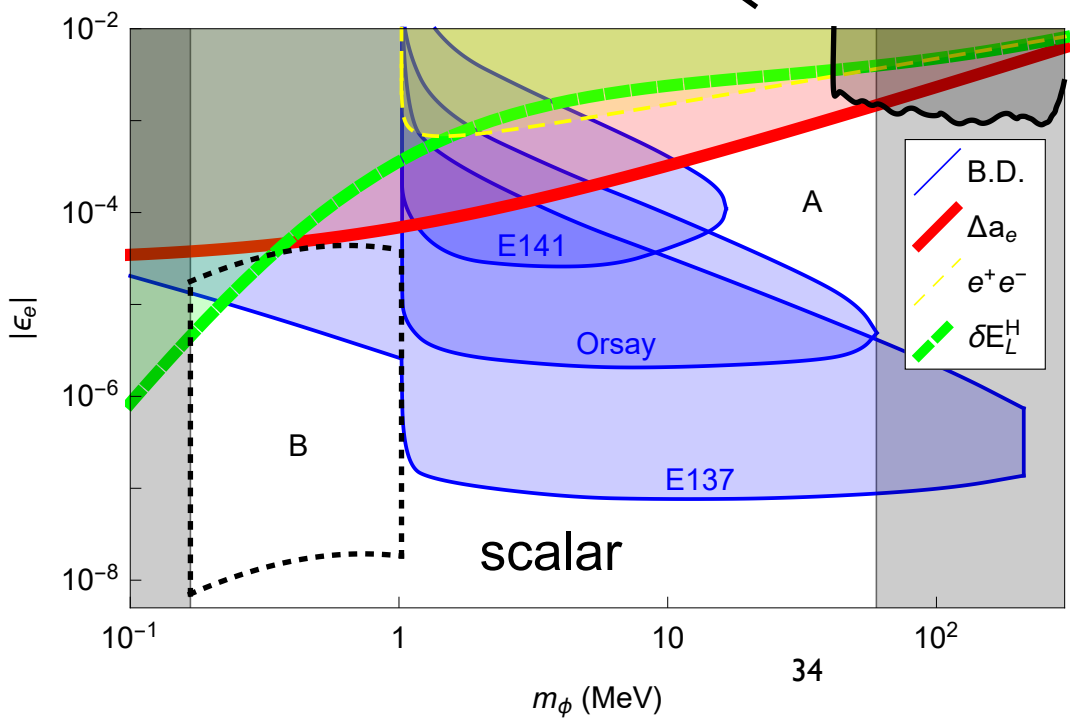
$\delta E_L^H < 45$ kHz

e^+e^- resonant (?) scattering



Tsertos et al PRD 40, 1397

claim experimental sensitivity of 0.5 b eV/sr



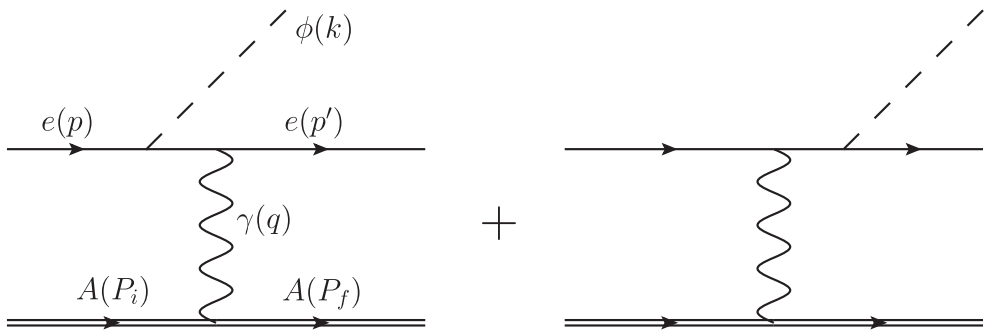
BD is beam dump discussed next

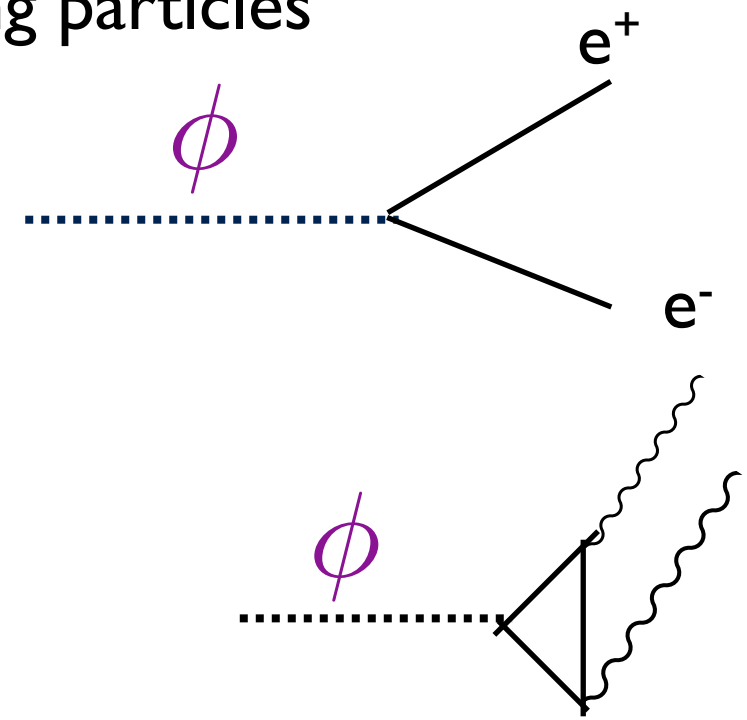
Beam dump experiments

Beam dumps absorb beam of charged particles

$e+^{27}Al$ to dissipate energy

$e+^{27}Al$ makes penetrating particles





PHYSICAL REVIEW D **95,** 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,[*] David McKeen,[†] and Gerald A. Miller[‡]

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Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

- previous cross sections obtained w. WW approximation
- cross sections not accurate
- exclusion plots changed substantially
- if discovery, WW gives wrong parameters
- not necessary to assume mass of new particle is much much greater than mass of electron

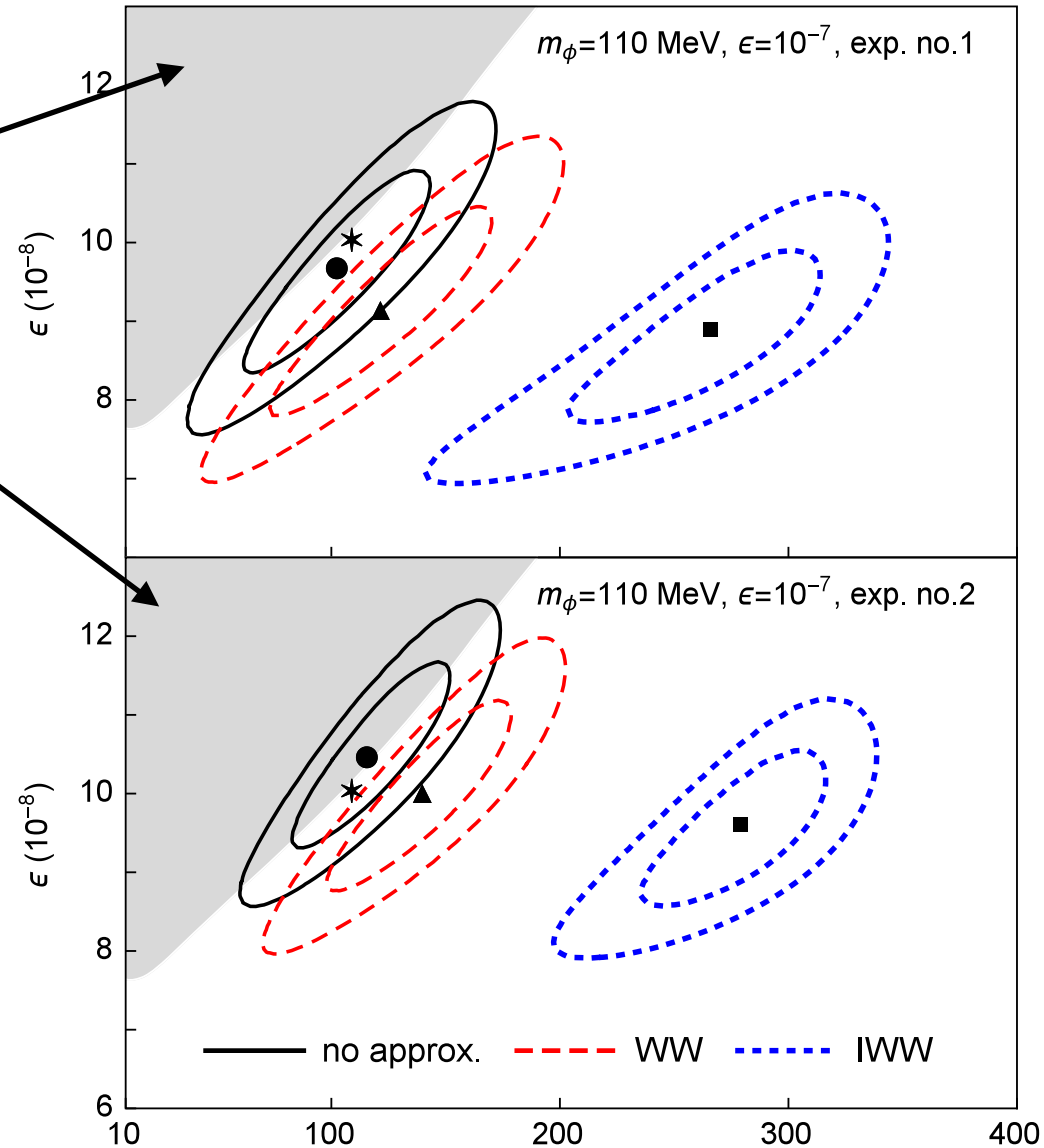
Generate pseudo data -E137 set up

We increase the incoming beam luminosities by 36, 36, and 137 times (increasing the total number of electrons dumped into the target), so that the expected total number of events is around 100, 100, and 400. We assume that the resolution of the detector is 1 GeV (which means that there are 18 bins) and generate the "observed" number of events using a Poisson distribution with the mean value from the complete calculation for each bin. Finally, we can fit the "observed" data with the calculation with no, WW, and IWW approximation

Pseudodata
luminosity of E137 increased by 36

excluded
E137

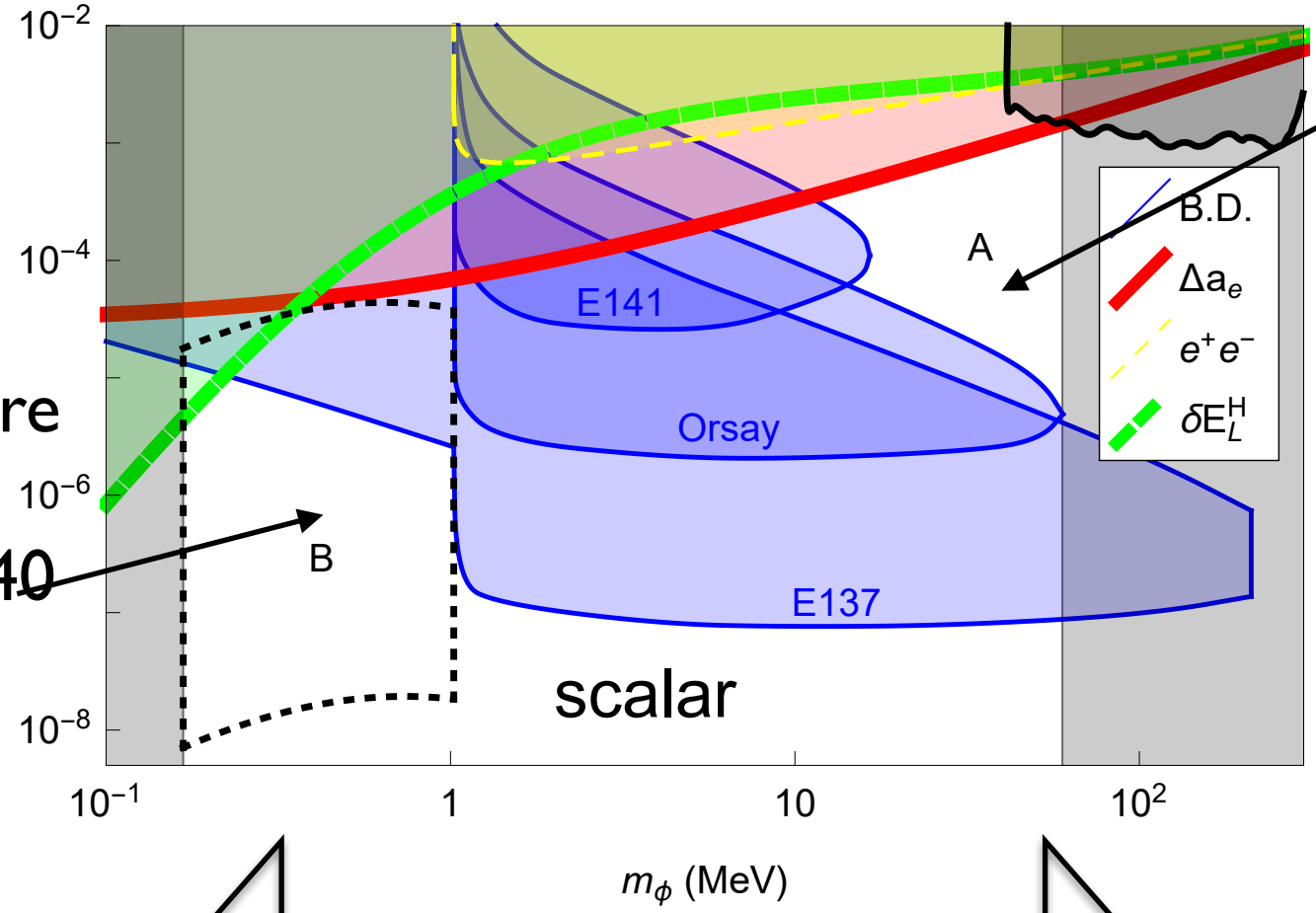
* is exact



Exclusion plot

APEX
HPS
DarkLight
VEPP-3
MESA

Izaguirre et al PLB740 61



allowed mass range

Parameter space

1. $\epsilon_p \neq \epsilon_n \rightarrow$ widens m_ϕ to between 130 keV to 73 MeV
2. $\epsilon_n \neq 0$: $\epsilon_n \epsilon_p < 0$ opens up the parameter space.
3. electron beam dump experiments $\rightarrow$ constraint on ϵ_e at $m_\phi = 1$ MeV improved by two orders of magnitude
4. Near maximum allowed $m_\phi \sim 70$ MeV,

   $|\epsilon_e| < 1.8 \times 10^{-3}$; $10^{-3} < \epsilon_\mu < 2 \times 10^{-3}$; $\epsilon_p \lesssim 0.4$; $-0.3 \lesssim \epsilon_p \lesssim 0$,

   large, testable

MUSE and scalar

$$V_\phi(r) = -1.7 \times 10^{-6}\, \alpha \frac{e^{-m_\phi\, r}}{r}$$

- No scattering experiment can detect a coupling this weak
- If this scalar exists (and other experiments correct) MUSE will find electrons/positrons see the same large radius and
- muons and anti-muons will see the same large radius

Other possible experiments

- $pp \to pp + \phi$
- muon beam dump experiments
- proton beam dump experiments
- improved muon g-2

Summary

- If all of the experiments, and their analyses, are correct a new scalar boson of mass must exist
- Direct detection is needed.

Does 4% matter?

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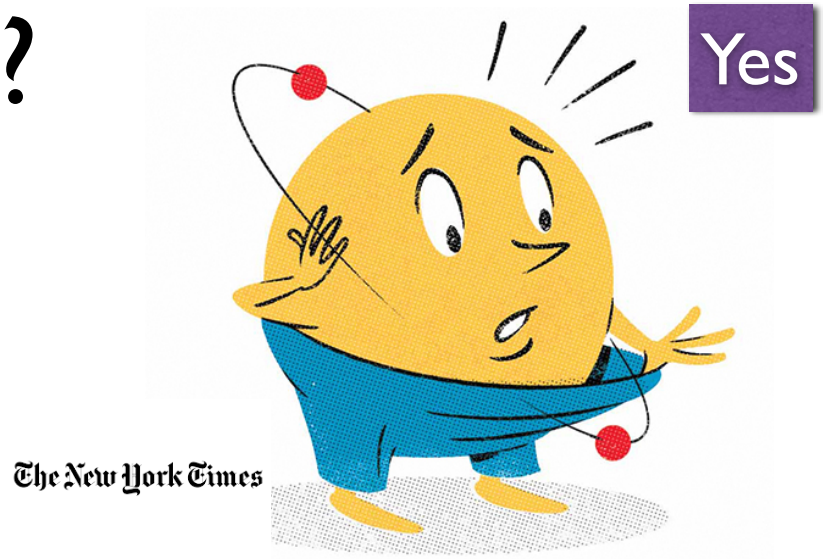
- Direct detection is needed.

Does 4% matter?

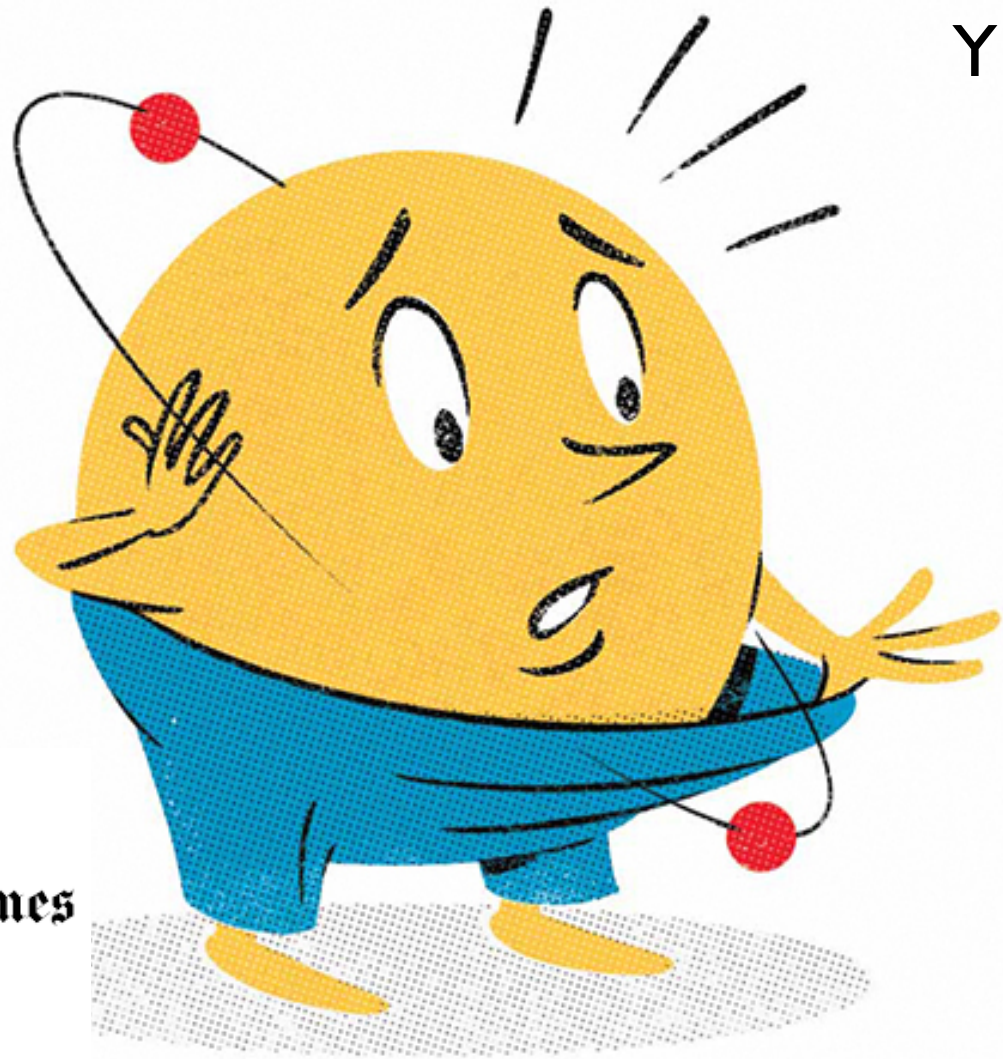


Spares follow

Does 4% matter?

Does 4% matter?

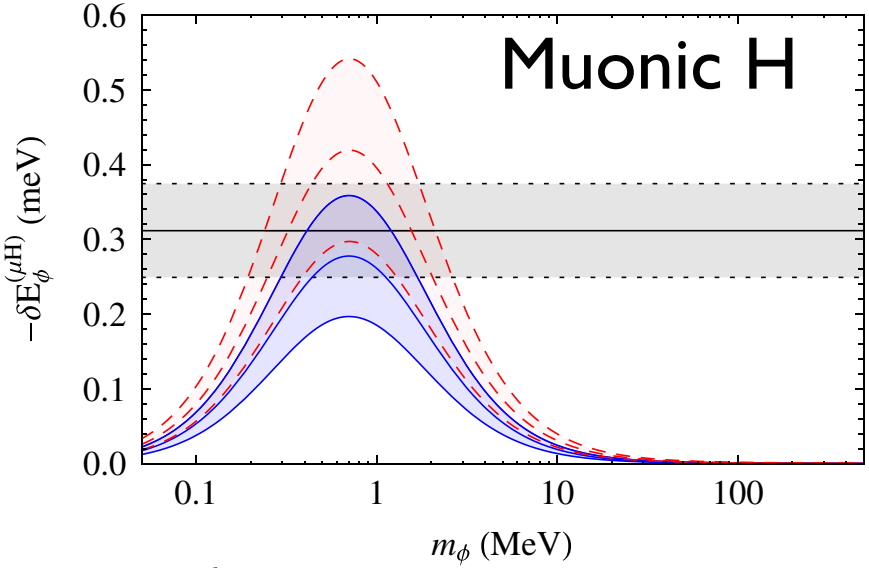
YES



The New York Times

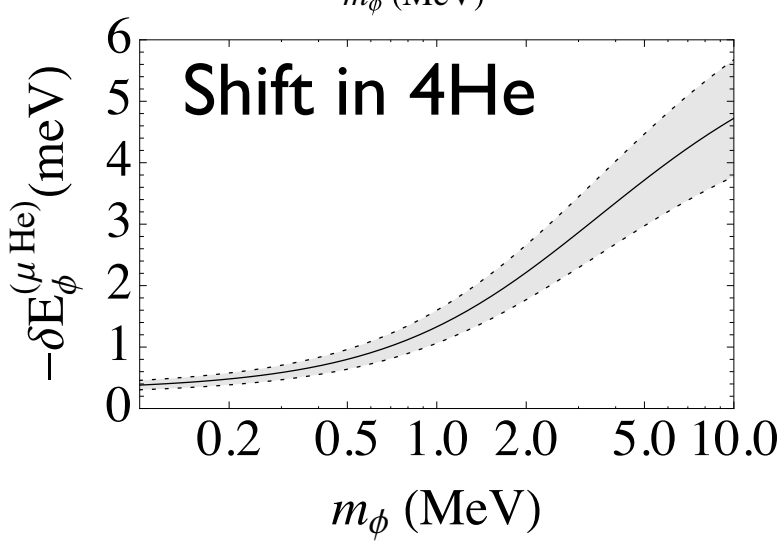
Tucker-Smith & Yavin PRD83, 101702(R)

ϕ couplings to μ, p greater than to e, n



Solid is scalar
dashed is vector
central curve gives
g-2 of muon

$$V_\phi(r) = -1.7 \times 10^{-6}\, \alpha \frac{e^{-m_\phi r}}{r}$$



1 meV shift is OK

Tucker-Smith & Yavin PRD83, 101702(R)

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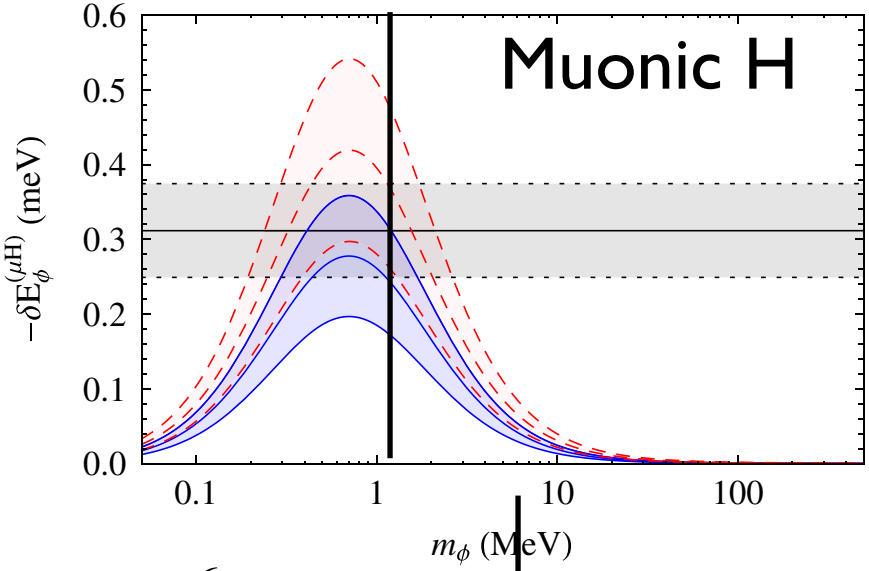

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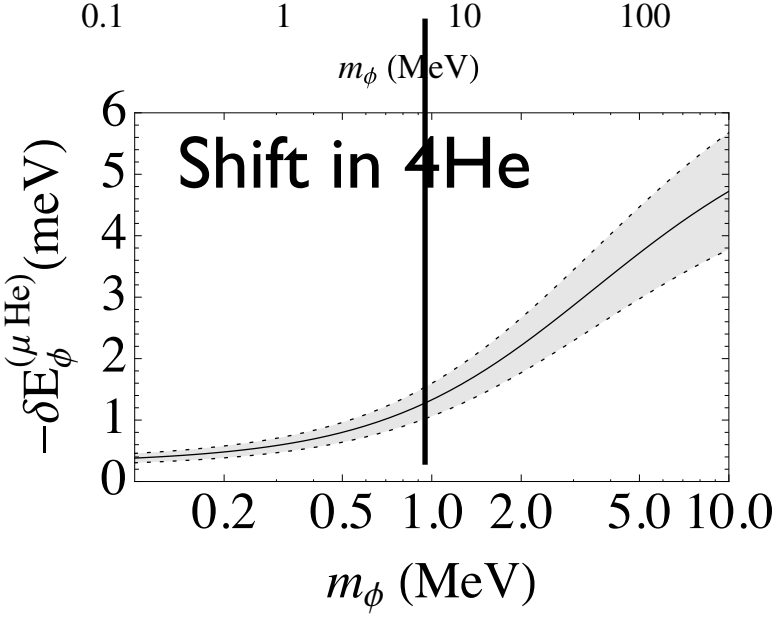


1 meV shift is OK

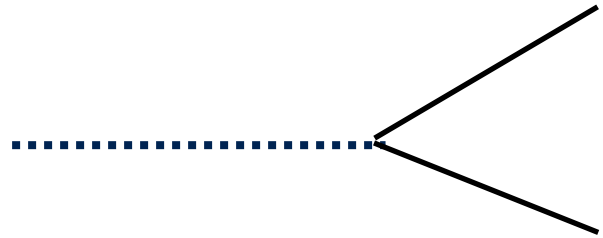
Vertical line- e^+e^- threshold

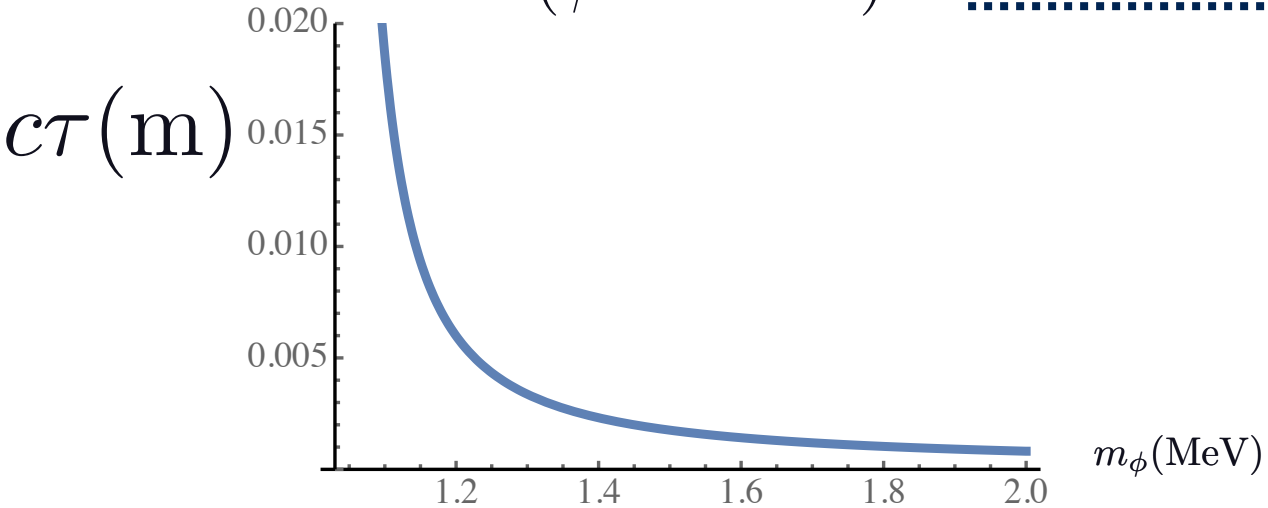
~1 MeV scalar consistent

- muonic Lamb shifts H,D, 4He
- no hyperfine
- K decays (Carlson 2015 review)
- Upsilon decay
- neutron scattering by model assumption
- g-2 of muon
- muonic atom (^{24}Mg ^{28}Si) transitions

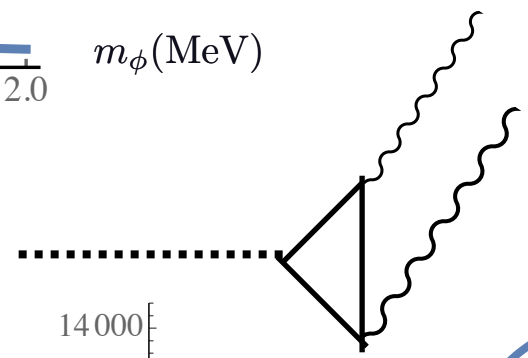
ϕ Decay modes

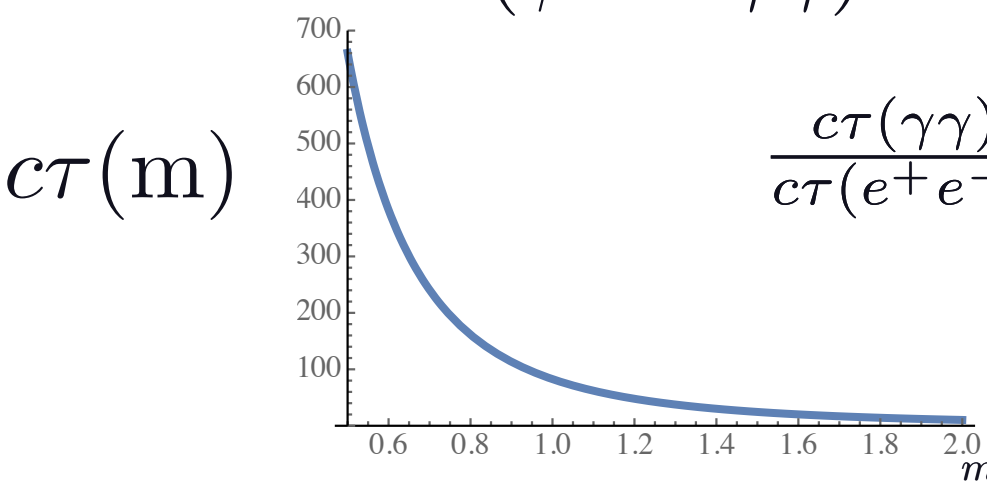
$\Gamma(\phi \to e^+e^-)$

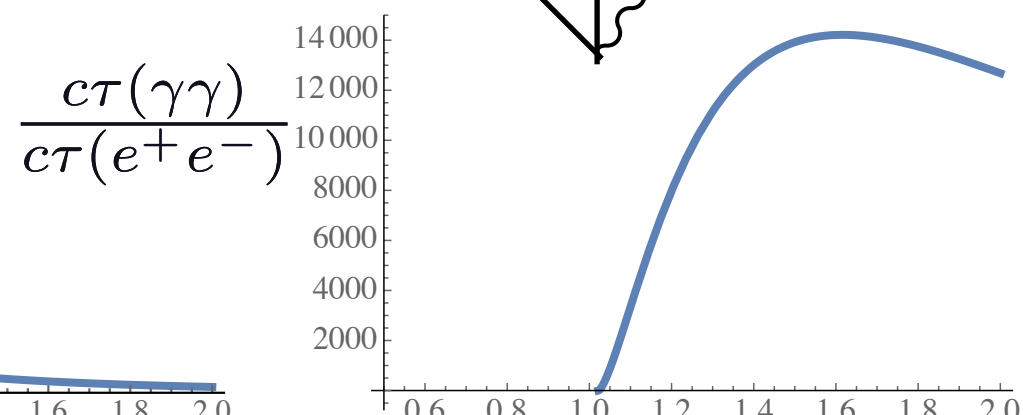




$\Gamma(\phi \to \gamma\gamma)$







Probing new physics with underground accelerators and radioactive sources

PLB 740, 61

Eder Izaguirre [a], Gordan Krnjaic [a,*], Maxim Pospelov [a,b]

Focus is on mass range ; $250\ \mathrm{keV} \leq m_\phi \leq 2m_e$

Discusses motivation and existing constraints BUT

The region with mass greater than 2 electron masses is NOT ruled out

- Focus here on detecting electron-positron decays of ϕ
- make ϕ with proton induced reaction or positron-electron scattering

$$^{16}O(6.05, 0^+) \rightarrow {}^{16}O(\mathrm{GS}, 0^+) + \phi, \quad \text{No single photon decay}$$

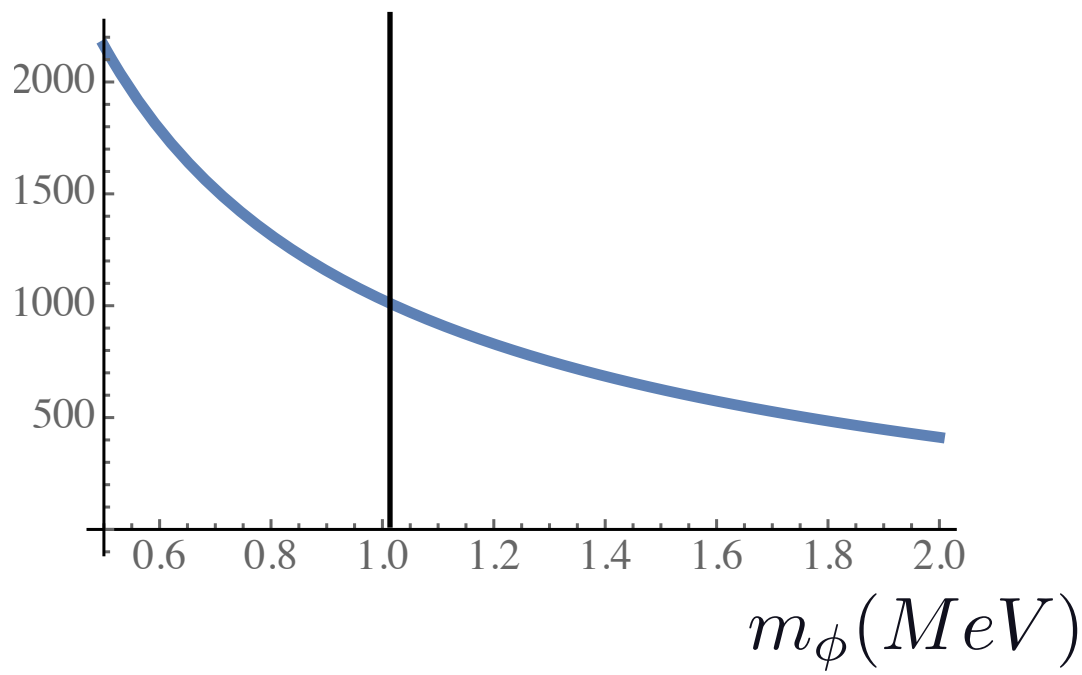
From electron g-2

$$\frac{\tau(A^* \rightarrow A + e^+e^-)}{\tau(A^* \rightarrow A + \phi} = 3.3 \times 10^3 \frac{g^2_{\phi e \bar{e}}}{e^2} \left(1 - \left(\frac{m_\phi}{6\mathrm{MeV}}\right)^2\right)^{5/2}$$

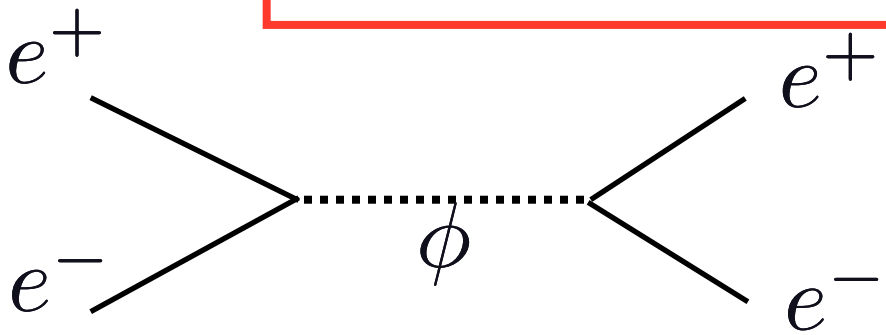
$$\tau(A^* \rightarrow A + e^+e^-): \quad \text{lifetime is } 10^{-10}\mathrm{s}$$

Decay length (m)

nuclear emission of scalar boson



e^+e^- resonant (?) scattering



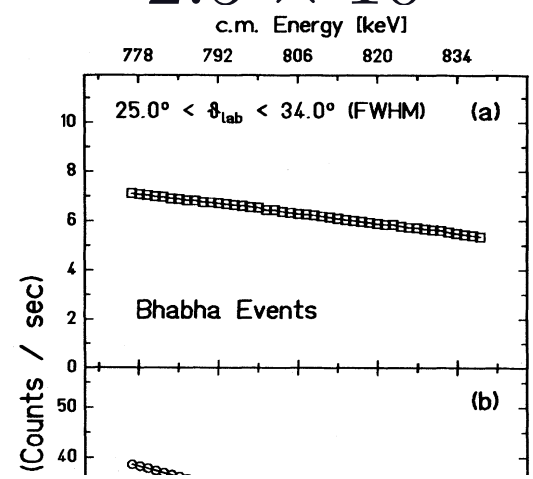
Tsertos et al PRD 40, 1397
claim experimental sensitivity
of 0.5 b eV/sr

rule out Mass 1.8 MeV

$$\text{Ratio to usual } \left(\frac{g^2_{\phi e}}{e^2}\right)^2 = (5.3 \times 10^{-8})^2 = 2.5 \times 10^{-15}$$

monoenergetic beam on Be

Our parameters



$$\int \frac{d\sigma}{d\Omega} dE = 0.14(1 - \frac{4m^2}{m_\phi^2})^{1/2} \leq 0.14 \text{ b eV}$$

Improve experiment by a factor of 4?

R Essig talk at APS '15

- Dark (heavy) photon is ruled out as explanation of muon g-2
- Complete parameter space has been searched and nothing is found
- But other scalar boson not searched completely

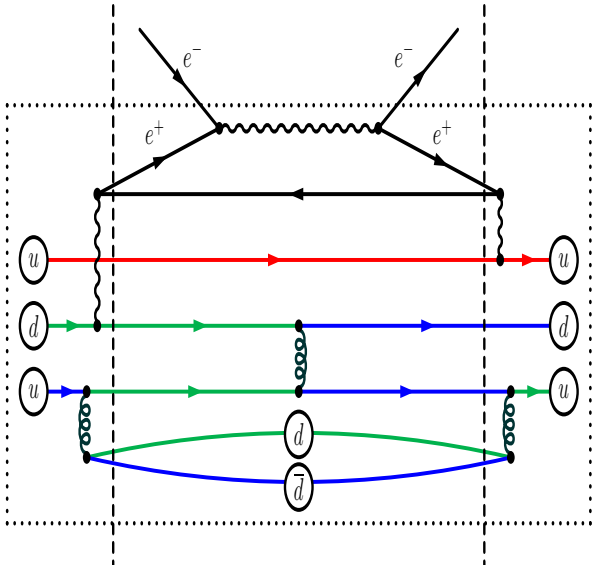
Yes it really is G_E

- Non-relativistic reduction of one-photon exchange leads to the spin independent interaction being $G_E(Q^2)/Q^2$
- All recoil effects properly accounted for:Breit-Pauli Hamiltonian computed for non-zero lepton and proton momentum

Light Sea Fermions in Electron–Proton and Muon–Proton Interactions

U. D. Jentschura Phys.Rev.A88 (2013) 062514

If we assume an average of roughly 0.7×10−7 light sea positrons per valence quark, then we can show that virtual electron-positron annihilation processes lead to an extra term in the electron-proton versus muon-proton interaction, which has the right sign and magnitude to explain the proton radius discrepancy.



Contribution for electron not muon

Non-perturbative lepton-pair exists in proton wave function. UDJ: energy shift $\propto 1/m_l^2$, from annihilation at rest. GAM: Shift $\propto 1/(\text{constituent quark mass})^2$

Any effect is small and same for electron and muon atoms

arXiv:1501.01036

Almost unknown $\overline{T}_1(0,Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$ Soft proton

$$\lim_{Q^2\to\infty} h(Q^2) \sim \frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha}Q^2 + \cdots$$

$\rightarrow$ Logarithmic divergence

Typo bleow

$\overline{T}_1(0,Q^2) \rightarrow \frac{\beta_M}{\alpha}Q^2 F_{\rm loop}(Q^2)$ Cuts off integral

Birse & McGovern assume dipole : $\Delta E^{\rm subt} = 0.004\,\text{meV}$ very small

Miller $F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N}, n \geq 2, N \geq$ n+3

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

Choose parameters so shift in proton mass <0.5 MeV (current uncertainty)

Recast in EFT- parameters seem natural

Arbitrary functions

$$\overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha} Q^2 F_{\rm loop}(Q^2)\,.$$

$$F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N},\ n \geq 2,\ N \geq n+3,$$

$$\overline{T}_1(0,Q^2) \sim \frac{1}{Q^4} \text{ or faster, } \beta_M \to \beta$$

$$\Delta E^{\rm subt} \approx 3\alpha^2 m \Psi_S^2(0) \frac{\beta}{\alpha} \gamma^n B(N,n), \gamma \equiv \frac{1}{M_0^2 a}$$

3 parameters: n, N, a $(M_0 = M_\beta)$

Choose parameters such that shift in proton mass < electromagnetic uncertainty of 0.5 MeV

Almost unknown $\overline{T}_1(0,Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$ Soft proton

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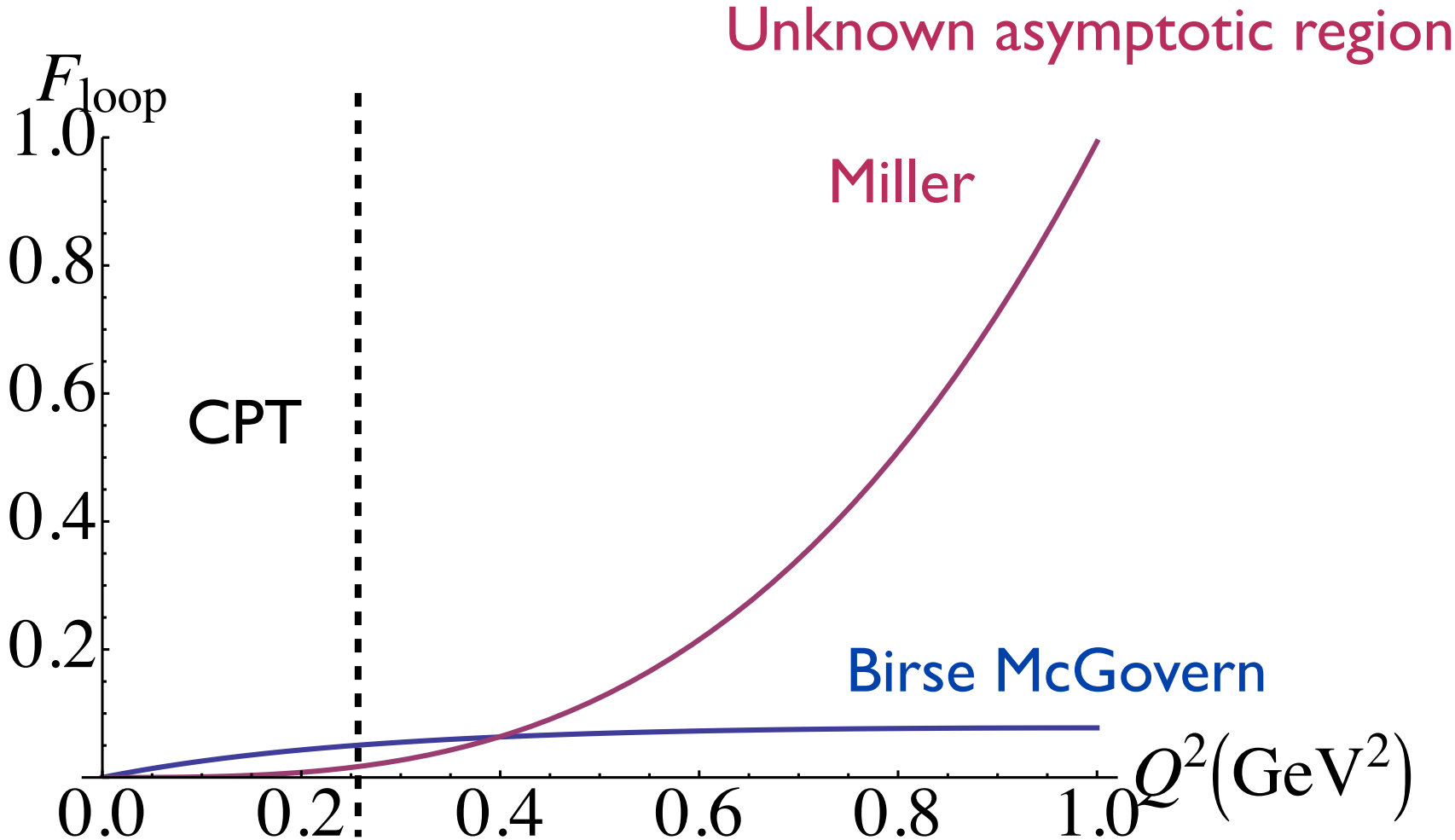
Choose parameters so shift in proton mass <0.5 MeV (current uncertainty)

Recast in EFT- parameters seem natural

New 1 MeV scalar boson

- give μ-p Lamb shift
- almost no hyperfine in μ proton
- consistent with g-2 of μ
- almost no effect for D, ^{4}He
- evade existing constraints
- be found

Form factors



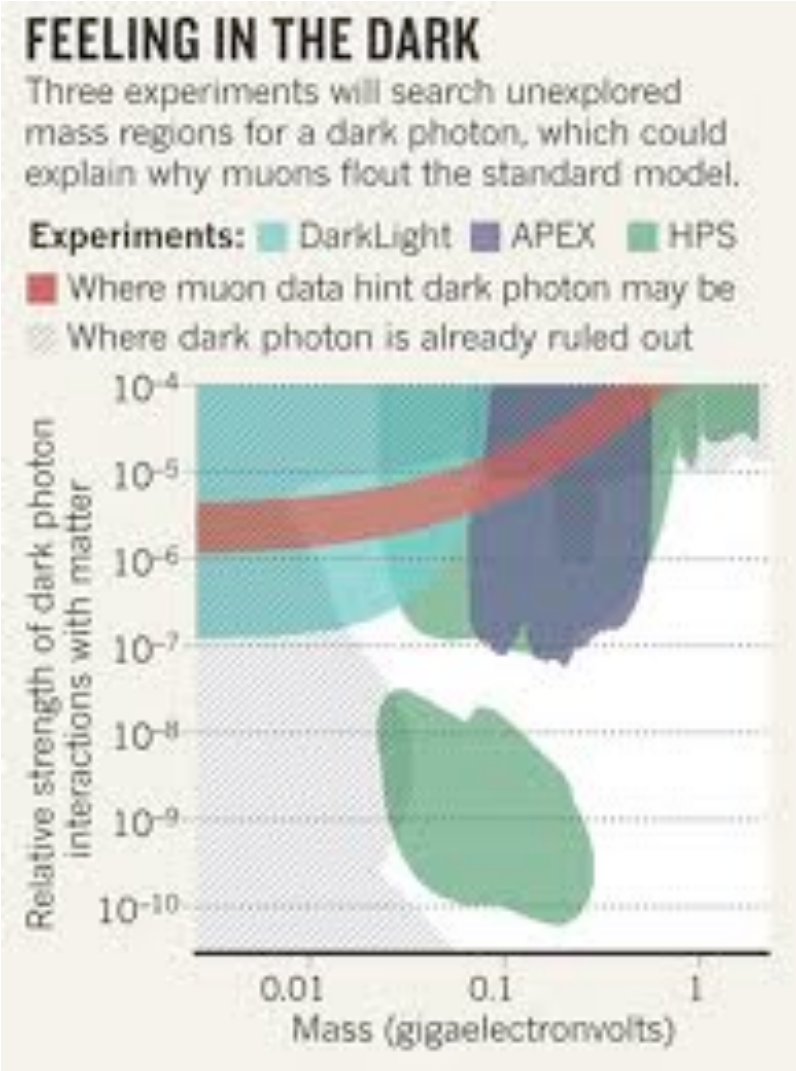
If recast into effective field theory strength seems natural

$$\mu \neq e$$

- Batell, McKeen, Pospelov **PRL 107,081802** New force differentiates between lepton species. Models with gauged right-handed muon number, contain new vector and scalar force carriers at the 100 MeV scale or lighter. Such forces would lead to an enhancement by several orders-of-magnitude of the parity-violating asymmetries in the scattering of low-energy muons on nuclei. Related to muon g-2--

- Karshenboim, McKeen Pospelov arXiv:1401.6154 Hyperfine effects in muonium> "completely disfavoring the remainder of the parameter space,

No BSM idea solves puzzle at this time,
but maybe



Three experiments at JLab

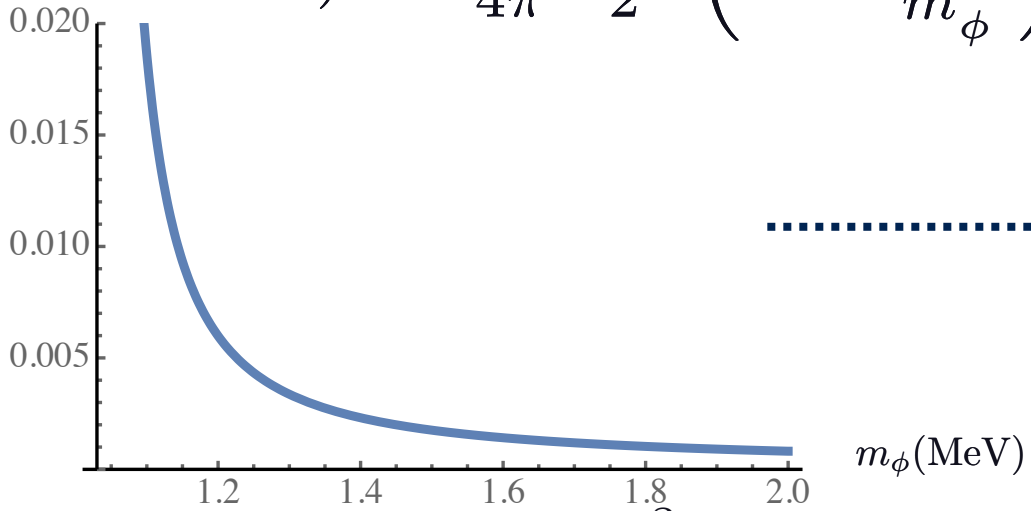
R. Essig

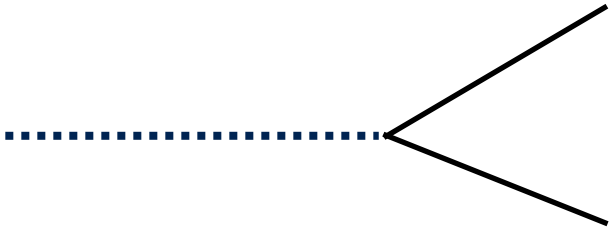
Muon data is g-2 - BNL exp't,
Hertzog- Kammel ...

ϕ Decay modes

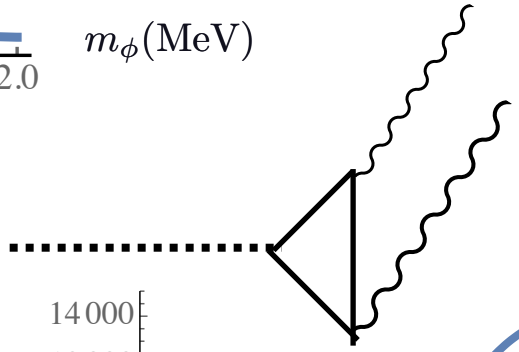
$$\Gamma(\phi \to e^+e^-) = \frac{g_{\phi e}^2}{4\pi}\frac{m_\phi}{2}\left(1 - \frac{4m_e^2}{m_\phi^2}\right)^{3/2}, m_\phi > 2m_e$$

$c\tau$(m)

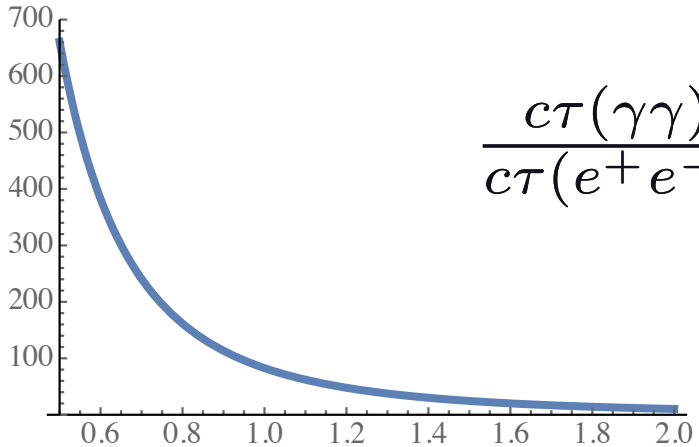




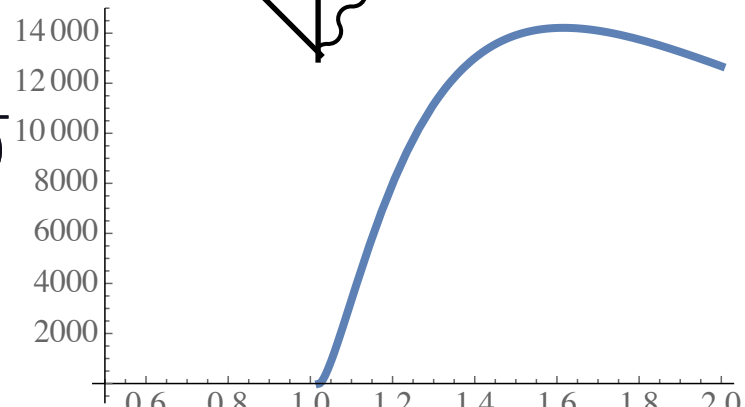
$$\Gamma(\phi \to \gamma\gamma) = g_{\phi e}^2 \frac{\alpha^2}{144\pi^3}\frac{m_\phi^3}{m_e^2}$$



$c\tau$(m)



$$\frac{c\tau(\gamma\gamma)}{c\tau(e^+e^-)}$$



m_ϕ(MeV)

2010 Experimental summary

Pulsed laser spectroscopy

"measure a muonic Lamb shift of 49,881.88(76) GHz. On the basis of present calculations[11–15] of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67)$ fm, which differs by 5.0 standard deviations from the CODATA value[3] of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by $-110\,\mathrm{kHz}/c$ (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient."

Jan. 2013, 7 st. dev
Antogini -Sci. 339,417

- Rydberg is known to 12 figures

$$R_\infty = \frac{m_e e^4}{8\varepsilon_0^2 h^3 c} = 1.097\,373\,156\,852\,5\,(73) \times 10^7\ \mathrm{m}^{-1},$$

- Puzzle- why muon H different than e H?

What theorists do

- make up new particles- compute shift
- study constraints -
- non-observation of new particles that couple mainly to muons

Constraints are obtained from the decay of the Y resonances;
neutron interactions with nuclei;
the anomalous magnetic moment of the muon
x-ray transitions in 24Mg and 28Mg, Si atoms;
J/Ψ decay;
neutral pion decay
eta decay

Any time a photon appears can also have a diagram with heavy photon

Pohl et al. Table of calculations

Lamb shift: vacuum polarization many, many terms

#	Contribution		Our selection		Pachucki[1–3]		Borie[5]	
		Ref.	Value	Unc.	Value	Unc.	Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*[7] Table 7.1. Item # 8 in Refs.[2,5] is the sum of items #6 and #7, without the recent correction from Ref.[12]. The error of #10 has been increased to 100% to account for a remark in Ref.[7]. Values are in meV and the uncertainties have been added in quadrature.

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Nuclear size correction of order $(Z\alpha)^6 < r_p^2 >$	1,27–29	−0.001243	$< r_p^2 >$		
Total $< r_p^2 >$ contribution		−5.22619	$< r_p^2 >$	−5.2256	−5.2244
Nuclear size correction of order $(Z\alpha)^5$	1,2	0.0347	$< r_p^3 >$	0.0363	0.0347

Table 2: All relevant radius-**dependent** contributions as summarized in Eides et al.[7], compared to Refs.[2,5]. Values are in meV and radii in fm.

Resolution 1-QED calcs not OK

α

Pohl et al. Table of calculations

Lamb shift: vacuum polarization many, many terms

Mostly irrelevant- theory replaced by experiment

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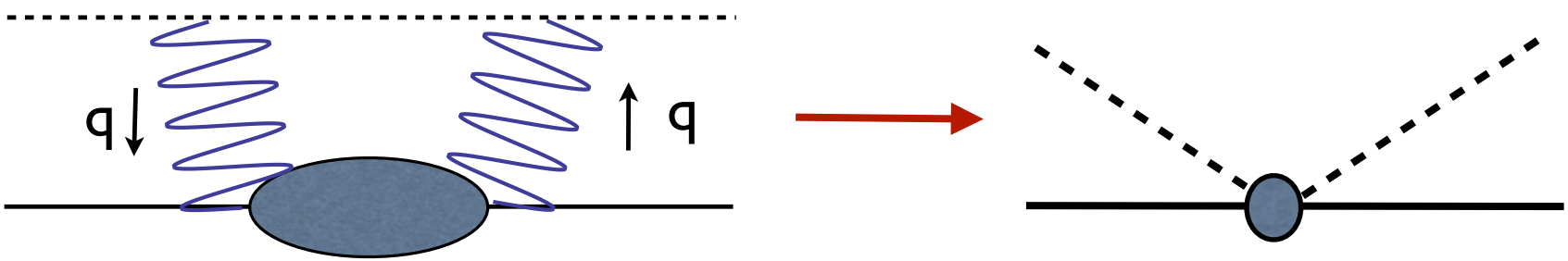
Resolution 1- QED calcs not OK

QED calcs expand in α

EFT of μp interaction

Caswell Lepage '86

- Compute Feynman diagram, remove log divergence using dimensional regularization
- include counter term in Lagrangian



$$\mathcal{M}_2^{DR} = \frac{3}{2} i\, \alpha^2 m \frac{\beta_M}{\alpha} \left[\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2} + \frac{5}{6} - \gamma_E + \log 4\pi\right] \overline{u}_f u_i \overline{U}_f U_i,$$

$$= i\, \alpha^2 m \frac{\beta_M}{\alpha} (\lambda + 5/4)\, \overline{u}_f u_i \overline{U}_f U_i$$

Choose λ to get 0.31 meV shift

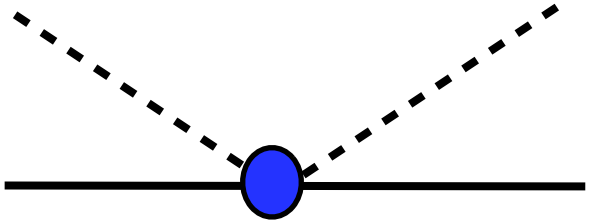
$$\Delta E^{\rm subt}(DR) = \alpha^2 m \frac{\beta_M}{\alpha} \Psi_S^2(0)(\lambda + 5/4)$$

$$\Delta E^{\rm subt}(DR) = 0.31 \text{ meV} \rightarrow \boxed{\lambda = 769}$$

β_M(magnetic polarizability) $= 3.1 \times 10^{-4}\text{fm}^3$ very small

Natural units $\beta_M/\alpha \sim 4\pi/(4\pi f_\pi)^3$ Butler & Savage '92

$$\mathcal{M}_2^{DR} = i\, 3.95\, \alpha^2 m \frac{4\pi}{\Lambda_\chi^3} \overline{u}_f u_i \overline{U}_f U_i.$$



3.95 =natural