Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

EWK interactions

EWK QE response

Outlook

Light-nuclei spectra and electroweak response: a status report

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October 20, 2017

Spectra of few-nucleon systems from LQCD

Beane *et al.* (2013)

Basic model

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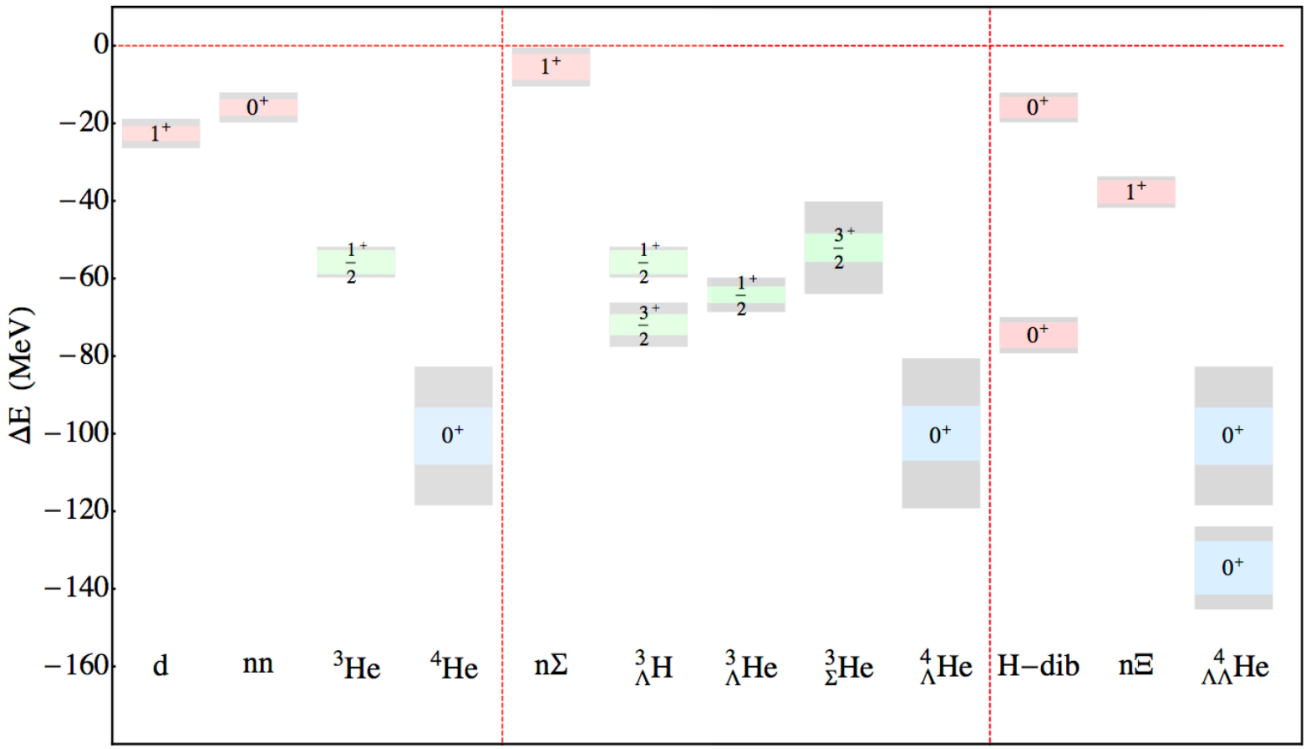
Outlook

- NPLQCD calculations ($m_\pi = 806$ MeV)



$2N$ potential from LQCD and nuclear spectra

Aoki *et al.* (2012); McIlroy *et al.* (2017)

Basic model

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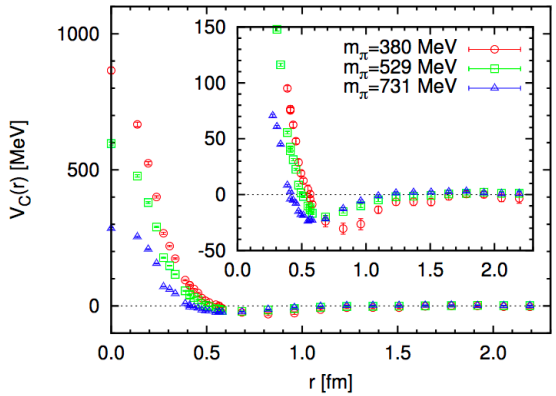
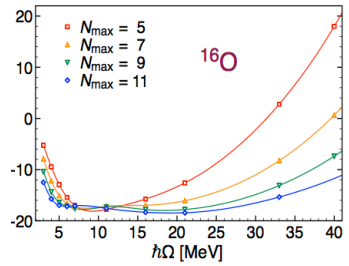
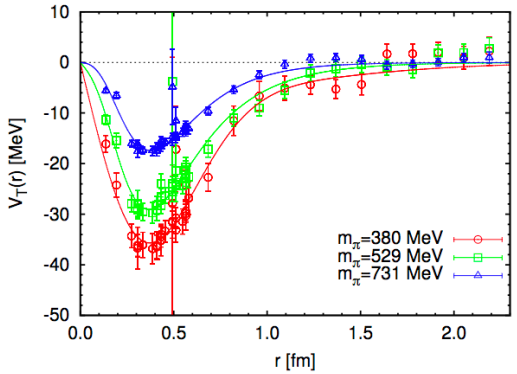
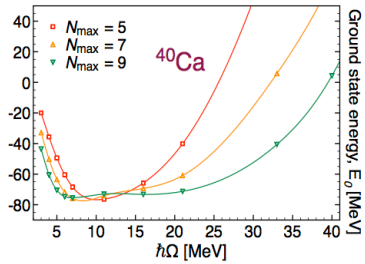
Outlook

- LQCD calculation of $2N$ potential by HAL collaboration









Outline

Basic model

Chiral $2N$ interactions

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EWK QE response

Outlook

- The basic model of nuclear theory
- Chiral $2N$ potentials
- Chiral $3N$ potentials and light-nuclei spectra
- Nuclear electroweak transitions
- Nuclear electroweak response in quasi-elastic regime
- Outlook

The basic model

Basic model

Chiral $2N$ interactions

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Outlook

- Effective potentials:

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2\,m_i} + \sum_{i<j=1}^{A} \underbrace{v_{ij}}_{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}} + \cdots$$

- Assumptions:
  - *Quarks in nuclei are in color singlet states close to those of N's (and low-lying excitations: Δ's, . . .)*
  - *Series of potentials converges rapidly*
  - *Dominant terms in v_{ij} and V_{ijk} are due to π exchange*

$$\text{leading } \pi N \text{ coupling} = \frac{g_A}{2\,f_\pi} \tau_a\, \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \phi_a(\mathbf{r})$$

- Effective electroweak currents:

$$j^{EW} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} j_{ij} + \sum_{i<j<k=1}^{A} j_{ijk} + \cdots$$

Yukawa potential in classical mechanics

Basic model

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Outlook

- What is the connection between meson-exchange interactions and their representation in terms of v_{ij}?
- A simple model: a classical scalar field $\phi(\mathbf{r}, t)$ interacting with static particles:

$$\mathcal{L} = \frac{1}{2}\left[\dot{\phi}^2 - |\boldsymbol{\nabla}\phi|^2 - \mu^2\,\phi^2\right] - g\,\phi\sum_{i=1}^{A}\delta\left(\mathbf{r}-\mathbf{r}_i\right)$$

- Lowest-energy configuration occurs in the static limit $\phi(\mathbf{r}, t) \to \phi(\mathbf{r})$ (Poisson-like equation of electrostatics)

$$\boldsymbol{\nabla}^2\,\phi - \mu^2\,\phi = g\sum_{i=1}^{A}\delta\left(\mathbf{r}-\mathbf{r}_i\right)$$

- Energy of the field (up to self energies) in this limit

$$E_\phi = \frac{1}{2}\int d\mathbf{r}\,\phi(\mathbf{r})\,g\sum_{i=1}^{A}\delta(\mathbf{r}-\mathbf{r}_i) = -\frac{g^2}{4\pi}\sum_{i<j=1}^{A}\frac{\mathrm{e}^{-\mu r_{ij}}}{r_{ij}}$$

Yukawa potential in quantum mechanics

Basic model

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Outlook

- Scalar-field Hamiltonian is

$$H = \underbrace{\sum_{\mathbf{k}} \omega_k \, a^\dagger_{\mathbf{k}} a_{\mathbf{k}}}_{H_0} + \underbrace{g \sum_{i=1}^{A} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\,\omega_k\, V}} \left(a_{\mathbf{k}} \, \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}_i} + a^\dagger_{\mathbf{k}} \, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}_i} \right)}_{H'}$$

- Set of shifted harmonic oscillators; exact eigenenergies of field given by

$$E_{\{n_{\mathbf{k}}\}} = \sum_{\mathbf{k}} \omega_k \, n_{\mathbf{k}} \underbrace{-g^2 \sum_{i<j=1}^{A} \overbrace{\int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathrm{e}^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}}{\omega_k^2}}^{\frac{1}{4\pi} \frac{\mathrm{e}^{-\mu r_{ij}}}{r_{ij}}}}_{\text{energy shift}}$$

- In CM and QM the scalar field energy in the presence of static particles can be replaced by a sum of $v_Y(r_{ij})$

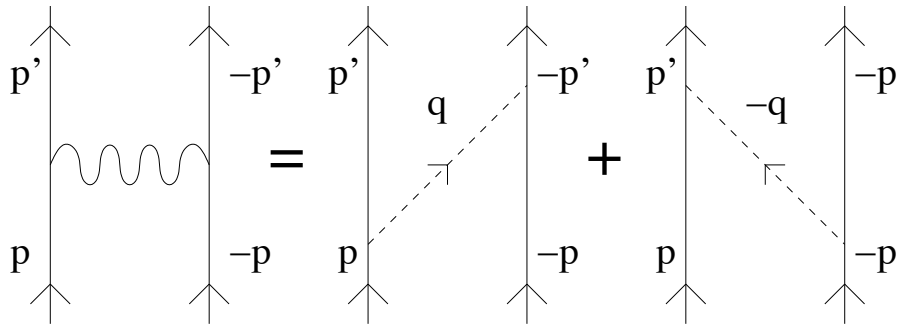
Scattering between slow-moving particles

- In potential theory to leading order

$$T_{fi}^{Y1} = \int d\mathbf{r}\, \mathrm{e}^{-i\mathbf{p}'\cdot\mathbf{r}}\, v_Y(r)\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} = \underbrace{-\frac{g^2}{\omega_q^2}}_{\widetilde{v}_Y(q)} \qquad (\mathbf{q} = \mathbf{p} - \mathbf{p}')$$

- In meson-exchange theory (to leading order)

$$T_{fi}^{M1} = \sum_I \frac{\overbrace{\langle \mathbf{p}', -\mathbf{p}'; 0|}^{\text{final state}}\, H'|I\rangle\langle I|H'\, \overbrace{|\mathbf{p}, -\mathbf{p}; 0\rangle}^{\text{initial state}}}{E_i - E_I} = -\frac{g^2}{\omega_q^2}$$



Beyond leading order

Basic model

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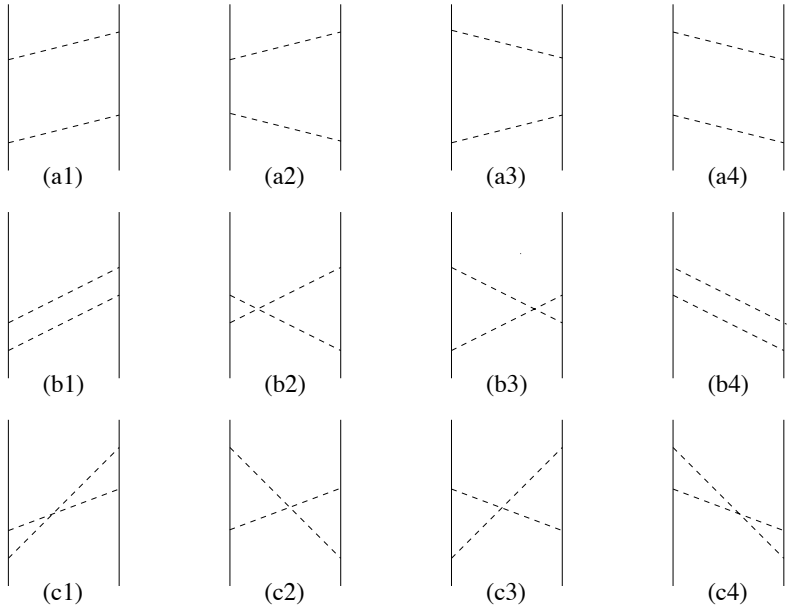
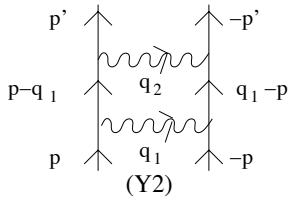
Outlook





$$T_{fi}^{M2} = T_{fi}^{Y2} + \text{correction}$$

- Obtain from $T_{fi}^{M2} - T_{fi}^{Y2}$ correction term $\widetilde{v}^{(2)}(\mathbf{q}, \overbrace{\mathbf{Q}}^{\mathbf{p}'+\mathbf{p}})$ such that $\widetilde{v}_Y(q) + \widetilde{v}^{(2)}(\mathbf{q}, \mathbf{Q})$ reproduces T_{fi}^{M2}

Many of the developers of the basic model . . .

Basic model

Chiral $2N$ interactions

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Outlook

χEFT formulation of the basic model

Basic model

Chiral $2N$ interactions

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Outlook

- χEFT is a low-energy approximation of QCD
- Lagrangians describing the interactions of π, N, ... are expanded in powers of Q/Λ_χ ($\Lambda_\chi \sim 1$ GeV)
- Their construction has been codified in a number of papers [1]

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \ldots \\ &\quad + \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \ldots\end{aligned}$$

- $\mathcal{L}^{(n)}$ also include contact $\left(\overline{N}N\right)\left(\overline{N}N\right)$-type interactions parametrized by low-energy constants (LECs)
- Initial impetus to the development of χEFT for nuclei in the early nineties [2]

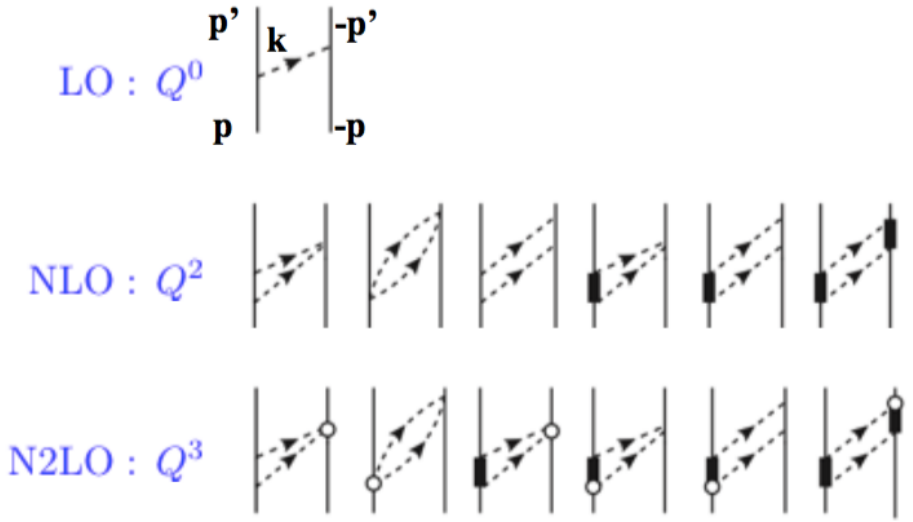
[1] Gasser and Leutwyler (1984); Gasser, Sainio, and Švarc (1988); Bernard *et al.* (1992); Fettes *et al.* (2000)

[2] Weinberg (1990)–(1992)

Chiral $2N$ potentials with Δ's

Piarulli *et al.* (2015); Piarulli *et al.* (2016)

- Two-nucleon potential: $v = v^{\rm EM} + v^{\rm LR} + v^{\rm SR}$
- EM component $v^{\rm EM}$ including corrections up to α^2
- Chiral OPE and TPE component $v^{\rm LR}$ with Δ's



- Short-range contact component $v^{\rm SR}$ up to order Q^4 parametrized by (2+7+11) IC and (2+4) IB LECs
- $v^{\rm SR}$ functional form taken as $C_{R_S}(r) \propto {\rm e}^{-(r/R_S)^2}$ with R_S=0.8 (0.7) fm for a (b) models

np ($T = 0$ and 1) and pp phase shifts

Basic model

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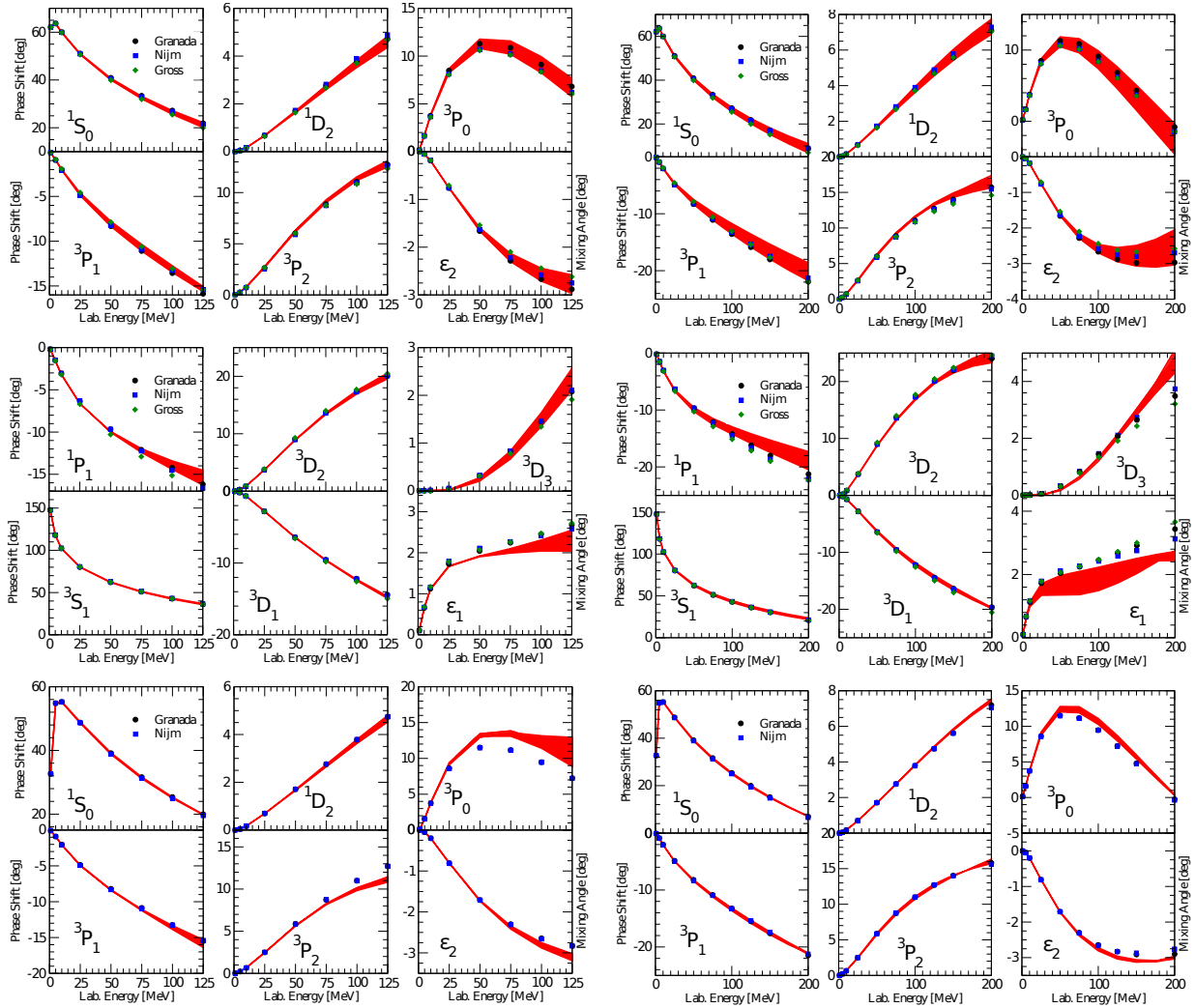
Outlook

Ia-Ib: $E_{\rm lab} = 125$ MeV IIa-IIb: $E_{\rm lab} = 200$ MeV



Deuteron wave functions

Basic model

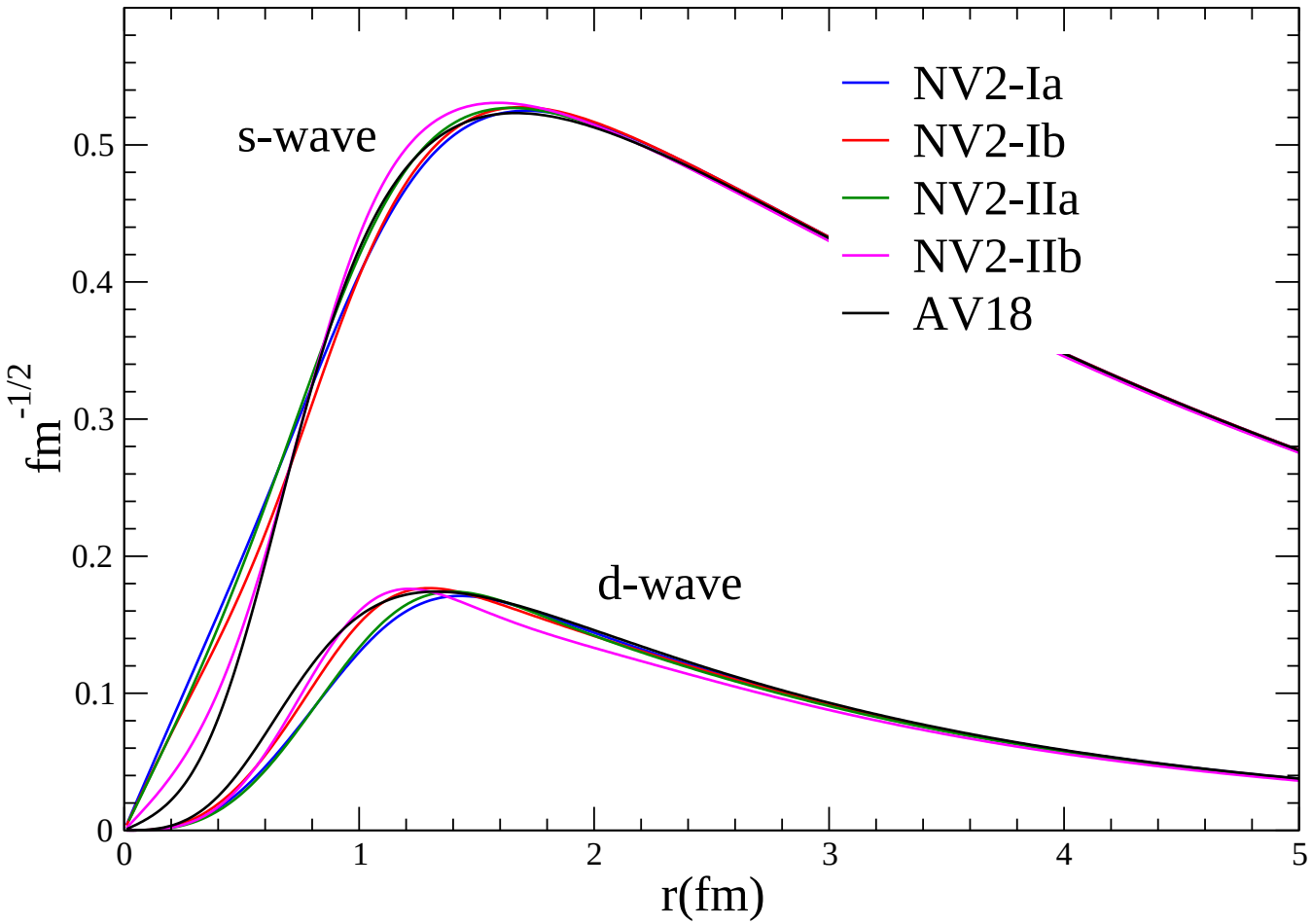
Chiral $2N$ interactions

Chiral $3N$ interactions

EWK interactions

EWK QE response

Outlook



Dominant features of $2N$ potential

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

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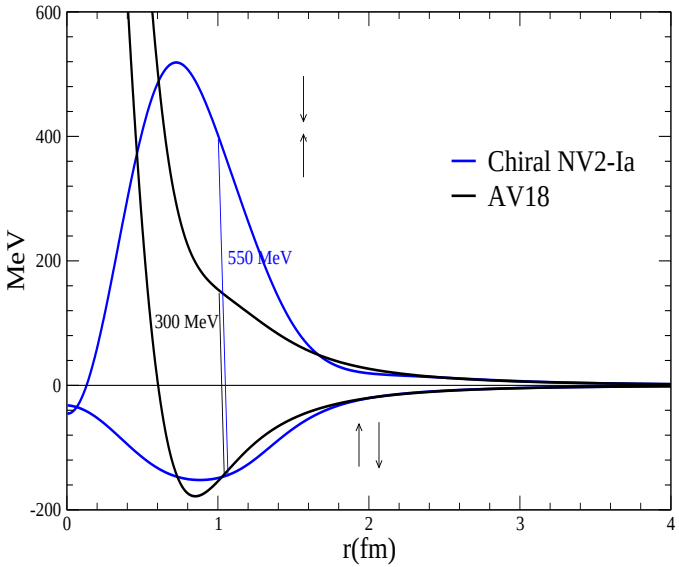
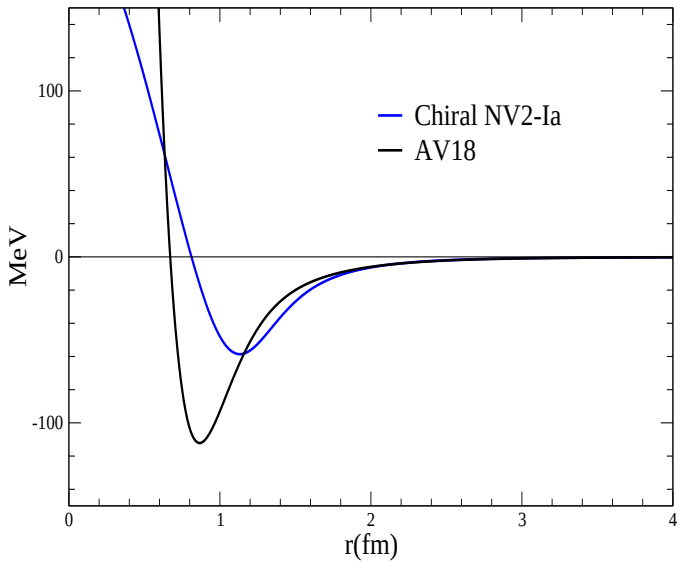
Outlook

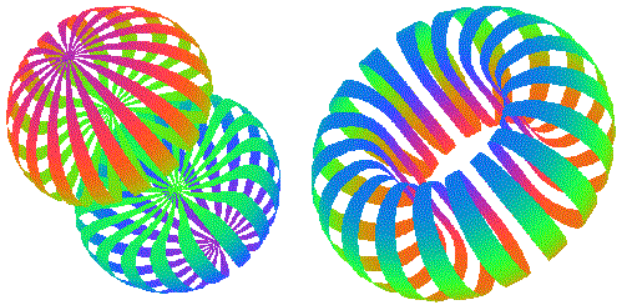
v in T,S=0,1 (d-like)



v in T,S=1,0 (quasi-bound)



- Strong state-dependent correlations



Impact on pair momentum distributions

Schiavilla *et al.* (2007); Wiringa *et al.* (2014); $^{12}\mathrm{C}(e, e'NN)$ JLab exp: Subedi *et al.* (2008)

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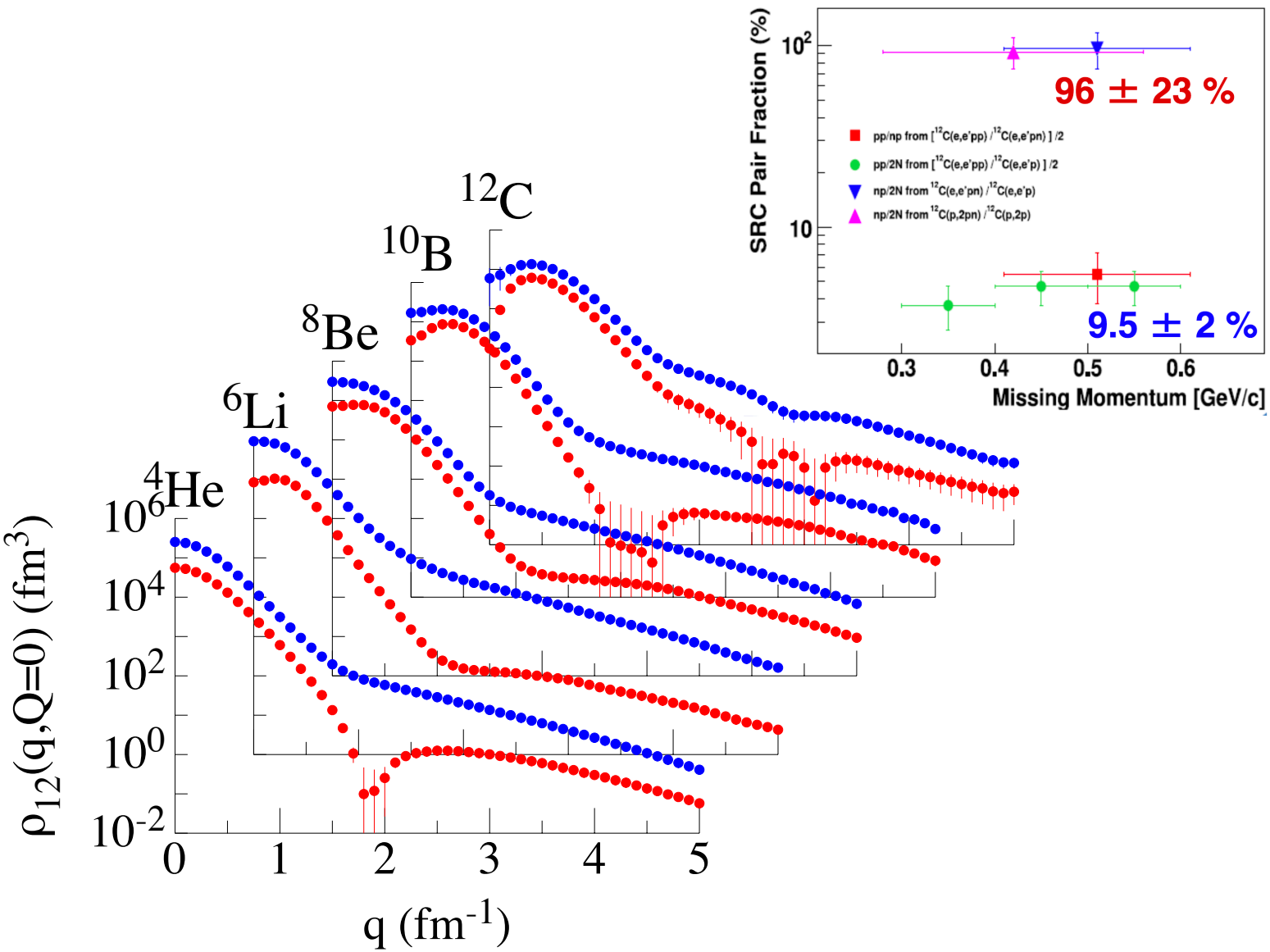
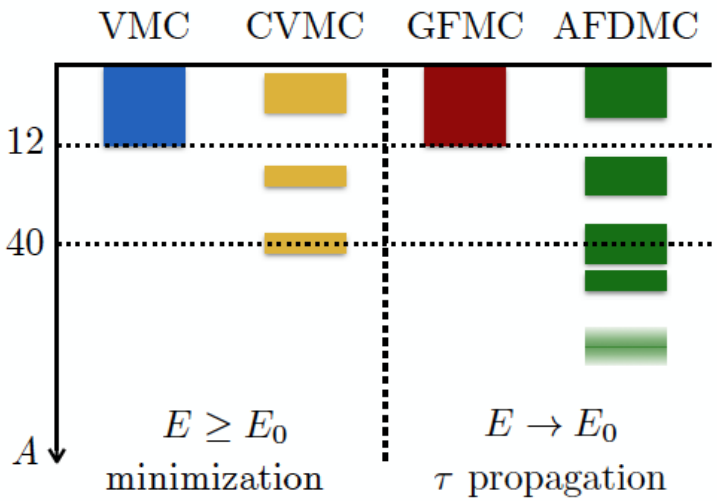


Ab initio methods

- Hyperspherical harmonics (HH) expansions for $A = 3$ and 4 bound and continuum states

$$|\psi_V\rangle = \sum_\mu c_\mu \underbrace{|\phi_\mu\rangle}_{\text{HH basis}} \quad \text{and} \quad c_\mu \text{ from } E_V = \frac{\langle\psi_V|H|\psi_V\rangle}{\langle\psi_V|\psi_V\rangle}$$

- Quantum Monte Carlo for $A > 4$ bound states



(C)VMC GFMC AFDMC	light systems	$A \leq 12$
CVMC AFDMC	light to medium-heavy nuclei	$A \sim 50$
AFDMC	infinite matter	$A \to \infty$

Figure by Lonardoni

GFMC for $A \leq 12$

RMP by Carlson *et al.* (2015)

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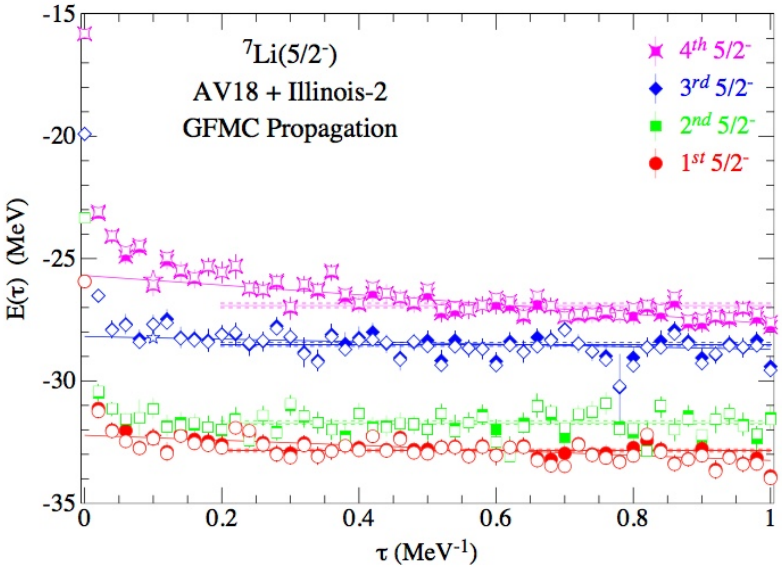
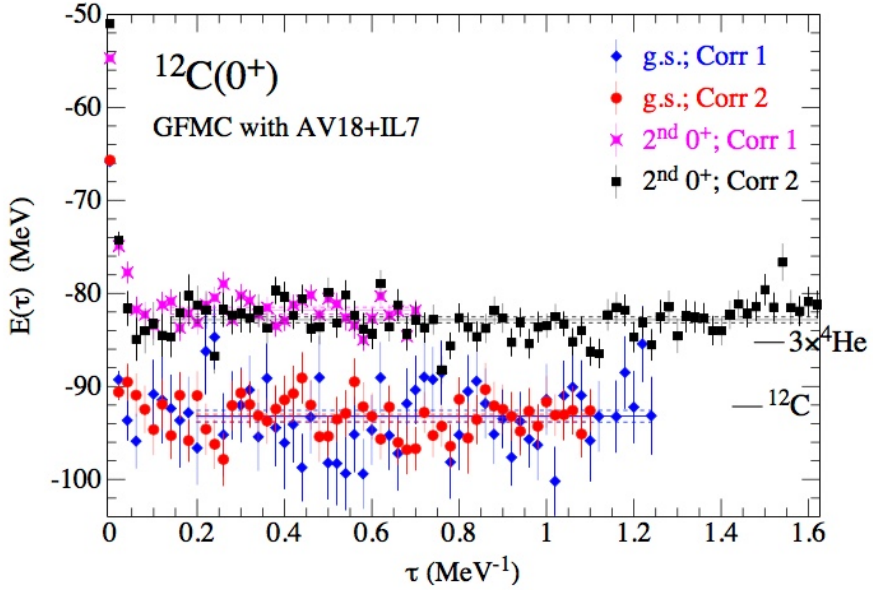
- Propagation in imaginary time

$$E_0 = \lim_{\tau \to \infty} \frac{\langle \psi_V | H \, \mathrm{e}^{-\tau H} | \psi_V \rangle}{\langle \psi_V | \mathrm{e}^{-\tau H} | \psi_V \rangle}$$

- Exponential growth with A (in ^{12}C st-states $\sim 4 \times 10^6$)

$$\psi_V = \sum_{s \leq 2^A} \sum_{t \leq 2^A} \phi_{st}(\mathbf{r}_1, \ldots, \mathbf{r}_A) \, \chi_{st}(1, \ldots, A)$$





Failures of basic model with $2N$ potentials only

Basic model

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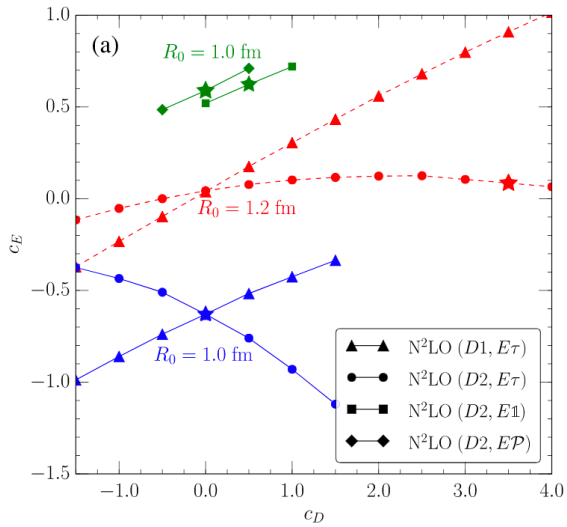
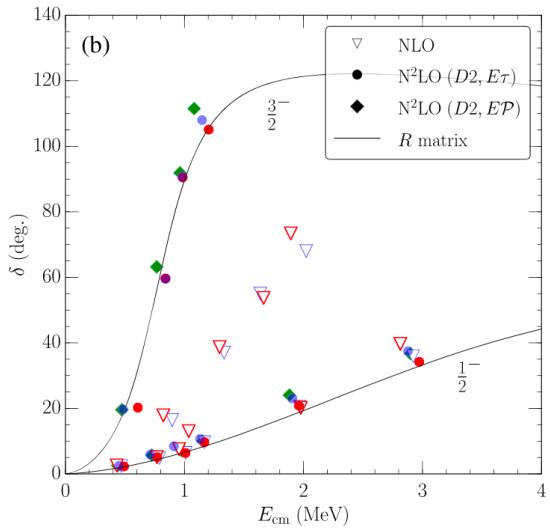
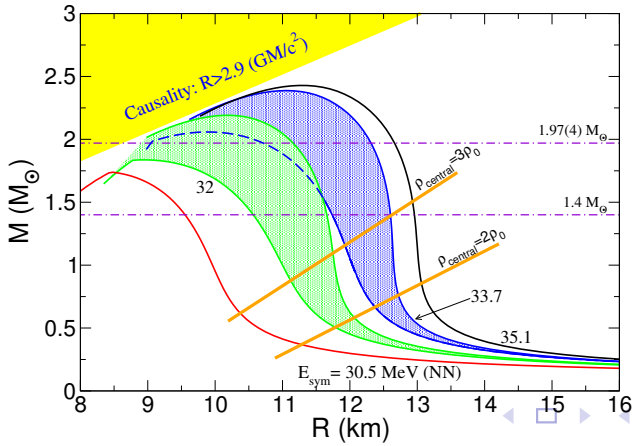




- spectra of light nuclei
- Intermediate-energy Nd and low-energy $n\alpha$ scattering
- nuclear matter $E_0(\rho)$ and neutron star masses



Chiral $3N$ potentials with Δ's

Basic model

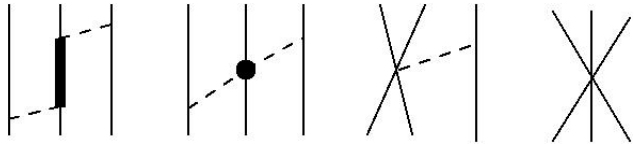
Chiral $2N$ interactions

Chiral $3N$ interactions

EWK interactions

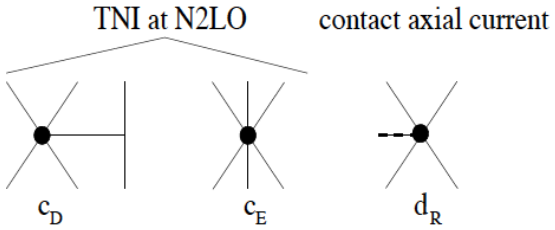
EWK QE response

Outlook

- $3N$ potential up to N2LO: 
- Short-range contact terms depend on LECs c_D and c_E
- c_D and c_E fixed by fitting $E_0^{\rm exp}(^3{\rm H}) = -8.482$ MeV and nd doublet scattering length $a_{nd}^{\rm exp} = (0.645 \pm 0.010)$ fm

			w/o $3N$				with $3N$	
Model	c_D	c_E	$E_0(^3{\rm H})$	$E_0(^3{\rm He})$	$E_0(^4{\rm He})$	$^2a_{nd}$	$E_0(^3{\rm He})$	$E_0(^4{\rm He})$
Ia	3.666	–1.638	–7.825	–7.083	–25.15	1.085	–7.728	–28.31
Ib	–2.061	–0.982	–7.606	–6.878	–23.99	1.284	–7.730	–28.31
IIa	1.278	–1.029	–7.956	–7.206	–25.80	0.993	–7.723	–28.17
IIb	–4.480	–0.412	–7.874	–7.126	–25.31	1.073	–7.720	–28.17

- Better strategy: fix c_D and c_E by reproducing $E_0^{\rm exp}(^3{\rm H})$ and the GT$^{\rm exp}$ matrix element in ^{3}H β-decay ...



Spectra of light nuclei

Piarulli *et al.* (2017)

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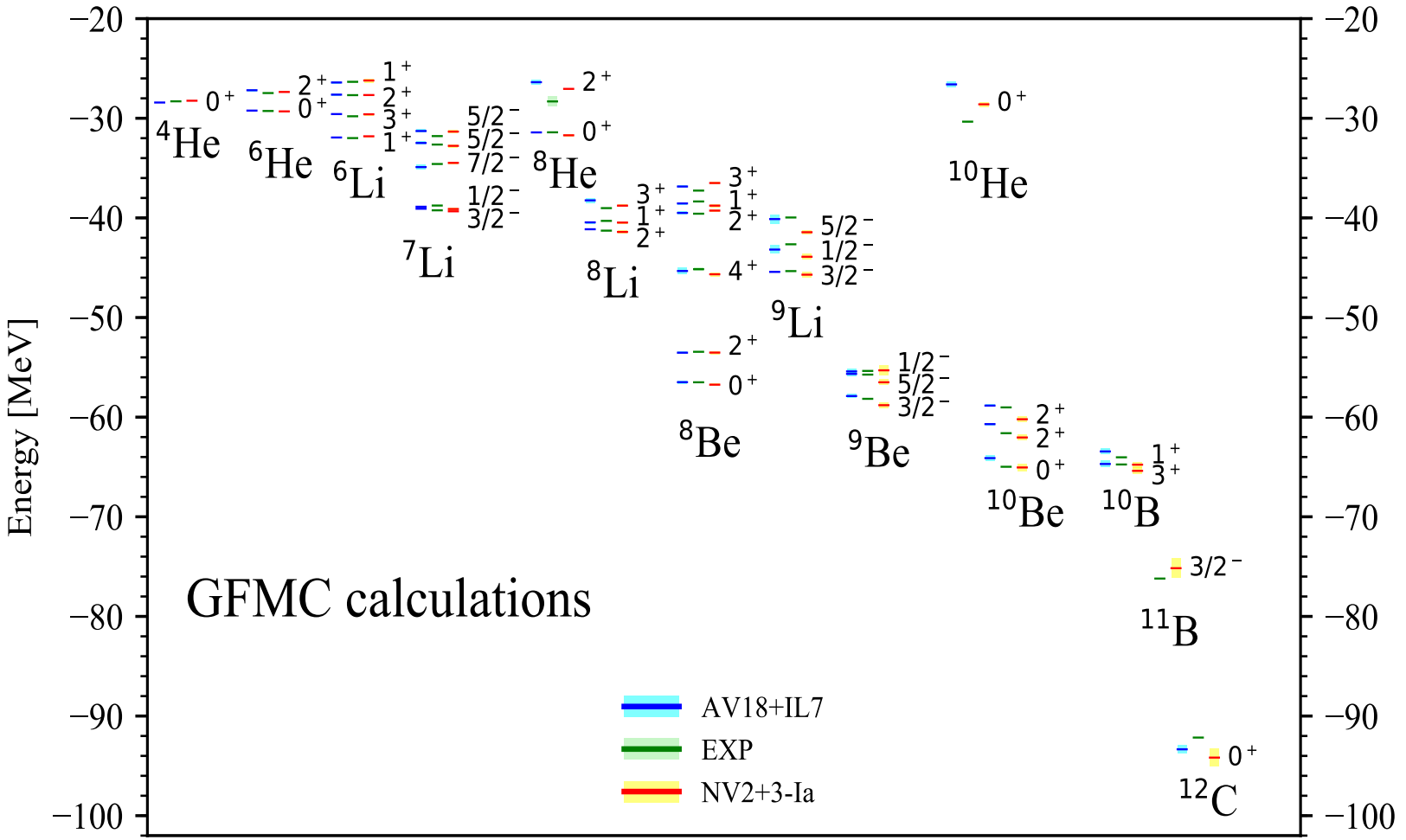

Nuclear axial currents at one loop

Baroni *et al.* (2016) & (2017); Pastore *et al.* (2017)

Basic model

Chiral $2N$ interactions

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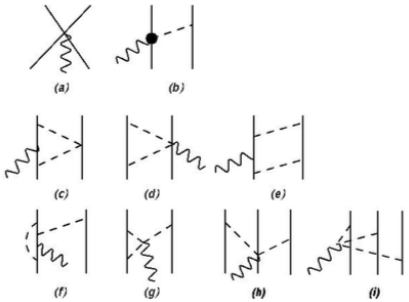
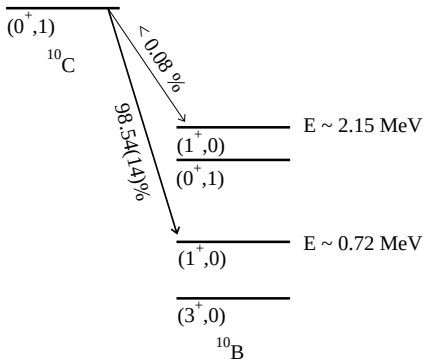
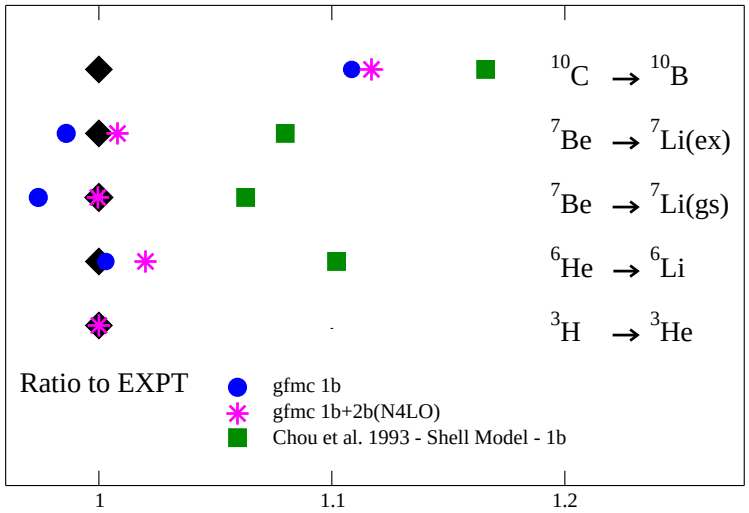
Outlook

- A single unknown LEC in $\mathbf{j}_5$ fixed in ^{3}H β-decay (only N2LO and N3LO terms shown below)



- GT m.e.'s in $A = 6$–10 nuclei (AV18/IL7 potential with χEFT axial current)





μ^- capture on ^{2}H and ^{3}He

Marcucci *et al.* (2012)

Basic model

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Outlook

- χEFT predictions with conservative error estimates:

$$\Gamma(^2\mathrm{H}) = (399 \pm 3)\,\mathrm{sec}^{-1} \qquad \Gamma(^3\mathrm{He}) = (1494 \pm 21)\,\mathrm{sec}^{-1}$$

- Errors due primarily to:
  - *experimental error on GT$^{\mathrm{EXP}}$ (0.5%)*
  - *uncertainties in EW radiative corrections*[1] *(0.4%)*
  - *cutoff dependence*
- Using $\Gamma^{\mathrm{EXP}}(^3\mathrm{He}) = (1496 \pm 4)$ sec^{-1}, one extracts

$$G_{PS}(q_0^2 = -0.95\, m_\mu^2) = 8.2 \pm 0.7$$

versus $G_{PS}^{\mathrm{EXP}}(q_0^2 = -0.88\, m_\mu^2) = 8.06 \pm 0.55$[2] and a χPT prediction of 7.99 ± 0.20[3]

- Upcoming measurement of $\Gamma(^2\mathrm{H})$ by the MuSun collaboration at PSI with a projected 1% error . . .

[1] These corrections increase rate by 3%, see Czarnecki *et al.* (2007); [2] From a measurement of $\Gamma^{\mathrm{EXP}}(^1\mathrm{H})$, Andreev *et al.* (2013); [3] Bernard *et al.* (1994), Kaiser (2003)

CC and NC ν-A scattering

Basic model

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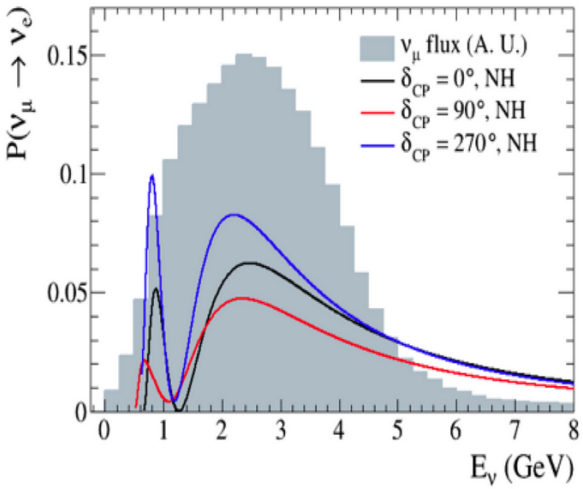
Outlook

- Large program in accelerator ν physics (MicroBooNE, NOνA, T2K, Minerνa, DUNE, . . .)

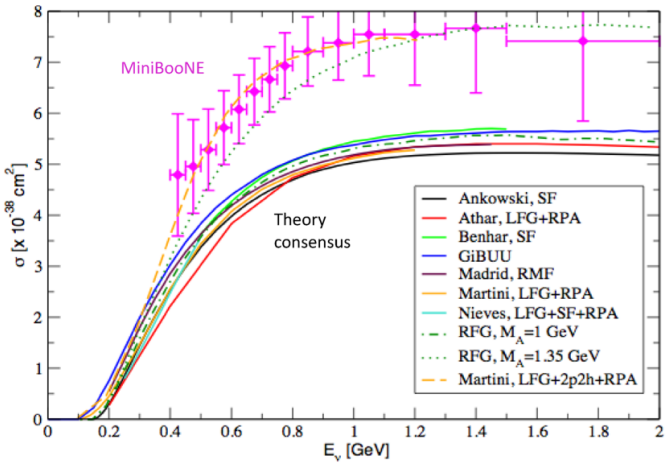
$$\text{rate} \propto \int dE\, \Phi_\alpha(E)\, P(\nu_\alpha \to \nu_\beta; E)\, \sigma_\beta(E, E')$$

- Determination of oscillation parameters depends crucially on our understanding of
  - ν *flux* $\Phi_\alpha(E)$
  - ν-A *cross section* $\sigma_\beta(E, E')$

DUNE ν_μ flux



MiniBooNE



GFMC calculation of EM response in ^{12}C

Carlson and Schiavilla (1992); Lovato *et al.* (2013–2016)

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

EWK interactions

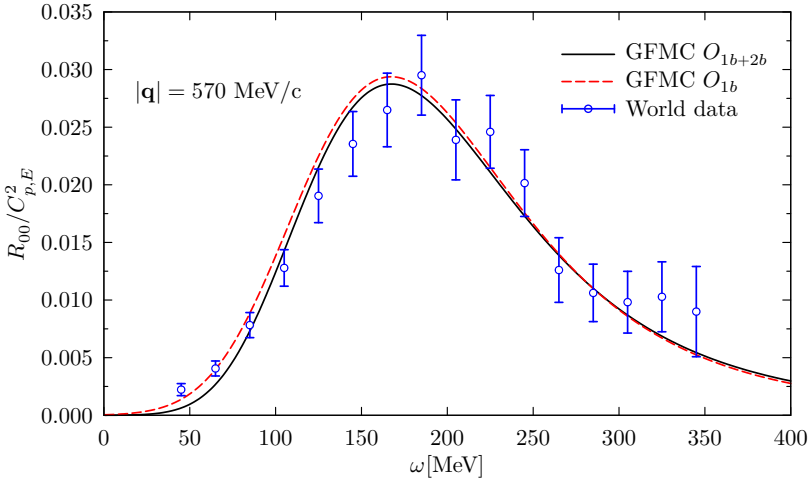
EWK QE response

Outlook

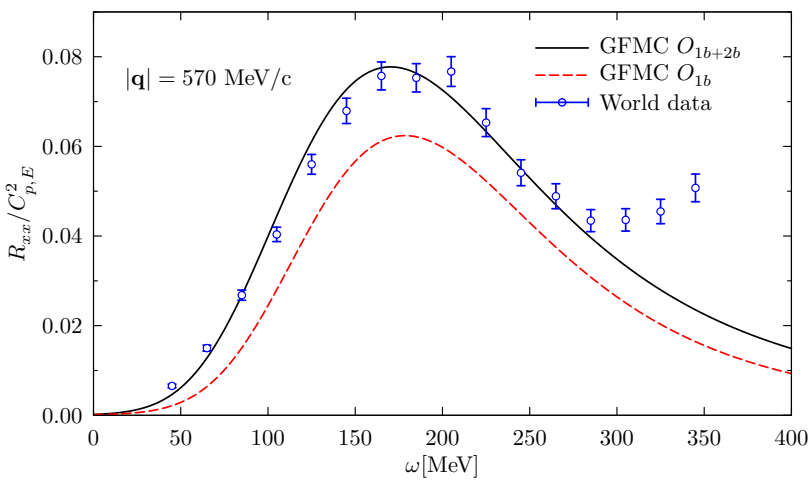
$$\int_0^\infty \mathrm{d}\omega\, \mathrm{e}^{-\tau\omega}\, R_{\alpha\beta}(q,\omega) = \langle i \mid j_\alpha^\dagger(\mathbf{q})\, \mathrm{e}^{-\tau(H-E_i)}\, j_\beta(\mathbf{q}) \mid i \rangle$$

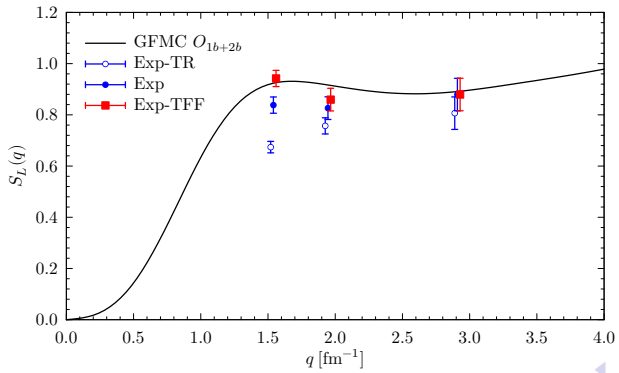
- Inversion back to $R_{\alpha\beta}(q,\omega)$ by maximum entropy methods

longitudinal



transverse





← Coulomb sum

NC responses and cross sections in ^{12}C

Lovato *et al.* in preparation

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

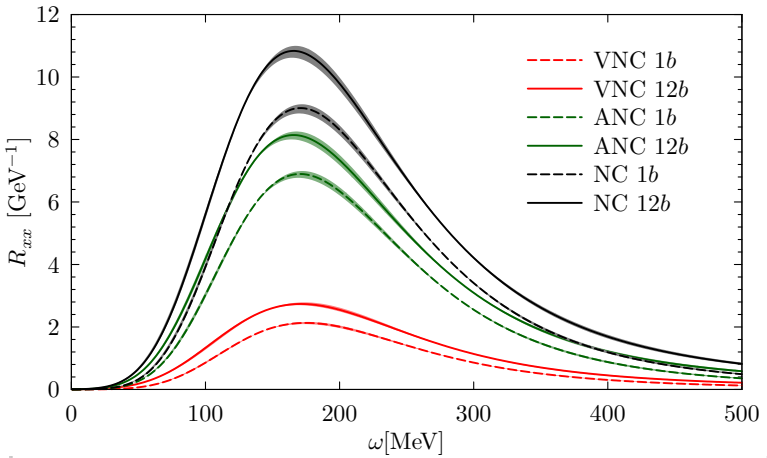
EWK interactions

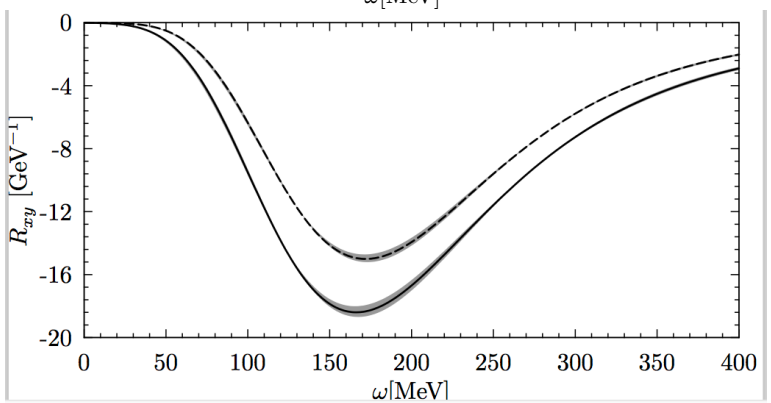
EWK QE response

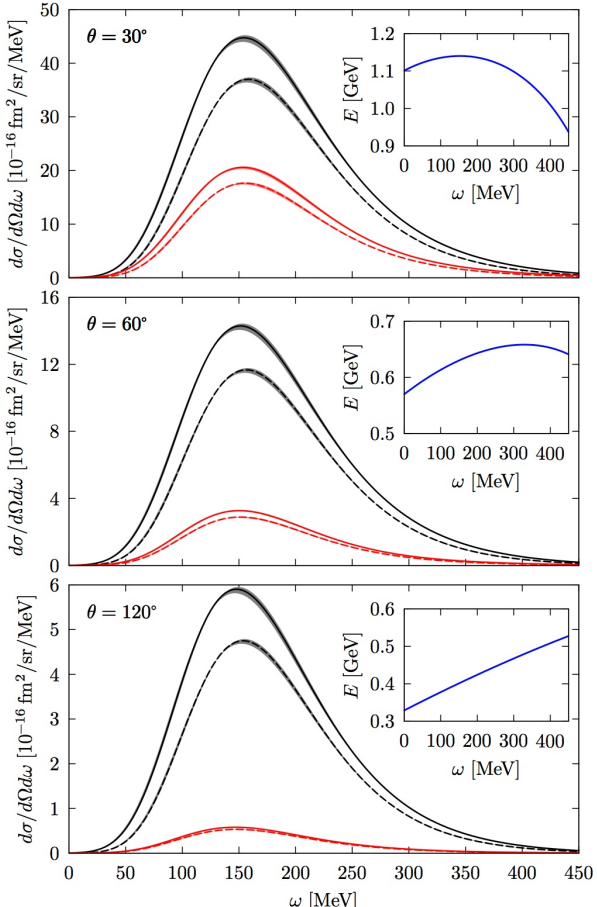
Outlook

- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section given in terms of five response functions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon_l' \mathrm{d}\Omega_l} \propto \left[v_{00}\, R_{00} + v_{zz}\, R_{zz} - v_{0z}\, R_{0z} + \overbrace{v_{xx}\, R_{xx} \mp v_{xy}\, R_{xy}}^{\text{dominant}} \right]$$







Outlook

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

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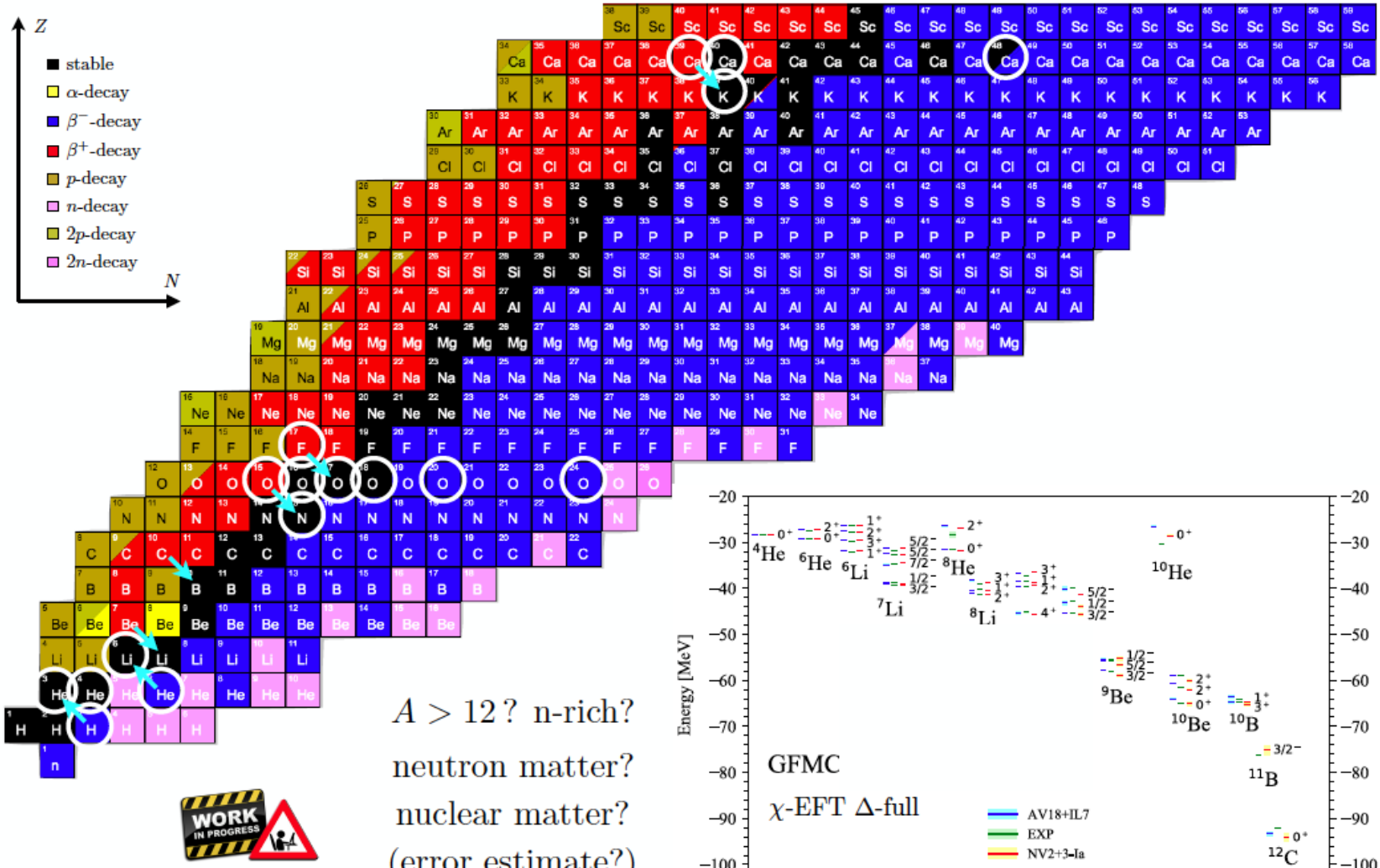
EWK QE response

Outlook



M. Piarulli *et al.*, arXiv:1707.02883

Slide by Lonardoni

Subleading $3N$ potentials

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

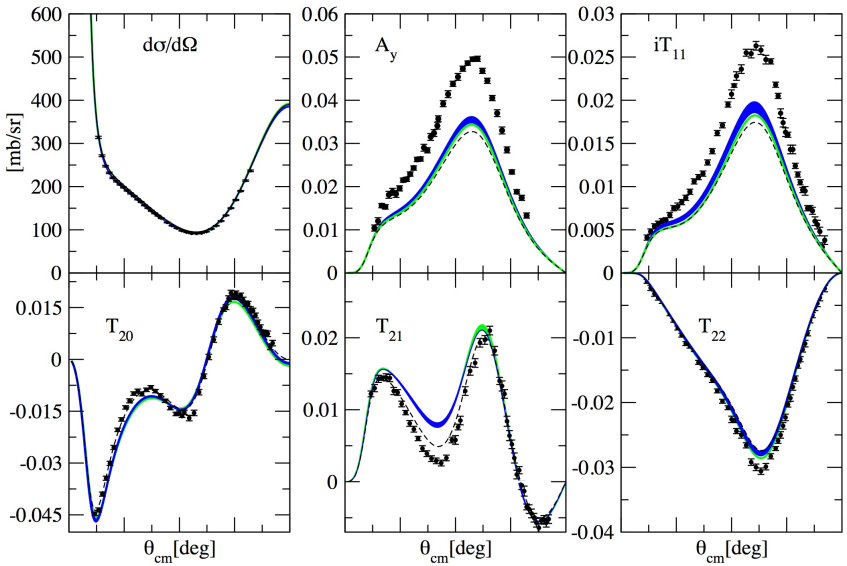
EWK interactions

EWK QE response

Outlook

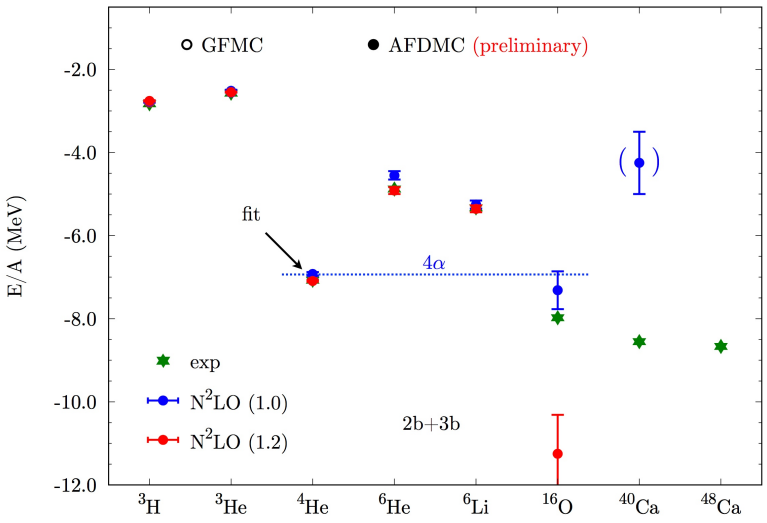
- Different strategies for constraining the (10) LECs in subleading contact $3N$ potential:
  - *Nd scattering observables at low energies*
  - *Spectra of light- and medium-weight nuclei and properties of nuclear/neutron matter*

pd scattering at 3 MeV



Piarulli *et al.* (2017)

AFDMC for $A = 16$ and 40



Lynn *et al.* (2016); Lonardoni *et al.* (2017)

Nuclear weak interactions: low energy regime

Basic model

Chiral $2N$ interactions

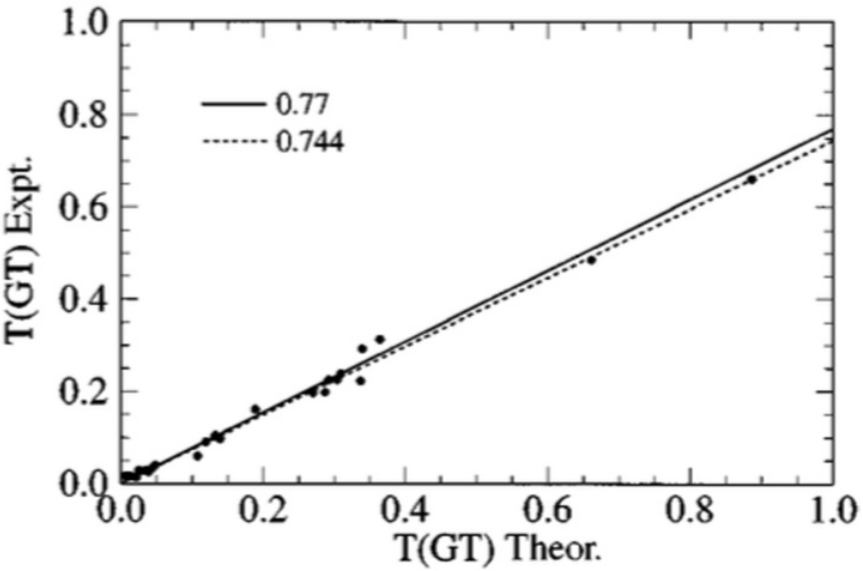
Chiral $3N$ interactions

EWK interactions

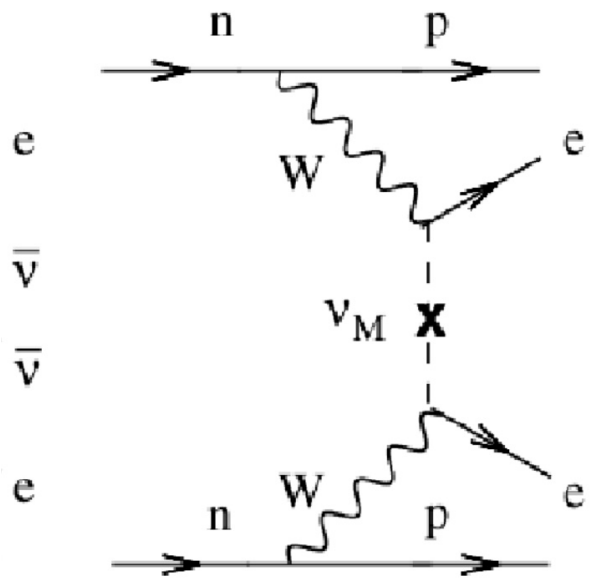
EWK QE response

Outlook

- Shell model in agreement with exp if $g_A^{\text{eff}} \simeq 0.7\, g_A$
- Understanding "quenching" of g_A in nuclear β decays
- Relevant for neutrinoless 2β-decay since rate $\propto g_A^4$



Martinez-Pinedo *et al.* (1996)



0ν–2β amplitude

Nuclear matrix element in 0ν–2β decay

Engel and Menendez (2017)

Basic model

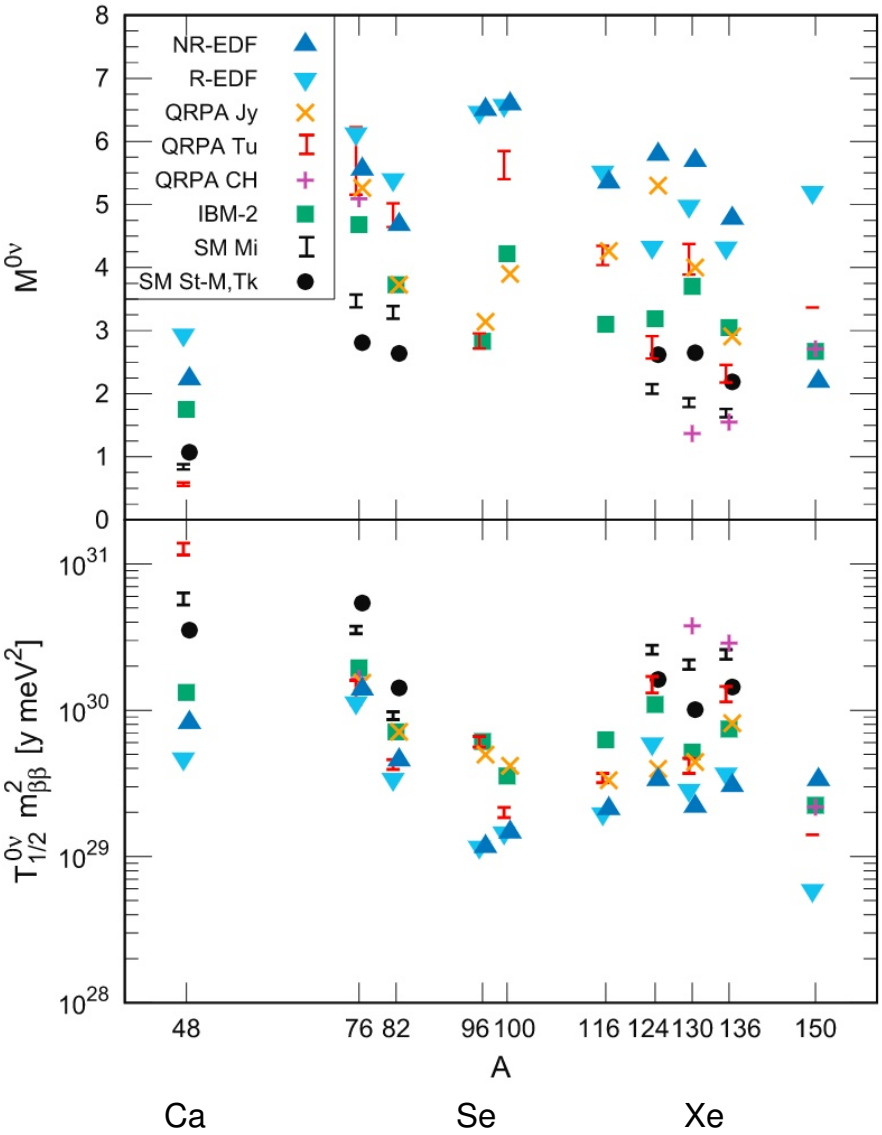
Chiral $2N$ interactions

Chiral $3N$ interactions

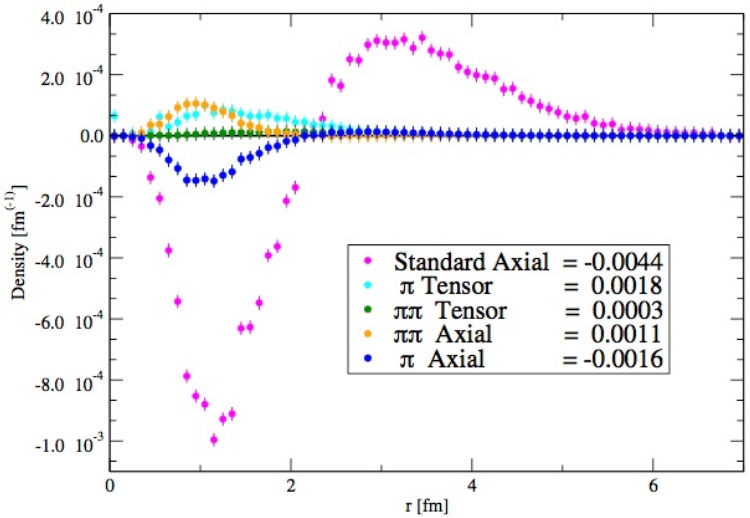
EWK interactions

EWK QE response

Outlook



Test case[1]: $^{8}\mathrm{He}(0^{+};2) \rightarrow {}^{8}\mathrm{Be}(0^{+};2)$



standard axial $\propto \tau_i^+ \, \tau_j^+ \, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$

[1] Mereghetti *et al.* (2017)

The HH/QMC team

Basic model

Chiral $2N$ interactions

Chiral $3N$ interactions

EWK interactions

EWK QE response

Outlook

- The ANL/JLAB/LANL/Pisa *quadri-axis*

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L. Girlanda (U-Salento)
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S. Pastore[3] (LANL)
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R.B. Wiringa (ANL)

- Computational resources from *ANL LCRC*, *LANL Open Supercomputing*, and *NERSC*

- Theory Center support for August 2017 collaboration meeting is gratefully acknowledged

[1] ODU Ph.D. 2017; [2] ODU Ph.D. 2000; [3] ODU Ph.D. 2010; [4] ODU Ph.D. 2015; [5] CEBAF theory postdoc 1994

EM operators up to one loop

Pastore *et al.* (2009) & (2011); Kölling *et al.* (2009) & (2011); Piarulli *et al.* (2013)

Basic model

Chiral $2N$ interactions

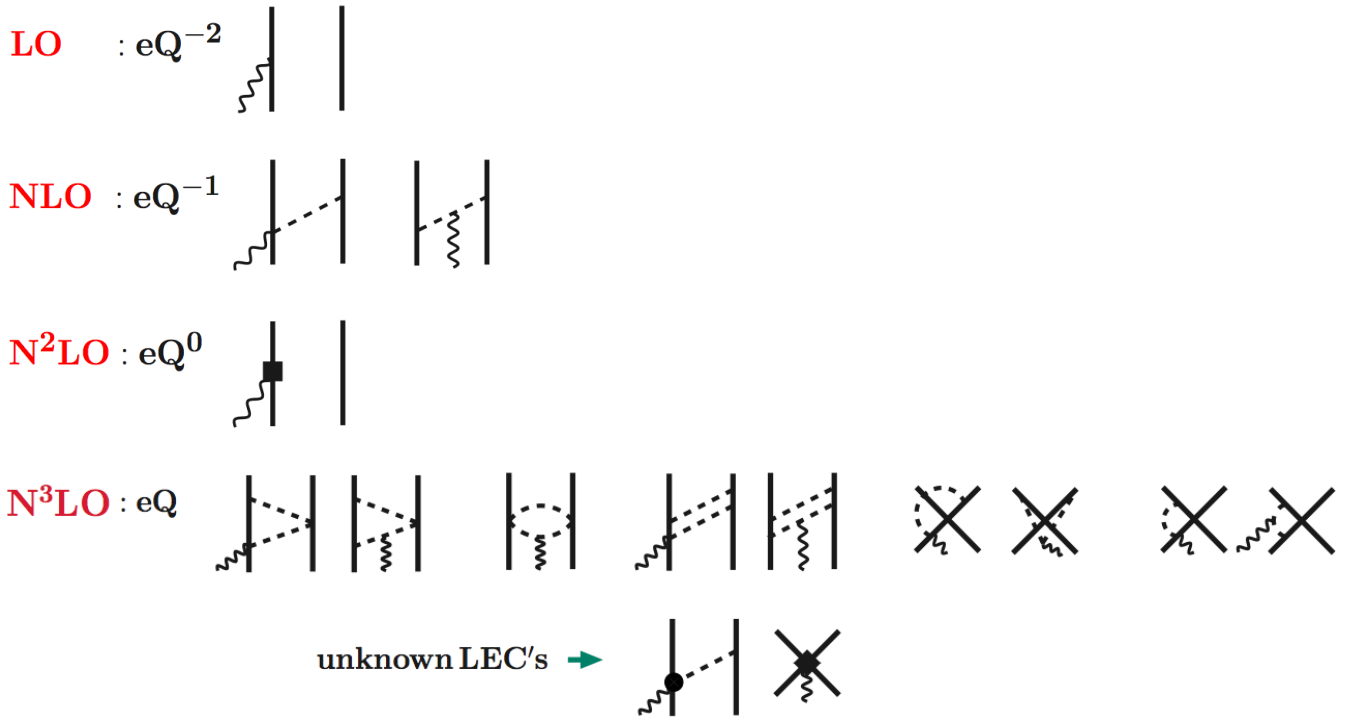
Chiral $3N$ interactions

EWK interactions

EWK QE response

Outlook

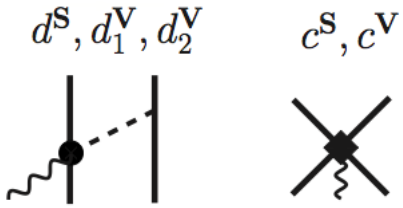
- Contributions to $\mathbf{j}_\gamma$ up to order $e\,Q$:



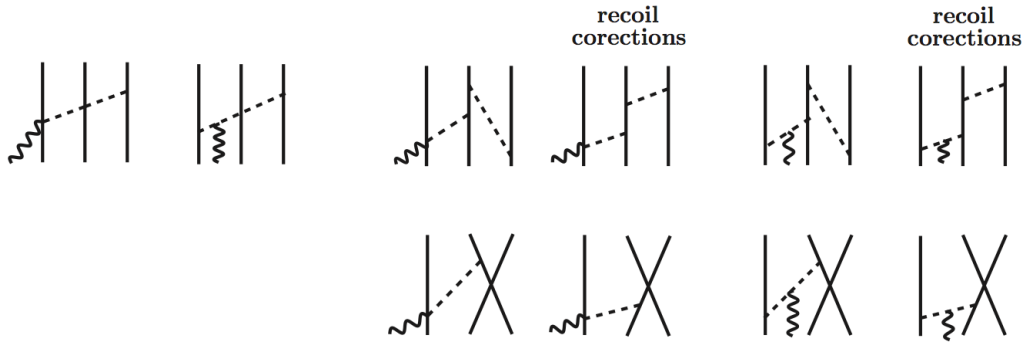
- Five unknown LEC's in $\mathbf{j}_\gamma$ at $e\,Q$
- LO for ρ_γ at $e\,Q^{-3}$ and no OPE corrections at $e\,Q^{-1}$
- No unknown LEC's in ρ_γ up to $e\,Q$

The LEC's characterizing $\mathbf{j}_\gamma$ at one loop

- d's could be determined by (γ, π) data on the nucleon



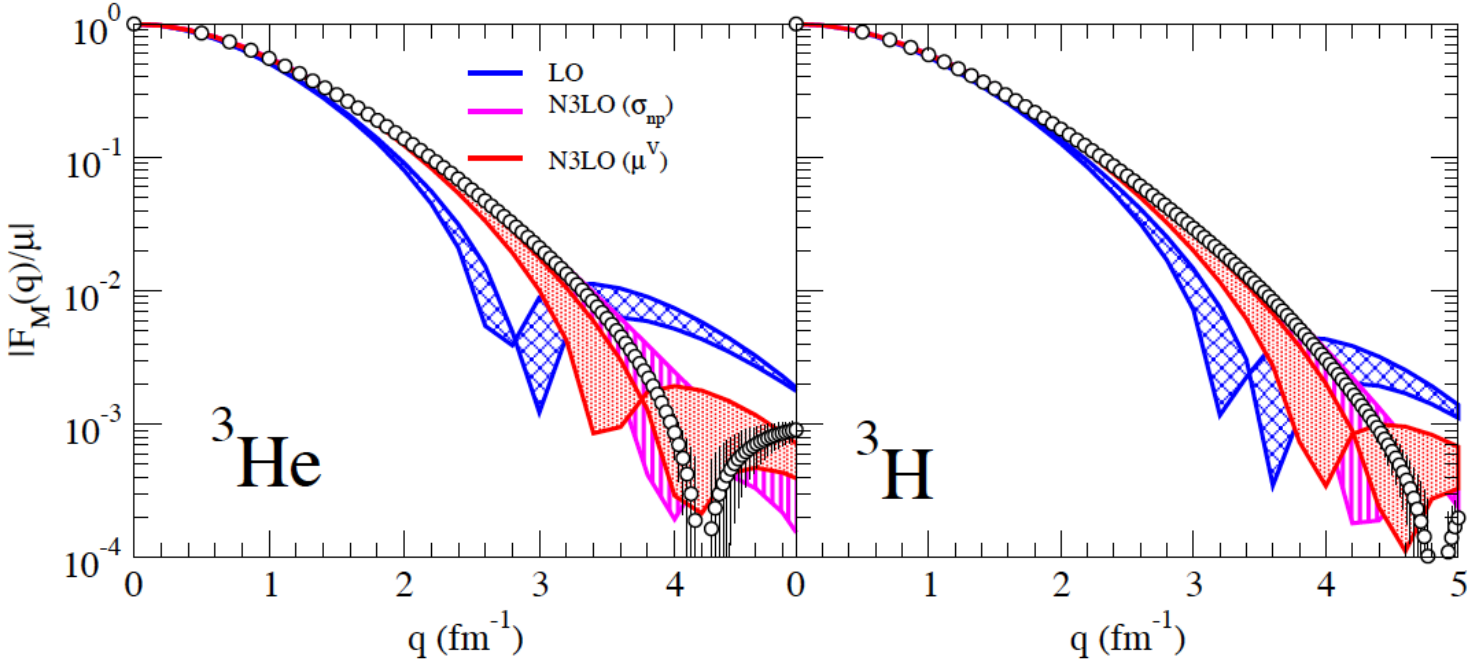
- d_1^V, d_2^V fixed by assuming Δ dominance, d_1^S, c^S, and c^V fixed by fitting $A = 2$ and 3 EM observables
- Three-body currents at N^{3}LO vanish:



^{3}He/^{3}H magnetic f.f.'s

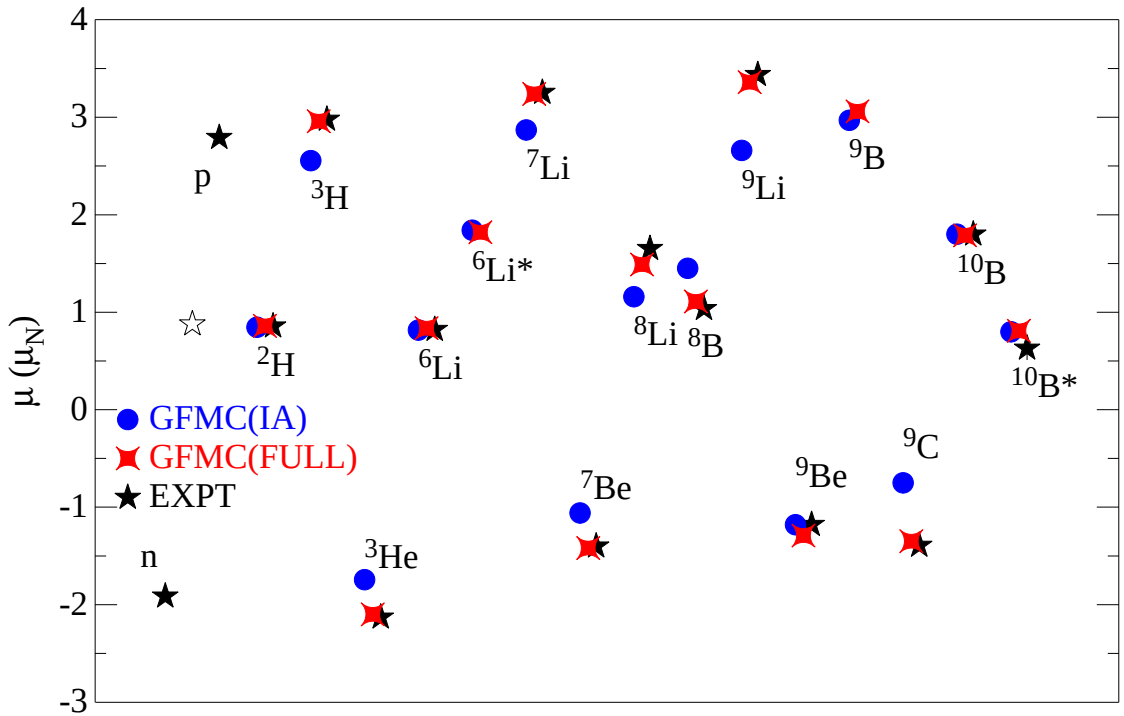
Piarulli *et al.* (2013)

- Two-body (isovector) contributions play crucial role



- Magenta shaded area: LEC c^V fixed by σ^{γ}_{np}
- Red shaded area: LEC c^V fixed by μ^V(^{3}He/^{3}H)

Magnetic moments in $A \leq 10$ nuclei

Pastore *et al.* (2013)

- GFMC calculations use AV18/IL7 (rather than chiral) potentials with χEFT EM currents
- Predictions for $A > 3$; about 40% of $\mu(^9\mathrm{C})$ due to corrections beyond LO