New Insights on the Drell-Yan Angular Distributions

Jen-Chieh Peng

University of Illinois at Urbana-Champaign

Seminar at
Jefferson Laboratory
April 20, 2018

First Dimuon Experiment

Lederman et al. PRL 25 (1970) 1523

Experiment originally designed to search for neutral weak boson (Z⁰)

Missed the J/Ψ signal!

"Discovered" the Drell-Yan process

\[ p + U \rightarrow \mu^+ + \mu^- + X \] 29 GeV proton
The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, \(s \to \infty\), \(Q^2/s\) finite, \(Q^2\) and \(s\) being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as \(Q^2/s \to 1\) is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function \(\nu W_2\) near threshold.

\[
\left( \frac{d^2 \sigma}{dx_1 dx_2} \right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_a e_a^2 \left[ q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1)q_a(x_2) \right]
\]
Naive Drell-Yan and Its Successor*

T-M. Yan
Floyd R. Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853

February 1, 2008

Abstract

We review the development in the field of lepton pair production since proposing parton-antiparton annihilation as the mechanism of massive lepton pair production. The basic physical picture of the Drell-Yan model has survived the test of QCD, and the predictions from the QCD improved version have been confirmed by the numerous experiments performed in the last three decades. The model has provided an active theoretical arena for studying infrared and collinear divergences in QCD. It is now so well understood theoretically that it has become a powerful tool for new physics information such as precision measurements of the W mass and lepton and quark sizes.

“... our original crude fit did not even remotely resemble the data. Sid and I went ahead to publish our paper because of the model’s simplicity...”

“... the successor of the naïve model, the QCD improved version, has been confirmed by the experiments...”

“The process has been so well understood theoretically that it has become a powerful tool for precision measurements and new physics.”
Complementarity between DIS and Drell-Yan

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)

Peng and Qiu, Prog. Part. Nucl. Phys. 76 (2014) 43
Fermilab Dimuon Spectrometer
(E605 / 772 / 789 / 866 / 906 / 1039)

1) Fermilab E772 (proposed in 1986 and completed in 1988)
   "Nuclear Dependence of Drell-Yan and Quarkonium Production"
2) Fermilab E789 (proposed in 1989 and completed in 1991)
   "Search for Two-Body Decays of Heavy Quark Mesons"
3) Fermilab E866 (proposed in 1993 and completed in 1996)
   "Determination of $\bar{d} / \bar{u}$ Ratio of the Proton via Drell-Yan"
4) Fermilab E906 (proposed in 1999, completed in 7/2017)
   "Drell-Yan with the FNAL Main Injector"
EXPERIMENT E789- Moving Cable at Meson. "The Snake".
(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1 + \cos^2 \theta)$ rather than $\sin^2 \theta$ as found in Sakurai’s vector-dominance model, where $\theta$ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.
Drell-Yan angular distribution

Lepton Angular Distribution of “naïve” Drell-Yan:

\[ \frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1 \]

Data from Fermilab E772
Why is the lepton angular distribution $1 + \cos^2 \theta$?

Helicity conservation and parity

Adding all four helicity configurations:

$$d\sigma \sim 1 + \cos^2 \theta$$
Drell-Yan lepton angular distributions

\[ \Theta \text{ and } \Phi \text{ are the decay polar and azimuthal angles of the } \mu^- \text{ in the dilepton rest-frame} \]

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

\[
\left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\Omega} \right) = \left[ \frac{3}{4\pi} \right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]
\]

Lam-Tung relation: \( 1 - \lambda = 2\nu \)

- Reflect the spin-1/2 nature of quarks
  (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections
Decay angular distributions in pion-induced Drell-Yan

\[
\left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\Omega} \right) = \left[ \frac{3}{4\pi} \right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]
\]

\[\nu \neq 0 \quad \text{and} \quad \nu \quad \text{increases with} \quad p_T\]

Dashed curves are from pQCD calculations.

NA10 \( \pi^- + W \)

Z. Phys.
37 (1988) 545
Decay angular distributions in pion-induced Drell-Yan

Is the Lam-Tung relation \((1-\lambda-2\nu=0)\) violated?

Data from NA10  (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins
(Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer, Väntinnen, Vogt, etc.)
Boer-Mulders function $h_1^\perp$

- Boer pointed out that the $\cos 2\phi$ dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^\perp$ can lead to an azimuthal dependence with $\nu \propto \left( \frac{h_1^\perp}{f_1} \right) \left( \frac{\overline{h}_1^\perp}{\overline{f}_1} \right)$

$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$\nu = 16 \kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4 M_C^2)^2}$$

$\kappa_1 = 0.47$, $M_C = 2.3$ GeV

$\nu > 0$ implies valence BM functions for pion and nucleon have same signs
With Boer-Mulders function $h_1^\perp$:

$\nu(\pi^-W\rightarrow\mu^+\mu^-X)\sim [\text{valence } h_1^\perp(\pi)] \ast [\text{valence } h_1^\perp(p)]$

$v(pd\rightarrow\mu^+\mu^-X)\sim [\text{valence } h_1^\perp(p)] \ast [\text{sea } h_1^\perp(p)]$

Sea-quark BM function is much smaller than valence BM function
Lam-Tung relation from CDF Z-production

\[ p + \bar{p} \rightarrow e^+ + e^- + X \] at \( \sqrt{s} = 1.96 \tev \)


- Strong \( p_T \) (\( q_T \)) dependence of \( \lambda \) and \( \nu \)
- Lam-Tung relation (1-\( \lambda = 2\nu \)) is satisfied within experimental uncertainties (TMD is not expected to be important at large \( p_T \))
Recent CMS (ATLAS) data for Z-boson production in $p+p$ collision at 8 TeV

- Striking $q_T$ dependencies for $\lambda$ and $\nu$ were observed at two rapidity regions

- Is Lam-Tung relation violated?  


- Striking $q_T$ dependencies for $\lambda$ and $\nu$ were observed at two rapidity regions

- Is Lam-Tung relation violated?
Recent data from CMS for Z-boson production in $p+p$ collision at 8 TeV

- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation?
Interpretation of the CMS Z-production results

\[
\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + \frac{A_0}{2}(1 - 3\cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\
+ \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\
+ A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi
\]

Questions:

• How is the above expression derived?

• Can one express \( A_0 - A_7 \) in terms of some quantities?

• Can one understand the \( q_T \) dependence of \( A_0, A_1, A_2, \) etc?

• Can one understand the origin of the violation of Lam-Tung relation?
How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame

1) Hadron Plane
   - Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
   - Angle $\beta$ satisfies the relation $\tan \beta = q_T / Q$

   - $Q$ is the mass of the dilepton ($Z$)
   - when $q_T \to 0$, $\beta \to 0^\circ$
   - when $q_T \to \infty$, $\beta \to 90^\circ$
How is the angular distribution expression derived?

**Define three planes in the Collins-Soper frame**

1) Hadron Plane
   - Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
   - Angle $\beta$ satisfies the relation $\tan \beta = q_T / Q$

2) Quark Plane
   - $q$ and $\bar{q}$ have head-on collision along the $\hat{z}'$ axis
   - $\hat{z}'$ and $\hat{z}$ axes form the quark plane
   - $\hat{z}'$ axis has angles $\theta_1$ and $\phi_1$ in the C-S frame
How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame

1) Hadron Plane
   - Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
   - Angle $\beta$ satisfies the relation $\tan \beta = q_T / Q$

2) Quark Plane
   - $q$ and $\bar{q}$ have head-on collision along the $\hat{z}'$ axis
   - $\hat{z}'$ axis has angles $\theta_1$ and $\phi_1$ in the C-S frame

3) Lepton Plane
   - $l^-$ and $l^+$ are emitted back-to-back with equal $|\vec{P}|$
   - $l^-$ and $\hat{z}$ form the lepton plane
   - $l^-$ is emitted at angle $\theta$ and $\phi$ in the C-S frame
How is the angular distribution expression derived?

What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0$$

Azimuthally symmetric!

How to express the angular distribution in terms of $\theta$ and $\phi$?

Use the following relation:

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1)$$
How is the angular distribution expression derived?

\[ \frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0 \]

\[ \cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1) \]

\[ \frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2}(1 - 3 \cos^2 \theta) \]

\[ + \left( \frac{1}{2} \sin 2\theta_1 \cos \phi_1 \right) \sin 2\theta \cos \phi \]

\[ + \left( \frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1 \right) \sin^2 \theta \cos 2\phi \]

\[ + \left( a \sin \theta_1 \cos \phi_1 \right) \sin \theta \cos \phi + \left( a \cos \theta_1 \right) \cos \theta \]

\[ + \left( \frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1 \right) \sin^2 \theta \sin 2\phi \]

\[ + \left( \frac{1}{2} \sin 2\theta_1 \sin \phi_1 \right) \sin 2\theta \sin \phi \]

\[ + \left( a \sin \theta_1 \sin \phi_1 \right) \sin \theta \sin \phi . \]
All eight angular distribution terms are obtained!

\[
\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2} (1 - 3 \cos^2 \theta)
\]

\[
+ \left( \frac{1}{2} \sin 2\theta_1 \cos \phi_1 \right) \sin 2\theta \cos \phi
\]

\[
+ \left( \frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1 \right) \sin^2 \theta \cos 2\phi
\]

\[
+ (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta
\]

\[
+ \left( \frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1 \right) \sin^2 \theta \sin 2\phi
\]

\[
+ \left( \frac{1}{2} \sin 2\theta_1 \sin \phi_1 \right) \sin 2\theta \sin \phi
\]

\[
+ (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.
\]

\[
\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta)
\]

\[
+ A_1 \sin 2\theta \cos \phi
\]

\[
+ \frac{A_2}{2} \sin^2 \theta \cos 2\phi
\]

\[
+ A_3 \sin \theta \cos \phi + A_4 \cos \theta
\]

\[
+ A_5 \sin^2 \theta \sin 2\phi
\]

\[
+ A_6 \sin 2\theta \sin \phi
\]

\[
+ A_7 \sin \theta \sin \phi
\]

\[A_0 - A_7\] are entirely described by \(\theta_1, \phi_1\) and \(a\)
Angular distribution coefficients $A_0 - A_7$

\[
A_0 = \langle \sin^2 \theta_1 \rangle \\
A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle \\
A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle \\
A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle \\
A_4 = a \langle \cos \theta_1 \rangle \\
A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle \\
A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle \\
A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle
\]
Some implications of the angular distribution coefficients $A_0 - A_7$

\begin{align*}
A_0 &= \langle \sin^2 \theta_1 \rangle \\
A_1 &= \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle \\
A_2 &= \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle \\
A_3 &= a \langle \sin \theta_1 \cos \phi_1 \rangle \\
A_4 &= a \langle \cos \theta_1 \rangle \\
A_5 &= \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle \\
A_6 &= \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle \\
A_7 &= a \langle \sin \theta_1 \sin \phi_1 \rangle
\end{align*}

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, $a$, is reduced by a factor of $\langle \cos \theta_1 \rangle$ for $A_4$
- $A_5, A_6, A_7$ are odd function of $\phi_1$ and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obtained
Some implications of the angular distribution coefficients $A_0 - A_7$

\[
A_0 = \langle \sin^2 \theta \rangle
\]
\[
A_1 = \frac{1}{2} \langle \sin 2\theta \cos \phi \rangle
\]
\[
A_2 = \langle \sin^2 \theta \cos 2\phi \rangle
\]
\[
A_3 = a \langle \sin \theta \cos \phi \rangle
\]
\[
A_4 = a \langle \cos \theta \rangle
\]
\[
A_5 = \frac{1}{2} \langle \sin^2 \theta \sin 2\phi \rangle
\]
\[
A_6 = \frac{1}{2} \langle \sin 2\theta \sin \phi \rangle
\]
\[
A_7 = a \langle \sin \theta \sin \phi \rangle
\]

Some bounds on the coefficients can be obtained:

\[
0 < A_0 < 1
\]
\[
-1/2 < A_1 < 1/2
\]
\[
-1 < A_2 < 1
\]
\[
-\alpha < A_3 < \alpha
\]
\[
-\alpha < A_4 < \alpha
\]
What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$?

1) $q\bar{q} \to \gamma^*(Z^0)g$

\[ \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2} \]
What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$?

2) $qg \rightarrow \gamma^*(Z^0)q$

In $\gamma^*$ rest frame (C-S)

In $\gamma^*$ rest frame (C-S)

$\theta_1 = \beta$ and $\phi_1 = 0$

$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$
Compare with CMS data on $\lambda$

(Z production in $p+p$ collision at 8 TeV)

$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$ for $q\bar{q} \rightarrow Zg$

$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$ for $qG \rightarrow Zq$

For both processes

$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^\circ$)

$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^\circ$)

Data can be well described with a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes
Compare with CMS data on $\nu$

$(Z$ production in $p+p$ collision at 8 TeV)

\[ \nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \rightarrow Zg \]

\[ \nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \rightarrow Zq \]

Dashed curve corresponds to a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes

Solid curve corresponds to

\[ \left\langle \sin^2 \theta_1 \cos 2\phi_1 \right\rangle / \left\langle \sin^2 \theta_1 \right\rangle = 0.77 \]

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane
Origins of the non-coplanarities

1) Processes at order $\alpha_s^2$ or higher

2) Intrinsic $k_T$ from interacting partons

(Boer-Mulders functions in the beam and target hadrons)
Compare with CMS data on Lam-Tung relation

Solid curves correspond to a mixture of 58.5% $qG$ and 41.5% $q\bar{q}$ processes, and

$$\frac{\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle}{\langle \sin^2 \theta_1 \rangle} = 0.77$$

Violation of Lam-Tung relation is well described
Compare with CDF data

(Z production in $p + \bar{p}$ collision at 1.96 TeV)

Solid curves correspond to a mixture of 27.5% $qG$ and 72.5% $q\bar{q}$ processes, and

$$\frac{\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle}{\langle \sin^2 \theta_1 \rangle} = 0.85$$

Violation of Lam-Tung relation is not ruled out
Compare with CMS data on $A_1$, $A_3$ and $A_4$

$$A_1 = r_1 \left[ f \frac{q_T Q}{Q^2 + q_T^2} + (1-f) \frac{\sqrt{5} q_T Q}{Q^2 + 5q_T^2} \right]$$

$$A_3 = r_3 \left[ f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5q_T^2}} \right]$$

$$A_4 = r_4 \left[ f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{Q}{\sqrt{Q^2 + 5q_T^2}} \right]$$

Rapidity of $A_1$, $A_3$ and $A_4$ are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Future prospects

• Extend this study to W-boson production
  – Preliminary results show that the data can be well described

• Extend this study to fixed-target Drell-Yan data
  – Extraction of Boer-Mulders functions must take into account the QCD effects

• Extend this study to dihadron production in $e^- e^+$ collision (inverse of the Drell-Yan)
  – Analogous angular distribution coefficients and analogous Lam-Tung relation
Future prospects

• Extend this study to semi-inclusive DIS at high $p_T$ (involving two hadrons and two leptons)
  – Relevant for EIC measurements

• Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients

• Comparison with pQCD calculations
Summary

• The lepton angular distribution coefficients $A_0$-$A_7$ are described in terms of the polar and azimuthal angles of the $q - \bar{q}$ axis.

• The striking $q_T$ dependence of $A_0$ (or equivalently, $\lambda$) can be well described by the mis-alignment of the $q - \bar{q}$ axis and the Collins-Soper $z$-axis.

• Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $q - \bar{q}$ axis and the hadron plane. This can come from order $\alpha_s^2$ or higher processes or from intrinsic $k_T$.

• This study can be extended to fixed-target Drell-Yan data.