Extraction and Parametrization of Isobaric Trinucleon Elastic Cross Sections and Form Factors

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Introduction
Brief Overview of JLab Work

• Graduate work at JLab since 2013.

• SEOP of polarized $^3$He targets at W&M.

• Gas Ring ImagiNg CHerenkov (GRINCH):
  – Built and tested the mirror system.
  – Characterized the PMTs and assembled the detector.
  – Developed preliminary DAQ.
  – Tested and implemented VETROC boards allowing for real-time high rate triggering.

• Hall A for the Tritium Experiments:
  – Maintained and prepared VDCs and EM calorimeters.
  – Counting house script maintenance and development.
  – Shift work and analysis shifts.

• Worked on many JLab experiments:
  – SRC X$^2$, A$^1$, GMp, Ar(e,e'p), DVCS, and the Tritium Experiments.
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Elastic Electron Scattering

Figure 2: Elastic Electron Scattering. An incident electron interacts with a target by exchanging a virtual photon causing the electron to scatter.

\[ \nu = E_0 - E' \]  

\[ E' = \frac{E_0}{1 + \frac{2E_0}{M} \sin^2 \left( \frac{\theta}{2} \right)} \]  

\[ Q^2 = -q^2 = 4E_0E'\sin^2 \left( \frac{\theta}{2} \right) \]  

\[ X_{Bj} = \frac{Q^2}{M(E_0 - E')} \]
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- Elastic scattering is completely determined by knowing two of \( E_0, \theta, \) or \( E' \).
Rutherford Cross Section

- The differential cross section describes the **likelihood of an electron interacting with a target.**
  - Measures the ‘size’ of an interaction.
  - Must be viewed within a detector’s solid angle acceptance.

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- Cross section falls off like \( \frac{1}{q^4} \).
  - Likelihood of electrons interacting with a target decreases rapidly with energy.
- Rutherford equation does not account for relativity, spin, or target recoil.
Now add a term to account for relativity obtaining the Mott Equation [1]:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott No Recoil}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} \left( 1 - \beta^2 \sin^2 \left( \frac{\theta}{2} \right) \right)
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- In the relativistic limit $\beta \to 1$ yielding [1]:

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(7)

• Now we have accounted for relativity, but we have also accounted for spin with the $\cos^2 \left( \frac{\theta}{2} \right)$ term.
  – Suppresses scattering through $180^\circ$ for a spinless target which is forbidden by conservation of helicity.
• At our $E_0$ of 3.356 GeV an electron has 1.48 GeV of energy and a $^3$He nucleus has a mass of 2.81 GeV.
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• Clearly neglecting recoil is no longer an option. Happily, the recoil term is easily found from Equation 2 describing elastic scattering

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\frac{E'}{E_0} = \frac{1}{1 + \frac{2E_0}{M} \sin^2 \left( \frac{\theta}{2} \right)} \quad (8)
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Mott Cross Section Cont.

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- Now we can add the recoil term and rewrite the Mott cross section with a few substitutions from earlier as [1]:

$$\left( \frac{d\sigma}{d\Omega} \right)_{Mott} = \frac{4Z^2\alpha^2 (\hbar c)^2 E'^3}{|qc|^4 E_0} \cos^2 \left( \frac{\theta}{2} \right) = Z^2 \frac{E'}{E_0} \frac{\alpha^2 \cos^2 \left( \frac{\theta}{2} \right)}{4E_0^2 \sin^4 \left( \frac{\theta}{2} \right)}$$

(9)
Now we have an equation that accounts for relativity, spin, and recoil off of a point mass.
Nuclear Form Factors

• Now we have an equation that accounts for relativity, spin, and recoil off of a point mass.

• **Nuclei are not point masses!** How do we describe the structure inside of a nucleus?

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} |F(q^2)|^2
\]  (10)

• The term \(|F(q^2)|^2\) is called a form factor. It contains all of the spatial and structural information about the target.

• If we assume the validity of the Born approximation (incident wave function \(\approx\) scattered wave function) and no recoil the form factor can be written as a Fourier transform of the charge distribution.
Now we have an equation that accounts for relativity, spin, and recoil off of a point mass.

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Form Factors Cont.

\[ F(q^2) = \int e^{\frac{iq\cdot x}{\hbar}} \rho(x)d^3x \xrightarrow{x \rightarrow r} 4\pi \int \rho(r) \frac{\sin(|q| r / \hbar)}{|q| r / \hbar} r^2 dr \quad (11) \]
Form Factors Cont.

\[ F(q^2) = \int e^{\frac{iq \cdot x}{\hbar}} \rho(x) \, d^3x \rightarrow 4\pi \int \rho(r) \frac{\sin(|q| r / \hbar)}{|q| r / \hbar} \, r^2 \, dr \quad (11) \]

- This procedure can be inverted to find the charge distribution, \( \rho \). [1]

\[ \rho(r) = \frac{1}{(2\pi)^3} \int F(q^2) e^{\frac{-iq \cdot x}{\hbar}} \, d^3q \quad (12) \]
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Figure 3: Hard Sphere Charge Distribution
Form Factors Cont.

\[
F(q^2) = \int e^{\frac{i\mathbf{q} \cdot \mathbf{x}}{\hbar}} \rho(\mathbf{x}) d^3\mathbf{x} \to 4\pi \int \rho(r) \frac{\sin(|q|r/\hbar)}{|q|r/\hbar} r^2 dr \tag{11}
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- Let’s approximate a nucleus as a hard sphere of charge.

- Yields oscillatory form factor.

- Charge radii can be estimated by minima location \[1\]!

\[
R \approx \frac{4.5\hbar}{q} \tag{13}
\]
• We can find charge radii by taking $r \to 0$ in the form factor.
  – The wavelength of the electron, $\frac{\hbar}{q}$, is assumed to be much larger than the charge radius: $R \ll \frac{\hbar}{q} \implies \frac{Rq}{\hbar} \ll 1$. 
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• Next we apply the Euler formula to the form factor equation and Taylor expand the cosine term [1]:

\[
F(q^2) = \int_0^\infty \int_{-1}^1 \int_0^{2\pi} \rho(r) \left(1 - \frac{1}{2} \frac{|q||r| \cos(\omega)}{\hbar}\right) r^2 \, d\phi \, d\cos(\omega) \, dr \quad (14)
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\[
F(q^2) = 4\pi \int_0^\infty \rho(r)r^2 dr - 4\pi \frac{q^2}{6\hbar^2} \int_0^\infty \rho(r)r^4 dr \quad (15)
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Charge Radius

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• Applying the normalization $4\pi \int_0^\infty \rho(r) r^2 \, dr = 1$, and defining the charge radius as $\langle r^2 \rangle = 4\pi \int_0^\infty r^2 \rho(r) r^2 \, dr$, we find:
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\[
F(q^2) = 1 - \frac{q^2}{6\hbar^2} \langle r^2 \rangle \quad \rightarrow \quad \langle r^2 \rangle = -6\hbar^2 \frac{dF(q^2)}{dq^2} \bigg|_{q^2=0} \tag{16}
\]
Now our cross section accounts for charge, relativity, spin, and recoil. Great! Is there anything still missing?
Rosenbluth Equation

- Now our cross section accounts for **charge**, **relativity**, **spin**, and **recoil**. Great! Is there anything still missing?
  - **Magnetic interactions** and **internal structure** are still not included.
  - Introduce a new **magnetic term** as was done for relativity [1].

\[ (d\sigma d\Omega) = (d\sigma d\Omega)_{Mott} \times \left[ G^2_E(Q^2) + \tau G^2_M(Q^2) \left(1 + \tau + 2\tau \tan^2(\theta^2)\right) \right] \]

\[ G^p_E(Q^2 = 0) = 1 \quad \text{and} \quad G^p_M(Q^2 = 0) = 2.79 \]

\[ G^n_E(Q^2 = 0) = 0 \quad \text{and} \quad G^n_M(Q^2 = 0) = -1.91. \]
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\left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \left[ \frac{G_E^2 (Q^2) + \tau G_M^2 (Q^2)}{1 + \tau} + 2\tau G_M^2 (Q^2) \tan^2 \left( \frac{\theta}{2} \right) \right] \tag{18}
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\]

\[
G_E^n (Q^2 = 0) = 0 \quad \text{and} \quad G_M^n (Q^2 = 0) = -1.91 \quad (20)
\]

\[
G_E^p (Q^2 = 0) = 1 \quad \text{and} \quad G_M^p (Q^2 = 0) = 2.79 \quad (19)
\]
• $G_E$ and $G_M$ are known as the Sach’s form factors. Several other form factors are commonly used.
Rosenbluth Equation Cont.

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  - $F_1$ and $F_2$ are called the Dirac and Pauli form factors respectively.

\[
G_E(Q^2) = F_1(Q^2) - \mu \tau F_2(Q^2) \quad G_M(Q^2) = F_1(Q^2) + \mu F_2(Q^2)
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  – There are also the $F_{ch}$ and $F_m$ form factors used in this analysis.

$$F_{ch}(Q^2) = G_E(Q^2) \quad (23) \quad F_m(Q^2) = \frac{G_M(Q^2)}{\mu} \quad (24)$$
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- Now let’s rewrite the cross section for the final Rosenbluth equation:

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \ast \frac{1}{1 + \tau} \left[ G_E^2 \left( Q^2 \right) + \frac{\tau}{\epsilon} G_M^2 \left( Q^2 \right) \right] \quad (25)$$

$$\epsilon = \left( 1 + 2(1 + \tau) \tan^2 \left( \frac{\theta}{2} \right) \right)^{-1} \quad (26)$$
Experimental Setup
Experiment E08-014

- Experiment E08-014 ran in Jefferson Lab’s Hall A in 2011 [2].
  - Measured inclusive cross sections of $^2$H, $^3$He, $^4$He, $^{12}$C, $^{40}$Ca, and $^{48}$Ca in the range of $1.1 \text{ GeV/c} < Q^2 < 2.5 \text{ GeV/c}$.
  - Compared heavy targets to two and three-nucleon targets to study short range correlations between two and three-nucleon clusters.
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- E08-014 mostly took quasielastic data, but through a happy coincidence, KIN 3.2 was able to view the $^3\text{He}$ elastic peak.
  - $E_0 = 3.356 \text{ GeV}$. Scattering angle of $20.51^\circ$. 
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Figure 4: Elastic Band for $^3\text{He}$. 
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- Experiment E08-014 ran in Jefferson Lab’s Hall A in 2011 [2].
  - Measured inclusive cross sections of $^2\text{H}$, $^3\text{He}$, $^4\text{He}$, $^{12}\text{C}$, $^{40}\text{Ca}$, and $^{48}\text{Ca}$ in the range of $1.1 \text{ GeV/c} < Q^2 < 2.5 \text{ GeV/c}$.
  - Compared heavy targets to two and three-nucleon targets to study short range correlations between two and three-nucleon clusters.
- E08-014 mostly took quasielastic data, but through a happy coincidence, KIN 3.2 was able to view the $^3\text{He}$ elastic peak.
  - $E_0 = 3.356 \text{ GeV}$. Scattering angle of $20.51^\circ$.

![Figure 4: Elastic Band for $^3\text{He}$.](image1)

![Figure 5: Elastic Peak in $x_{Bj}$.](image2)
Hall A Configuration

- E08-014 used the standard Hall A configuration and detector packages.
  - Main trigger \((S_1 \& S_{2m} \& GC)\).

Figure 6: Hall A Top View. Standard Hall A configuration. Image from [3].
Cross Section Extraction
• We can rewrite the cross section in a more useful form:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \frac{ps * N_e}{N_{in} * \rho * LT * \epsilon_{det}} \frac{1}{\Delta\Omega \Delta P \Delta Z}
\]  

(27)
Extracting a Cross Section

• We can rewrite the cross section in a more useful form:

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\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \frac{ps \ast N_e}{N_{in} \ast \rho \ast LT \ast \epsilon_{det}} \frac{1}{\Delta\Omega \Delta P \Delta Z}
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\]  \hspace{1cm} (27)

- Essentially, this is a game of electron counting.
  - To count electrons we need the beam charge.

\[
Q = \langle I_{\text{beam}} \rangle \ast \text{time}
\]  \hspace{1cm} (28)

\[
N_{in} = \frac{Q}{e}
\]  \hspace{1cm} (29)
Particle Identification

- How many elastic electrons, $N_e$, were detected?
  - We need a pure electron sample. Pions can contaminate the sample.
Particle Identification

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  - The EM calorimeters and GC can discriminate electrons from pions.
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  - We need a pure electron sample. Pions can contaminate the sample.
  - The EM calorimeters and GC can discriminate electrons from pions.

Figure 8: PID with the Pion Rejectors. Image from [2].

Figure 9: PID with the Gas Cherenkov. Image from [2].
Pion Rejectors PID

- For our six runs there appear to be very few pions.
- Some delta (knock-on) electrons are in the sample.
  - These delta electrons and few pions can be removed with a simple diagonal cut on the pion rejectors.

![PID PR](image)
Gas Cherenkov PID

- Does the gas Cherenkov agree with the pion rejectors?
Gas Cherenkov PID

- Does the gas Cherenkov agree with the pion rejectors?
- Again there appear to be almost no pions at our kinematics.

Figure 11: PID with the Gas Cherenkov.
Now we need to know how many scattering centers are in the target.
Now we need to know how many scattering centers are in the target. Let’s look at the target’s density profile to find $\rho$.

**Figure 12:** $^3$He Boiling Effect. Image from [2].
Target Density

- Now we need to know how many scattering centers are in the target.
- Let’s look at the target’s density profile to find $\rho$.

![Figure 12: $^3$He Boiling Effect. Image from [2].](image)

- The density is not constant along the cell due to boiling effects.
- CFD calculations by Silviu Covrig allowed $\rho$ to be calculated [4].
  - $0.013\ \text{g/cm}^3 \pm 0.0004\ \text{g/cm}^3$. 

Simulating Elastic Electrons

- How do we know how many events were elastic?

- Find the elastic electron yield.
- Count electrons in the elastic peak ($X_{\text{Bj}} = 3$).

- Before finding the experimental elastic electron yield we want a point of comparison.
- We can simulate what the elastic electron spectrum is expected to look like at our kinematics using SIMC.

- SIMC:
  - Monte Carlo generates events randomly in given kinematic ranges.
  - Transports electrons from scattering vertex through spectrometers to detector stack.
  - Contains old $^3$He elastic scattering model from Amroun et al [5].
  - Shape of form factors and cross section should be accurate.
  - Cross section magnitude is expected to be off at our kinematics.
  - Can calculate radiative effects.
Simulating Elastic Electrons

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  - Shape of form factors and cross section should be accurate.
  - Cross section magnitude is expected to be off at our kinematics.
  - Can calculate radiative effects.
SIMC Output

Figure 13: $\phi$. In plane angle.

- **Red** is SIMC and **blue** is data.
- $\phi$ and $\theta$ both look decent.
- There is a shift in dP.
  - Known issue with SIMC.
  - Little impact on final measurement.

Figure 14: $\theta$. Out of plane angle.

Figure 15: dP Momentum fraction.
Aluminium Background Subtraction

- Take a closer look at the $X_{Bj}$ plot from earlier (blue histogram).
  - What are the events above the $^3$He elastic peak?

![Production Runs Compared to Scaled Al Background](image)

**Figure 16:** Scaled Aluminium Background and $X_{Bj}$.
Aluminium Background Subtraction

- Take a closer look at the $X_{Bj}$ plot from earlier (blue histogram).
  - What are the events above the $^3$He elastic peak?

![Production Runs Compared to Scaled Al Background](image)

**Figure 16: Scaled Aluminium Background and $X_{Bj}$.**

- Aluminium contamination from target cell (red histogram).
  - Use dummy cell to subtract Al events.
  - Scale dummy by charge, thickness, and radiative corrections.
• What happens when we subtract off the aluminium contamination from the $X_{Bj}$ spectrum?

**Figure 17:** Scaled Aluminium Background and $X_{Bj}$. 
Aluminium Background Subtraction Cont.

- What happens when we subtract off the aluminium contamination from the $X_{Bj}$ spectrum?

**Figure 18: Aluminium Subtracted $X_{Bj}$.**

- Now most of the events above the elastic peak disappear!
Fitting the Elastic Peak

- How do we count the number of electrons in the elastic peak, $N_e$?

Figure 19: Combined Fit of $X_{Bj}$ for E08-014.
Fitting the Elastic Peak

How do we count the number of electrons in the elastic peak, $N_e$?
- **Fit the peak and integrate** over the elastic peak.
Fitting the Elastic Peak

- How do we count the number of electrons in the elastic peak, $N_e$?
  - Fit the peak and integrate over the elastic peak.
- What kind of fit might be appropriate?

$$F_{\text{combined}} = e^{(P_0 + P_1 \times x)} + P_2 \times e^{-\frac{1}{2}(x - P_3)^2}$$ (30)

Figure 19: Combined Fit of $X_{Bj}$ for E08-014.
Fitting the Elastic Peak

- How do we count the number of electrons in the elastic peak, $N_e$?
  - Fit the peak and integrate over the elastic peak.
- What kind of fit might be appropriate?
  - QE background looks exponential and elastic peak looks Gaussian.
  - Create a fit equation summing an exponential and a Gaussian:

$$F_{\text{combined}} = e^{(P_0 + P_1 \times x)} + P_2 \times e^{(-\frac{1}{2} (x - P_3 \times P_4)^2)}$$

Figure 19: Combined Fit of $X_{Bj}$ for E08-014.
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- What kind of fit might be appropriate?
  - QE background looks exponential and elastic peak looks Gaussian.
  - Create a fit equation summing an exponential and a Gaussian:

$$F_{combined} = e^{(P_0 + P_1*x)} + P_2 * e\left(\frac{-1}{2} \left(\frac{x-P_3}{P_4}\right)^2\right)$$ (30)

**Figure 19:** Combined Fit of $X_{Bj}$ for E08-014. 

\[\text{Counts} \]
\[1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5\]
\[1 \quad 10 \quad 10^2 \quad 10^3\]

$X_{Bj}$ for $^3$He Production Runs
Comparing SIMC and Experimental Yields

- Now let’s look at the elastics from SIMC.

![SIMC $^3$He Elastic Electrons](image)

**Figure 20:** SIMC Elastically Scattered Electrons.
Comparing SIMC and Experimental Yields

• Now let’s look at the elastics from SIMC.

![SIMC $^3$He Elastic Electrons](image)

**Figure 20:** SIMC Elastically Scattered Electrons.

• The spectrum looks reasonable.
  – Prominent elastic peak at $X_{Bj} = 3$.
  – Nice radiative tail as expected.
How do we compare simulated elastic events to experimental events that are mostly quasielastic?

- We can fit only the QE background and then sum the fit with the SIMC elastics.
- The QE fit is made in the region where few elastics are expected.

Figure 21: Histogram Binned to Fit of Quasielastic Background.
Comparing SIMC and Experimental Yields Cont.

- How do we compare simulated elastic events to experimental events that are mostly quasielastic?
  - We can fit only the QE background and then sum the fit with the SIMC elastics.
  - The QE fit is made in the region where few elastics are expected.
Comparing SIMC and Experimental Yields Cont.

- How do we compare simulated elastic events to experimental events that are mostly quasielastic?
  - We can fit only the QE background and then sum the fit with the SIMC elastics.
  - The QE fit is made in the region where few elastics are expected.

Figure 21: Histogram Binned to Fit of $^3$He Quasielastic Background.

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Finally we can compare the elastic electron yield from SIMC and experimental data.
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- The area under the Gaussian of the two combined fits, but above the QE background, represents the elastic electron yield.
- The elastic electron yield is directly proportional to the cross section.
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- Note that the real data yield is increased slightly to correct for live-time and various efficiencies.
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Recall that at our kinematics the old $^3$He model in SIMC has the correct cross section shape but incorrect magnitude [5].

SIMC produces the average cross section for this experiment based on the previous model.
Finally we can compare the elastic electron yield from SIMC and experimental data.

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- The elastic electron yield is directly proportional to the cross section.
- Note that the real data yield is increased slightly to correct for live-time and various efficiencies.

Recall that at our kinematics the old $^3$He model in SIMC has the correct cross section shape but incorrect magnitude [5].

SIMC produces the average cross section for this experiment based on the previous model.

If the SIMC cross section is scaled by a constant until the SIMC yield matches the experimental yield then the average cross section SIMC produces will equal the experimental cross section.
Yields plots for the experimental data and SIMC scaled to match.

- Red is SIMC + QE background fit and blue is experimental data.

**Figure 22: SIMC + QE Scaled to Experimental Elastic Electron Yield.**

New $^3$He cross section value is $1.335 \times 10^{-10}$ fm$^{-2}$/sr.
Cross Section Value

- Yields plots for the experimental data and SIMC scaled to match.
  - Red is SIMC + QE background fit and blue is experimental data.

Figure 22: SIMC + QE Scaled to Experimental Elastic Electron Yield.

- Yield shapes are similar. Slight shift is likely a Z offset issue.
- SIMC scale factor to match data = 1.01984.
- New $^3$He cross section value is $1.335 \times 10^{-10}$ fm$^{-2}$/sr.
Where to Place the Data Point

- We now have the cross section’s magnitude. Great! We’re done right?

- Wrong!

- Where do we put our point?

- What is the uncertainty?

- Can’t we just calculate $Q^2$ from the center of our bin and be done?
Where to Place the Data Point

- We now have the cross section’s magnitudes. Great! We’re done right?
  - Wrong!
Where to Place the Data Point

• We now have the cross section’s **magnitude**. Great! We’re done right?
  – Wrong!
  – Where do we put our point?
  – What is the **uncertainty**?
Where to Place the Data Point

- We now have the cross section’s magnitude. Great! We’re done right?
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- The bin center is only correct if the function is linear [6].
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![Figure 23: $^3$He Elastic Cross Section at 3.356 GeV.](image-url)
• Now let’s zoom in and remove the log scale to see the true shape.
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Figure 24: $^3$He Elastic Cross Section $Q^2$ Bin at 3.356 GeV.
Where to Place the Data Point Cont.

• Now let’s zoom in and remove the log scale to see the true shape.

Figure 24: $^3$He Elastic Cross Section $Q^2$ Bin at 3.356 GeV.

• Clearly not linear.
• Weight the $Q^2$ values in the bin by the cross section magnitude.
Now let’s zoom in and remove the log scale to see the true shape.

*Figure 24: $^3$He Elastic Cross Section $Q^2$ Bin at 3.356 GeV.*

- Clearly not linear.
- Weight the $Q^2$ values in the bin by the cross section magnitude.
- The bin center would place the $Q^2$ at 35.90 fm$^{-2}$.
- The weighted bin center is at $Q^2 = 34.19$ fm$^{-2}$. 
Lastly, we need to quantify the uncertainty on our point.

### Table 1: Table of Uncertainties

<table>
<thead>
<tr>
<th>Uncertainty Source</th>
<th>Cross Section Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Statistical Sources</strong></td>
<td></td>
</tr>
<tr>
<td>Electron Yield</td>
<td>4.21%</td>
</tr>
<tr>
<td>Al Background Subtraction</td>
<td>1.1%</td>
</tr>
<tr>
<td><strong>Total Statistical</strong></td>
<td>4.36%</td>
</tr>
<tr>
<td><strong>Systematic Sources</strong></td>
<td></td>
</tr>
<tr>
<td>Target Density</td>
<td>3.08%</td>
</tr>
<tr>
<td>Optics Calibration</td>
<td>2.248%</td>
</tr>
<tr>
<td>GC Efficiency</td>
<td>1.32%</td>
</tr>
<tr>
<td>Beam/Target Offsets</td>
<td>1.1%</td>
</tr>
<tr>
<td>Radiative Corrections</td>
<td>1%</td>
</tr>
<tr>
<td>Beam Charge</td>
<td>1%</td>
</tr>
<tr>
<td>VDC Single-Track Efficiency</td>
<td>1%</td>
</tr>
<tr>
<td>Trigger Efficiency</td>
<td>1%</td>
</tr>
<tr>
<td>Beam Energy</td>
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</tr>
<tr>
<td>SIMC Model Comparison to Reality</td>
<td>0.5%</td>
</tr>
<tr>
<td>PR Cut</td>
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</tr>
<tr>
<td>$Y_{\text{target}}$ Position</td>
<td>0.045%</td>
</tr>
<tr>
<td>Live-time</td>
<td>0.01145%</td>
</tr>
<tr>
<td><strong>Total Systematic</strong></td>
<td>4.72%</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td></td>
</tr>
<tr>
<td>Statistical &amp; Systematic</td>
<td><strong>6.42%</strong></td>
</tr>
</tbody>
</table>
Comparison to Other Measurements

- Our cross section measurement is now $1.335 \pm 0.086 \times 10^{-6} \, \mu b/sr$
at $Q^2 = 34.19 \, fm^{-2}$.

- JLab took high $Q^2$ data for $^3$He [7].
  - $E_0 = 3.304 \, GeV$. Scattering angle of $20.83^\circ$.
  - Cross section $= 1.57 \pm 0.10 \times 10^{-6} \, \mu b/sr$ at $Q^2 = 34.1 \, fm^{-2}$.
  - This analysis' cross section is $\approx 15\%$ smaller.
  - Accounting for our higher $Q^2$ the error bars should nearly overlap.

- Cross section estimate from older SIMC model [5].
  - $E_0 = 3.356 \, GeV$. Scattering angle of $20.51^\circ$.
  - Cross section $= 1.887 \times 10^{-6} \, \mu b/sr$ at $Q^2 = 34.19 \, fm^{-2}$.
  - This analysis' cross section is $\approx 30\%$ smaller.
  - Old model had very little high $Q^2$ data so we expect the magnitude to not be extremely accurate.

- In sum this new point seems reasonably consistent with other data.
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- In sum this new point seems reasonably consistent with other data.
Global Fits
We now have a new cross section!
• We now have a new cross section! But what use is a single point?
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We need context to learn anything.
We now have a **new cross section**! But what use is a **single point**?

*We need context to learn anything.*

For context we can gather the **world data** for $^3\text{He}$ elastic cross sections.

- We can also gather the $^3\text{H}$ world data so we have mirror nuclei.
- The world data for $^3\text{H}$ and $^3\text{He}$ stretches back over **50 years**!
- Experiments were done at at least nine different facilities.
- Many experiments used different methodologies.
• We now have a new cross section! But what use is a single point?
• We need context to learn anything.
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  – The world data for $^3$H and $^3$He stretches back over 50 years!
  – Experiments were done at at least nine different facilities.
  – Many experiments used different methodologies.
• This analysis used as many of the data sets as possible.
  – Some papers did not publish their data.
  – Some published only form factors and/or did not publish scattering angles and energies.
• This analysis adds new JLab high $Q^2$ data for $^3$He as well as the new cross section from SRC $X>2$. 
What do we want to get out of the world data?

...
Sum of Gaussians Parametrization

- What do we want to get out of the world data?
  - Form factors, charge densities, and charge radii.

\[
\rho(r) = \frac{Z e^2}{\pi^3/2 \gamma^3} \sum_{i=1}^{N} Q_i \left( e^{-\left(\frac{r-R_i}{\gamma}ight)^2} + e^{-\left(\frac{r+R_i}{\gamma}ight)^2} \right)
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Sum of Gaussians Parametrization

- What do we want to get out of the world data?
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- How do we find these quantities?
Sum of Gaussians Parametrization

- What do we want to get out of the world data?
  - Form factors, charge densities, and charge radii.
- How do we find these quantities?
  - We need some parametrization to fit the world data.

\[
\rho(r) = \frac{Z e^2}{\pi^3/2 \gamma^3} \sum_{i=1}^{N} Q_i \left( 1 + 2\frac{r}{\gamma} \right) \left( e^{-\left(r-R_i\right)^2/\gamma^2} + e^{-\left(r+R_i\right)^2/\gamma^2} \right)
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Sum of Gaussians Parametrization

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- Select a sum of Gaussians (SOG) parametrization [8].

\[ \rho(r) = \frac{Ze}{\gamma^3N} \sum_{i=1}^{Q} \left( e^{-\left( r - R_i \right)^2/\gamma^2} + e^{-\left( r + R_i \right)^2/\gamma^2} \right) \]

- With normalization:
  \[ 4\pi \int_0^{\infty} \rho(r) r^2 dr = Ze \]
Sum of Gaussians Parametrization

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  - We need some parametrization to fit the world data.
- Select a sum of Gaussians (SOG) parametrization [8].
  - Parametrizes form factors using multiple Gaussians placed at different radii.
  - Disallows unphysically small structures in the charge density.
  - $q_{\text{max}}$ based on limited data: $\lambda = \frac{2\pi}{q_{\text{max}}}$. 

\[ \rho(r) = \frac{Ze}{2\pi^{3/2}} \gamma^3 \sum_{i=1}^{N} Q_i \left( e^{-\left( r - R_i \right)^2/\gamma^2} + e^{-\left( r + R_i \right)^2/\gamma^2} \right) \]
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How do we find these quantities?
- We need some parametrization to fit the world data.

Select a sum of Gaussians (SOG) parametrization [8].
- Parametrizes form factors using multiple Gaussians placed at different radii.
- Disallows unphysically small structures in the charge density.
- $q_{max}$ based on limited data: $\lambda = \frac{2\pi}{q_{max}}$.

\[
\rho(r) = \frac{Ze}{2\pi^{3/2}\gamma^{3}} \sum_{i=1}^{N} \frac{Q_{i}}{1 + \frac{2R_{i}^{2}}{\gamma^{2}}} \left( e^{-(r-R_{i})^{2}/\gamma^{2}} + e^{-(r+R_{i})^{2}/\gamma^{2}} \right) \tag{31}
\]

With normalization:

\[
4\pi \int_{0}^{\infty} \rho(r) r^{2} dr = Ze \tag{32}
\]
• When using the plane wave born approximation the form factors can be written as [8]:

\[
F_{(ch,m)}(q) = \exp \left( -\frac{1}{4} q^2 \gamma^2 \right) \sum_{i=1}^{N} \frac{Q_{i(ch,m)}}{1 + 2 R_i^2 / \gamma^2} \left( \cos(q R_i) + \frac{2 R_i^2}{\gamma^2} \sin(q R_i) \right)
\]  

(33)
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(33)

• The \(Q_i\) are the fit parameters.
  
  – Represent the **fraction of charge held by each Gaussian**.
  
  – \(Q_i > 0\) and \(\sum Q_i = 1\).
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• The \(R_i\) represent the radii at which each Gaussian is placed.
  – Selected pseudorandomly then optimized.

• The differential cross section can be written as [5]:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \frac{1}{\eta} \left[ \frac{q^2}{2} F_{ch}^2(q) + \frac{\mu^2 q^2}{2M^2} \left( \frac{1}{2} \frac{q^2}{q^2} + \tan^2 \left( \frac{\theta}{2} \right) \right) \right] F_m^2(q)
\]

(34)

\[
\eta = 1 + \frac{q^2}{4M^2}
\]

(35)
• To account for the Born approximation we utilize $Q_{\text{eff}}$ in place of a full phase shift code.

$$Q_{\text{eff}}^2 = Q^2 \left( 1 + \frac{1.5\alpha}{E_0 \times 1.12 \times A^{\frac{1}{3}}} \right)^2$$ (36)
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Next we need to decide how to generate our starting radii, $R_i$.
- There is an $R_{\text{max}}$ beyond which there is almost no charge density.
- $R_{\text{max}} \approx 5 \text{ fm}$ for our nuclei.
- Consecutive $R_i$ spacing is closer at smaller radii.
- $R_i < R_{\text{max}}/2 \approx$ half as far apart as spacing of $R_i > R_{\text{max}}/2$. 

Essentially, the set of $R_i$ constitute a model of the charge distribution.

$R_i$ are generated within pseudorandom ranges initially to span the model space.

$R_i$ are then adjusted up and down by 0.1 fm until $\chi^2$ is minimized.
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• $R_i$ are generated within pseudorandom ranges initially to span the model space.

• $R_i$ are then adjusted up and down by 0.1 fm until $\chi^2$ is minimized.
Selecting the Number of Gaussians

- How do we select how many Gaussians, $N_{Gaus}$, to use in our fit?

\[ \chi^2 = \sum_{n=1}^{N} \left( \sigma_{\text{exp}} - \sigma_{\text{fit}} \right)^2 \Delta^2 \quad (37) \]

\[ \chi^2 = \chi^2 / N - N_{\text{var}} - 1 \quad (38) \]

- Akaike and Bayesian information criterion are more powerful [9].

\[ \text{AIC} = N \ln \left( \chi^2 / N \right) + 2 N_{\text{var}} \quad (39) \]

\[ \text{BIC} = N \ln \left( \chi^2 / N \right) + \ln \left( N / N_{\text{var}} \right) \]

- The $Q_i$ are not forced to sum to unity.

- By not forcing $\sum Q_i = 1$ the $\sum Q_i$ becomes a measure of how well our fit and the current data approach physical requirements.

- Lastly the fits can be visually inspected for physicality.
Selecting the Number of Gaussians

- How do we select how many Gaussians, \( N_{Gaus} \), to use in our fit?
  - \( \chi^2 \) is useful, but it can be misleading. Reduced \( \chi^2 \) can help.

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  \]  
  
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Selecting the Number of Gaussians Cont.

- Compare the metrics for different values of $N_{Gaus}$ for $^3\text{He}$ and $^3\text{H}$.
Selecting the Number of Gaussians Cont.

- Compare the metrics for different values of $N_{Gaus}$ for $^3\text{He}$ and $^3\text{H}$.

<table>
<thead>
<tr>
<th>$N_{Gaus}$</th>
<th>Avg. $\chi^2$</th>
<th>$r\chi^2$</th>
<th>BIC</th>
<th>AIC</th>
<th>$\sum Q_{ich}$</th>
<th>$\sum Q_{im}$</th>
<th>$\chi^2_{max}$</th>
<th>‘Good’ Fits</th>
</tr>
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<tbody>
<tr>
<td>8</td>
<td>584.902</td>
<td>2.41695</td>
<td>255.440</td>
<td>223.228</td>
<td>1.00769</td>
<td>1.11065</td>
<td>765</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>470.435</td>
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<td>56</td>
</tr>
</tbody>
</table>

Table 2: Determination of $N_{Gaus}$ for $^3\text{He}$
Selecting the Number of Gaussians Cont.

- Compare the metrics for different values of $N_{Gaus}$ for $^3\text{He}$ and $^3\text{H}$.

<table>
<thead>
<tr>
<th>$N_{Gaus}$</th>
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</tr>
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</table>

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<th>Avg.</th>
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</thead>
<tbody>
<tr>
<td>7</td>
<td>611.690</td>
<td>2.79310</td>
<td><strong>263.039</strong></td>
<td>238.851</td>
<td></td>
<td><strong>1.08373</strong></td>
<td>1.32730</td>
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<td>1</td>
</tr>
<tr>
<td>8 close</td>
<td>601.836</td>
<td>2.77344</td>
<td>264.694</td>
<td>237.051</td>
<td></td>
<td>1.09013</td>
<td>1.32859</td>
<td>603</td>
<td>32</td>
</tr>
<tr>
<td>8 wide</td>
<td>601.752</td>
<td>2.79892</td>
<td>264.661</td>
<td><strong>237.018</strong></td>
<td></td>
<td><strong>1.08970</strong></td>
<td>1.33270</td>
<td>603</td>
<td>39</td>
</tr>
<tr>
<td>9</td>
<td>601.768</td>
<td>2.82579</td>
<td>270.123</td>
<td>239.025</td>
<td></td>
<td>1.08849</td>
<td>1.31982</td>
<td>604</td>
<td><strong>95</strong></td>
</tr>
<tr>
<td>10</td>
<td>601.893</td>
<td>2.84416</td>
<td>275.627</td>
<td>241.074</td>
<td></td>
<td>1.09248</td>
<td><strong>1.29611</strong></td>
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<tr>
<td>11</td>
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<td><strong>2.77305</strong></td>
<td>280.637</td>
<td>242.629</td>
<td></td>
<td>1.08699</td>
<td>1.34100</td>
<td>602</td>
<td>88</td>
</tr>
</tbody>
</table>

Table 3: Determination of $N_{Gaus}$ for $^3\text{H}$
$^3$He Fits

- Let’s look at the $N_{Gaus} = 12$ $^3$He form factor plots.
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Figure 25: Charge Form Factors from 1352 $^3$He Fits with no $\chi^2_{max}$ cut.
3He Fits

- Let's look at the $N_{Gaus} = 12$ $^3$He form factor plots.

![3He Charge Form Factor](image)

**Figure 25:** Charge Form Factors from 1352 $^3$He Fits with no $\chi^2_{max}$ cut.

- Many of these fits look nonphysical. How do we remove them?
  - Apply a cut on $\chi^2$. 
### Table 4: Metrics for Final $^3$He Fits

<table>
<thead>
<tr>
<th>$N_{Gaus}$</th>
<th>Avg. $\chi^2$</th>
<th>$r\chi^2$</th>
<th>BIC</th>
<th>AIC</th>
<th>$\sum Q_{ch}$</th>
<th>$\sum Q_{im}$</th>
<th>$\chi^2_{max}$</th>
<th>Below Cut</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>523.743</td>
<td>2.23822</td>
<td>249.063</td>
<td>184.771</td>
<td>1.01018</td>
<td>1.04558</td>
<td>No Cut</td>
<td>1352</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>436.564</td>
<td>1.86566</td>
<td>201.908</td>
<td>159.223</td>
<td>1.00840</td>
<td>1.02235</td>
<td>500</td>
<td>852</td>
<td></td>
</tr>
</tbody>
</table>

Figure 26: Charge Form Factors from 1352 $^3$He Fits with no $\chi^2_{max}$ cut.
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<table>
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<th>Avg. $\chi^2$</th>
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<th>BIC</th>
<th>AIC</th>
<th>$\sum Q_{ch}'$</th>
<th>$\sum Q_{im}'$</th>
<th>$\chi^2_{max}$</th>
<th>Below Cut</th>
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</tbody>
</table>

$^3$He Charge Form Factor

![Graph of $^3$He Charge Form Factor with New $^3$He $|F_{ch}(q^2)|$ Fits and $^3$He $|F_{ch}(q^2)|$ Fit from Amroun et al.](image-url)
Now we can Fourier transform $F_{ch}$ to find the **charge density**.

Figure 28: Charge Densities from 1352 $^3$He Fits with no $\chi^2_{max}$ cut.
• Now we can Fourier transform $F_{ch}$ to find the charge density.
  – As expected the density falls off by $r = 5$ fm.
  – Density turns over slightly and plateaus at small $r$.

Figure 29: Charge Densities from 852 $^3$He Fits surviving a $\chi^2_{max} = 500$ cut.
Using the derivative of $F_{ch}$ we can obtain the charge radius.
Using the derivative of $F_{ch}$ we can obtain the charge radius. 
- Higher radii disappear completely with the cut.
- Avg. $^3$He charge radius = 1.90 fm, SD = 0.00144 fm.

**Figure 31: Charge Radius from $^3$He Fits surviving a $\chi^2_{max} = 500$ cut.**
3\textsuperscript{He} Charge Radius

- Using the derivative of $F_{ch}$ we can obtain the charge radius.
  - Higher radii disappear completely with the cut.
  - Avg. $3\textsuperscript{He}$ charge radius = 1.90 fm, SD = 0.00144 fm.
  - Saclay $1.96 \pm 0.03$. Bates $1.87 \pm 0.03$ [10].
  - GFMC $1.97 \pm 0.01$. $\chi$EFT $1.962 \pm 0.004$ [10].
New $^{3}\text{He} \ F_{ch}$ Fits in Context

- We can compare the new $^{3}\text{He} \ F_{ch}$ fits to older fits as well as theory [11].

Figure 32: $^{3}\text{He}$ Charge Form Factors.
The $F_{ch}$ fits are very tightly grouped due to an abundance of low $Q^2$ data.

- All $R_i$ models strongly agree up to 55-60 fm$^{-2}$.

The new fits almost perfectly overlap the fits of Amroun et al.
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**Conventional Approach**: Simulates 2 and 3-body nucleon interactions and relativistic corrections [11].
- Describes the $F_{ch}$ minima and magnitude very well.
New $^3$He $F_{ch}$ Fits in Context Cont.

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  - Underestimates magnitude of $F_{ch}$. $\chi$EFT500 finds first minima.
New $^3\text{He} F_{ch}$ Fits in Context Cont.

- The $F_{ch}$ fits are very tightly grouped due to an abundance of low $Q^2$ data.
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  - Underestimates magnitude of $F_{ch}$. $\chi$EFT500 finds first minima.
- **Covariant spectator theorem (CST):** Covariant FT where nucleons and light mesons are effective DOF (fully relativistic) [11].
  - Misses $F_{ch}$ minima and underestimates magnitude.
$^3$He Magnetic Form Factor

Figure 33: Magnetic Form Factors from 1352 $^3$He Fits with no $\chi^2_{\text{max}}$ cut.
Figure 34: Magnetic Form Factors from 852 $^3$He Fits surviving a $\chi^2_{max} = 500$ cut.
New $^3\text{He}$ $F_m$ Fits in Context

- We can compare the new $^3\text{He}$ $F_m$ fits to older fits as well as theory.

**Figure 35**: $^3\text{He}$ Magnetic Form Factors.
**New $^3\text{He} F_m$ Fits in Context Cont.**

- $F_m$ fits more loosely grouped. Lacking high $Q^2$ data.
  - The $R_i$ models take divergent paths above 40 fm$^{-2}$.
- The first minima is shifted back from Amroun et al.
- Magnitude decreased between minima.
New $^3\text{He}$ $F_m$ Fits in Context Cont.

- $F_m$ fits more loosely grouped. Lacking high $Q^2$ data.
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- Conventional Approach [11]:
  - Minima shifted too low. Appropriate $F_m$ magnitude above 25 fm$^{-2}$.
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  - $\chi$EFT500 misses minima. $\chi$EFT600 closest to minima, but underestimates $F_m$ magnitude.
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- CST [11]:
  - Very poor description of the data.
New $^3$He $F_m$ Fits in Context Cont.

- $F_m$ fits more loosely grouped. Lacking high $Q^2$ data.
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  - Minima shifted too low. Appropriate $F_m$ magnitude above 25 fm$^{-2}$.
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  - $\chi$EFT500 misses minima. $\chi$EFT600 closest to minima, but underestimates $F_m$ magnitude.
- **CST [11]:**
  - Very poor description of the data.
- **Data first minima moved further away from all predictions.**
  - Theory is having difficulty predicting the $^3$He $F_m$. 

![Graph showing $^3$He Magnetic Form Factor]
$^3$He Representative Cross Section Fit Statistics

- 259 $^3$He points. $\chi^2 = 436$. $r\chi^2 = 1.86$.

Figure 36: $^3$He Representative Form Factors and World Data Distribution.
$^3$He Representative Cross Section Fit Statistics

- 259 $^3$He points. $\chi^2 = 436$. $r\chi^2 = 1.86$.

**Figure 37:** $^3$He Representative Fit $\chi^2$ vs. $Q^2$. 
$^3$He Representative Cross Section Fit Statistics

- 259 $^3$He points. $\chi^2 = 436$. $r\chi^2 = 1.86$.

Residual of Representative Fit vs. $Q^2$

![Residual Plot](image)

Figure 38: $^3$He Representative Fit Residual vs. $Q^2$. 
259 $^3$He points. $\chi^2 = 436$. $r\chi^2 = 1.86$.

Figure 39: $^3$He Representative Fit Residual vs. $Q^2$ Zoomed. Two large residual points from not shown.
### 3H Charge Form Factor

<table>
<thead>
<tr>
<th>$N_{Gaus}$</th>
<th>Avg. $\chi^2$</th>
<th>$r\chi^2$</th>
<th>BIC</th>
<th>AIC</th>
<th>$\sum Q_{ch}$</th>
<th>$\sum Q_{im}$</th>
<th>$\chi^2_{max}$</th>
<th>Below Cut</th>
<th>Cut</th>
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<tr>
<td>8</td>
<td>611.385</td>
<td>2.81744</td>
<td>266.175</td>
<td>238.532</td>
<td>1.08866</td>
<td>1.33481</td>
<td>2600</td>
<td>No Cut</td>
<td></td>
</tr>
<tr>
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<td>601.840</td>
<td>2.77346</td>
<td>264.695</td>
<td>237.053</td>
<td>1.08991</td>
<td>1.32926</td>
<td>603</td>
<td>908</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Metrics for Final $^3$H Fits

### Figure 40: Charge Form Factors from $^3$H Fits with no $\chi^2_{max}$ cut.
**3H Charge Form Factor**

<table>
<thead>
<tr>
<th>$N_{Gaus}$</th>
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</tr>
</tbody>
</table>

Table 5: Metrics for Final $^3$H Fits

Figure 41: Charge Form Factors from $^3$H Fits surviving a $\chi^2_{max} = 603$ cut.
Again we Fourier transform $F_{ch}$ to find the charge density.


- Again we Fourier transform $F_{ch}$ to find the charge density.
  - Plateaus at small $r$ like $^3$He. Unclear if the density turns over.
  - Magnitude at $r = 0$ has much more uncertainty than $^3$He.

**Figure 43:** Charge Densities from 908 $^3$H Fits surviving a $\chi^2_{max} = 603$ cut.

---
3^H Charge Radius

- Again, using the derivative of $F_{ch}$ we can obtain the charge radius.

Figure 44: Charge Radius from 3^H Fits with no $\chi^2_{\text{max}}$ cut.
\( ^3\text{H} \) Charge Radius

- Again, using the derivative of \( F_{ch} \) we can obtain the charge radius.
  - Avg. \( ^3\text{H} \) charge radius = 2.02 fm, SD = 0.0133 fm.
\[ ^3\text{H} \] Charge Radius

- Again, using the derivative of \( F_{ch} \) we can obtain the charge radius.
  - Avg. \(^3\text{H}\) charge radius = 2.02 fm, SD = 0.0133 fm.
  - Saclay 1.76 ± 0.09. Bates 1.68 ± 0.03 [10].
  - GFMC 1.77 ± 0.01. \( \chi \)EFT 1.756 ± 0.006 [10].
Again, using the derivative of $F_{ch}$ we can obtain the charge radius.

- Avg. $^3$H charge radius = 2.02 fm, SD = 0.0133 fm.
- Saclay $1.76 \pm 0.09$. Bates $1.68 \pm 0.03$ [10].
- GFMC $1.77 \pm 0.01$. $\chi$EFT $1.756 \pm 0.006$ [10].
- This is the result of not forcing $Q_{i_{ch}} = 1$. 
New $^3\text{H}$ $F_{ch}$ Fits in Context

- We can compare the new $^3\text{H}$ $F_{ch}$ fits to older fits.

Figure 46: $^3\text{H}$ Charge Form Factors.
● Results are comparable with Amroun et al.
  – No new $^3$H data added.
  – Above $Q^2 \approx 25 \text{ fm}^{-2}$ the fits diverge greatly.

● Demonstrates the consistency of our method.
New $^3H$ $F_{ch}$ Fits in Context Cont.

- Results are comparable with Amroun et al.
  - No new $^3H$ data added.
  - Above $Q^2 \approx 25 \text{ fm}^{-2}$ the fits diverge greatly.

- Demonstrates the consistency of our method.

- Conventional Approach [11]:
  - Describes minimum well. $F_{ch}$ magnitude a bit large.
New $^3$H $F_{ch}$ Fits in Context Cont.

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- $\chi$EFT [11]:
  - $\chi$EFT500 misses minima and magnitude. $\chi$EFT600 close to minimum, and slightly large $F_{ch}$ magnitude.
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  - $\chi$EFT500 misses minima and magnitude. $\chi$EFT600 close to minimum, and slightly large $F_{ch}$ magnitude.

- CST [11]:
  - Poorly describes the data.

- Theory predicts data relatively well.
  - Better understanding of $F_{ch}$ magnitude needed.
Figure 47: Magnetic Form Factors from 2600 $^3$H Fits with no $\chi_{max}^2$ cut.
Figure 48: Magnetic Form Factors from 908 $^3\text{H}$ Fits surviving a $\chi^2_{max} = 603$ cut.
New $^3$H $F_m$ Fits in Context

- We can compare the new $^3$H $F_m$ fits to older fits.

Figure 49: $^3$H Magnetic Form Factors.
New $^3\text{H} F_m$ Fits in Context Cont.

- Results are comparable with Amroun et al.
  - No new $^3\text{H}$ data added.
  - Very little understanding of $F_m$ above $Q^2 = 35 \text{ fm}^{-2}$.
- Need more high $Q^2$ data.
Results are comparable with Amroun et al.
- No new $^3\text{H}$ data added.
- Very little understanding of $F_m$ above $Q^2 = 35 \text{ fm}^{-2}$.

Need more high $Q^2$ data.

Conventional Approach [11]:
- Early first minimum. If minimum shifts right $F_m$ magnitude looks close.
New $^3\text{H} F_m$ Fits in Context Cont.

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CST [11]:
- Poorly describes the data.

Theory struggles to predict data.
- Magnitude may be close to correct if minimum shifts up in $Q^2$. 
$^3$H Representative Cross Section Fit Statistics

- 234 $^3$H points. $\chi^2 = 602$. $r\chi^2 = 2.77$.

Figure 50: $^3$H Representative Form Factors and World Data Distribution.
$^{3}\text{H}$ Representative Cross Section Fit Statistics

- 234 $^{3}\text{H}$ points. $\chi^2 = 602$. $r\chi^2 = 2.77$.

**Figure 51:** $^{3}\text{H}$ Representative Fit $\chi^2$ vs. $Q^2$.  

Representative Fit $\chi^2$ vs. $Q^2$
$^3$H Representative Cross Section Fit Statistics

- 234 $^3$H points. $\chi^2 = 602$. $r\chi^2 = 2.77$.

Figure 52: $^3$H Representative Fit Residual vs. $Q^2$. 
Conclusions
Conclusions

- New $^3$He elastic cross section of $1.335 \pm 0.086 \times 10^{-6}$ $\mu$b/sr.

- Modern SOG fits with new JLab and this analysis' data point were performed.
  - $^3$He $F_{ch}$ and $^3$H $F_{ch}$ and $F_{m}$ relatively unchanged.
  - $^3$He $F_{m}$ first minimum shift up several fm in $Q^2$.
  - $^3$He charge radii agrees with past data.
  - $^3$H charge radii disagrees with past data ($\sum Q_i \neq 1$).

- A conventional theoretical approach using 2 and 3-body nucleon interactions and relativistic corrections reproduces $F_{ch}$ well. $\chi$EFT also performs decently.
  - Theory predictions struggle with predicting $F_{m}$.

- Need more high $Q^2$ data to understand form factors beyond the first minimum.
  - JLab is well positioned to make these measurements!
  - Hall A back angle max of 150$^\circ$ with 12 GeV available. Rates fall extremely fast, but very high $Q^2$ could be accessed.
  - Probe transitional region where scattering off hadrons and mesons → scattering off quarks and gluons.
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Acknowledgements

- This work was made possible by DOE grant 742481 as well as a JSA Graduate Fellowship.
- Thanks to Douglas Higinbotham for his knowledge of fitting best practices and XS extractions as well as his invaluable mentorship.
- Thanks to Todd Averett for his guidance as my advisor, and the freedom he has allowed me in my research.
- Special thank you to Dien Nguyen for pioneering this data set and saving me countless hours of confusion.
- And thanks to my other JLab mentors like Bob Michaels and Bogdan Wojtsekhowski, and the many others at JLab who have supported me in my graduate work.
Questions?
5. A. Amroun et al., “$^3$H and $^3$He EM Form Factors”, Nuclear Physics A 579, 596 (1994).